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Agent-Based Modeling and Tax Evasion: Theory and Application

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1.1 Introduction

While the formal study of tax evasion began with a seminal paper by M.G. Allingham and A. Sandmo titled "Income Tax Evasion: A Theoretical Analysis" in 1972, scholars and practitioners continue to be challenged with designing and implementing policies and incentives to mitigate tax evasion. While early theoretical studies provide a baseline from which to evaluate hypotheses, the methodology underlying the classical formulation; that is, the use of utility functions and the assumptions of taxpayer homogeneity and rationality, fall short in characterizing taxpayer behaviors observed in practice. In this book, we seek to advance the state of the art in the study of tax evasion by presenting an alternative computational approach based on simulating individual agents. These so-called *agent-based models (ABM)* aim to take into account individual preferences and can accommodate a larger variety of intrinsic and extrinsic variables to help explore a broader space of compliance outcomes. In this introductory chapter, we present a formal definition of tax evasion in Section 1.2 and outline the case for why its analysis is a priority not only for tax administrators, but also for society at large. The classical theoretical models of tax evasion are then summarized

in Section 1.3 to provide historical context as well as an understanding of the assumptions and abstractions that have helped formalize the preliminary studies in this field. The ABM paradigm is introduced in Section 1.4 with an outline of the Overview, Design Concepts, and Details plus Decision-Making (ODD+D) protocol (Müller *et al*., 2013) that we propose as a standard to guide both the development and presentation of simulation results. Here we stress the need to calibrate and replicate ABMs to help further advance our collective efforts. A literature review follows in Section 1.6 describing the tax evasion agent-based methods developed to date. In conclusion, Section 1.7 provides an overview of the edited volume by summarizing the remaining chapters that further explore the nuances in this field.

1.2 Tax Evasion, Tax Avoidance and Tax Noncompliance

What constitutes tax evasion? Alm (2012, p. 55) defines tax evasion as the "illegal and intentional actions taken by individuals to reduce their legally due tax obligations." The most common "strategies" for tax evasion involve intentionally underreporting income, claiming fake deductions and credits, exploiting loopholes in the tax regulations, and engineering artificial losses. The US Internal Revenue Service (IRS) links tax evasion with tax fraud, which it defines as "… an intentional wrongdoing, on the part of the taxpayer, with the specific purpose of evading a tax known or believed to be owing" (IRS, 2016b). Note that tax evasion should be distinguished from legal tax avoidance that allows individuals to reduce their tax liability through legitimate means. A grey area exists when individuals exploit tax loopholes as a means for tax avoidance. In certain countries (such as Germany) this may fall within the definition of legal tax avoidance. Notably, Slemrod and Yitzhaki (2002) distinguish tax avoidance and evasion, where the former is the legal usage of tax loopholes to lower the tax burden while the later is illegal. Thus the law draws the line between tax avoidance and evasion.

An additional term "tax noncompliance" also appears in the literature and is sometimes used interchangeably with tax evasion. We consider tax noncompliance as a more general term that includes *both* intentional evasion *and* unintentional errors. However, the tax gap studies by the IRS reports overall tax noncompliance (estimated at $458 billion for 2008–2010, see IRS, 2016a) without attempting to consider this distinction (GAO, 2012, p. 3). Hence, strictly speaking, interchangeable use of tax evasion and tax noncompliance is inconsistent. To be clear, our discussions center on characterizing intentional acts by individuals to reduce their tax liability rather than on factors that may lead to mistakes in tax reporting.

One may ask, why analyze tax evasion? First and foremost, tax is essential to fund public expenditures. Tax evasion is therefore not only a concern for the tax authorities, but also for society at large. Given a nation's investment in healthcare, education, defence, social security, transportation, infrastructure, science and

technology are derived substantially from public financing, tax evasion causes misallocations of public goods (Andreoni *et al.*, 1998; Slemrod and Yitzhaki, 2002; Torgler, 2002; Kirchler, 2007; Slemrod, 2007; Alm, 2012, provide surveys, and James and Edwards, 2010, present a bibliography of the literature on tax compliance). Moreover, given the nonnegligible extent of tax evasion as indicated by the tax gap estimates by the IRS (2016a), tax authorities can incur significant expenses to enact deterrence efforts. An analysis of tax evasion can therefore be used to identify factors that influence compliance rates and ultimately help governments reach revenue targets. Secondly, by exploring issues of tax evasion across different tax systems, there is an opportunity to inform and share insight about the effectiveness of different policy initiatives. As an example, even though the tax rate, tax base, and tax-exempt amounts may differ among nations, cross-border comparison of countries with flat income taxes might yield valuable lessons. Now that we have briefly discussed the meaning of terms used to describe tax evasion, tax avoidance, and tax noncompliance, we will now address theories of tax evasion.

1.3 Standard Theories of Tax Evasion

Research on tax evasion started in the 1970s with the seminal works by Allingham and Sandmo (1972) and Srinivasan (1973), who applied the economics-of-crime approach by Becker (1968, 1993) to tax reporting behavior.[1] Comparing these tax reporting models reveals various distinctions and similarities, that can be grouped based on (i) mathematical modeling, (ii) taxpayer's optimal choice, (iii) comparative statics, (iv) framework extensions, and (v) model critique.

To begin with, we briefly describe standard versions of the tax evasion models. Allingham and Sandmo (1972) consider an individual, a taxpayer, who is faced by a decision problem about how much income X of the true income I to declare to the tax authority given an audit probability α, a tax rate θ, and a penalty rate π. Further, the taxpayer is assumed to be risk averse, so that the marginal utility $\mathcal{U}'$ is strictly decreasing;, that is, the resulting utility function $\mathcal{U}$ is concave. To solve the decision problem, the taxpayer maximizes his expected utility

$$\mathcal{E}\mathcal{U}[X] = (1 - \alpha)\mathcal{U}[I - \theta X] + \alpha\mathcal{U}[(1 - \pi)I + (\pi - \theta)X] \qquad (1.1)$$

such that the necessary condition for a maximum is

$$(1 - \alpha)(-\theta)\mathcal{U}'[I - \theta X] + \alpha(\pi - \theta)\mathcal{U}'[(1 - \pi)I + (\pi - \theta)X] = 0 \qquad (1.2)$$

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[1] On the one hand, the economics-of-crime approach allows for a rational point of view on individual illegal activities. On the other hand, such an analysis takes into account social welfare maximization and related problems to design optimal policies for combating illegal behavior. At the Nobel Lecture Becker (1993, p. 391) highlighted that the economics-of-crime theory and its analytical tools have allowed the examination of a variety of topics, for instance, tax evasion.

The taxpayer has an incentive toward tax evasion if his marginal expected utility is positive for total evasion, that is, $X = 0$, and negative for full tax compliance, that is, $X = I$; mathematically speaking the first derivative of the expected utility with respect to income declaration X needs to have a sign change, and the second derivative must be negative. The latter condition is fulfilled because of the concavity of the utility function and the former condition leads to

$$\frac{\partial \mathcal{EU}[X]}{\partial X}\Big|_{X=0} = (1 - \alpha)(-\theta)\mathcal{U}'[I] + \alpha(\pi - \theta)\mathcal{U}'[(1 - \pi)I] > 0 \qquad (1.3)$$

and

$$\frac{\partial \mathcal{EU}[X]}{\partial X}\Big|_{X=I} = (1 - \alpha)(-\theta)\mathcal{U}'[(1 - \theta)I] + \alpha(\pi - \theta)\mathcal{U}'[(1 - \theta)I] < 0 \quad (1.4)$$

Rearranging Eqs (1.3) and (1.4), yields the condition which guarantees an interior solution for the income maximization problem

$$\theta > \alpha\pi > \theta\left(\alpha + (1 - \alpha)\frac{\mathcal{U}'[I]}{\mathcal{U}'[(1 - \pi)I]}\right) \qquad (1.5)$$

If the tax rate θ changes, then a fixed penalty rate π on undeclared income $I - X$ might cause a conflict between two effects on the optimal income declared X^*: an income effect and a substitution effect. Yitzhaki (1974) shows that modeling a fine on the evaded tax via a sanction tax rate $\zeta = \pi/\theta > 1$ resolves this conflict.

