

Dyscalculia, Dyslexia and Mathematics

Introduction

In 1981, when we moved from working in mainstream schools and began teaching in schools for dyslexic learners, our initial expectation was that teaching mathematics would be much the same as before. At that time we could not find any source of guidance to confirm or contradict this expectation. We thought dyslexia meant difficulties with language, not mathematics. Experience would, very quickly, change this impression.

Over the last 35 years, and the 23 years since we published the first edition of this book, we have accumulated experience, tried out new (and old) ideas, researched, read what little appropriate material was available (there is still far less published on learning difficulties in mathematics than on language (Gersten *et al.*, 2007)), learned from our learners and have become convinced that difficulties in mathematics go hand in hand with the difficulties of dyslexia and, especially, that a different teaching attitude and approach is needed.

The first four chapters of this book look at some of the background that influenced the evolution of these teaching methods and continues to underpin their ongoing development. This requires a look at the learner, the subject (mathematics), the teacher and the pedagogy. The main mathematical focus of this book is number, primarily because this is the first area of mathematics studied by children and thus provides the first opportunity to fail. Our experience suggests that number remains the main source of difficulty for most of the learners we have worked with, even in secondary education. We also know that the foundations for all

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work to GCSE (the national examination for 16-year-old students in England), and beyond, are based in these early learning experiences. The evaluations and expectations of a child's mathematical potential are often based, not always correctly, on performance in early work on number (e.g. Desoete and Stock, 2011). The remaining chapters describe some of the methods we use to teach our dyslexic learners, with the ever-present caveat, that no one method will work for all learners.

One of the main reasons for the first four chapters is to address the complexity of learning profiles. This will explain why the methods described in the subsequent chapters are effective, but still will not meet the needs of every single child, and why teachers need the skill of responsive reactivity. There are now a number of researchers who have referred to this complexity and from a number of perspectives. Watson (2005) states:

There is no standard recipe for mathematical success. The joyous range of characteristics that make each child an individual ensure that this is true, so teachers need an understanding of the child and the subject to be able to adjust methods and improvise, from secure foundations and principles, to meet those individual needs.

Mabbott and Bisanz (2008) note that, 'Children who experience difficulties in mathematics are a heterogeneous group' and as Zhou and Cheng (2015) express so elegantly and succinctly, 'mathematical competence is a constellation of abilities'. Kaufmann and a collection of international researchers (2013) writing together say that heterogeneity is a feature of developmental dyscalculia. Chapter 2 provides more detail on some of the reasons for this heterogeneity.

We also believe that a greater understanding of the ways dyslexic and dyscalculic students learn and fail mathematics will illuminate our understanding of how other children learn and fail mathematics. In other words, the reasons for failure are unlikely to be specific to dyslexic and dyscalculic learners. Poor performance in maths spreads beyond students identified as dyscalculic, for example Rashid and Brooks (2010) found low levels of attainment in a significant percentage of the population of 13–19-year-old students in England. The extrapolation from this is that many, if not all of the methods advocated in this book will also help many non-dyslexic and non-dyscalculic students to learn mathematics. We have long been advocates of the principle of learning from the 'outliers' (Murray *et al.*, 2015).

Our aim has always been to teach mathematics in a mathematical way rather than seek out patronising collections of mnemonics and one-off tricks.

Definitions of Dyslexia

The year 2016 marks the 120th anniversary of the publication of the first paper (Pringle-Morgan, 1896, reproduced in the *BDA Handbook* 1996) describing a 14-year-old student with specific difficulties with reading, which Pringle-Morgan labelled, based on Kussmaul's study in 1878, as 'congenital word blindness'. Pringle Morgan also described idiosyncratic difficulties for the young student in maths: 'Interestingly he could multiply 749 by 867 quickly and correctly as well as working out $(a+x)(a-x) = a^2 - x^2$, yet failed to do $4 \times \frac{1}{2}$.'

The issue of mathematics disappeared from definitions of dyslexia for a while, for example in 1968 the World Federation of Neurology defined dyslexia as: 'A disorder manifested by a difficulty in learning to read, despite conventional instruction, adequate intelligence and socio-cultural opportunity. It is dependent upon fundamental cognitive difficulties that are frequently of a constitutional character.'

But, by 1972 the Department of Education and Science for England and Wales included number abilities in its definition of specific reading (sic) difficulties. In the USA, the Interagency Conference's (Kavanagh and Truss, 1988) definition of learning disabilities included 'significant difficulties in the acquisition of mathematical abilities' and, in the UK, Chasty (1989) defined specific learning difficulties as: 'Organising or learning difficulties, which restrict the students competence in information processing, in fine motor skills and working memory, so causing limitations in some or all of the skills of speech, reading, spelling, writing, essay writing, numeracy and behaviour.'

In 1992 Miles and Miles, in their book *Dyslexia and Mathematics*, wrote: 'The central theme of this book is that the difficulties experienced by dyslexics in mathematics are manifestations of the same limitations which also affect their reading and spelling.'

In 1995 Light and Defries (1995) highlighted the comorbidity of language and mathematical difficulties in dyslexic twins, one of the earliest mentions of the possibility of comorbid dyslexia and dyscalculia.

