CHAPTER 1

Voltage and Capacitors

OVERVIEW

This first chapter describes the electric field that is basic to all electrical activity. The electric or E field represents forces between charges. The basic charge is the electron. When charges are placed on conductive surfaces, these forces move the charges to positions that store the least potential energy. This energy is stored in an electric field. The work required to move a unit of charge between two points in this field is the voltage between those two points.

Capacitors are conductor geometries used to store electric field energy. The ability to store energy is enhanced by using dielectrics. It is convenient to use two measures of the electric field. The field that is created by charges is called the D field and the field that results in forces is the E field. A changing D field represents a displacement current in space. This changing current has an associated magnetic field. This displacement current flows when charges are added or removed from the plates of a capacitor.

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1.1 INTRODUCTION

Every person that has designed a circuit has considered issues of grounding and shielding. Every person that has used electronics to make measurements has encountered interference of some sort. The problems vary as the technology evolves. Searching for ways to deal with these issues consumes a lot of engineering time. It is the intent of this book to provide the reader with some insight as to what is happening. The sensing of signal takes place in the analog world. The computations involving data take place in the digital world. The energy to operate electronics comes from the power utility world. It makes

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sense that the engineer must be familiar with all three areas if he wants to understand what is happening.

The parallel with automobile travel is interesting. A fine automobile makes sense only if there is an infrastructure. We need highways, bridges, repair centers, and gas stations or the system cannot work. The pieces of the system must work together for cars to be effective. Fuels must match engine needs, curves must match driving speeds, and the number of lanes must match traffic requirements.

This book makes an attempt to bring several disciplines together so that working in electronics is a lot easier. These disciplines involve the physics of electricity, the nature of the digital world, the shielding of the analog world, and finally the distribution of power in the utility world. To fully understand grounding, shielding, and interference, we must spend time in all of these areas. A discussion of utility power is important but unless transmission lines and radiation are understood, the subject of interference will make no sense. Shielding makes no sense unless the analog world is explained. There is a tendency to specialize in electronics. This book is an attempt to broaden the view and add to the general understanding of how nature functions.

How does a circuit work? One answer is to do a sinusoidal analysis using Kirchhoff's laws. Another answer is to write a set of logic statements. These responses provide a small part of the answer. The full answer is buried in a mountain of details. In this book, we are going to look at some of this detail but in a non-circuit way. We will take this approach because circuit diagrams and circuit theory by their very nature must leave out a lot of pertinent detail. This detail is important for quality performance whether it be for wide bandwidth or amplifying very low-level signals. It is also important when radiation, interference, or susceptibility is involved. Wire size, connection sequences, component orientation, and lead dress are often critical details. I like to call these details "circuit geometry." These details in geometry are important in analog circuits, power circuits, and especially in digital circuits, where clock rates rise year by year.

When a circuit is put to practice, there are many details that we take for granted. The components will most likely be connected together by strips or cylinders of copper. They will be soldered into eyelets or onto copper- or gold-plated pads. Traces will go between layers on a printed circuit board using vias. These are a few of the details in a design that are not questioned. There are details of a more subtle nature such as the thickness of a trace or ground plane or the dielectric constant of an epoxy board. In most cases, we do not question how things are done because we tend to rely on "accepted practice." Circuits built this way in the past have worked, so why make changes?

Taking things for granted is not always good engineering. Note that digital clock rates have changed from 1 MHz to 1 GHz in 20 years. That is three orders of magnitude! Imagine what would happen if automobile speeds went up one order of magnitude. That is 600 mph or jet aircraft speed. Even a modest increase in automobile speed would require extensive changes to the design of our roads and cities not to mention extensive driver training.

In electronics, an increase in speed does not pose a safety hazard. There are however differences and limitations in performance that should be understood. Often an effect is not sensed until the next generation in design is introduced. Understanding and correcting these effects requires an understanding of basic principles. The details we will look into do not appear on a circuit diagram. We will address these details because through understanding we can improve performance, reduce costs, and hopefully stay out of trouble.

Electronics often makes use of power from the local utility. For reasons of safety, the utility connects one of its power conductors to earth. Electronic hardware must often interface with this power and share this same earth connection. The result can be interference. I will discuss the relation between power distribution and circuit performance throughout the book.