Implicitly assuming a linear utility function, Srinivasan (1973) investigates the taxpayer's expected income after taxes and penalties

$$\mathcal{EI}[I] = (1 - \alpha)(I - \mathcal{T}[(1 - \lambda)I]) + \alpha(I - \mathcal{T}[I] - \lambda\mathcal{P}[\lambda]I) \qquad (1.6)$$

where $\mathcal{T}[I]$ denotes the taxes as function of true income, and $\mathcal{P}[\lambda]$ the penalties as function of $\lambda = 1 - X/I$ denoting the fraction of true income not declared to the tax authority. Hence, the first-order condition for a maximum is

$$(1 - \alpha)\mathcal{T}'[(1 - \lambda)I]I - \alpha(\mathcal{P}[\lambda] + \lambda\mathcal{P}'[\lambda])I = 0 \qquad (1.7)$$

Table 1.1 summarizes the mathematical syntax and its meaning in these standard tax reporting theories. The essential distinction in the tax evasion theories is that Allingham and Sandmo (1972) employ expected utility maximization while Srinivasan (1973) maximizes an expected income after tax and penalties without an utility function modeled explicitly. The strict concavity of the utility functions permits Allingham and Sandmo (1972) to explore the tax reporting behavior of risk averse taxpayers. In particular, Allingham and Sandmo (1972) rule out linear utility functions because of the need for a strict decline of marginal utility. To put it differently, Allingham and Sandmo (1972) provide an all-or-nothing solution for the borderline case of a linear utility function, that is, a risk neutral individual. Yet,

Table 1.1 Mathematical syntax for standard theories of tax evasion

Mathematical syntax	Meaning
α	Audit probability
θ	Tax rate
π	Penalty rate
ζ	Sanction tax rate
λ	Fraction of true income not declared to the tax authority
I	True income
X	Income declaration
X^*	Optimal income declaration
$\mathcal{U}$	Utility
$\mathcal{U}'$	Marginal utility
$\mathcal{EU}$	Expected utility
$\mathcal{EI}$	Expected income (after tax and penalties)
$\mathcal{T}$	Tax function
$\mathcal{P}$	Punishment function

risk neutral subjects are implicitly considered by Srinivasan (1973).[2] Furthermore, taxpayers considered in the Allingham and Sandmo (1972) model chose an optimal amount of the income to declare while Srinivasan (1973) assumes the taxpayers to declare an optimal fraction understating the income. Both models suppose taxpayers can report between the true value, that is, full tax compliance, and nothing that is, full tax evasion. A fixed audit probability is the standard assumption in Allingham and Sandmo (1972). Thus, Srinivasan (1973) represents a broader framework in the sense of implementing an audit probability, which varies and may depend on income declaration. Srinivasan (1973) makes use of progressive tax schemes without any kind of tax allowances or deductions while the fixed tax rate in Allingham and Sandmo (1972) reflects a flat tax scenario. Further, a taxpayer faces a fixed penalty rate on nondeclared income if his cheating behavior is uncovered. Srinivasan (1973) varies a penalty rate on nondeclared income. To summarize, the mathematical features in Allingham and Sandmo (1972) and Srinivasan (1973) are quite similar but differ with respect to optimization procedures, perceived taxpayer's risk attitudes, decision variables, audit probabilities, tax tariffs, and penalty functions.

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[2] Risk neutral taxpayers rather fit to Srinivasan (1973) than suit to Allingham and Sandmo (1972). In line with this Sandmo (2012, p. 6, Note 1) remarks that individual's risk neutrality of Srinivasan (1973) is due to an implicit linear utility function. However, the other way around, Srinivasan (1973, pp. 341–342) emphasizes that applying strictly concave utility functions and, therefore, examining risk averse taxpayers would not necessarily change his theoretical findings.

Second, to find the taxpayer's optimal reporting behavior both theories of tax evasion take into account a first-order condition, which refers either to maximizing the expected utility (Allingham and Sandmo, 1972) or the expected income after tax and penalties (Srinivasan, 1973). Allingham and Sandmo (1972) derive taxpayer's evading conditions (Eq. (1.5)) when to declare no income at all, full income or a fraction of true income, that is, an interior solution. The Allingham and Sandmo (1972) conditions (Eq. (1.5)) depend on the characteristics of taxpayer's utility function and the "tax enforcement variables," audit probability, penalty, and tax rate (e.g., see Cowell, 1992, p. 527). The interior solution vanishes if taxpayer's utility function reflects risk neutrality. Likewise, Srinivasan (1973) proves the existence of an interior solution depending on audit probability and the characteristics of prevailing tax and penalty functions. It is remarkable that this study does not allow for a corner solution, which is quite a converse result to Allingham and Sandmo (1972).

Third, while comparative statics bring up a variety of findings, both tax evasion models show a coinciding result; that is, increasing the audit probability *ceteris paribus* (c.p.) leads to less tax evasion. Allingham and Sandmo (1972) find that higher penalty rates, ceteris paribus, rise the optimal income declared and, thus, reduce the theoretical extent of tax evasion. The remaining comparative static results require additional assumptions. For instance, Srinivasan (1973) considers simultaneously (i) a progressive tax scheme and (ii) an income-independent audit probability. Then, raising the income, ceteris paribus, increases the declared fraction of income understatement and, therefore, tax evasion. To put it differently, richer taxpayers evade more tax – in relative and absolute terms – if a progressive income tax tariff applies. Further, the author examines an increase in income given (i) constant marginal tax rates and (ii) increasing audit probabilities. Here, the opposite effect occurs; that is, a higher income, ceteris paribus, decreases the magnitude of tax evasion. Allingham and Sandmo (1972) consider an increase of income and simultaneously analyze (i) a reduction of absolute risk aversion and (ii) a constant penalty rate, which is equal or greater than one. In this case, a larger income, ceteris paribus, triggers a higher optimal income declared and, therefore, leads to less tax evasion. Next, in addition to optimal income declared, a fraction of income declared is studied. In this context, it is worth noting that fractions of taxpayer's true income (Allingham and Sandmo, 1972) and income understatement (Srinivasan, 1973) are counterparts and add up to unity. The group of authors finds variations of fractions declared to tax authorities depending on taxpayer's relative risk aversion. Allingham and Sandmo (1972) introduce an income and substitution effect to the tax evasion theory, which can be traced back to their modeling of tax and penalty rates. The authors observe negative substitution effects since rising tax rates enhance taxpayer's incentives to evade tax. The sign of the income effect is ambiguous and depends on taxpayer's absolute risk aversion. For instance, decreasing absolute risk aversion yields strictly positive income effects such that the model fails to provide a clear-cut hypothesis. In summary, ceteris

paribus, lower extents of tax evasion are caused by separately increasing the two enforcement parameters (i) audit probability and (ii) penalty rate, whereas all other comparative statics results by Allingham and Sandmo (1972) or Srinivasan (1973) require further assumptions.

Fourth, the authors present various extensions of their theoretical frameworks. Allingham and Sandmo (1972) examine three additional features, which are (i) lapse of time effects (or back auditing), (ii) nonpecuniary factors and (iii) variable audit probabilities. Concerning the last feature, comparative statics support the notion that more audit efforts, ceteris paribus, reduce tax evasion if and only if the audit function is monotone decreasing and strictly concave with respect to income. Nonpecuniary factors are added to taxpayer's utility function, for example, to reflect shame and guilt of identified and punished tax evaders. Allingham and Sandmo (1972) show the impact of such a modeling on taxpayer's evading constraints (Eq. (1.5)), even though the signs are not clear. With respect to back auditing, the authors find tipping points when a representative taxpayer starts to declare almost everything honestly.[3] Srinivasan (1973) studies two other extensions, which are (i) progressive versus proportional tax tariffs and (ii) optimization of audit probabilities. Considering various feasible tax tariffs, the author obtains that flat tax, ceteris paribus, yields less tax evasion than progressive tax schemes. With respect to an audit optimization procedure, he deduces a precondition that depends on marginal governmental expenditures due to the costs of raising audit probabilities. To sum up, Allingham and Sandmo (1972) and Srinivasan (1973) find that variable audit probabilities, lapse of time effects and changes of tax tariffs, for example, to implement flat tax, reduce tax evasion while the remaining extensions, for instance nonpecuniary factors, yield no clear-cut conclusions.