In the new millennium, it seems that the definitions of dyslexia are moving back to focus solely on language. This is likely to be due to the current interest in and awareness of dyscalculia and comorbidity and the trend in the UK to see 'specific learning difficulties' used as an umbrella term to cover dyslexia, dyscalculia, dyspraxia (developmental coordination disorder) and dysgraphia, rather than a label that was solely interchangeable with dyslexia. This is relevant for our perceptions of dyscalculia and mathematical learning difficulties. So, recently in the UK, the Rose Report's (2009) definition of dyslexia focused on reading and spelling, with no

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mention of arithmetic or numeracy skills: ‘Dyslexia is a learning difficulty that primarily affects the skills involved in accurate and fluent word reading and spelling. Characteristic features of dyslexia are difficulties in phonological awareness, verbal memory and verbal processing speed.’ However, within the report, there are discussions on co-occurring issues, which include difficulties with mental calculation.

In the USA, the International Dyslexia Association adopted a definition of dyslexia (2002), which also focused on language:

Dyslexia is a specific learning difficulty that is neurobiological in origin. It is characterised by difficulties with accurate and/or fluent word recognition and by poor spelling and decoding abilities. These difficulties typically result from a deficit in the phonological component of language that is often unexpected in relation to other cognitive abilities and the provision of effective classroom instruction. Secondary consequences may include problems in reading comprehension and reduced reading experience that can impede growth of vocabulary and background knowledge.

If dyslexia and dyscalculia are now to be defined as separate, distinct specific learning difficulties, then the concept of comorbidity (e.g. Cirino *et al.*, 2015; Shin and Bryant, 2015) becomes very relevant. An important question for researchers is to decide whether the comorbidity is causal, independent or a different outcome resulting from the same neurological basis. The study by Moll *et al.* (2014) suggests that deficits in number skills are due to different underlying cognitive deficits in children with reading disorders compared to children with mathematics disorders. These deficits are, for reading disorders, a phonological deficit and, for mathematics disorders, a deficit in processing numbers.

Our classroom experience is that most of the dyslexics we have taught have had difficulties in at least some areas of mathematics. It should be noted that, in our school, the results from our specifically designed intervention, in terms of grades achieved in GCSE (the national exam for 16-year-old students in England) were from A* to D and with one ex-student, who was severely dyslexic, obtaining a degree in mathematics. The theme of this book is of positive prognosis.

In her seminal book, Yeo (2002) looked at the issues surrounding dyspraxia, dyslexia and mathematics difficulties. The specific learning difficulty, dyspraxia (developmental coordination disorder) brings another set of issues to a pupil’s attempts to learn mathematics.

Finally in this section, we should be aware that dyslexia is a problem internationally (as dyscalculia certainly is). Although the English language is probably the most challenging language to learn, especially for the

mastery of spelling, dyslexia occurs in many languages. For example, the Yemen Dyslexia Association (Al Hakeemi, 2015) defines dyslexia as: ‘A functional disorder of the left side of the brain. It causes difficulty in reading, writing or mathematics associated with other symptoms such as weakness in short-term memory, ordering, movements and directions awareness.’

The Evolution of Definitions of (Developmental) Dyscalculia

At the time (2015) of writing this, the fourth edition of our book, the idea of a specific mathematics disability, now known as dyscalculia in the UK, had slipped out of common usage in our government documents, whereas at the time of the third edition it had recently slipped in. This observation draws attention to the influence of governments on the recognition of and provision for learning disabilities. A search for ‘dyscalculia’ on the gov.uk website on 10 April 2015 yielded no results, suggesting instead that we try to search for ‘calculi’.

The term dyscalculia remains not well defined, or at least without a consensus, though there have been some recent proposals as to what the definition should be (e.g. Kaufmann *et al.*, 2013). However, it does now seem agreed that it is a specific learning difficulty that is solely related to mathematics, that is, there is no mention of a comorbid language difficulty. As one would expect, the prevalence of dyscalculia will be dependent on how it is defined.

It should be stated at this stage that, erroneously, for some people ‘dyscalculia’ suggests a dire prognosis, that of a *permanent inability* to do mathematics. This would be ‘acalculia’, a complete loss of the ability to work with numbers and caused by a stroke or a traumatic injury to the brain. The two terms are not interchangeable.

It remains the situation that much less research exists in comparison to dyslexia. When David Geary spoke at the 2002 IDA conference he compared our knowledge of dyslexia to being close to adulthood and our knowledge of maths learning difficulties to being in its early infancy. Gersten *et al.* (2007) give data on the ratio of papers on reading disability to mathematical learning disability for five subsequent decades. For 1966–1975 the ratio was 100:1 and for 1996–2005 it was 14:1. Desoete *et al.* (2004) note that from 1974 to 1997 only 28 articles on maths learning difficulties were cited in Psyc-Info, whereas there were 747 articles on reading disabilities.

The work of Kosc, a pioneer in the field of dyscalculia, plus a review of the early literature on dyscalculia can be found in *Focus on Learning Difficulties in Mathematics* (Kosc, 1986). Ramaa and Gowramma (2002) provide a comprehensive review of the literature to that time and Gersten *et al.* (2007) provide a more recent review, taking in a range of different perspectives. The various authors in *The Routledge International Handbook of Dyscalculia and Mathematical Learning Difficulties* (Chinn, 2015) provide a wealth of references. The international nature of the research tells us that dyscalculia is an international problem (e.g. Faber, 2014).

The earliest reference to a specific learning difficulty in maths that we could find is by Bronner (1917), referred to in Buswell and Judd (1925): ‘Frequent references have been made to children whose ability seems to be normal or even superior as far as general mental capacity is concerned, but who have special difficulties in arithmetic. Bronner has proposed the hypothesis that there are special disabilities in such subjects as arithmetic.’

