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Time Reversal: A Different Perspective

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1.1 Introduction

In this introductory chapter, we will present the theoretical background of the time reversal in electromagnetism. We will start by discussing the notion of time in physics. The time-reversal invariance of physics laws will then be described. Special attention will be devoted to the time-reversal invariance of Maxwell's equations. The concept of time-reversal cavity and the use of time reversal as a means of refocusing electromagnetic waves will be described. Finally, the chapter will end by briefly presenting application areas of electromagnetic time reversal.

1.2 What is Time?

Most of the words that define very complex concepts are rarely used in our daily language and writings.[1] There are, however, exceptions, *time* being a particularly notable one. Even though *time* appears to be a quite familiar notion that carries the feeling

1 For instance, the word "inchoate" (partly in existence or imperfectly formed).

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of an obvious reality to everyone, there is nothing harder than giving a definition that truly captures its essence using concepts other than our intuitive idea of time itself. Let us look at some entries given in dictionaries:

> Indefinite, unlimited duration in which things are considered as happening in the past, present, or future; every moment there has ever been or ever will be.
>
> *(Webster's New World College Dictionary)*

> The indefinite continued progress of existence and events in the past, present, and future regarded as a whole.
>
> *(Oxford Dictionaries)*

Etienne Klein, a French physicist, has also proposed an interesting definition for time [1]: a jail on wheels. Why a jail? Because we are not free to choose our position along the timeline. We are in the present instant and we cannot get away from it. Why on wheels? Because time moves forward. *Time* takes us from the present to the future.

One can find hundreds of other definitions, proposed by philosophers, physicists, and linguists. Many of them use metaphors to describe this concept, as in the one proposed by Klein. However, none of these tells us about the nature of *time* since some idea of *time* is used in the definition itself [1]. This paradox was noticed by Saint Augustine in the fourth century: "If I am not asked, I know what time is; but if I am asked, I do not."

Physicists have managed to consider *time* as an operative concept. The first mathematical expression of physical time was enunciated by Galileo and formalized by Newton, assuming that time has one dimension and is expressed by a real number. Furthermore, in that definition, time is absolute in the sense that there is only one time associated with any given moment, and that time is the same everywhere in the universe. In 1905, Einstein's special relativity theory showed that time is not an absolute quantity and should be considered in relation to space. Quantum mechanics changed again the time paradigm [2] with the Heisenberg uncertainty principle, which applies not only to position and momentum, but also to time and energy.

1.3 Time Reversal or Going Back in Time

In this section, using a simple example, we will present three approaches that can be used to effectively make a system go back in time, in the sense that it retraces the path it came from in the immediate past. The three methods that will be discussed are (i) the recording of the state of the system throughout its evolution from time $t = 0$ to a time $t = t_1$, (ii) the use of expressions that describe the evolution of the system as a function of time, and (iii) the imposition of initial conditions so that the system regresses, following its own natural defining equations, towards the states it went through in its past. We will also discuss the conditions under which each one of the approaches can be applied and the implications for practical applications.

The simple example we will consider is that of an object of a given mass launched at an angle near the surface of the earth, as shown in Figure 1.1. The resistance of the air is considered to be zero. We will concentrate on the position of the object as the physical quantity of interest. Since the position can be represented pictorially, the success of an approach will be illustrated by the possibility of producing a movie of the trajectory of the object in reverse.

Note that, since we are using classical mechanics, we are using the Newtonian definition of time and not that of Einstein's relativity.

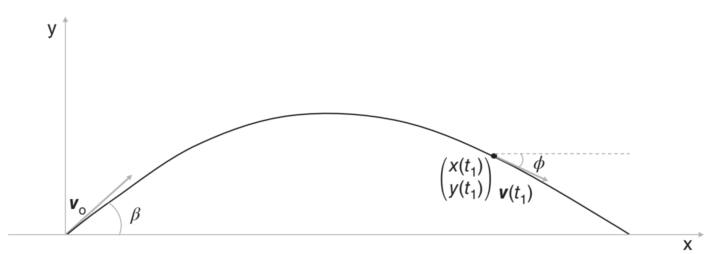

Figure 1.1 Projectile launched at an angle over a flat earth. The speed and position at time t_1 after the launch are shown after the highest point in the trajectory.

1.3.1 Approach 1: Recording of the State of the System Throughout Its Evolution

If the position of the object is recorded using, for instance, calibrated, synchronized video cameras, then the evolution of the system can be viewed both in the forward and, by reversing the order of the frames, in reverse. Clearly, the only condition that needs to be imposed for this approach to be applicable is that the physical quantity of interest be observable in principle at all times, although a limited number of samples may be sufficient depending on the intended application. No conditions need to be imposed on the properties of the underlying physical equations.

1.3.2 Approach 2: Use of Expressions that Describe the Evolution of the System as a Function of Time

The current state of knowledge in physics, based on observation, experimentation, and the application of the appropriate mathematical tools, allows us in a number of fields to write expressions that can be used to predict as a function of time, with known accuracy, the values of physical quantities associated with bounded physical systems.

Using classical mechanics, we can write the following function in Cartesian coordinates to describe the position of an object with a constant gravitational acceleration g and neglecting any friction with the air:

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} x_0 + v_{x_0}(t - t_0) \\ y_0 + v_{y_0}(t - t_0) - \frac{1}{2}g(t - t_0)^2 \end{pmatrix} \tag{1.1a}$$

in which x_0 and y_0 represent the initial x and y coordinates of the object, v_{x_0} and v_{y_0} are the components of the initial velocity of the object and t_0 is the reference time for the launch of the object.

