1
Introduction

Abstractly, one can consider time-periodic groundwater flow to be the response of a physical system to a stimulus or excitation. The groundwater system consists of a subsurface porous medium and a resident pore fluid such as fresh or saline water, oil, air, or natural gas. The stimulus is some kind of time-periodic forcing, and the response is the time-periodic variation of hydraulic head and specific discharge. Thus, a possible alternative title for this book is "Introduction to the Theory of Periodically Forced Groundwater Systems."

1.1. TERMINOLOGY

To clearly articulate ideas about periodic flow, it will be useful to first clarify and standardize some basic related terminology.

For this book, we define *functions of practical interest* as those mathematical functions that are capable of representing real physical phenomena. We will assume that every function of practical interest is either periodic or aperiodic.

If $f(t)$ is a *periodic* function of time, then there exists a nonzero real number (period) T for which

$$f(t+T) = f(t) \quad \forall\, t \in \mathbb{R}.$$

Thus, periodicity is a type of global translational symmetry. Synonyms for the term *periodic* include the terms *fully periodic*, *purely periodic*, and *strictly periodic*.

We can represent every periodic function of time as the sum of one or more distinct *harmonic constituents* (also *frequency components* or *modes*), each of which is a purely sinusoidal function of time, wherein the constituent frequencies are rational multiples of one another. This description coincides with the classical Fourier series representation of a periodic function.

An *aperiodic* function (also *nonperiodic* function) is any function that is not periodic. Aperiodic functions include an important class of functions that are closely related to the periodic functions—the *almost-periodic* functions. An almost-periodic function (also, *quasiperiodic* function) is a function composed of (i.e., formed by summing) two or more harmonic constituents, at least two of which have frequencies that are not rational multiples of one another. A simple example of an almost-periodic function having two distinct harmonic constituents is

$$f(t) = 3\sin(2t) + 7\cos(3\sqrt{2}t) \quad \forall\, t \in \mathbb{R}.$$

Equivalently, an almost-periodic function is an aperiodic function that we can represent as the sum of two or more periodic functions and thus as a generalized Fourier series.

Throughout this book we generally use the term *periodic function* to represent that broad class of functions that includes both the strictly periodic functions and the almost-periodic functions. What these functions have in common is that we can represent both types by generalized Fourier series. Similarly, we use the term *nonperiodic function* to represent the class of functions that are neither strictly periodic nor almost periodic.

In the literature readers may encounter numerous terms that have meanings related to concepts of periodicity. For instance, some authors use the terms *cyclic* (or *cyclical*) and *rhythmic* as synonyms for *periodic*. In some contexts the terms *oscillating* (or *oscillatory*) and *undulating* also can have meanings similar to that of the term *periodic*; in other contexts these terms might be used to describe types of variation that are more irregular. Similarly, the term *fluctuating* is commonly used to describe variations that are less regular or less predictable than those described by the term *periodic*; in fact, the term *fluctuating* is frequently used to describe variations controlled by random processes.

Hydrodynamics of Time-Periodic Groundwater Flow: Diffusion Waves in Porous Media, Geophysical Monograph 224,
First Edition. Joe S. Depner and Todd C. Rasmussen.

Other terms that may reflect temporally periodic flow include *alternating* (e.g., *alternating flow*; see *Stewart et al.* [1961]), *pulsatile* (also, *pulsating*, *pulsed*, *pulsing*), and *reciprocating*.

To distinguish between periodicity in time and periodicity in space, some authors use the terms *time periodic* (also *temporally periodic* or *steady periodic*) and *space periodic* (or *spatially periodic*). In the literature, the term *periodic media* generally refers to (porous) media that are spatially periodic with respect to material properties.

1.2. PERIODIC FORCING

Time-periodic flow occurs in a groundwater system only if the system undergoes periodic forcing. Based on system geometry, periodic forcing can be of two basic types, which can occur either alone or in combination. In *boundary forcing*, a system boundary is subject to time-periodic conditions. An example is the time variation of hydraulic head at an aquifer's seaward boundary. In *internal forcing*, an internal water source/sink is time periodic. An example is time-periodic water injection/pumping at an injection well.

We can classify periodic forcing of groundwater systems according to various additional criteria as well. In summary, criteria for the classification of periodic forcing include the following:

Origin Natural versus artificial.

Frequency High frequency (short period) versus low frequency (long period).

Geometry Boundary versus internal.

Periodicity Purely periodic versus almost periodic.

Other Hydraulic versus nonhydraulic (e.g., periodic water pressurization at a vertical boundary versus periodic mechanical loading on a horizontal upper boundary).

This book addresses both purely periodic and almost-periodic forcing. This choice is largely a matter of convenience—we can represent the time behavior of both types mathematically using general trigonometric series.

1.3. POTENTIAL AREAS OF APPLICATION

The following is a summary of potential areas of application for the theory of time-periodic groundwater flow. While this summary is broad, it is not comprehensive; there likely are additional applications, and we expect the number of applications to grow with time.