Little happened for dyscalculia and specific learning difficulties in the next 60 years. Indeed, it was not until the third edition of this book (2007) that the word ‘dyscalculia’ was included in the title. However, there have been a few definitions of dyscalculia proposed over the past 50 years, with one of the earliest from Kosc (1974) who defined it in terms of brain abnormalities: ‘Developmental dyscalculia is a structural disorder of mathematical abilities which has its origin in a genetic or congenital disorder of those parts of the brain that are the direct anatomico-physiological substrate of the maturation of the mathematical abilities adequate to age, without a simultaneous disorder of general mental functions.’

Weinstein (1980), quoted in Sharma (1986) considered dyscalculia as, ‘A disorder of the abilities for dealing with numbers and calculating which is present at an early age and is not accompanied by a concurrent disorder of general mental functions.’

Magne (1996) published a bibliography of the literature on dysmathematics.

The definition of dyscalculia from the UK’s Department for Education and Skills booklet (DfES, 2001, now archived) on supporting learners with dyslexia and dyscalculia in the National Numeracy Strategy is: ‘Dyscalculia is a condition that affects the ability to acquire mathematical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence.’

There does seem to be a long-standing consensus that dyscalculia should be perceived as a specific difficulty, for example:

The World Health Organisation (2010) uses the term ‘Specific disorder of arithmetical skills’ which ‘involves a specific impairment in arithmetical skills that is not solely explicable on the basis of general mental retardation or of grossly inadequate schooling. The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication and division rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry or calculus.’

The American Psychiatric Association (2013) also uses ‘specific’ in their definition of Developmental Dyscalculia (DD) as: ‘A specific learning disorder that is characterised by impairments in learning basic arithmetic facts, processing numerical magnitude and performing accurate and fluent calculations. These difficulties must be quantifiably below what is expected for an individual’s chronological age, and must not be caused by poor educational or daily activities or by intellectual impairments.’

It is of interest that an international team of experts (Kaufman *et al.*, 2013) proposed a definition that suggests two sub-types of Developmental Dyscalculia (DD), ‘Primary DD is a heterogeneous disorder resulting from individual deficits in numerical or arithmetic functioning at behavioral, cognitive/neuropsychological and neuronal levels. The term secondary DD should be used if numerical/arithmetic dysfunctions are entirely caused by non-numerical impairments (e.g. attention disorders).’

Also working with a hypothesis of sub-types, Karagiannakis and Cooreman (2015) propose a classification model of mathematical learning difficulties:

Core number. Difficulties in the basic sense of numerosity and subitising (Butterworth, 2005; 2010).

Visual-spatial. Difficulties in interpreting and using spatial organisation and representation of mathematical objects.

Memory. Difficulties in retrieving numerical facts and performing mental calculations accurately.

Reasoning. Difficulties in grasping mathematical concepts, ideas and relations and understanding multiple steps in complex procedures/algorithms.

These may be taking us onwards from Butterworth’s 2005 single core deficit model. The interactions between the complexity of maths and the heterogeneous nature of individuals makes the situation highly complex.

Thus, we may have to seek some key patterns whilst being open to variations on these core themes. We should not risk failing children (and adults) by simply saying, ‘If people can’t agree to the definition, then it doesn’t exist.’

There seem to be two components to the various definitions that have been proposed. One is a description of the mathematical difficulties. This component tends to focus on basic maths, that is, numeracy and arithmetic, sometimes with a focus on the very basic skills, for example numerosity (Butterworth, 2010), core number deficits (e.g. Reeve and Gray, 2015) and number sense. Even here there are complexities. Berch (2005) found 30 alleged components of number sense in the literature.

The other component focuses on the neurological causes. As technology grows at an exciting pace, then the possibilities of watching the brain at work (e.g. Bugden and Ansari, 2015; Reigosa-Crespo and Castro, 2015) are in stark contrast to the early days of examining the brains of dyslexics post-mortem (Gallaburda, 1989). However, even with this amazing capacity to watch brains at work, the situation remains complex, as Bugden and Ansari (2015) observe: ‘Neuroimaging studies investigating the cognitive mechanisms that contribute to DD deficits have yielded an inconsistent and hard to interpret pattern of data. Given the early stages of functional MRI and EEG research, it is difficult to interpret from the current set of data what neurobiology underlies cognitive deficits in children with DD.’

Returning to the mathematical behaviours that might contribute to that aspect of definitions of dyscalculia, our experience and the relevant research, suggests that the list below covers many of these:

- Difficulty when counting backwards.
- A poor sense of number and estimation.
- Difficulty in remembering ‘basic’ facts, despite many hours of practice/rote learning.
- The only strategy used to compensate for lack of recall is to count in ones.
- Difficulty in understanding place value.
- No sense of whether any answers that are obtained are right or nearly right.
- Slow to perform calculations.
- Forgets mathematical procedures, especially when complex, for example ‘long’ division.
- Addition is often the default operation.

- Avoids tasks that are perceived/predicted as likely to result in a wrong answer.
- Weak mental arithmetic skills.
- High levels of mathematics anxiety.

As a footnote to this section, we suggest that if one views dyslexia and dyscalculia as similar in nature, then it would follow that many of the problems of learning maths can be circumvented. They may well still persist into adulthood, with the danger of regression if hard-won skills are not regularly practised. This optimistic view would not preclude great success in maths for some ‘dyscalculics’ in the same way that dyslexia has not held back some great writers and actors.

Comorbidity

An awareness of the co-occurrence (termed comorbidity in the medical field) of two or more educationally relevant disorders in the same individual, has grown in the past 30 years. The first edition of this book was titled *Mathematics for Dyslexics*. It was the third edition that saw the introduction of ‘Dyscalculia’ into the title. We now have a wider recognition of learning difficulties, including Asperger syndrome (though that term may soon be subsumed into autistic spectrum disorders), dyspraxia/developmental coordination disorder (Yeo, 2003; Pieters *et al.*, 2015), attention deficit hyperactivity disorder, hearing impairment (Gowramma, 2015) and, indeed, dyscalculia.