Finally, both tax evasion models are subject to notable criticism. One of the outstanding problems is that both theoretical frameworks in their standard version simply over-predict the amount of income tax evasion, which is estimated in reality. This is inconsistent with the real-world magnitudes of tax enforcement variables, which are obviously too low to guarantee deterrence levels that are theoretically necessary for taxpayers to become fully tax compliant. Therefore, the emerging crucial research question is the taxpayer's compliance mystery, why are they so tax compliant or intend to be totally compliant? To put this differently, the challenging issue is not, why tax avoidance, evasion, and the like exist but on the contrary it is the well-known question: "Why do people pay taxes?" (Alm *et al.*, 1992b). Moreover, on the technical side, the Yitzhaki (1974) criticism deals with the problem of ambiguous income and substitution effects in the standard setting by Allingham and Sandmo (1972). Yet, Yitzhaki (1974) himself

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[3] Hokamp and Pickhardt (2010) consider lapse of time effects via an expected utility approach within their agent-based tax evasion model, which represents the neoclassical (Allingham and Sandmo, 1972) model, if populated with 100% expected utility maximizing a-type agents. Thus, computational simulations might be regarded as numerical approximations for theories.

has suggested a solution; that is, the penalty rate has to depend on evaded tax rather than on nondeclared income. Under these circumstances the assumption of decreasing absolute risk aversion, with increasing income, yields pure income and no substitution effects so that higher tax rates, ceteris paribus, reduce the extent of tax evasion. Obviously, Srinivasan (1973) is not subject to the Yitzhaki (1974) critique because of penalty rates that are based on income understatement.

In addition to the critics of the components of the models, there are also a number of critics of these models based upon missing elements. These critics are not unique to these foundational tax evasion models and relate to issues of the heterogeneity of taxpayer behavior, potential taxpayer learning and adaptation over time, the potential impact of social networks, and the role of intermediaries such as tax preparers.

Despite the aforementioned critics, the standard theoretical model proposed by Allingham and Sandmo (1972) has become the standard theoretical model to analyze income tax evasion. The approach is still a fertile origin for rich work concerning tax evasion and related issues (e.g., for empirical observations see Kleven *et al.*, 2011, for applications to the shadow economy see Buehn and Schneider, 2012; Slemrod and Weber, 2012). Since the early 1970s, considerable extensions have been added to correct, enlarge, and adjust (Allingham and Sandmo, 1972), for instance misperceptions of tax enforcement variables, psychological effects and social norms (e.g., see Myles and Naylor, 1996; Fortin *et al.*, 2007, for a survey on psychological incentives of tax evasion see Kirchler, 2007). Yet, model extensions frequently generate such a complexity that clear-cut hypotheses are not available (e.g., see Alm, 2012, pp. 60–64). Of course, without any doubt, income tax evasion dynamics clearly influence tax revenues and public budgets. The above discussion outlined the canonical economic foundations for the analysis of tax paying behavior; next we will discuss the use of agent-based modeling in this endeavor.

1.4   Agent-Based Models

The rapid increase in computing power over the past several decades has led to a widespread usage of computers for conducting research via simulation. The social sciences in particular have turned to computer simulations as a technique to investigate complex real-world situations. Garson (2009, p. 267) identifies the main research branches of computational social simulation as: (i) ABMs, (ii) network models, (iii) spatial models, and (iv) systems dynamics models. The last mentioned consider digital-programmed systems of equations (or rules) that allow for illuminating complex socio-economic systems, for example, "*The limits of economic growth*" by Meadows *et al.* (1972). Spatial models simulate the interaction between a geographical (or physical) environment and, for instance, human behavior patterns. Network models cover a variety of topics; for example, from neural networks to queuing theory and its application to traffic problems

(Gilbert and Troitzsch, 2005). ABMs, the focus of this book, consist of discrete autonomous agents that behave according to prescribed decision rules. Interaction of agents at the micro-level often leads to complex emergent phenomena at the macro-level.

Individual decisions on tax evasion are embedded in a highly complex environment of behavioral rules, social norms, and tax enforcement (Kirchler, 2007). Our focus on the use of ABMs is driven by their ability to explore the heterogeneous behaviors of large populations that can result from this complexity. This is in rich contrast to the standard neoclassical approach based on a single representative agent (see Alm, 2010; Bloomquist, 2011a). Furthermore, ABMs investigate the interaction of adaptive agents and allow for the use of tools from related branches; for example, systems dynamics models to describe the behavior patterns and the interaction rules of the agents and spatial and network models to describe an agent's physical environment and social neighborhood, respectively (Carley and Maxwell, 2006). Hence, agent-based modeling seems to be a promising tool for research on complex social systems such as tax evasion in societies of behaviorally heterogeneous agents (for overviews on agent-based simulations see Garson, 2009; Heath *et al.*, 2009; Schinckus, 2013, for surveys on agent-based tax evasion models see Bloomquist, 2006; Hokamp, 2013; Pickhardt and Seibold, 2014; Bazart *et al.*, 2016, the insights from tax evasion theory, income declaration experiments and field studies are surveyed by Alm, 2012, the behavioral dynamics of tax compliance are reviewed by Pickhardt and Prinz, 2014). In particular, Garson (2009, p. 271) employs tax evasion as an example to point out that "[…] simulation is a way to explore assumptions, not necessarily to find a "correct" solution or optimal set of parameters […]." The above discussion focused on the "why" of agent-based modeling, next we discuss how to report ABMs to aid in the creation of a cumulative science.

1.5   Standard Protocols to Describe Agent-Based Models

In this book we focus on model-based science, specifically the use of ABMs to explore more realistic, or higher resolution, representations of taxpayers and the tax payment system. Here, one uses the ABM to generate "sufficiency theorems" (Axtell, 2000a): for example, a population of agents acting under certain behavioral rules and specific conditions are sufficient to produce the observed dynamics and structures. In this work we stress the importance of clearly and thoroughly explaining the simulations used because we feel that this is a science. This being the case, we must ensure that the work can be understood, evaluated, and extended by others; and not seen as a "black box" (Lorscheid *et al.*, 2012). In order to make the claim that we are engaged in science we must create "theories." For our purposes, theories are statements about the world that are sufficiently specific as to be falsifiable, general enough to be comprehensive, and simple enough to be parsimonious (the ratio of predictions to assumptions should be large) (Harte, 2011).

Here, the output of a run of the model serves as a prediction, it is a strict deduction (Epstein, 2006). Ideally, this prediction is sufficiently specific as to be falsifiable. One can then use collections of output (deductions) to inductively build up theories, here, of human behavior.

As with most scientific endeavors, our work should be cumulative and build upon a foundation of previous research. This requires that enough information be reported to allow for replication, and then extension, of previous work. To that end, one may say that all researchers should be required to post their code. Arguably, that could be even better than a well expressed formulation (narrative), and certainly more efficient from a cumulative perspective. However, that would mean that researchers may understand less of the foundation they are using and, more importantly, could perpetuate unexpressed biases in implementation choices and bugs that may go unnoticed. If prior simulation-based work is re-implemented, then these assumptions are more likely to be discovered and previous bugs are less likely to be perpetuated (which is not to imply that potentially new bugs will not be introduced).

A second reason to stress the importance, and use, of a standard protocol to document computational simulations is for validation and verification purposes. These two concepts become increasingly important as the use of the models and simulation move from academic thought experiments to decision support systems used by policy makers. Verification is the process of deciding if the model matches its specification. Validation is the process of deciding how well a model matches the "real-world" phenomena under study. It is through this process that one can develop and document an understanding of how the model relates to the real world and how much "faith" to place in its predictions (a general process typically referred to as accreditation). One process to use in this endeavor is that of Docking (Axtell *et al*., 1996) and categorizing the ABM's empirical relevance (Axtell, 2005). Docking refers to the process of comparing your model to a referent system (usually another model). There are three degrees of docking: Identity, where the two models produce numerically identical results; Distributional, where the models produce statistically indistinguishable results; and Relational, where the models produce qualitatively similar results that are statistically distinct. The empirical relevance of a model is a way to characterize how well the model represents both the micro-level (individuals) and macro-level (emergent structure) of the real world under study. Level 0 is micro-level qualitative correspondence; Level 1 is macro-level qualitative correspondence; Level 2 is macro-level quantitative correspondence, and finally, Level 3 is micro-level quantitative correspondence. The required level of empirical relevance is a function of how one plans to use the model. This process should be done with the model's intended purpose and audience in mind. For example, if one is creating an ABM simply to help think through a particular generating mechanism, then Level 0, Relational correspondence to the real world is likely to be sufficient. Here, the

agents behave plausibly and, as a whole, the system changes in an appropriate manner when inputs are adjusted. However, if one is creating a model to be used by policy makers as they reform a taxation system then a higher standard should be used. Under these circumstances one would likely want to achieve at least Level 2, Empirical Relevance and Distributional Equivalence. Meaning, if income in the real world is Zipf distributed, the income distribution within the simulation is also Zipf distributed and is statistically indistinguishable form that of the real world.