For the particular example of Figure 1.1, Equation (1.1a) can be written as

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} |\boldsymbol{v}_0| \cos(\beta)t \\ |\boldsymbol{v}_0| \sin(\beta)t - \frac{1}{2}gt^2 \end{pmatrix} \tag{1.1b}$$

where x_0 and y_0 were set to zero since the projectile is launched from the origin of the coordinate system. Also, time is counted from $t = 0$ and, therefore, $t_0 = 0$. The speed can also be found as a function of time using Equation (1.2), which is simply the derivative of (1.1):

$$\begin{pmatrix} v_x(t) \\ v_y(t) \end{pmatrix} = \begin{pmatrix} |\boldsymbol{v}_0| \cos(\beta) \\ |\boldsymbol{v}_0| \sin(\beta) - gt \end{pmatrix} \tag{1.2}$$

If we let t increase from 0 to t_1, Equation (1.1) literally allows us to draw the trajectory of the projectile the same way we would observe it in a physical setup under the conditions posed above. Since the position is calculable at any time, it is possible to create a movie by drawing a point at the appropriate location in each of the movie frames based on the desired number of frames per second and on Equation (1.1). Of course, we could now stop the movie while the projectile is in flight and we could run it in reverse. We would then see the particle fly back to the point from where it was launched.

This approach can be used to produce movies that simulate forward-in-time and backward-in-time mechanical movement in more complex scenarios, as long as we are able to write the equations of movement for the complete time period of interest.

1.3.3 Approach 3: System Regressing Through Its Own Natural Defining Equations

In the first two approaches, the prior states of the system are found by looking at times in the past. In this approach, *the previous behavior of the system is reproduced in the future.* To achieve this, appropriate initial conditions are imposed on the original system so that it begins to retrace its previous states. An advantage of this approach is that there is no need to record the behavior of the system for the whole time interval of interest, since only the final conditions are required (they are the basis for the determination of the required new initial conditions for the system to regress). A disadvantage of this approach is the fact that the underlying physical equations of the particular system have to satisfy conditions that we will investigate later on, after we have illustrated the approach using the projectile example of

Figure 1.1. Another advantage of this approach, and the reason why we will concentrate on it in the remainder of the chapter, is that this is the only approach that can be applied both by simulation and experimentally.

Let us assume that we have measured the speed and the position of the projectile at time t_1 after the launch as illustrated in Figure 1.1.

Using (1.1) and (1.2), we can get the position and the speed that would be measured at time t_1:

$$\begin{pmatrix} x(t_1) \\ y(t_1) \end{pmatrix} = \begin{pmatrix} |\boldsymbol{v}_0| \cos(\beta) t_1 \\ |\boldsymbol{v}_0| \sin(\beta) t_1 - \frac{1}{2} g {t_1}^2 \end{pmatrix} \tag{1.3}$$

$$\begin{pmatrix} v_x(t_1) \\ v_y(t_1) \end{pmatrix} = \begin{pmatrix} |\boldsymbol{v}_0| \cos(\beta) \\ |\boldsymbol{v}_0| \sin(\beta) - g t_1 \end{pmatrix} \tag{1.4}$$

Using the approach described here, after having halted the evolution of the system at $t = t_1$, we now use the same system with appropriately modified initial conditions as follows.

The initial position for the regressing projectile will be the position of the object at time t_1 given by the right-hand side of (1.3):

$$\begin{pmatrix} x'_0 \\ y'_0 \end{pmatrix} = \begin{pmatrix} |\boldsymbol{v}_0| \cos(\beta) t_1 \\ |\boldsymbol{v}_0| \sin(\beta) t_1 - \frac{1}{2} g {t_1}^2 \end{pmatrix} \tag{1.5}$$

The new initial speed is given by the additive inverse[2] of the speed of the object at time t_1, which can be obtained by multiplying the right-hand side of (1.4) by –1:

$$\begin{pmatrix} v'_{x_0} \\ v'_{y_0} \end{pmatrix} = \begin{pmatrix} -|\boldsymbol{v}_0| \cos(\beta) \\ -|\boldsymbol{v}_0| \sin(\beta) + g t_1 \end{pmatrix} \tag{1.6}$$

To see if these initial conditions make the system behave in such a way that the future trajectory, starting at $t = t_1$, will actually retrace the path that the object had followed from $t = 0$ to

2 Note that we are reversing the direction of the speed vector based only on our intuition. A more rigorous procedure to determine the initial conditions will be discussed later on.

$t = t_1$, let us plug (1.5) and (1.6) into the underlying equations for the position of the object in the system, given by (1.1a):

$$\begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} |\boldsymbol{v}_0|\cos(\beta)t_1 - |\boldsymbol{v}_0|\cos(\beta)(t - t_1) \\ |\boldsymbol{v}_0|\sin(\beta)t_1 - \frac{1}{2}gt_1^2 + (-|\boldsymbol{v}_0|\sin(\beta) + gt_1)(t - t_1) - \frac{1}{2}g(t - t_1)^2 \end{pmatrix} \tag{1.7}$$

If Equation (1.7) indeed represents the reversion of the system starting at time t_1, then the position at time $2t_1$ should be the origin of the coordinate system, a fact that can be readily verified by introducing $t = 2t_1$ in (1.7).

Let us now investigate the conditions that need to be satisfied by the equations of the system so that it is possible to make it behave in a time-reversed manner after a selected point in time, which, as in the example above, we will call t_1.