But the authors’ interest in learning difficulties was born out of our experiences of working with children who had been diagnosed as dyslexic, so that will be a starting point for this section.

In terms of the co-occurrence of difficulties in language and maths, Joffe’s pioneering paper (1980a) on maths and dyslexia included a statistic that has been applied over enthusiastically and without careful consideration of how it was obtained, that is, ‘61% of dyslexics are retarded in arithmetic’ and thus, many have since assumed, 39% are not). The sample for this statistic was quite small, some 50 dyslexic learners. The maths test on which the statistic was primarily based was the British Abilities Scales Basic Arithmetic Test, which is, as its title suggests, predominantly a test of arithmetic skills. Although the test was untimed, Joffe noted that the group that achieved well would have done less well if speed had been a consideration. She also stated the extrapolations from this paper would have to be cautious. Other writers seem to have overlooked Joffe’s own cautions and detailed observations. For example she states, ‘Computation

was a slow and laborious process for a large proportion of the dyslexic sample.’ The results from mathematics tests can depend on many factors and speed of working will be one of the most influential of these factors for a population that is often slow at processing written information.

Joffe (1980a; 1980b; 1983) provided an excellent overview of the relationship between dyslexia and mathematics. Within these three relatively short papers Joffe provided many observations that create a clearer understanding of difficulties in mathematics. Most notably Joffe drew attention to a deficit in the essential skill of generalising, an observation rarely seen in other research.

Miles (Miles and Miles, 1992) suggests that mathematical difficulties are likely to occur concurrently with language difficulties. Lewis *et al.* (1994) provide data on co-occurrence from a large sample of 9 and 10-year-old pupils. More recently Landerl and Moll (2010) concluded that: ‘Comorbidities of learning disorders are not artificial. They are the result of a complex interplay between both general and disorder-specific aetiological factors.’

It has been our combined experience of 50 years of teaching maths to dyslexics that the percentage of co-occurrence is close to 100, though obviously with a range of levels of impact on learning.

One of the key beliefs for interventions for dyslexia is that the teaching and learning are multisensory. However, there is an inclination in maths teaching in UK schools, as compared to, say teaching physics (SC’s initial teaching role), to drop experiments and demonstrations in the early years and move to the sole use of symbols and ‘talk and chalk’. One of the earliest papers to suggest a multisensory approach to the teaching of mathematics to dyslexics was from Steeves (1979), a pioneer in this field. Steeves advocated the same teaching principles for mathematics as Samuel Orton had suggested for language.

There are many other parallels at many levels between dyslexia and dyscalculia and all that surrounds these specific learning difficulties, for example prevalence, definition, teaching methods, aetiology, perseveration, attitude of academics and governments and so forth.

Prevalence

Perhaps it is not surprising, given that we do not have a clear agreed definition of the problem, that there is a range of figures given for the prevalence of dyscalculia. For example, in the study by Lewis *et al.* (1994) of 1200 children aged 9–12, only 18 were identified as having specific mathematics difficulties in the absence of language difficulties. Lewis *et al.*

did not find any one pattern or reason why this was so. The same distinction is made by Ramaa and Gowramma (2002) in a fascinating study of children in India. Ramaa and Gowramma used both inclusionary and exclusionary criteria to determine the presence of dyscalculia in primary school children. Both experiments suggest that the percentage of children identified as potentially dyscalculic was between 5.5% and 6%. Ramaa and Gowramma also list 13 observations from other researchers about the nature and factors associated with dyscalculia, including persistent reliance on counting procedures and extra stress, anxiety and depression. Sutherland (1988) states that on the basis of his study, few children have specific problems with number alone. Badian (1999) has produced figures for the prevalence of persistent arithmetic, reading or arithmetic and reading disabilities, from a sample of over 1000 children, suggesting that for grades 1–8, 6.9% qualified as low in arithmetic, which included 3.9% low only in arithmetic.

Hein *et al.* (2000) studied samples from rural and urban areas in Germany and found that 6.6% of their third grade sample performed significantly worse in arithmetic than in spelling tests. Shalev *et al.* (2001) working in Israel, have suggested that developmental dyscalculia, taking a discrepancy model, has a significant familial aggregation. They estimate the prevalence of developmental dyscalculia to be between 3% and 6.5% of children in the general school population and conclude that there is a role for genetics in the evolution of this disorder. Inevitably this will raise a mathematical learning disabilities version of the nature/nurture debate. A study which offers a further perspective on the nature/nurture aspect was conducted by Ramaa (2015), who has investigated arithmetic difficulties among socially disadvantaged children and children with dyscalculia.

Desoete *et al.* (2004) found prevalence rates of dyscalculia, from a study of a large sample of pupils in Belgium, that were 2.2% of second graders, 7.7% of third graders and 6.6% of fourth graders.

Reigosa-Crespo *et al.* (2012) take an interesting perspective, looking at what they term as arithmetical dysfluency (AD) alongside developmental dyscalculia (DD). They estimated the prevalence and gender ratio of arithmetical dysfluency and dyscalculia in the same cohort. The estimated prevalence of DD was 3.4%, and the male to female ratio was 4:1. However, the prevalence of AD was almost three times as great (9.35%) and with no gender differences found (the male to female ratio was 1.07:1). They conclude that, based on these contrasting findings, DD, defined as a defective sense of numbers, could be a distinctive disorder that affects only a portion of children with AD. The difference in these findings could also be explained by the restriction of the definition of DD to a defective sense of numbers.