1.5.1   The Overview, Design Concepts, Details, and Decision-Making Protocol

Originally, the Overview, Design Concepts, and Details (ODD) protocol was introduced by Grimm *et al*. (2006) and later refined in Grimm *et al*. (2010). It was created to address the need for a standardized way to adequately document individual-based and ABMs within Ecology. Recently, the ODD protocol has gained increasing acceptance within the computational social sciences as a way to document ABMs focused on human complex systems (e.g., the Computational Social Science Society of the Americas now requires the use of the ODD protocol for papers discussing ABMs). While the ODD protocol is an improvement over ad hoc discussions of ABMs it was not specifically designed around human systems, which meant that important elements of human systems (such as individual decision-making) could be lost or not discussed in adequate detail to allow for complete understanding or replication. Therefore, Müller *et al*. (2013) introduced the ODD+D protocol. In addition to the standard ODD protocol, the authors specifically added Decision-making (+D) to the protocol to ensure this critical component was not lost or under-specified. It is this ODD+D protocol that we use for this book. We would like to note that additional protocols termed the "transparent and comprehensive ecological modeling (TRACE) documentation" (Grimm and Schmolke, 2011) and the "Dahlem ABM documentation guidelines" (Wolf *et al*., 2013) were proposed as an alternative or add-on to the ODD protocol to better ease description of large-scale ABMs. While the basic structure of the ODD protocol was retained, certain subsections were removed and others added to ensure that the model description could be made more concise. We do not attempt to compare and contrast these three approaches here.

For our purposes, we will adopt the basic definitions used by Müller *et al*. (2013). Therefore, an "agent" is a bundle of "data and behavioral methods representing an entity constituting part of a computationally constructed world" (Tesfatsion, 2006). Moreover, "decision-making" is defined as: "the methods agents use to make decisions about their behavior" (Dibble, 2006). Given that human

agents often have dynamic behaviors within a model (Müller *et al*., 2013) go on to distinguish learning and adaptation. For the authors, adaptation is more "passive" meaning, an agent that changes its behavior under different circumstances while using the same rules is defined as adapting. However, if an agent changes its rules over time, then it is defined as learning. Most other concepts within the ODD+D protocol are adopted from the original ODD protocol. In summary, the protocol is defined using the following taxonomy:

1.5.1.1   Overview of the Model

Purpose
This section discusses why the model was made and for whom. For example, is the model designed as a thought experiment to understand one very specific component of a tax regime; or is it designed to help inform policy makers as they work to reform an existing tax system? It is very important to explain clearly the purpose of the model so that readers will understand the motivation and apply the correct amount of scepticism and rigor to their understanding and evaluation.

Entities, State Variables, and Scales
This section discusses what entities (types of agents) are in the model and what makes each type unique (its state variables), what exogenous forces are included, and the temporal and spatial extent of the model and how it relates to the agents. For example, if there is only one kind of tax paying agent in the model, is heterogeneity introduced via differences by variable values or by different types of agents? Is the taxation authority explicitly represented? Is a time step a year or is a finer resolution of time used? It should be noted that this section should highlight what makes agents unique even if agents use the same submodels but with different parameter values. For example, if all taxpayers have the same submodel for deciding how much income to declare but agents that are expected to declare less income are given a lower risk aversion to induce that behavior; that sort of distinction should be highlighted.

Process Overview and Scheduling
This section discusses what goes on during a model run and in what order. How are agents activated? Are all agents activated in the same order or randomized? Are different types of agents activated together? Will every agent be activated at each time step or is activation probabilistic, or are agents only activated when they have something to do (discrete event)? It is very important to be as explicit as possible in this section. The activation regime can have significant impacts on model results (Axtell, 2000b). Researchers should be very thorough when describing these model elements. Pseudo-code can be particularly useful in this section when describing the structure of the simulation's activation regime.

1.5.1.2   Design Concepts

Theoretical and Empirical Background

This section should cover the theoretical and empirical underpinnings of the model, and the rational and assumptions used in the design choices for the structure of the model. Is the model based on rational choice theory or bounded rationality? How do agents deal with uncertainty, are they based upon behavioral data or psychology? Do agents have an objective or sense of better and worse outcomes?

Individual Decision-Making

Here, one should discuss agent decision-making, how decisions are made; what information is used; how sociocultural norms, uncertainty, spatiality, and time are dealt with; and how decision-making changes over time. What decisions do the agents make? For example, do agents decide how much of their income to report or is that simply a function of their type or class? Does experience over time, or the experience of their friends impact their decisions? What is the influence of social norms, do agents know what the majority of other agents are doing or do they assume (or not care)?

Learning

This section deals with individual learning and, if implemented, collective learning. How do agents incorporate experiences into their decision-making? Do agents have a memory that is used for decision-making?

Individual Sensing

Here, one discusses agent sensing, what variables are updated via sensing, what state variables are exposed for other agents to sense, how is error introduced, the temporal and spatial scale of sensing, and what costs are associated with sensing. How do agents know about their environment? Does the taxing authority make public statements? Do agents exchange information with each other, or is communication indirect?

Individual Prediction

This section covers the methods of prediction used by agents, how they are implemented, what is it the agents are predicting, and how error is introduced. For example, do agents make predictions about changes to the tax code or a likelihood to be caught if they cheat on their taxes?

Interaction

Here one discusses the direct and indirect interactions that take place among the agents, if the interactions depend upon communication, and any coordination

networks that may exist. How do agents interact? Is interaction based upon random connections or do agents have specific networks?

Collective

This section discusses the existence of collectives, how they emerge or are imposed, and how the collective impacts the individual and vice versa. For example, do preparer agents belong to a group that sets standards for itself that are a function of the current knowledge, skills, and abilities of its members? In this case, when an agent becomes part of the collective of tax preparers it must then ensure that it meets the standards of the collective. Over time, with changes in the population of preparers, these standards could change as the make up of the preparers change.

Heterogeneity

Here, one deals explicitly with heterogeneity among agents, what variables it impacts, how it changes, and if it includes agent behavior and decision-making. As mentioned earlier, this section should explicitly and thoroughly discuss what makes an agent (or group of agents) unique. This could come from different sets of variables or different values for the same variables. This heterogeneity could also come from the unique "experiences" that an agent has over the course of the simulation run.

Stochastic

This section specifies where randomness is used within the model, including both runtime and initialization. What stochasticity is in the model? Does it come into the order agents are activated? Do agents also use randomness within their decisions? Does the entire system use a single random number generator or does each agent have their own?

Observation

Here, the modeler discusses how the simulation is observed, what data is collected and when it is collected, and should also include how emergent phenomena are observed and measured. What data is gathered from the simulation? Do agents dump their data or are aggregates measured?

1.5.1.3   Details

Implementation Details

This section specifies how the model was implemented, including simulation framework, hardware needs, and where the source code may be found. What agent-based modeling platform was used to build the simulation, Repast, NetLogo, MASON, and so on? Did you create your own framework? If so, why? Is

the framework that was used available to others? What computational hardware did you use? Is the memory footprint of your simulation large enough that specialized hardware is needed?

Initialization

Here, the design choices made about how the model will start are specified, what values are used and how were they chosen, is there a variation, and what is the overall state in which the model starts. How is the model initialized? Does the model require a "burn-in" period before data is collected? Did you do multiple runs for each given collection of parameter values? Was the random number generator initialized with different values for each run?

Input Data

This sections discusses what input data the model uses, where it came from or how it was derived, and what its form/format is. If the model is not based solely on theory, then it is using some sort of input data. Where did the data that it uses come from? Was data cleaning or transformation necessary? This is an important section as it should also relate to the motivation and audience for your model. Different uses and motivations will call for more or less inclusion of "real world" data.

Submodels

Finally, one details all of the submodels in the simulation, how they were created and parametrized, what variables they act upon, and how they were tested during development. For example, if agents use a specific routine to calculate how much income to declare, one should explain how the agent thinks through this problem, how data is used, and where it comes from. The submodel descriptions should include a narrative of it, the rational for it, and should include equations (if appropriate).

1.5.2   Concluding Remarks on the ODD+D Protocol

The ODD+D protocol provides a comprehensive guide to structure assumptions and to deliberate the level of detail required to address the research question at hand. In the context of developing models to characterize individual tax evasion, this deliberation is key to ensure that outputs from the model are interpretable and can be defended. As will be evident from the discussion of the literature on agent-based tax evasion modeling in the upcoming section, there exists a broad spectrum of prior studies and corresponding findings that cannot be considered in isolation of the underlying modeling methodologies. As we attempt to advance the state of the art, the ODD+D protocol additionally provides a common framework for the basis of extracting findings that might apply under only specific circumstances versus those that might be more broadly applicable. This

has implications for tax authorities, taxpayer service providers, and legislative bodies that are potential consumers of this research. This discussion outlined our preferred method for reporting ABMs within the scientific literature, the next section will review the extant tax evasion literature making use of ABMs.