A circuit diagram is only a plan or an organization of ideas. Circuit theory applied to a circuit diagram provides a basic overview of circuit performance. Circuit symbols are a part of the problem. They are necessarily very simple representations of complex objects. Every capacitor has a series resistance and an inductance. It can also be considered a transmission line stub. At some level it has its nonlinearities. Every inductor has series resistance and shunt capacitance. These considerations only begin to tell the entire story. For example, at high frequencies, dielectrics are nonlinear. For magnetic materials, permeability falls off with frequency. Thus, circuit symbols can only convey limited information. Further, we do not have symbols for skin effect, transit time, radiation, or current flow patterns. A straight line on a diagram may actually be a very complex path in the actual circuit. In short, a schematic diagram provides little information on physical structure and this can limit our appreciation of what is actually happening. If we had all this detail, we would be overwhelmed. It is important to be able to back off and take the broad view and still be aware that many details have been intentionally disregarded. A good designer walks a fine line, always aware that there are details in his field that he is not yet familiar with.

The performance of many circuits or systems is closely related to how they are built. It is not a question of whether there is an electrical connection but where the connection is to be made. In an analog circuit, it is often important to know which end of a shield is grounded not whether it is grounded. Here is a good question. How should a digital circuit board ground plane be connected to the surrounding chassis? The answer to this question is not available from a schematic diagram. Here is the answer: If possible, the connection should be arranged so that ground current does not flow in the ground plane of the board.

A repeated theme discussed in this book relates to how signals and power are transported in circuits. This approach will lead to an understanding of many issues that are often poorly understood. In order to discuss the transport of electrical power and signals, the electric and magnetic fields related to voltages and currents must be discussed. To begin this discussion, we introduce the electron. Don't despair. The time spent reviewing basic physics will make it much easier to understand the ideas presented in this book. Even if you do not follow the mathematics, the ideas will be clearly stated.

1.2 CHARGES AND ELECTRONS

Circuit theory allows us to relate circuit voltages and the flow of current in a group of interconnected components. For RLC networks (resistor–inductor–capacitor), this analysis is straightforward using Kirchhoff's laws. The processes I want to discuss do not involve this approach. To understand the fundamentals of circuit performance, we will use basic physics to explain many details that are often ignored. Our starting point may seem a bit remote but please read on.

Atoms are composed of a nucleus of protons and neutrons surrounded by electrons. For our purposes, the electrons can be considered negatively charged particles located in shells around the atom. The quantum mechanical view of electrons in atoms is that they are overlapping waves described by quantum numbers and a probability function. We will use the shell viewpoint of the atom as we are not involved in nuclear physics or quantum mechanics. We can treat electrons as particles and not get into trouble. The electrons have a negative charge and the matching protons in the nucleus have a positive charge. In a neutral atom, the positive and negative charges are exactly equal. Each electronic shell is limited to a fixed number of electrons. The number of electrons in the outer shell says a lot about the character of the atom. As an example, copper has just one electron in its outer shell. This outer electron has a great deal of mobility and is involved in electrical conductivity. Because protons are comparatively heavy and the shells of electrons shield them, they are not directly involved in the electronics we are going to consider.

Molecules are formed from atoms that bond together. Bonding really means that electrons from one atom share spaces in the shells of other atoms. For an insulator, this bonding greatly restricts the activity of outer shell electrons. Typical insulators might be nylon, air, epoxy, or glass. This bond does vary between insulators. If two insulators are rubbed together, such as a silk cloth against a rubber wand, some of the shared electrons on the wand will transfer to the cloth. In this case, the silk cloth with extra electrons is called a negatively charged body. We will call the absence of negative charge a positive charge. The rod is said to be positively charged. In reality, the positive charge stems from the immobile protons in the nucleus of atoms that do not have matching outer shell electrons. The absence of negative charges behaves the same as if there were fictitious positive charges on the surface of the insulator. An analogy can help. Consider an auditorium full of people. There is one empty seat. When a person moves left into this seat, the vacant seat has moved to the right. If people keep changing seats this way, we can see the flow as moving empty seats. Now consider the empty seat as a positive charge.

Experiments with charged bodies can demonstrate the nature of the forces that exist between charges (electrons). These same forces exist for the absence of charge that we will call a positive charge. If one charged body repels another, it is actually the fields of electrons that are involved. If you remember your physics class, these forces can be demonstrated using pith balls that hang by a string. Here the charges are attached to small masses and we can see the pith balls attract or repel each other. For there to be an attraction, one pith ball needs to have extra electrons and the other an absence of electrons.