What is mathematics?

Mathematics is not just arithmetic or manipulating numbers. It is possible that a person could be good at some topics in maths and a failure in other topics? Does dyscalculia imply an inability to succeed in all of the many topics that make up mathematics?

In terms of subject content, early maths is primarily about numbers and thus about our number system. Later it becomes more varied, with new topics introduced such as measure, algebra, and shape and space. So the demands of maths can be quite varied. This can be very useful from the perspective of intervention. We believe that intervention for maths difficulties should also include some time doing parts of maths that the learner can do, so that intervention sessions are not all about the things the learner cannot do. It is a problem that number is a disproportionate part of early learning experiences (and of ‘everyday’ maths). So it seems logical that poor number skills are a key factor in dyscalculia. It also seems logical that we have to consider the match between the demands of the task and the skills of the learner.

In terms of approach, maths can be a written subject or a mental exercise. It can be formulaic or it can be intuitive. It can be learnt and communicated in either way, or in a combination of ways by the learner and it can be taught and communicated in either way or a combination of ways by the teacher. Maths can be concrete and visual, but fairly quickly moves to the abstract and symbolic. It has many rules and a surprising number of inconsistencies, particularly in the early stages. In terms of judgement, feedback and appraisal, maths is unique as a school subject. Work is usually a blunt ‘right’ or ‘wrong’ and that judgement is a consequence of the mathematics itself, not of how the teacher chooses to appraise the work.

And one has to ask, ‘Why is it such an entrenched part of mathematics culture that it has to be done quickly?’

What is the role of memory?

We often pose the question in lectures: ‘What does the learner bring (to maths)?’ We have already mentioned some factors such as anxiety. But what about memory? We know that Krutetskii (1976) lists mathematical memory as a requirement to be good at maths. We are certain that short-term and working memory are vital for mental arithmetic, particularly for those sequential, formula based maths thinkers, but can a learner compensate for difficulties in some of these requirements and thus ‘succeed’ in maths?

However excellent a maths curriculum, it is virtually impossible for it to meet the needs of every learner, for example in the dictated pace of progress. It is certain that a component of the curriculum will be mental arithmetic. This activity needs effective memories, long, short and working. So a learner with poor short-term and working memories could fail maths when it is mental maths, even though he may have the potential to become an effective mathematician. If failure is internalised as a negative attributional style by the learner then that potential may never be realised.

It is possible that Krutetskii's *mathematical memory* draws a parallel with Gardner's (1999) multiple intelligences. Perhaps there are multiple long-term memories. That would explain some of the discrepancies we see in children's memory performances. Like any subject, there is a body of factual information for maths and if a learner can remember and recall this information then he will be greatly advantaged and, if he can't, then failure is likely. Just how much that is the case depends on the curriculum and how it is taught.

So, good memories may be required for doing maths in general. Short-term and working memories may be essential for mental maths and mathematical long-term memory will be very important for the number facts and formulae needed when doing mental arithmetic. Geary considers memory to be a key contributing factor in mathematics learning difficulties (Geary, 2004).

There is an accumulation of evidence in the UK that the teaching of maths is heavily reliant on pupils having good memories, often at the expense of developing understanding. We suspect that this is not unique to the UK.

Counting

The first number test on Butterworth's Dyscalculia Screener (2003) is for subitising. Basically, this means an ability to look at a random cluster of dots and know how many are there, without counting. Most adults can do this at six dots plus or minus one.

A person who has to rely entirely on counting for addition and subtraction is severely handicapped in terms of speed and accuracy. Such a person is even more handicapped when trying to use counting for multiplication and division. Often their page is covered in endless tally marks and often they are just lined up, usually untidily, not grouped as, for example HHH , that is, the gate pattern for five. Maths for them is done by counting in steps of one. If you show them patterns of dots or groups, they prefer rows and rows of tallies.

But maths is not just the ability to ‘see’ and use five. It’s the ability to see other chunks, patterns and inter-relationships, for example to see nine as one less than ten, to see $6+5$ as $5+5+1$, to count on in twos, fives and tens, especially if the pattern is not the basic one of 10, 20, 30, 40... but, for example, 13, 23, 33, 43.... It seems to us from our teaching experiences that the ability to work with numbers in chunks is vital for progression.

An over-reliance on counting puts a much greater load on working memory and makes computations much more difficult, especially for mental arithmetic.

It seems to be an assumption that, because a child can count forwards, then they can, with equal facility, count backwards, too. This is not the case for many children and the ability to reverse a procedure extends to other areas of maths, too. For example, we teachers often instruct children to reverse a process in algebra in order to ‘solve’ an equation. Counting backwards, reversing a procedure requires an effective working memory capacity.

However, at the root of all this is the need for students to progress beyond the ‘counting in ones’ strategy (Chinn and Ashcroft, 2004).

What distinguishes the dyscalculic learner from the garden-variety poor mathematician?

Stanovich (1991) asked: ‘How do we distinguish between a “garden variety” poor reader and a dyslexic?’ The equivalent key question to ask for maths is ‘How do we distinguish between a “garden variety” poor mathematician and a dyscalculic?’ Of course, part of the answer will depend on how dyscalculia is defined. However, in the classroom situation, we would suggest that the answer to this latter question has a lot to do with perseveration of the difficulty in the face of skilled, varied and conventional intervention and the stage in the curriculum at which that intervention is targeted.