Figure 1.2a represents a plot of the evolution of a physical quantity $f(t)$ as a function of time in a system. The quantity could be the position of a particle along a particular axis, the temperature, one of the components of the electric field intensity, or any other observable. In Figure 1.2a, the system has been frozen in time at $t = t_1$. We would like for the system's physical quantity to retrace the same values it just exhibited prior to t_1. The desired resulting behavior is shown by the dashed line in Figure 1.2b. The dashed curve can be obtained from the original function $f(t)$ by first translating it by t_1 to the left, then applying the T-symmetry transformation $T : t \to -t$, and finally translating it back to the right by t_1. The resulting function is $g(t) = f(-t + 2t_1)$.

From Figure 1.2, it can be concluded that the condition that needs to be satisfied by the underlying physical equations is that, given a solution $f(t)$ to the underlying differential equations governing a physical phenomenon, $g(t) = f(-t + 2t_1)$ must also be a solution. In the next section, we will formally use this condition to define time-reversal invariability in the strict sense.

1.3.4 Time Reversal in the Strict Sense and in the Soft Sense

We concluded the last section with a statement of the condition that the underlying equations governing the behavior of a

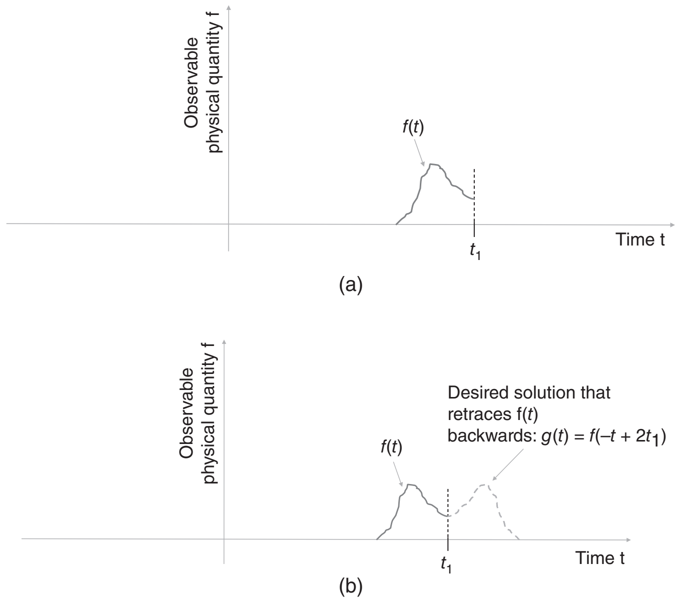

Figure 1.2 Plot of the evolution of the physical quantity f. (a) Evolution of $f(t)$ up to a time t_1. (b) Function $f(t)$ followed by the desired behavior $g(t)$, the dashed curve, of the system after t_1.

physical quantity must satisfy so that the system can be made to retrace the path followed in the immediate past by setting its initial conditions and letting it evolve into the future. We call this property of a physical system *time-reversal invariance in the strict sense*, as opposed to *time-reversal invariance in the soft sense*, which will be introduced later on in this section. The condition for time-reversal invariance in the strict sense is:

Strict Time-Reversal Invariance Condition

A system is time-reversal invariant with respect to a physical quantity in the strict sense if, given a solution $f(t)$ of its underlying equations, the time-reversed function $g(t)$, given by Equation (1.8) hereunder, is also a solution:

$$g(t) = f(-t + 2t_1) \tag{1.8}$$

The time t_1 is the time at which we freeze the system state before letting it regress. For simplicity, we will replace twice that constant time by k. The parameter k could also be set to zero since one can shift the origin of the time coordinate so that the observation period starts at a time $t = -2t_1$ and freeze the time at time $t = 0$.

In the case of the example of Figure 1.1, the differential equation that governs the movement of the object satisfies the condition for time-reversibility in the strict sense, as we will show in what follows.

The equations of the movement of the projectile are given by

$$\begin{pmatrix} \dfrac{\partial^2 x(t)}{\partial t^2} \\ \dfrac{\partial^2 y(t)}{\partial t^2} \end{pmatrix} = \begin{pmatrix} 0 \\ -g \end{pmatrix} \tag{1.9}$$

Assuming that $x(t)$ and $y(t)$ satisfy (1.9), we will now show that $x(-t+k)$ and $y(-t+k)$ also satisfy the equation:

$$\begin{pmatrix} \dfrac{\partial^2 x(-t+k)}{\partial t^2} \\ \dfrac{\partial^2 y(-t+k)}{\partial t^2} \end{pmatrix} = \begin{pmatrix} 0 \\ -g \end{pmatrix} \tag{1.10}$$

Note now that, since the right-hand side of (1.9) is a constant vector, its left-hand side must also be independent of time. Therefore, the left-hand sides of (1.9) and (1.10) must be equal since (1.10) is just (1.9) evaluated at time $-t + k$. The example in Figure 1.1 is thus time-reversal invariant in the strict sense.

When we originally wrote Equation (1.7), we used our intuition when, by inspection of the specific problem of a projectile in flight, the speed vector direction at $t = t_1$ was reversed and the appropriate initial conditions were selected and imposed. According to the developments presented here, it is possible to obtain Equation (1.7) by plugging Equation (1.1b) (as $f(t)$) into Equation (1.8). We will not include the details here but the result is indeed identical to (1.7).

The equations governing the example discussed up to now involved additive terms containing (1) the second derivative of the quantity of interest (which was the position) and (2) constant terms (see, for instance, Equation (1.9)). As we will see, the presence of other types of terms may violate the conditions for strict reversibility. We will now investigate time reversal in a system whose underlying equations involve such terms. This will lead to the definition of time-reversal invariability in the soft sense.