1.6   Literature Review of Agent-Based Tax Evasion Models

In this section, we trace the evolution of agent-based tax compliance modeling and briefly outline recent advances. The agent-based frameworks highlight the need to model tax noncompliance decisions not in isolation, but rather embedded within a complex environment. The characteristics of the different methodologies used are summarized in Tables 1.2–1.4 presenting two highlights (or main areas) per agent-based tax noncompliance methodology and categorizing them with respect to (i) lead authors and research groups, (ii) software, (iii) domain, (iv) population size, (v) public goods, (vi) governmental tasks, (vii) back auditing, (viii) replication and docking studies, (ix) calibration, and (x) model highlights. The literature review is based on six surveys; (i) Bloomquist (2006) on the three early agent-based tax noncompliance frameworks by Mittone and Patelli (2000), Davis *et al*. (2003), and Bloomquist (2004a,b, 2008), (ii) Alm (2012) on tax evasion theories, income declaration experiments, and field studies, (iii) Hokamp (2013) on agent-based tax evasion and noncompliance models (iv) Pickhardt and Prinz (2014) on the behavioral dynamics of tax compliance, (v) Oates (2015) on tax evasion literature, and (vi) Bazart *et al*. (2016) on the calibration of agent-based tax evasion models.

   Inspection of Tables 1.2–1.4 permits us to identify more than 30 agent-based settings of tax noncompliance, although Kim (2003), Meacci *et al*. (2012), Bertotti and Modanese (2014a,b, 2016a,b), and Nicolaides (2014) might be considered as borderline cases. The sequential arrangement of methodologies is alphabetic by the lead author of the very first contribution with respect to each research group. Thus, the first letter of the lead author ranges from A to C, in Table 1.2, from D to MA, in Table 1.3, and from MB to W, in Table 1.4. The research groups are not strictly disjoint, for instance L. Antunes, K. M. Bloomquist, S. Hokamp, M. Koehler, L. Mittone, M. Pickhardt, and F. W. S. Lima provide more than one agent-based tax noncompliance methodology. Moreover, seven research groups provide an abbreviation of their ABM: (i) Tax Compliance Simulator (TCS, Bloomquist, 2004a,b, 2008), (ii) Networked Agent-based Compliance Model (NASCM, Korobow *et al*., 2007), (iii) Agent-based Tax Evasion Simulator (TAXSIM, Szabó *et al*., 2009, 2010; Gulyás *et al*., 2015), (iv) Small Business Tax Compliance Simulator (SBTCS, Bloomquist, 2011a), (v) Individual Reporting Compliance Model (IRCM Bloomquist, 2011b, 2013; Bloomquist and Koehler, 2015), (vi) Model C (Méder *et al*., 2012) and (vii) SIMULFIS (Llacer *et al*., 2013; Noguera *et al*., 2014).

Table 1.2 Overview of agent-based tax evasion settings, research groups A–C

Research group	Software	Domain	Population size	Public goods	Governmental tasks	Back auditing	Replication/ docking study	Calibration	Highlights
Andrei et al. (2014) Chapter 8	NetLogo	Economics	441	–	–	–	Korobow et al. (2007) Hokamp and Pickhardt (2010)	–	• Docking study • Network structures
Antunes et al. (2006, 2007a,b) Balsa et al. (2006)	NetLogo	Economics	500	×	–	×	–	–	• Back auditing • Social interactions
Arsian and İcan (2013a,b)	NetLogo	Economics	10,000	–	–	–	Bloomquist (2011a)	Turkish Revenue Administration	• Audit perception • Social networks
Bertotti and Modanese (2014a,b, 2016a,b)	–	Economics	25^a	–	×	–	–	–	• Income classes • Trade
Bloomquist (2004a,b, 2008)	NetLogo	Economics	(i) 300 (ii) 1,000	–	–	–	–	–	• Life span • Social networks
Bloomquist (2011a)	NetLogo	Economics	10,000	–	–	–	–	Experiments / Internal Revenue Service	• Bankruptcy • Calibration
Bloomquist (2011b, 2013) Bloomquist and Koehler (2015) Chapter 7	RePast	Economics	85,000	–	–	–	–	Internal Revenue Service	• Artificial county • Third-party reporting
Carley and Maxwell (2006)	Construct	Economics	–	–	–	–	–	–	• Advertising campaign • Synthetic city
Cline et al. (2014) Chapter 8	RePast HPC	Economics	12,000,000	–	–	–	Bloomquist (2011b) Andrei et al. (2014)	U.S. synthesized population dataset	• Massive-scale • Social networks
Crokidakis (2014)	–	Econophysics	10,000	–	–	–	Zaklan et al. (2009)	–	• Opinion exchange dynamics • Three states of tax declaration

"Research Group" allows for assigning the agent-based tax evasion frameworks to teams of scholars in alphabetic order that refers to the leading author regarding the first contribution. "Software" shows computational tools. "–" denotes no information. "Domain" classifies the settings with respect to the economics or the econophysics modeling branch. "Population Size" lists the number of agents for scenarios in order of appearance, denoted as (i), (ii), and (iii). ^{a} The income classes in Bertotti and Modanese (2014a,b, 2016a,b). "X" represents an agent-based setting of tax noncompliance that incorporates "Public Goods," "Governmental Tasks," and "Back Auditing," respectively. "Replication/Docking Study" permits to figure out the replicated frameworks. "Calibration" presents the data source used. "Highlights" provides two key features per setting.

Table 1.3 Overview of agent-based tax evasion settings, research groups D–MA

Research group	Software	Domain	Population size		Public goods	Governmental tasks	Back auditing	Replication/ docking study	Calibration	Highlights
Davis et al. (2003)	Mathematica	Economics		500	–	–	–	–	–	• Compliance periods • Social norms
Garrido and Mittone (2013) Chapter 3	–	Economics		180	–	×	–	–	Experiments	• Audit frequency • Optimal audit programs
Hashimzade et al. (2014, 2015, 2016) Hashimzade and Myles (2017) Chapter 4	MATLAB	Economics	(i) (ii)	1,000 6,000	–	×	–	–	–	• Audit strategy • Occupational choice
Hokamp (2014) Chapter 9	RePast	Economics		2,000	×	×	×	Hokamp and Pickhardt (2010)	–	• Pareto-optimality • Social norm updating
Hokamp and Pickhardt (2010) Chapter 9	MATLAB	Economics		150,000	–	×	×	–	–	• Back auditing • Political cycle
Kim (2003)	–	Economics		–	–	×	–	–	–	• Audit policy • Social coordination
Korobow et al. (2007)	–	Economics		1,600	–	–	–	–	–	• Heat maps • Social neighborhoods
Lima (2010, 2012a,b) Zaklan et al. (2008, 2009)	–	Econophysics	(i) (ii) (iii)	1,000,000 400 4,000	–	×	–	Zaklan et al. (2009)	–	• Majority voting • Replication study
Lima and Zaklan (2008)	Fortran	Econophysics	(i) (ii)	400 1,000,000	–	×	–	–	–	• Ising-model • Network structures
Llacer et al. (2013) Noguera et al. (2014) Chapter 5	NetLogo	Economics		–	–	×	–	–	Spanish empirical data	• Learning mechanisms • Social benefits
Magessi and Antunes (2013a,b, 2015)	NetLogo	Economics	(i) (ii)	20 1,000	–	–	–	–	Portuguese survey data	• Greed • Risk perception
Manhire (2015)	NetLogo	Economics	(i) (ii)	19[a]:1,986[b] 0[a]–125[a]:1,986[b]	–	×	–	–	–	• Audit effects • Tax compliance puzzle

[a] The number of tax examiners Manhire (2015).

[b] The taxpayers in Manhire (2015). For a brief description of the syntax see Table 1.2.