This leads to further questions, such as, ‘Can you be a good reader and still be a dyslexic? Can you be good at some areas of maths and still be dyscalculic?’ Our hypothesis is that the answer to both questions is ‘Yes’, but that is partly because maths is made up of many topics, some of which make quite different demands (and for both these questions, good and appropriate teaching can make such a difference). It is also to do with this difficulty being a continuum and it is the interaction between a learner’s position on that spectrum and the way he is taught that creates the potential to move forwards or backwards along that spectrum of achievement.

The temptation is to return to the thought that problems with numbers are at the core of dyscalculia. And it is numbers that will prevail in real life, when school algebra is just a distant memory. And it is likely that the main problem is in accessing these facts accurately and quickly, usually straight from memory, rather than via inefficient strategies such as counting. There is also the practice among some educators to hold learners at the number stage in the mistaken belief that mastery of number, often judged in terms of mechanical recall of facts and procedures, is an essential prerequisite for success in mathematics.

Not all factors involved in learning difficulties are solely within the cognitive domain and the child. A difficulty may be exacerbated by a bureaucratic decision. For example, some bureaucrats stipulate a level of achievement, often specifying this level precisely, that defines whether or not a child's learning difficulties may be addressed in school or even assessed. This decision may be influenced, at least in part, by economic considerations. But, even then, is a child's dyslexia or dyscalculia defined solely by achievement scores? Is there room to consider the individual and what he brings to the situation? For example, an 11-year-old pupil I assessed in 2015 had been scored on the Number Skills Test of the British Ability Scales as 1 y 10m behind chronological age when she was nine years old. This did not even come near to quantifying her current difficulties. By the age of 11 years, she was below the 5th percentile in low-stress tests of the four operations. She was in the top 5% of students of her age for maths anxiety. She struggled to repeat four digits forward and reversed under ideal and quiet lab conditions. Her standardised score on a 15-minute maths test (Chinn, 2017a) placed her at the 2.5th percentile. She struggled to match the symbols for the operations with the appropriate vocabulary. She had little understanding of place value.

If we take an assumption that maths learning difficulties are on a normal distribution or even a spectrum, then our view is that it is the severity and the multi-manifestations of contributing difficulties coupled with very limited impact from individualised instruction (which may not always be as skilled as is necessary) that distinguishes a child with dyscalculia from a child with mathematical learning difficulties.

What are the predictors?

We have to keep in mind the fact that children develop at different rates. So when should we have concerns about learning and what signs should we be looking for? A number of researchers have identified potential predictors.

Keeler and Swanson (2001) found that significant predictors of maths achievement are verbal and visuospatial working memory and knowledge of strategies (e.g. clustering or rehearsal) to enhance working memory.

Gersten *et al.* (2005) identify fluency and proficiency with number combinations (also known as basic number facts).

Gathercole and Alloway (2008) found that working memory capacity at age four years can be predictive of low attainment levels in maths. Children in the low working memory group in their study were more than seven times more likely not to reach expected levels in maths 30 months later.

Desoete (2011; 2015) has studied predictive indicators in children in kindergarten. Whilst finding that 87.5% of children at risk for dyscalculia/MLD can be detected, it seemed easier to screen the children who are not at risk. The central executive component of working memory and digit recall were important predictors. Siegler *et al.* (2012) found that primary schoolchildren's knowledge of fractions and division predicted their overall achievement in maths in secondary school five or six years later. Since many of the children with dyscalculia are unlikely to have mastered fractions and division in primary school, this negative prediction is likely to be apposite for them.

Geary's (2013) warning is that: 'Children's quantitative competencies upon entry to school can have lifelong consequences. Children who start behind generally stay behind.' He suggests that explicit direct instruction of core numerical relations may be particularly important. We have to confess that our agreement with this recommendation on direct instruction is wholehearted.

Chan *et al.* (2014) found that for Hong Kong children first graders' place value understanding in the first semester was the strongest predictor of their mathematical achievement at the end of first and second grades.

The first of the three Key Findings of The National Research Council's study on *How People Learn* (Bransford *et al.*, 2000) supports our use of the Buswell and Judd (1925) observation on the first learning experience: 'Students come to the classroom with preconceptions about how the world works. If their initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught, or they may learn them for the purpose of a test, but revert to their preconceptions outside the classroom.'

The message is that predictors matter, but we must not underestimate the power of the first learning experiences to influence later learning.

(Note: There is a special volume of the *Journal of Learning Disabilities*, 38 (4), 2005, dedicated to early identification and intervention for students with difficulties in mathematics.)

What is appropriate teaching?

For many teachers the first reaction to hearing of a child's diagnosis will be, 'So he's dyscalculic, how can I teach him?' We are certain that using the range of methods and strategies we developed and used during our years working with students with dyslexia and mathematical difficulties will also be effective with dyscalculic learners. Indeed we have probably taught many, many learners who have the comorbid problems of dyslexia and dyscalculia. What we address as teachers is the way the learner presents, not a learner defined solely by some stereotypical attributes or, even more summarily, by a label.

The majority of the chapters in this book are about methods for teaching maths to students who have maths learning difficulties and dyscalculia.

A good question to ask is, 'Where do I begin? How far back in maths do I go to start the intervention?' This may be a difference, should we need one, between the dyscalculic and the dyslexic who is also bad at maths. It may be that the starting point for the intervention is further back in the curriculum for the dyscalculic than for the dyslexic. (This is yet another topic needing research.) It may also be that the subsequent rates of progress are different. Kaufmann *et al.* (2003) advocate a numeracy intervention programme that involves both basic numerical knowledge and conceptual knowledge, and suggest that there is a need for explicit teaching of numerical domains that often have been neglected in school mathematics. In other words they are asking, 'How far back do you start to explain mathematics?' The answer is almost always, 'At the beginning.'