Let us consider now an object that is subjected to a force that is proportional to the elapsed time. For simplicity, we will assume the force's proportionality constant with respect to time to be m, the value of the mass of the object. Under these conditions, Newton's force equation for the object can be written as

$$a(t) = \frac{d^2x(t)}{dt^2} = t \tag{1.11}$$

We will now find a solution of (1.11) and test its time-reversibility.

The speed and the position are obtained by two successive integrations to be

$$v(t) = \frac{t^2}{2} + v_0 \tag{1.12}$$

$$x(t) = \frac{t^3}{6} + v_0 t + x_0 \tag{1.13}$$

Let us assume, to facilitate the calculations, that $v_0 = x_0 = 0$. With these assumptions, (1.12) and (1.13) can be written as

$$v(t) = \frac{t^2}{2} \tag{1.14}$$

$$x(t) = \frac{t^3}{6} \tag{1.15}$$

Let us now see if the function $g(t) = x(-t + 2t_1)$ satisfies Equation (1.11). First, let us write $g(t)$,

$$g(t) = x(-t + 2t_1) = \frac{(-t + 2t_1)^3}{6} \tag{1.16}$$

Plugging (1.16) into (1.11), and checking if the left- and right-hand sides are equal, we get

$$\frac{d^2\left[\dfrac{(-t + 2t_1)^3}{6}\right]}{dt^2} = -t + 2t_1 \neq t \tag{1.17}$$

The equality does not hold unless we apply the time-reversal transformation also to the right hand side of (1.11).

By applying the T-symmetry + translation transformation $t \to -t + 2t_1$ to the acceleration on the right-hand side of (1.11) we are modifying the behavior of the force in this example.

Indeed, the force, equal to mt in the original system in which the object evolved up to $t = t_1$, becomes $m(-t + 2t_1)$ after that time. It is therefore no longer sufficient to choose the appropriate initial conditions for the system to evolve into the future by retracing the original trajectory as if time were moving backwards. It is also necessary to change the behavior of the force after t_1 so that its functional dependence is $m(-t + 2t_1)$. In other words, the time-reversal transformation $T : t \to -t$ is also applied to the time-dependent terms that do not contain the physical quantity. This change is possible if the evolution of the system is carried out by simulation after $t = t_1$.

Phenomena that require changes in their defining equations such as the ones described here in order for their state to retrace in the future the path it followed in its past are referred to as *time-reversal invariant in the soft sense.*

Soft Time-Reversal Invariance Condition

A system is time-reversal invariant with respect to a physical quantity in the soft sense if, given a solution $f(t)$ of its underlying equations, the time-reversed function $g(t)$, given by Equation (1.18) hereunder, is also a solution of the underlying equation after appropriate changes to the equation are applied:

$$g(t) = f(-t + 2t_1) \tag{1.18}$$

The basic equations of electricity and magnetism, as well as equations in quantum mechanics appear to be time-reversal invariant in the soft sense, as will be shown in the next subsections.

1.3.5 Schrödinger Equation

Let us consider the Schrödinger equation in quantum mechanics that describes the wave nature of particles:

$$j\hbar \frac{\partial \psi(r,t)}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi(r,t) + V(r,t)\psi(r,t) \tag{1.19}$$

where j is the square root of -1, $\hbar$ is the reduced Planck constant, obtained by dividing h by 2π, m is the particle's reduced mass, $V(r,t)$ its potential energy and $\psi(r,t)$ the wave function.

It can be readily seen that replacing t by $t' = -t + 2t_1$, the term on the left-hand side changes sign and, therefore, $\psi(r, -t + 2t_1)$ is not a solution of the Schrödinger equation. However, time-reversal symmetry is achieved when substituting t by $t' = -t + 2t_1$ and taking the complex conjugate of Equation (1.19):

$$j\hbar\frac{\partial\psi^*(r,t')}{\partial t'} = -\frac{\hbar^2}{2m}\nabla^2\psi^*(r,t') + V(r,t')\psi^*(r,t') \tag{1.20}$$

where the term on the left-hand side remains unchanged since two sign changes, one due to the change of variables variables $t' = -t + 2t_1$ and another to the conjugate transformation $j^* = -j$, cancel.

Equation (1.20) shows that if $\psi(r, t)$ is a solution of the Schrödinger equation, then $\psi^*(r, -t + 2t_1)$ is also a solution. Therefore, the Schrödinger equation is invariant under time reversal in the soft sense because of the applied conjugate complex operation.[3]

1.3.6 Maxwell's Equations

Let us consider classical electromagnetism, in which the electric and magnetic fields obey Maxwell's equations:

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\mu(\mathbf{r})\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} \tag{1.21}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \varepsilon(\mathbf{r})\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} + \mathbf{J}(\mathbf{r},t) \tag{1.22}$$

$$\nabla \cdot (\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t)) = \rho(\mathbf{r},t) \tag{1.23}$$

$$\nabla \cdot (\mu(\mathbf{r})\mathbf{H}(\mathbf{r},t)) = 0 \tag{1.24}$$

Let us express the current density as the sum of two parts, one corresponding to a source, imposed by an external agent, and one being the consequence of a non-zero conductivity in the medium:

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\mu(\mathbf{r})\frac{\partial \mathbf{H}(\mathbf{r},t)}{\partial t} \tag{1.25}$$