Table 1.4 Overview of agent-based tax evasion settings, research groups MB–Z. See Table 1.2 for a brief description

Research group	Software	Domain		Population size	Public goods	Governmental tasks	Back auditing	Replication/ docking study	Calibration	Highlights
Meacci et al. (2012)		Economics		1,600	–	×	–	–	–	• Cellular automata • Cross-border policy
Méder et al. (2012)	NetLogo	Economics		1,000	×	×	–	–	–	• Laffer-curves • No enforcement
Mitone and Patelli (2000) Chapter 3	SWARM	Economics		300	×	–	–	–	–	• Public goods • Strategic auditing
Nicolaides (2014)	–	Economics		–	×	×	–	–	–	• Institutional quality • Social norms
Nordblom and Žamac (2012)	Java	Economics		10,000	–	–	–	–	Swedisch Tax Agency Survey	• Black market • Life cycle
Pellizzari and Rizzi (2014)	–	Economics		1,000	×	×	–	–	–	• Individual preferences • Slippery-slope framework
Seibold and Pickhardt (2013) Hokamp and Seibold (2014b) Pickhardt and Seibold (2014) Bazart et al. (2016) Chapter 11	Fortran	Econophysics	(i) (ii)	1,000,000 150,000	×	×	×	Zaklan et al. (2009) Hokamp and Pickhardt (2010)	Experiments	• Magnetic fields • Temperature
Szabó et al. (2009, 2010) Gulyás et al. (2015) Chapter 6	RePast	Economics	(i) (ii)	200[a]; 40[b] 500[a]; 50[b]	×	×	–	–	–	• (Closed) Economy • Job market
Vale (2015)	–	Economics	(i) (ii)	10 100	–	–	×	–	–	• Social networks • Varying evasion probability
Warner et al. (2015) Rosen et al. (2015) Hemberg et al. (2016) Chapter 10	–	Economics		–	–	–	–	–	–	• Genetic algorithms • Tax evasion schemes

In Szabó et al. (2009, 2010) [a]denotes the number of workers and [b]denotes the number of employers.

Additional columns describe the software utilized, the domain classifications as an economics or econophysics setting[4] and the magnitude of population considered. The software utilized ranges from open source (for instance, NetLogo, RePast HPC) to restricted and commercial software (e.g., Mathematica, MATLAB). With respect to the classification in a domain, the majority of frameworks belongs to economics (28 out of 32), but econophysics seems to gain more and more attractiveness since the seminal paper by Zaklan *et al.* (2009). S. Hokamp and M. Pickhardt contribute to both, the economics and econophysics branch (Hokamp and Pickhardt, 2010; Seibold and Pickhardt, 2013; Hokamp, 2014; Hokamp and Seibold, 2014b; Pickhardt and Seibold, 2014; Bazart *et al.*, 2016). The population size ranges from 10 taxpayers (see Vale, 2015) to 12,000,000 agents (see Cline *et al.*, 2014), while econophysics ABMs deal with up to 1,000,000 taxpayers (e.g., see Zaklan *et al.*, 2009). Notably, Cline *et al.* (2014) were the first to attempt a massive-scale agent-based tax evasion model, without roots in econophysics, to investigate a society of 12,000,000 taxpayers.

Furthermore, Tables 1.2–1.4 allow for jointly discussing and visualizing public goods provision, governmental tasks, back auditing, replication and docking issues, calibration and two highlights for each model. We provide a detailed overview of these issues in the next subsections.

1.6.1   *Public Goods, Governmental Tasks and Back Auditing*

Mittone and Patelli (2000) proposed the seminal ABM taking tax evasion into account from a micro-perspective. The authors analyze the interplay of tax compliance and public goods provision (Cowell and Gordon, 1988; Myles and Naylor, 1996) through a factor that reflects an agent's group conformity and show that strategic monitoring does not work to repress tax evasion behavior. Antunes *et al.* (2006, 2007a,b) and Balsa *et al.* (2006) present a family of eight agent-based tax compliance settings; Antunes *et al.* (2006) discuss the level of trust in a government (Wintrobe and Gërxhani, 2004) and, in their Ec2 model, they relate such an ethical attitude to a personal surplus received from a central authority taking into account back audits. The authors find that taxpayers "keep a good ethical attitude longer" because of lapse of time effects (Antunes *et al.*, 2006, p. 158). Szabó *et al.* (2009, 2010) and Gulyás *et al.* (2015) draw on Szabó *et al.* (2008) describing a one-product economy along with the shadow economy, tax evasion, and governmental services. The authors consider employees, employers, a tax authority, and a government; the taxpayers base their compliance decision on their experience of

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[4] Hokamp and Pickhardt (2010) introduced the classification by an individual interaction process to the economic and the econophysics domain of agent-based tax evasion models. If the interaction among taxpayers has roots in physics, we have an econophysics ABM (Hokamp and Pickhardt, 2010; Hokamp and Seibold, 2014a).

satisfaction with the level of governmental services provided. Gulyás *et al.* (2015) analyze the effects of unemployment and figure out that an improvement of governmental services leads to a lower extent of tax evasion.

Pellizzari and Rizzi (2014) investigate a society of citizens and a government, where public expenditures finance public goods, so that the taxpayers take into account the behavior of their neighbors for their own compliance decision. Considering social beliefs the authors discover that individual preferences significantly drive tax evasion behavior. Nicolaides (2014) assumes strategic interaction among taxpayers (Blume, 1993) when contributing to public goods via taxes. The author finds that social norms on tax compliance serve as a substitute for deterrence via an effective audit ability. Méder *et al.* (2012) show in their utopian world without any pecuniary fine (i.e., the implicit audit probability is zero) that the presence (or absence) of prisoner's dilemma situations in the context of public goods provision turns out to be the crucial factor for long-run tax evasion dynamics.[5] The authors find that "higher tax rates induce more evasion" (Méder *et al.*, 2012, p. 187).

The econophysics branch of agent-based tax noncompliance models was invented by Lima and Zaklan (2008) and Zaklan *et al.* (2008, 2009), who assume two states for tax declaration, compliant and full evasion, corresponding to the spins in the Ising model of ferromagnetism (Ising, 1925)[6] and not allowing to consider tax rates. Lima and Zaklan (2008) build upon Wintrobe and Gërxhani (2004) and interpret an external magnetic field as a taxpayer's confidence in governmental institutions. The authors find that enforcement always works to trigger tax compliance; this finding is robust for Barabási-Albert and Voronoi-Delaunay networks.

Hokamp and Pickhardt (2010) demonstrate that back auditing and the evolution of social norms strongly influence tax evasion. Hokamp (2014) shows the counter-intuitive effect that an improvement in public goods provision leads to a higher extent of tax evasion (Alm *et al.*, 1992a; Alm, 2010). Seibold and Pickhardt (2013), Hokamp and Seibold (2014b), and Pickhardt and Seibold (2014) perform an econophysics agent-based approach to tax evasion which rests on the Ising model (cf. Ising, 1925; Zaklan *et al.*, 2009; Hokamp and Pickhardt, 2010). Seibold and Pickhardt (2013) conclude that an increase of the tax relevant periods subject to back auditing, ceteris paribus, lowers the extent of tax evasion. Hokamp and Seibold (2014b) implement public goods via a feedback depending on the rate of fully compliant taxpayers and they find that providing more public goods

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[5] Garay *et al.* (2012) consider the analytical results by Méder *et al.* (2012) concerning global stability in a nontrivial steady state and provide the related mathematical arguments.

[6] The Ising model is a mathematical model of atomic spins that take one of two states corresponding to the magnetization status (Ising, 1925). In the econophysics tax evasion settings these two states are identified with tax compliance and noncompliance. Furthermore, econophysics models pick up the punishment idea by Davis *et al.* (2003); that is, the notion of penalizing through pre-announced time periods during which the taxpayers are restricted to be fully tax compliant.

decreases tax noncompliance. Picking up the econophysics notion to model the two extreme states of tax compliance, Meacci *et al*. (2012) investigate cross-border effects and the allocation of a budget to combat tax evasion making use of a cellular automata and ordinary differential equations. The authors find that an extra amount spent by the society induces "relevant changes in system behavior both in time and in space" (Meacci *et al*., 2012, p. 608).

Bertotti and Modanese (2014a,b, 2016a,b) consider social inequality (measured by the Gini index), income redistribution, and tax compliance and they take into account governmental tasks. The authors simulate the direct and indirect interaction of taxpayers by a kinetic model with nonlinear ordinary differential equations. Bertotti and Modanese (2014a) conclude that (i) a fair fiscal policy and (ii) a population with tax compliant behavior for the most part is needed to overcome social inequalities. Bertotti and Modanese (2014b) find that the rich taxpayers benefit most from tax noncompliance, which yields the Gini index to be larger in the presence of tax evasion. Bertotti and Modanese (2016a) show that audits increase tax revenues, which are not affected by an agent's audit experience. Bertotti and Modanese (2016b) confirm the Gini index increases when the level of tax evasion rises (Bertotti and Modanese, 2014b), but it is independent of the spread of tax evasion performed among the income classes. Vale (2015) employs difference equations to investigate social networks within an ABM of heterogeneous taxpayers. The author utilizes back auditing and finds that a significant proportion of agents turn to be either fully compliant or evading. Kim (2003) makes use of decision heuristics combined with numerical simulations to consider tax tariffs and revenue schemes at the macro-level. The author finds that an optimal audit strategy is a concentration of efforts on auditing the lower and middle class taxpayers, if the tax burden is higher for the poor than for the rich. Manhire (2015) investigates tax noncompliance via modeling the role of tax examiners. The author finds that "the audit probability influences individual compliance decisions, it has negligible effects on system-level compliance patterns" (Manhire, 2015, p. 623).