And for a final thought in this section, we ask, 'What is the influence of the style of curriculum?' We know, for example, from a European study in which S.C. was involved (Chinn *et al.*, 2001), that the pedagogy behind the maths curriculum certainly affects thinking style in maths for many pupils.

What are the interactions and factors? (See also Chapter 2)

There are many reasons why a child or an adult may fail to learn maths skills and knowledge. For example, a child who finds symbols confusing may have been successful with mental arithmetic, but finds written arithmetic very challenging. There may be other examples of an onset of failure at different times, which will most likely depend on the match between the demands of the curriculum and the skills and deficits of the learner, for example a dyslexic will probably find word problems especially difficult (though good reading skills do not solve many of the issues with word

problems). Even a child who is not dyslexic or dyscalculic, but is learning at the concrete level, may find the abstract nature of algebra difficult. A child who is an holistic learner may start to fail in maths if his new teacher uses a sequential and formula-based inchworm teaching style (see Chapter 3). A learner may have a poor mathematical memory and the demands on memory may build to a point where they exceed his capacity. For example, Skemp (1971) commented on rote learning:

The problem here is that a bright and willing child can memorise so many of the processes of elementary maths so well that it is difficult to distinguish it from learning based on comprehension. Sooner or later, however, this must come to grief, for two reasons. The first is that as maths becomes more advanced and more complex the number of different routines to be memorised imposes an impossible burden on the memory. Second, a routine only works for a limited range of problems.

This quote is not exclusive to dyslexics and dyscalculics. It illustrates the commonality of many of the difficulties across a broad spectrum of learner, but, of course, not the commonality of the severity of these difficulties.

Among the many hypotheses we have generated between us as to why our dyslexic students have such extraordinary difficulty in retaining basic multiplication facts in long-term memory is the powerful influence of the first learning experience. Buswell and Judd (1925) explained how the first experience of learning something new is a dominant entry to the brain, then going further by suggesting that re-learning the information, even to the point of apparent mastery, will only revert to the original erroneous recall. This critical finding, now 90 years old, can be related to recent work on understanding the role of inhibition in learning maths, that suppressing distracting information and unwanted responses plays a critical role in the development of mathematics proficiency (Borst and Houde, 2014; Cragg and Gilmore, 2014).

For many people the perception of maths is that it is a consistent and logical subject. The report, *Key understandings in mathematics learning* (Nunes *et al.*, 2007) states: ‘The evidence demonstrates beyond doubt that children must rely on logic to learn mathematics.’ The reality is that there are many instances where there are challenges to consistency and logic. Children build beliefs about maths, for example that four is bigger than two. Fractions appear to contradict this belief when $\frac{1}{2}$ is bigger than $\frac{1}{4}$. Morsanyi and Szucs (2015) note that ‘when we reason about belief-inconsistent materials we have to actively inhibit the effect of our beliefs.’ In his fascinating book, *Influence: The Psychology of Persuasion*, Cialdini (2007) maintains that the desire for consistency is a central motivator of our behaviour.

What happens in the classroom with learning difficulties will depend on the interactions between the demands of the task, the skills of the teacher, and the skills and attitudes of the learner. Sometimes the interaction is harsh in its impact on the learner, for example if one of the demands of mental arithmetic is that it be done quickly, then any learner who retrieves and processes facts slowly will fail as a consequence of this one factor alone.

However, none of the underlying contributing factors discussed above and in Chapter 2 are truly independent. Anxiety, for example, is a consequence of many influences. Our hypothesis is that the factors mentioned earlier in this chapter and in the next chapter are the key ones. There may well be others and the pattern and interactions will vary from individual to individual, and possibly even from day to day in that individual, but these are what we consider to be the difficulties at the core of dyscalculia and mathematical learning difficulties.

The Nature of Mathematics and the Ways it is Taught

In order to teach successfully, you need a knowledge of the learner and a knowledge of the subject. You may not need to be a degree level mathematician, but to teach mathematics effectively, even at its earliest stages, you must have a deep understanding of the nature of mathematics and its progression beyond the immediate topics being taught and an ability to communicate that understanding. Mathematics is a subject that builds on previous knowledge to extend knowledge. We are convinced of the need for teachers to be flexible and responsive in their ways of teaching and doing maths and to recognise and accept this flexibility in their pupils, too. Teaching is a hugely complex skill. At its best it is an art.

Number and arithmetic are the first experience of mathematics for most children and are the areas of mathematics most people use in later life. Early experience of success or failure at this stage sets the scene for later, both academically and emotionally (Geary, 1990; 1994; Desoete and Stock, 2011). Some learners learn competence in limited areas of arithmetic, for example they are comfortable with addition, but cannot carry out subtractions. Indeed SC's recent research (Chinn, 2012) has suggested that addition is the default operation for many learners. The developmental nature of maths is key to success and failure for learners. Any gaps in the precursors for new topics will make learning unsuccessful.

As a strategy to address this issue of pre-requisite knowledge, there is currently in the UK a keen interest in 'mastery'. Whilst much depends on

the interpretation of this approach, there is always a danger that a new pedagogy for teaching can be over-applied. What can create significant problems for many learners are programmes that require mastery before progression (e.g. Kumon mathematics), because mastery, especially of rote learning tasks, and even more especially under the pressure of working quickly, is, despite its name, a transient stage for many dyslexics. There is no doubt that the developmental nature of maths makes it essential for children to understand each new concept as they build their knowledge of maths. However, we have to be careful about what has to be ‘mastered’ and whether that involves understanding as well as recall, and how that will be used to dictate subsequent progress.