3 It should be noted that only $|\psi^2(r,t)|$ leads to observable effects [3].

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \varepsilon(\mathbf{r})\frac{\partial \mathbf{E}(\mathbf{r},t)}{\partial t} + \mathbf{J}_s(\mathbf{r},t) + \sigma(\mathbf{r})\mathbf{E}(\mathbf{r},t) \tag{1.26}$$

$$\nabla \cdot (\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t)) = \rho(\mathbf{r},t) \tag{1.27}$$

$$\nabla \cdot (\mu(\mathbf{r})\mathbf{H}(\mathbf{r},t)) = 0 \tag{1.28}$$

Let us now assume that we have found solutions $\mathbf{E}(\mathbf{r},t)$ and $\mathbf{H}(\mathbf{r},t)$ for the electric and the magnetic fields, respectively. We are interested in the conditions under which the time-reversed functions $\mathbf{E}(\mathbf{r},-t+2t_1)$ and $\mathbf{H}(\mathbf{r},-t+2t_1)$ satisfy Maxwell's equations.

Let us write the same equations for the time-reversed functions. Note that we have added a question mark at the end of each equation to indicate that we are, at this time, not sure that the equations are satisfied.

$$\nabla \times \mathbf{E}(\mathbf{r},-t+2t_1) = -\mu(\mathbf{r})\frac{\partial \mathbf{H}(\mathbf{r},-t+2t_1)}{\partial t}? \tag{1.29}$$

$$\begin{aligned}\nabla \times \mathbf{H}(\mathbf{r},-t+2t_1) = \varepsilon(\mathbf{r})\frac{\partial \mathbf{E}(\mathbf{r},-t+2t_1)}{\partial t} + \mathbf{J}_s(\mathbf{r},-t+2t_1)\\ + \sigma(\mathbf{r})\mathbf{E}(\mathbf{r},-t+2t_1)?\end{aligned} \tag{1.30}$$

$$\nabla \cdot (\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},-t+2t_1)) = \rho(\mathbf{r},-t+2t_1)? \tag{1.31}$$

$$\nabla \cdot (\mu(\mathbf{r})\mathbf{H}(\mathbf{r},-t+2t_1)) = 0? \tag{1.32}$$

Note also that, in (1.30) and (1.31), we also applied the T-symmetry transformation to the sources (as discussed in Section 1.3.4).

Let us pose $t' = -t + 2t_1$ and rewrite Equations (1.29) to (1.32) as a function of t':

$$\nabla \times \mathbf{E}(\mathbf{r},t') = \mu(\mathbf{r})\frac{\partial \mathbf{H}(\mathbf{r},t')}{\partial t'} \tag{1.33}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t') = -\varepsilon(\mathbf{r})\frac{\partial \mathbf{E}(\mathbf{r},t')}{\partial t'} + \mathbf{J}_s(\mathbf{r},t') + \sigma(\mathbf{r})\mathbf{E}(\mathbf{r},t') \tag{1.34}$$

$$\nabla \cdot (\varepsilon(\mathbf{r})\mathbf{E}(\mathbf{r},t')) = \rho(\mathbf{r},t') \tag{1.35}$$

$$\nabla \cdot (\mu(\mathbf{r})\mathbf{H}(\mathbf{r},t')) = 0 \tag{1.36}$$

Since the first derivatives will change the sign of the terms involved, we can see that Equations (1.33) and (1.34), as they are shown here, are not time-reversal invariant. Thus, the transformation $t \rightarrow -t + 2t_1$ does not satisfy time reversibility alone. However, by inspection, we can see that changing the sign of the magnetic field, of the source current density, and of the conductivity will result in the time reversibility of the Maxwell equations.[4]

A discussion is in order on the necessity of the sign change for the magnetic field and the current density, which has created some controversy in the literature (see, for example, [4] to [7]). Indeed, it was argued by Albert [6] that Maxwell's equations are not time-reversal invariant and the sign change of the magnetic field was just a mathematical trick to save the time-reversal invariance (see the discussion in [5]). Snieder [3] argued that there is a physical reason for which the electric current density should change sign under time reversal. When the direction of time is reversed, the velocity of the charges changes sign. As a result, the associated current density should change sign as well. Finally, a change of sign of the current density should result in a change of sign of the associated magnetic field.

The fact that changes are required in the equations means that Maxwell's equations are time-reversal invariant in the soft sense.

1.3.7 Time-Reversal Process: Summary

We have seen that in order to be able to reproduce the past behavior of a system in the future, the condition that needs to be satisfied by the underlying physical equations is that, given a solution $f(t)$ to the underlying differential equations related to a physical phenomenon, $g(t) = f(-t + k)$ must also be a solution. Since k is a constant related to the arbitrary reference origin for the time, this condition can be simply checked by verifying that the underlying equations remain unchanged by making the T-symmetry substitution $t \rightarrow -t$. If this condition is satisfied, the physical law described by the equations is called invariant under time reversal in the strict sense. We have also seen that this

4 A second possibility is the change the signs of the electric field, the conductivity and the charge density.

condition alone might not be enough to satisfy time reversibility. For the case of Maxwell's equations, for instance, the sign of the magnetic field, of the source current density, and of the conductivity should also be changed. We have defined phenomena that require these additional conditions as time-reversal invariant in the soft sense.

It is interesting to observe that most laws of nature feature invariance under time reversal, either in the strict or in the soft sense. There are a few exceptions, an example of which is the weak force governing radioactive decay [3].