Hashimzade *et al*. (2014, 2015, 2016) and Hashimzade and Myles (2017) consider the interplay of occupational choice and tax evasion in an ABM. Hashimzade *et al*. (2014) demonstrate that via occupational choice a society of heterogeneous taxpayers segregate in homogeneous groups with respect to their risk attitudes.[7] Hashimzade *et al*. (2015) develop two agent-based tax evasion models concerning both, (i) occupational choice associated to risk-taking, and (ii) social networks related to beliefs. The authors claim that the most effective strategy (in the sense of first-order stochastic dominance) is auditing a fixed number of taxpayers from each field of work. Hashimzade *et al*. (2016) show that random auditing generates less tax revenues than predictive analytics based on occupational choice.

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[7] The segregation behavior due to occupational choice by Hashimzade *et al*. (2014) is in line with racial segregation (Schelling, 1969, 1971).

Hashimzade and Myles (2017) confirm that predictive analytics thrive in raising tax compliance.

1.6.2 *Replication, Docking, and Calibration Studies*

Andrei *et al*. (2014) and Crokidakis (2014) provide independent replication and docking studies of ABMs on tax compliance, in the economics and econophysics domain, respectively. Andrei *et al*. (2014) employ Korobow *et al*. (2007) and Hokamp and Pickhardt (2010) to show that Erdös-Rényi- and Power-Law-distributed networks reflect the impacts on tax compliance particularly strongly. Crokidakis (2014) draws on Zaklan *et al*. (2009) and makes use of a three-state kinetic opinion exchange model to show tax enforcement works to successfully fight against tax evasion, when a critical threshold is transgressed for the coupling of agents. Furthermore, in this section we draw on Bazart *et al*. (2016), who provide a literature overview on calibration studies in the field of tax evasion and fill in the gap of calibrating an econophysics agent-based tax evasion model.

Bloomquist (2011a) calibrated the very first study of an agent-based tax evasion model (Bazart *et al*., 2016); his calibration combines experimental data on tax compliance and empirical data by the IRS National Research Program. The author finds that "based on the calibration and simulation results, it appears that both subjects in lab experiments and actual small business taxpayers are risk averse (i.e., overweight the probability of an audit) and taxpayers in the real world do not base their reporting compliance decisions on their neighbors' behavior" (Bloomquist, 2011a, p. 46). Arsian and İcan (2013a,b) draw on Bloomquist (2011a) to analyze tax evasion via an ABM for Turkey. The authors make use of data by the Turkish Revenue Administration and find that "both von Neumann and Moore neighborhoods are reducing compliance behavior of taxpayers considerably" (Arsian and İcan, 2013b, p. 337). Bloomquist and Koehler (2015) calibrate Bloomquist (2011b) with data by the IRS National Research Program and they are "able to model the complexities of real-world tax systems, such as differences in reporting compliance at the line item level and taxpayers' heterogeneous response behaviors" (Bloomquist and Koehler, 2015). Cline *et al*. (2014) draw on Korobow *et al*. (2007), Hokamp and Pickhardt (2010), Bloomquist (2011b, 2012), and Andrei *et al*. (2014) and they employ the US Synthesized Population dataset to present a massive-scale ABM of California. The authors argue that "running models at this (i.e., national) scale will provide the ability to produce policy insights without the fear of hidden scale effects or other issues of drawing national-scale conclusions from city-scale analyzes" (Cline *et al*., 2014, p. 6).

Nordblom and Žamac (2012) employ a survey of buying black market services in Sweden to reaffirm that tax evasion by the elderly is substantially less as compared to younger taxpayers. Garrido and Mittone (2013) calibrate their ABM with experimental data on tax compliance from Chile and Italy. Given income inequality, the authors point out that the tax authorities might optimize tax revenues by

auditing those taxpayers who more likely behave according to the bomb crater effect[8] (Krauskopf and Prinz, 2011).

Magessi and Antunes (2013a,b, 2015) apply to Portugal an agent-based tax evasion model, which rests on Antunes *et al*. (2006, 2007a,b) and Balsa *et al*. (2006).Magessi and Antunes (2015) make use of empirical data from a survey to consider the interplay of tax evasion and risk perception; in particular, the authors find that the subjective perception of the audit probability strongly influences the extent of tax evasion. Miguel *et al*. (2012), Llacer *et al*. (2013), and Noguera *et al*. (2014) present an agent-based tax evasion framework for Spain, which allows the investigation of social networks and behavioral aspects. Miguel *et al*. (2012) describe the basics of the agent-based framework to simulate tax noncompliance. Llacer *et al*. (2013) find that a consideration of a rational society (consisting of only rational taxpayers) overestimates tax evasion while social interaction allows for generating more plausible extents of tax compliance. Noguera *et al*. (2014) calibrate the ABM, called SIMULFIS, with empirical data from Spain. The group of authors conducts computational experiments to show that social norms do not always optimize tax compliance.

To fill the gap of calibrating an econophysics ABM of tax compliance[9] Bazart *et al*. (2016) make use of experimental data of a tax declaration game by Alm, *et al*. (2009) and Bazart and Bonein (2014). The authors find rather mixed behavior, for example, the taxpayers mix randomness and imitation, than the pure taxpayer behavioral archetypes assumed by Hokamp and Pickhardt (2010). Pickhardt and Seibold (2014) fill the gap of linking econophysics and economics agent-based tax evasion models by a successful replication of the two basic frameworks, Zaklan *et al*. (2009) and Hokamp and Pickhardt (2010). Lima (2010) provides a replication study of the econophysics agent-based tax evasion model by Zaklan *et al*. (2009) and adds the majority-vote-model and an Apollonian network (Lima, 2010, 2012a,b) to confirm robustness that enforcement works to trigger tax compliance.

1.6.3   *Concluding Remarks on Agent-Based Tax Evasion Models*

Agent-based tax evasion models show a broad flexibility to handle the complexity of tax compliance behavior. This is documented by the "highlights" column in Tables 1.2–1.4 presenting two main aspects per agent-based tax evasion

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[8] Soldiers recognized it is very unlikely that a bomb hits the same place twice in a short time window, and, therefore, a strategy to survive is to hide in a bomb crater. Transferred to tax evasion, taxpayers behave according to the bomb crater effect, if they again evade after being detected as a tax cheater, they assume it is very unlikely to be audited again.

[9] Hokamp and Seibold (2014a) present an econophysics ABM of the shadow economy. The authors utilize experimental data on tax compliance by Bazart and Pickhardt (2011) to figure out that, as compared with Germany, France seems to have a larger fraction of subjects rationally engaged in the shadow economy.

methodology. The highlights cover a variety of topics; for example, social neighborhoods, third-party reporting, political cycles and majority voting. Davis *et al*. (2003) investigate social norms and obtain nearly full tax compliance for comparatively low objective audit probabilities. Bloomquist (2004a,b, 2008) claims that social networks have a significant influence on the extent of tax evasion. Carley and Maxwell (2006) consider a synthetic city to demonstrate that a short-term publicity campaign has the potential to increase the overall participation rates in tax declaration schemes. Korobow *et al*. (2007) conclude that taxpayers with limited knowledge of their neighbor's payoffs show higher tax compliance levels. Warner *et al*. (2015) make use of genetic algorithms from biology to help policy-makers anticipate tax evasion schemes.

Further, the literature review of agent-based tax evasion models reveals a lack of research on public goods and back auditing as well as a lack of studies presenting replications and calibrations. Indeed, some ABMs consider public goods provision but the crucial question remains unclear: how does an improvement of public goods provision change the extent of tax evasion? So far, agent-based tax compliance models have not presented a single clear-cut hypothesis with respect to public goods provision; the findings are even contradictory (Szabó *et al*., 2009, 2010; Hokamp, 2014). In contrast, the effects of back auditing seem to be clear: a larger lapse of time period leads to a lower extent of tax evasion. Many ABMs neglect back auditing because of its large effect which then may outshine the other effects under consideration. However, back auditing may help solve the tax evasion puzzle "Why do people pay taxes?" (Alm *et al*., 1992b). For these reasons, ABMs should incorporate back auditing or at least be able to successfully deal with lapse of time effects.