Atmospheric Pressure Natural. Diurnal, annual (seasonal), etc. Mechanical effect of barometric forcing on the upper surface of the capillary fringe, on the upper surface of a confining unit, or on the free surface within a well. Here the theory is used both to understand aquifer response to atmospheric forcing and to investigate aquifer hydrologic properties. Examples: *Furbish* [1991], *Hanson* [1980], *Hobbs and Fourie* [2000], *Merritt* [2004], *Neeper* [2001, 2002, 2003], *Rasmussen and Crawford* [1997], *Rinehart* [1972], *Ritzi et al.* [1991], *Rojstaczer* [1988], *Rojstaczer and Agnew* [1989], *Rojstaczer and Riley* [1990, 1992], *Seo* [2001], *Toll and Rasmussen* [2007], *van der Kamp and Gale* [1983], *Weeks* [1979]. Recently the theory has been used to assess the effectiveness of subsurface energy resource exploitation efforts [e.g., *Burbey and Zhang*, 2010].

Infiltration/Recharge Mass flow (hydraulic) effect at recharge boundaries. Here the theory is used to model the effects of periodic recharge cycles on groundwater systems.

- **Artificial Recharge** Seasonal and other cycles. Examples: *Latinopoulos* [1984, 1985].
- **Natural Infiltration/Recharge** Associated with seasonal cycles of precipitation and evapotranspiration. Examples: *Latinopoulos* [1984], *Maddock and Vionnet* [1998], *Rasmussen and Mote* [2007].

Plant Water Uptake/Transpiration Mass flow effect at or near the water table. Seasonal and diurnal cycles. Here the theory is used for modeling the interaction of the biosphere with groundwater systems. Examples: *Butler et al.* [2007], *Kruseman and de Ridder* [2000], *Lautz* [2008a,b].

Tides Natural. Multiple periods, from semidiurnal to monthly and longer. Both hydraulic and mechanical effects.

- **Earth Tides** The theory is used to infer aquifer and petroleum reservoir physical properties [e.g., *Bredehoeft*, 1967; *Chang and Firoozabadi*, 2000; *Cutillo and Bredehoeft*, 2011; *Hsieh et al.*, 1987, 1988; *Kümpel et al.*, 1999; *Marine*, 1975; *Morland and Donaldson*, 1984; *Narasimhan et al.* 1984; *Ritzi et al.*, 1991], to assess the effectiveness of subsurface energy resource exploitation efforts [e.g., *Burbey and Zhang*, 2010], and to understand geyser eruption timing [e.g., *Rinehart*, 1972].
- **Ocean Tides** The theory is used to infer aquifer hydraulic properties [e.g., *Carr and van der Kamp*, 1969; *Erskine*, 1991; *Ferris*, 1951; *Jacob*, 1950; *Jha et al.*, 2008; *Trefry and Bekele*, 2004; *Trefry and Johnston*, 1998], to correct nonsinusoidal hydraulic test results for tidal influence [e.g., *Chapuis et al.*, 2006; *Trefry and Johnston*, 1998], to assess groundwater fluxes in coastal aquifers [e.g., *Serfes*, 1991], and to assess groundwater–surface water fluxes in coastal environments [e.g., *Burnett et al.*, 2006; *Taniguchi*, 2002].

Sinusoidal Hydraulic Tests Artificial. Variable period(s). The theory is used to design and interpret the results of tests to infer material hydraulic properties.

Field (Pumping) Tests Hydraulic effect at face of well. The testing is conducted in situ. Examples: *Black and Kipp* [1981], *Cardiff et al.* [2013], *Hvorslev* [1951], *Mehnert et al.* [1999], *Rasmussen et al.* [2003], *Renner and Messar* [2006].

Laboratory Tests Hydraulic effect at opposite faces of material sample. The testing is conducted on material samples in the laboratory. Examples: *Adachi and Detournay* [1997], *Bernabé et al.* [2006], *Fischer* [1992], *Kranz et al.* [1990], *Rigord et al.* [1993], *Song and Renner* [2006, 2007].

Periodic Groundwater Pumping/Injection Artificial. Variable period(s). Hydraulic effect at face of well(s). Here the theory could be used to design subsurface environmental remediation systems [e.g., *Zawadzki et al.*, 2002], to design aquifer recharge systems [e.g., *Latinopoulos*, 1984, 1985], or to evaluate hydraulic connectivity in functioning geothermal well fields [e.g., *Becker and Guiltinan*, 2010; *Yano et al.*, 2000].

This list represents only a sample of the available literature.

1.4. CHAPTER SUMMARY

In this chapter we introduced basic terminology on the time behavior of periodically forced groundwater systems and proposed criteria for the classification of time-periodic forcing. We also briefly listed some potential areas of application for the theory of time-periodic groundwater flow. The list illustrates that time-periodic groundwater flow is relevant across multiple fields:

Earth sciences: geophysics, groundwater hydrology and hydrogeology, and oceanography.

Engineering fields: civil (environmental and geotechnical) and energy (geothermal and petroleum).