1.4 Application of Time Reversal in Practice

Assume a steel ball is propelled at a time $t = 0$ into the playfield of a pinball machine [8]. The ball will follow a complex trajectory moving in different directions, striking different targets and bumping at kickers and slingshots. Now, let us apply the third approach presented in Section 1.3.3 and let us freeze the event at a given time t and apply the time-reversal process as described in the previous section. The ball will then converge back towards the plunger, from which it was originally launched, as if we were watching the film of the game backward. Interestingly, this thought experiment is in reality infeasible in practice, even using computer simulations due to the general impossibility of representing real numbers with a finite number of bits.[5] After a few collisions, the ball will miss a barrier it should have hit and the rest of its route will be changed. As a consequence, the ball will never reach back to its origin. The reason why time reversal is practically impossible to realize in dynamical systems is the deterministic chaos, namely the fact that these systems are very sensitive to the initial conditions. Small differences in the initial conditions, in this case the measurement of the position and speed at time t, would result in a diverging behavior of the system. The perturbations in the solution in chaotic dynamical systems grow exponentially with time, following a function $\exp(\mu t)$, where μ is the so-called Lyapounov exponent [3].

5 Note that it would be possible to make the ball retrace the path to the launch position if the complete process of forward and back-propagation were made using idealized analytical expressions for Newtonian mechanics.

Furthermore, the presence of friction breaks the time-reversal symmetry of the laws of motion.

Unlike dynamical systems, the application of the time-reversal process to wave phenomena appears to be very effective. Fields of waves are not very sensitive to the initial conditions. Perturbations in the solution in wave systems grow at a much smaller rate compared to dynamical systems [3]. Intuitively, this can be explained by the fact that waves travel along all possible scattering paths while particles travel along a unique path [3]. Furthermore, the dispersion effect is, in many practical situations, negligible in wave phenomena. As a result, and following the pioneering work of Fink and co-workers (e.g., [9]), the time-reversal process has found a great number of applications, first in acoustics and later in electromagnetics.

In the next section, we will discuss the use of the time-reversal process as a way of refocusing electromagnetic fields towards their source. Note that in the remainder of this chapter, the time reversal process is defined according to the third approach presented in Section 1.3.3.

1.5 Refocusing of Electromagnetic Waves Using Time Reversal

1.5.1 Time-Reversal Cavity

The concept of a time-reversal cavity was proposed by Cassereau and Fink [10] for acoustic waves and later extended to electromagnetic waves, using the Lorentz reciprocity principle [11]. A time-reversal cavity extends the concept of a time-reversal mirror (TRM) [9, 12], a technique used to refocus an emitted wave back to its source.

Consider the situation depicted in Figure 1.3a, in which a source emits an impulsive electromagnetic field in a linear and non-magnetic medium.[6] Consider a closed surface S surrounding the source and suppose that we are able to determine the tangential fields generated by the source at any point on this surface.

6 The medium can be inhomogeneous.

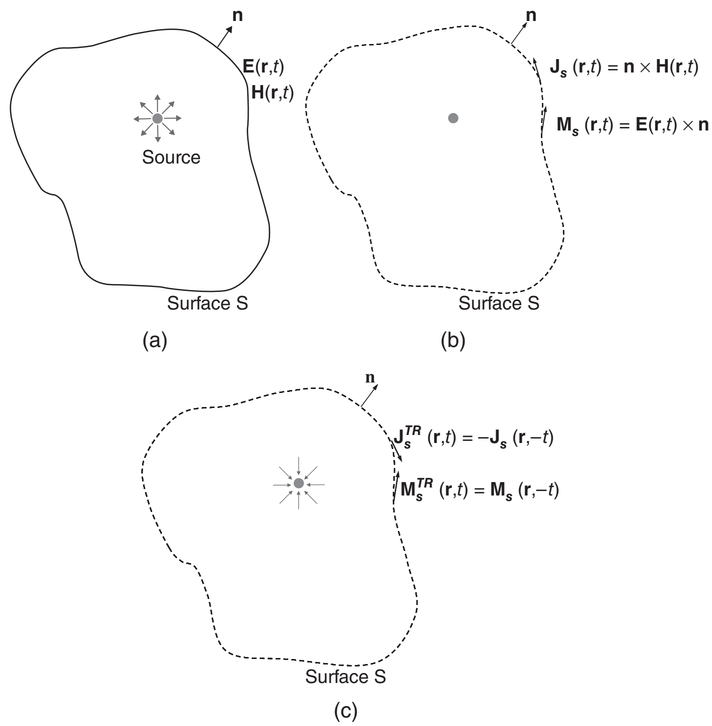

Figure 1.3 (a) A source emits an impulsive electromagnetic field in the medium at time $t = 0$. We determine the tangential fields generated by the source at any point on the closed surface S as a function of time. (b) Making use of the equivalence theorem, it is possible to replace the source in (a) by equivalent electric and magnetic current sources $\mathbf{J}_s$ and $\mathbf{M}_s$. (c) Time-reversing the equivalent sources on the surface will result in time-reversing the electromagnetic fields in all the medium within that surface, converging back to the source location.

Making use of the equivalence theorem, it is possible to replace the source in Figure 1.3a by equivalent electric and magnetic current sources $\mathbf{J}_s$ and $\mathbf{M}_s$ on the surface S (Figure 1.3b), which are given by

$$\mathbf{J}_s(\mathbf{r}, t) = \mathbf{n} \times \mathbf{H}(\mathbf{r}, t) \tag{1.37}$$

$$\mathbf{M}_s(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, t) \times \mathbf{n} \tag{1.38}$$

The fields outside the surface can be calculated either by computing the fields from the original source in Figure 1.3a or by calculating the fields from the new electric and magnetic currents on S, shown in Figure 1.3b. In other words, according to the equivalence theorem, the fields generated by the equivalent sources on the surface will be the same as the fields generated by the original source.