Furthermore, calibration and replication is necessary for ABMs to gain credibility and to avoid the perception of being a "black box"; in particular, the communication of results and the design of experiments is crucial (Lorscheid *et al.*, 2012). While there are now limited calibrations of agent-based tax evasion models, there are much fewer replication attempts. Notably, the very first independent replication of an agent-based tax evasion setting was performed by Andrei *et al*. (2014). Finally, a common protocol for describing agent-based tax evasion models may help provide data needed for independent calibration and replication.

The next section provides an outlook on the structure and presentation of the book and surveys the upcoming chapters that deal with the various aspects of complexity in tax evasion research.

1.7   Outlook: The Structure and Presentation of the Book

Our book is structured as follows. Part I includes this introductory chapter that provides a conceptual background, a review of agent-based modeling, and the literature on tax evasion. It also discusses "dark" economic behavior in general and the methodological link between experiments and agent-based modeling. Part II

presents various applications of agent-based modeling to tax evasion, constructed for or calibrated to various countries (the United Kingdom, the United States, Spain, Hungary, and Germany). This is completed by two chapters extending the horizon of computational models of tax evasion. One applies genetic algorithms to co-evolve tax audit strategies with clandestine behavior, in order to "breed" possible novel forms of tax evasion. The other brings an "econophysics" approach to the problem by developing a variant of a classic model of ferromagnetism adapted to tax evasion.

1.7.1 Part I Introduction

The chapter by Prinz (Chapter 2) discusses how the scientific study of and the discourse about clandestine activities should be examined. Since research on such clandestine activities is of great importance for the implementation of new effective policies, not only theoretical speculations, but also experiments and empirical studies are necessary to understand these activities of hidden nature. Theories are abstract and they aim to identify few generalizable behavioral drivers, while empirical or experimental applications try to address specific real-world issues. Thus, according to the author, better scientific understanding and thus better policies need theories counterbalanced by empirical and experimental studies.

The author enumerates a number of different approaches to study clandestine activities and discusses their advantages and disadvantages (e.g., computer simulations calibrated with human-based experiments and empirical data, network analysis, and social media analysis in general). The chapter argues for a hybrid approach, a combination of methods according to the specific question of interest.

The next chapter by Mittone and Saredi (Chapter 3) complies with Prinz's proposal. It discusses model calibration efforts that are aimed to validate ABMs via laboratory experiments. After a summary of the literature on tax evasion modeling, the chapter highlights the recent advances made toward integrating human-subject laboratory experiments with computational agent-based simulations (or Agent-based Computational Economics, ACE). The discussion is put in context by an extensive review of the author's previous work in the same domain, separating three approaches: the macroeconomic, the microeconomic, and the approach of micro-level dynamics for macro-level interactions among behavioral types.

1.7.2 Part II Agent-Based Tax Evasion Models

The chapter by Hashimzade and Myles (Chapter 4) is the first among the ones in Part II reporting on specific ABMs of tax evasion. It provides a summary of the authors' work on ABMs of tax noncompliance. The model presented is based on a basic framework of occupational choice: employment versus self-employment,

where the latter is understood as a strategy to optimize taxes. Agents make their choices based on their beliefs formed during social interactions, thus forming endogenous attitudes and beliefs about tax compliance. Computational results show that the strategy of audit target selection is an essential ingredient in order to maximize the effectiveness of audit resources. Moreover, it is demonstrated that predictive analytics dominates random audits in terms of raising tax revenue. This is an important observation, since random audit strategy is a frequently used baseline case in agent-based tax evasion models.

In the following chapter, Llacer *et al.* (Chapter 5) present SIMULFIS, an agent-based simulator, where agent behavior is driven by a set of decision rules ("filters") with the aim of generating realistic behavioral outcomes. Possible "filters" include, for example, rational evaluations, beliefs about the fairness of state policies, and social influence by peers. The focus is on personal choices regarding income declarations, where the incomes themselves and the opportunities for evasion are exogenous. The authors calibrate SIMULFIS to reproduce real-world traits of Spain with a view to investigate scenarios to fight against tax evasion. Their computational results show that audits are more effective than fines to prevent tax noncompliance. On the other hand, the contribution of the social network on tax compliance is less clear, albeit its influence is clearly demonstrated. This underlines the importance of SIMULFIS' capability to analyze the effects of social influence at the micro-level.

The chapter by Gulyás *et al.* (Chapter 6) provides a comprehensive description of TAXSIM, a family of ABMs of tax noncompliance. In TAXSIM the level of noncompliance is a calculated decision of employees and employers, made during contract negotiation, in order to increase net income and to reduce costs, respectively. The agents base their decisions on their estimations of the expected costs of noncompliance and on their satisfaction with government services. The estimations are driven by the agents' individual experiences and by information received through the agents' social networks. The authors discuss computational results to illustrate the capabilities of the model and it provides a brief analysis of the model's behavior space. Among other findings, the importance of adaptive audit strategies is confirmed.

This is followed by the chapter by Bloomquist (Chapter 7) that introduces the IRCM, an ABM to investigate tax (non)reporting behavior taking into account various sources of income, social networks, taxpayer learning, and enforcement methods. Following the introduction, the author presents results considering a community of 85,000 US taxpayers, calibrated using real-world tax reporting behavior based on random audits. The study focuses on the change in taxpayer compliance due to a partial shift in the workforce composition from full-time employees to contingent workers. This application of agent-based tax evasion modeling to the "Gig" economy (i.e., a novel trend, where temporary positions are common and organizations contract with independent workers for short-term engagements) is a unique and interesting contribution to the literature.

The next chapter by Koehler *et al.* (Chapter 8) applies another scale to the computational modeling of tax evasion. It replicates existing and well-known ABMs of tax evasion to test the response of the model to different kinds of networks and scales (Hokamp and Pickhardt, 2010; Andrei *et al.*, 2014). The effect of changing the scale (i.e., the number of agents) is analyzed in connection with the various networks tested. In addition, the authors provide a proof-of-concept demonstration that massive (i.e., realistic) scales can be incorporated and handled within economic ABMs. Using their custom made simulation framework (based on python and capable of multiprocessor runs) to study models with one million agents or more, the authors conclude that the qualitative results are scale-independent for the model under study.

The chapter by Hokamp and Cuervo Díaz (Chapter 9) discusses the agent-based tax evasion framework by Hokamp and Pickhardt (2010) and Hokamp (2013, 2014) providing a comprehensive summary and adding some novel insights. The study considers taxpayers to be classified into four behavioral archetypes (neo-classical A-types, social interacting B-types, ethical C-types and erratic D-types), a tax authority, and a government. As with most ABMs, the focus is on the effects of micro-level composition and interaction on the various evasion and compliance behaviors at the macro-level. The authors calibrate their model to experimental data from a tax declaration game and consider several different situations construed by the presence of the following six model assumptions: lapse of time, subjective audit probability, age heterogeneity, social norm updating, public goods provision, and checking for Pareto-optimality. Computational results show that lapse of time has a significant effect on the extent of tax evasion; a feature frequently neglected in agent-based tax evasion models.

The last two chapters widen the horizon of agent-based modeling of tax evasion and compliance. The chapter by Wijesinghe *et al.* (Chapter 10) deals with the complex transaction networks of tax shelters. These constructs combine multiple business entities, for example, partnerships, trusts, and are designed to reduce and obscure tax liabilities. The complex networks of intricate transactions are manufactured in such a way that their sequences inflate the basis of particular assets under their control. The authors propose an approach to co-evolve the audit actions of the tax authorities with the behaviors of tax evaders, in order to anticipate the likely forms of emerging evasion schemes and thus, to give enforcement agencies a tactical advantage. Computational results reported in the chapter indicate that the proposed combination of agent-based modeling with a heuristic search based on a genetic algorithm is indeed capable of identifying potentially new tax evasion schemes.

The last chapter by Seibold (Chapter 11) adds the flavor of "econophysics" to our collection by discussing the classic Ising model as a means to describe tax evasion dynamics (Ising, 1925). This is done by extending the tax evasion Ising model developed by Zaklan *et al.* (2009). The main contribution is to consider the four behavioral archetypes proposed by Hokamp and Pickhardt

(2010) (discussed earlier) as opposed to the two agent types of the original model (compliant, noncompliant). In addition, the author also proposes an interpretation of the temperature variable in the Ising model as the standard deviation of the "unobserved utility" that corresponds to the "spread in nonmeasurable taste or attitude."

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