In terms of subject content, early maths is primarily numbers. Much of the work on dyscalculia has focused on very early number sense, ‘approximate number sense’ (ANS), for example Landerl *et al.* (2004). Later, maths becomes more varied, with new topics introduced such as measure, data and spatial topics. Up to GCSE (the national examination in England for children aged 16 years), despite the different topic headings, the major component remains as number. So although the demands of maths can be quite broad, which can be very useful for many learners, number can remain a disproportionate part of early learning experiences.

Numbers can be exciting, challenging tools (McLeish, 1991), or the cause of great anxiety (Ashcraft *et al.*, 2007; Chinn, 2009; Devine *et al.*, 2012; Lyons and Beilock, 2012). Mathematics is a sequential, interrelating subject, building on early skills and knowledge to take the student on to new skills and knowledge whilst reflecting on previous learning. It is a subject of organisation and patterns (Ashcroft and Chinn, 2004), of abstract ideas and concepts. Gaps in the early stages of understanding can only handicap the learner in later stages, for example in the speed of processing number problems.

Mathematics is a subject where the child learns the parts; the parts build on each other to make a whole; knowing the whole enables the learner to reflect with more understanding on the parts; that in turn strengthens the whole. Knowing the whole also enables one to understand the sequences and interactions of the parts and the way they support each other, so that the getting there clarifies the stages of the journey. Teachers are (usually) in the fortunate position of being conversant with the subject and can bring to their work knowledge and experience beyond the topic they are teaching. The learner is rarely in this position and thus is vulnerable to assumptions about his levels of knowledge and experience, often made unconsciously by the teacher or based on the child’s fluency in reciting maths facts and procedures.

It is important that the learner develops a clear, broad and flexible understanding of number and processes at each stage, that he begins to see the interrelationships, patterns, generalisations and concepts clearly and without anxiety. To teach a child to attain this understanding of mathematics requires in teachers a deep understanding of mathematics and numbers to a level where their communication is effective with children of a wide range of abilities. They also need to understand where mathematics is going beyond the level at which they teach, as well as where it has come from, so that what they teach is of benefit to the child at the time and helps, not hinders, him later on as his mathematics develops. Teachers need to be mindful of what is coming after what they have taught, because the development of a concept starts long before it is addressed directly. There is a vital role for teacher training here and that role will depend on how it is defined and executed. We would wish to see significant elements on the manifestations and consequences of difficulties in learning maths in all initial teacher training.

To illustrate the point of where a topic is rooted and where it is leading, consider the strategy advocated in this book for teaching the nine times table (see Chapter 6). The method uses previous information (the ten times table), subtraction, estimation, refinement of the estimation and patterns. Although a child may not need to realise that he is doing all these things when he learns how to use a strategy to work out 6×9 , the processes are being used, concepts are being introduced and foundations are being laid. We agree with Madsen *et al.* (1995) that instruction should be conceptually oriented. Two key observations from a 2008 Ofsted Report (Ofsted are the official inspectors of schools in England) were, 'Their (the pupils) recall of knowledge and techniques was stronger than their understanding.... The pupils' view that mathematics is about having correct written answers rather than about being able to do the work independently, or understand the method, is holding back pupils' progress.' These are from observations of mainstream students. Many of the problems of maths are not exclusive to dyslexic and dyscalculic students.

A second illustration of the influence of early ideas involves a subtraction such as:

$$\begin{array}{r} 93 \\ -47 \\ \hline \end{array}$$

A frequent error is the answer 54, which occurs when the child subtracts 3 from 7. This is an easier process than the correct one, but can also be the

consequence of earlier subtraction experience where the child is told to: 'Take the smaller number from the larger number.' Dyslexics have a tendency to take instructions literally and feel safer when procedures are consistent. There is also (again) the problem that a first learning experience is often a dominant learning experience (Buswell and Judd, 1925), which means that the consequences of that experience being incorrect are very detrimental.

Rawson (1984) said of teaching English to dyslexics, 'Teach the language as it is to the child as he is.' Chasty (1989) said, 'If the child does not learn the way you teach, then you must teach the way he learns.' This advice is apposite for teaching mathematics to all children, but most especially to those with maths learning difficulties. One of the attributes of an effective teacher is the ability to communicate clearly. This is usually a consequence of knowing the child, usually enhanced by listening to the child, and presenting work in a way that pre-empts as many of the potential difficulties as is possible. Thus, the teacher needs to understand the way each child learns and fails to learn, though the different ways individuals learn can be frustrating in that a lesson which works superbly with one child may not work at all with another (see Chapter 2). This combined understanding of the child and all his strengths, weaknesses and potentials plus a knowledge of the nature, structure and concepts of mathematics will help to pre-empt many of the potential learning problems. It can often sustain the child whilst at the lowest stage of school intervention.

We believe that there are certain key concepts in the maths taught to most children up to the age of 16 years and that these concepts reappear regularly to be developed throughout a child's progression through his school years. The benefit of this is that the child may strengthen that concept as each new manifestation appears. The drawback is that the child may never develop the concept if he has not generalised and internalised all or even some of the preceding experiences. It is a vital part of the teacher's role to ensure that as many children as possible develop a sound understanding of these concepts, rather than a rote learned regurgitation of a mass of unconnected memories.

Finally, it should be remembered that an insecure learner values consistency. This characteristic is linked to automaticity, in that automaticity allows the brain to devote more capacity to what is different or an extension of what is known. Consistency will also reduce anxiety.