In addition, according to the uniqueness theorem, the specification of the electric and/or magnetic field on the surface S corresponds to a unique distribution of the electromagnetic field in the volume surrounded by S.

Now assume that we time-reverse the electric and magnetic fields in all the points of the volume surrounded by S, namely

$$\mathbf{E}(\mathbf{r}, t) \rightarrow \mathbf{E}(\mathbf{r}, -t) \tag{1.39}$$

$$\mathbf{H}(\mathbf{r}, t) \rightarrow -\mathbf{H}(\mathbf{r}, -t) \tag{1.40}$$

In this case, the new equivalent sources on the surface S become

$$\mathbf{J}_s^{TR}(\mathbf{r}, t) = -\mathbf{n} \times \mathbf{H}(\mathbf{r}, -t) \tag{1.41}$$

$$\mathbf{M}_s^{TR}(\mathbf{r}, t) = \mathbf{E}(\mathbf{r}, -t) \times \mathbf{n} \tag{1.42}$$

Obviously, the new equivalent sources corresponding to the time-reversed electric and magnetic fields are time-reversed versions of the direct-time sources (1.37) and (1.38):

$$\mathbf{J}_s^{TR}(\mathbf{r}, t) = -\mathbf{J}_s(\mathbf{r}, -t) \tag{1.43}$$

$$\mathbf{M}_s^{TR}(\mathbf{r}, t) = \mathbf{M}_s(\mathbf{r}, -t) \tag{1.44}$$

Likewise, time-reversing the equivalent sources on the surface will result in time-reversing the electromagnetic fields in all the medium within that surface (Figure 1.3c).

Starting from this observation, the concept of a time-reversal cavity allows refocusing a wave back to its source, using a two-step process. In the first step, the electromagnetic fields generated by the source are determined over the surface S forming the cavity. In the second step, the internal source is removed from the medium and equivalent time-reversed sources on the surface are considered. The resulting field in the second step will be a time-reversed copy of the fields in the first step and thus will converge back to the source, before diverging again. In order to eliminate the diverging field, the initial source must be replaced by a time-reversed sink [12].

1.5.2 Use of a Limited Number of Sensors

Obviously, a time-reversal cavity cannot be realized experimentally [10] because it requires an infinite number of transducers covering a closed surface around the medium to obtain information about all wavefronts propagating in all directions [13]. In practice, the fields can be measured using a limited number of sensors and the question arises whether the focusing property of time reversal remains intact. Draeger *et al.* [13, 14] have shown that, in the case of acoustic waves, focusing is possible using a single element in a closed reflecting cavity with negligible absorption. The same focusing property was obtained with electromagnetic waves (e.g., [15]). Derode *et al.* [16] have shown that, compared to a homogeneous medium, a higher focusing quality can be achieved in an inhomogeneous medium, as a result of multiple reflections and scattering.

Consider the example of a source that emits an electromagnetic field impulse in free space (triangle at the center of the cylindrical wavefront in Figure 1.4a). This could be, for example, a cloud-to-ground lightning discharge. Suppose we have three sensors that record the fields generated by the source (shown in Figure 1.4a on the top, left and right parts of the domain). Figure 1.4a shows the cylindrical wave generated by the source at a given time. Time-reversing the received waveforms captured by each sensor and re-emitting them back into the medium will result in a maximum peak field at the location of the source, at which the injected time-reversed waveforms contributing to the total field are in phase (Figure 1.4b).

Now let us consider the case where the the medium contains two perfectly conducting reflecting walls, as shown in Figure 1.5a [17]. In this case, it will be possible to locate the source with only one sensor. Indeed, the source point (triangle at the center of the cylindrical wavefront) generates a field, which is reflected by the two walls (Figure 1.5a), which, by virtue of image theory, can be replaced by three mirrored sources. Hence the sensor (triangle shown on the left part of the domain) receives four successive

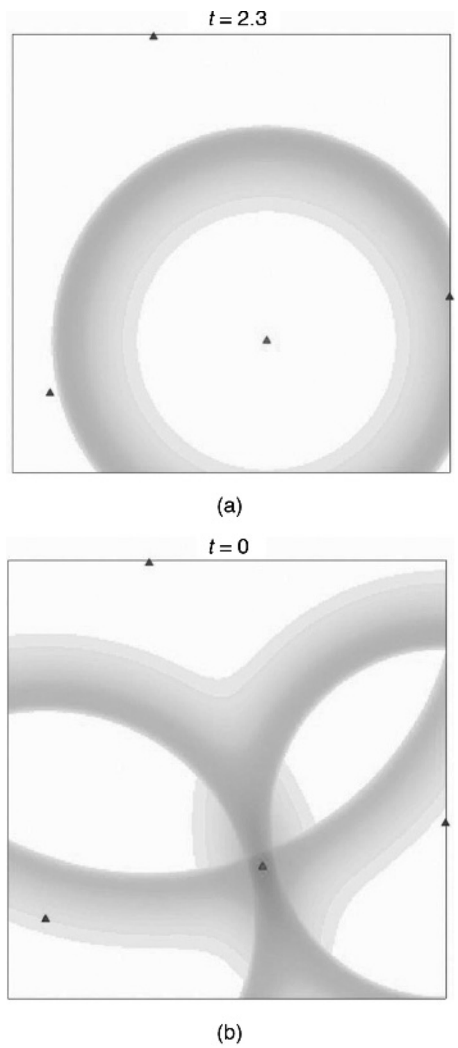

Figure 1.4 (a) A source emitting an electromagnetic field impulse in free space (triangle at the center of the cylindrical wavefront). Three sensors (triangles on the top, left and right parts of the domain) record the emitted field. (b) Time-reversed waveforms transmitted by the three sensors converge back to the source. (Adapted from [17].) An animated version of this figure can be found on www.wiley.com/go/rachidi55.

waves. In the second step, the sensor re-emits the time-reversed fields into the medium, which are in turn reflected by the two walls (Figure 1.5b). As can be seen in Figure 1.5b, the maximum amplitude field is obtained at the source location and, therefore, a single sensor makes it possible to locate the source in this case.

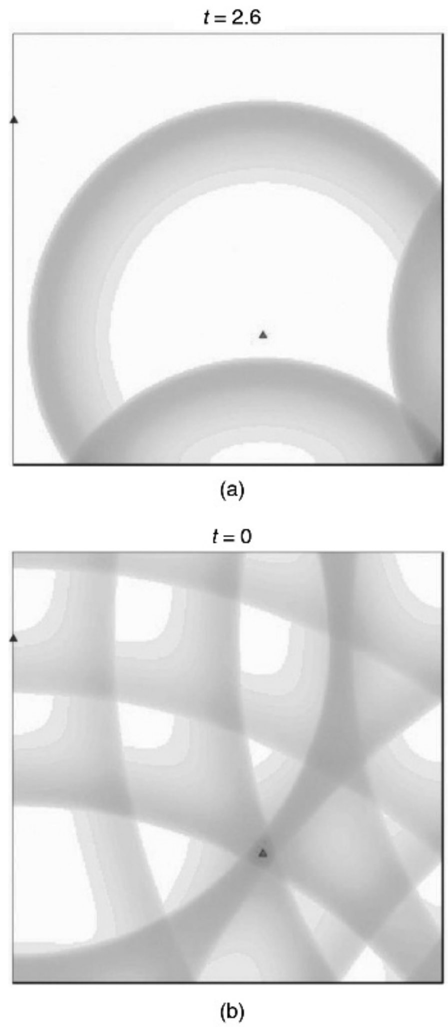

Figure 1.5 (a) A source (triangle at the center of the cylindrical wavefront) emitting an electromagnetic field impulse in a medium containing two reflecting walls (bottom and right). One sensor (triangle on the left part of the domain) records the emitted field. (b) Time-reversed waveform received by the sensor converges back to the source. (Adapted from [17].) An animated version of this figure can be found on www.wiley.com/go/rachidi55.

Another important point to be mentioned is that the field distribution resulting from the injection of the time-reversed fields from a finite number of sources back into the medium will obviously not be a time-reversed copy of the fields in direct time [18, 19].

1.5.3 Time Reversal and Matched Filtering

Time reversal is similar to the concept of matched filtering used in signal processing. Consider, as an example, an antenna A in a linear, time-invariant medium. A signal $x(t)$ is applied to antenna A. Suppose that we have N receiving antennas $B_1, \ldots, B_N$. The signal received at the output of antenna B_i can be written as

$$y_i(t) = x(t) \otimes h_i(t) \tag{1.45}$$

where $h_i(t)$ is the system impulse response and $\otimes$ denotes the convolution product. Now, let us time-reverse the received signals and feed each one of them back to its corresponding antenna B_i. Using reciprocity, we can express the signal received at antenna A as

$$\begin{aligned} z(t) &= \sum_{i=1}^{N} y(-t) \otimes h_i(t) = \sum_{i=1}^{N} x(-t) \otimes h_i(-t) \otimes h_i(t) \\ &= \sum_{i=1}^{N} R_{ii}(t) \otimes x(-t) \end{aligned} \tag{1.46}$$

in which $R_{ii}(t)$ is the autocorrelation function of the system impulse response $h_i(t)$, the Fourier transform of which is given by

$$R_{ii}(\omega) = H_i(\omega)H_i^*(\omega) = \left|H_i(\omega)\right|^2 \tag{1.47}$$

Thus, the signal $z(t)$ received at antenna A has the same phase as the input signal $x(t)$, but is conjugated and its amplitude is modified by the autocorrelation function. Therefore, it cannot be considered as a perfect replica of the emitted signal.

1.6 Applications of Time Reversal in Electrical Engineering

The first experiment using time reversal in electromagnetism was reported by Bogert [20] in 1957, in which a time-reversal technique was used to compensate delay distortion on a slow-speed picture transmission system.

The time-reversal technique was popularized in the scientific community by Fink and his colleagues in the 1990s in various studies related to acoustics (e.g., [8] to [10], [13], [16], and [21]), and later to electromagnetism (e.g., [12] and [15]).

In the twenty-first century, the time-reversal technique has emerged as a very interesting technique with potential applications in various fields of engineering, leading to mature technologies with unprecedented performance compared to classical techniques. Applications of time reversal include

– Focusing and amplification of electromagnetic waves (e.g., [15] and [22])
– Biomedical engineering (e.g., [23])
– Imaging (e.g., [24] to [27])
– Wireless communications (e.g., [28])
– EMC testing (e.g., [29])
– Fault detection and location (e.g., [30] to [32])
– Earthquake detection [33]
– Landmine detection (e.g., [34] and [35])
– Communications and radar (e.g., [36])
– Lightning location [37, 38].

It is expected that the fields of application of electromagnetic time reversal (EMTR) will continue to grow in the near future. In the following chapters of this book, some of the applications of the electromagnetic time reversal to power systems and electromagnetic compatibility will be described.

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