The percentage of electrons involved in any of these experiments is extremely small. To illustrate this point, I want to paraphrase the writing of Dr. Richard Feynman.[1] If two people are standing a few feet apart, what would be the force of repulsion if 1% of the electrons in each body were to repel each other? Would it be a few pounds? More! Would it be greater than their weight? More! Would it lift a building? More! Would it lift a mountain? The answer is astounding. The force would be great enough to lift the earth out of orbit. This is

[1] The Feynman Lectures on Physics Volume 2 page 1-1. Addison Wesley Publishing Company Inc. Copyright 1964, California Institute of Technology.

why gravity is called a weak force and the force between electrons is called a strong force. This also tells us something about nature. The percentage of electrons involved in electrical activity is extremely small. We know that the forces in a circuit do not move the components or the traces. Obviously, since electrical forces are so large, electrical activity in a circuit involves an extremely small percentage of the available electrons. Yet there are so many electrons on the move, we can think of any typical current flow as being continuous and seamless.

1.3 THE ELECTRIC FORCE FIELD

When we encounter forces at a distance, we use the expression force field. We experience a force field at all times as we live in the gravitational force field of the earth. Every mass has a force field including the earth. The earth has the dominant field because the earth is so massive. The result is that each mass on earth is attracted toward the center of the earth. The forces of attraction between individual objects on the earth are so small that they are very difficult to measure. On the earth's surface, the force field is nearly constant. We would have to go out thousands of miles into space to see a significant reduction in the force of gravity.

The electrical and gravitational force fields are similar in many ways. Every electron carries with it an associated force field. This force field repels every other electron in the area. If a group of extra electrons are located on an isolated mass, we call this mass a charged body. We refer to the extra electrons as a charge. If this mass is a conductor, the extra electrons will move apart until there is a balance of forces. On a conducting isolated sphere, the extra electrons will move until they are evenly spaced over the entire outer surface. None of these excess electrons will remain on the inside of the conductor. For a perfect insulator, extra electrons are not free to move about. Extra electrons on the inside of this material are called trapped electrons. It is also possible to have trapped absences of electrons.

1.4 FIELD REPRESENTATIONS

The electric force field in a volume of space can be measured by noting the forces on a small test charge in that space. A test charge can be formed using a small mass with a small excess of electrons on its surface. The force on this test charge has a magnitude and direction at each

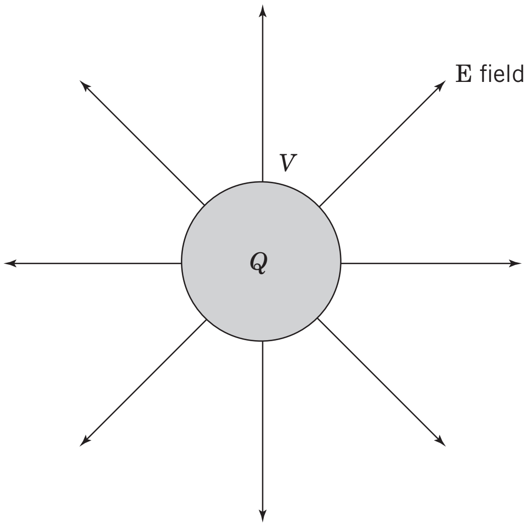


Figure 1.1 The force field lines around a positively charged conducting sphere

point in space. Having direction, the force field is called a vector field. To be effective, this test charge must be small enough so that it does not influence the charge distribution on the objects being measured. Performing this experiment is difficult but fortunately we can deduce the field pattern without performing an actual test.

It is convenient to represent a force field by lines that follow the direction of the force. For an isolated conducting charged sphere, the lines of force are shown in Figure 1.1.

Note that the field exists everywhere between the lines. The lines are simply a way of showing the flow or shape of the field. As pointed out, the number of extra electrons that form the charge Q on the surface of a conductor is small compared to the number of electrons in the conductor. In spite of this fact, the number of electrons is still so large that we can consider the charge as being continuously distributed over the surface of interest. This is the reason we will not consider the force field as resulting from individual electrons. From here on, we will consider all charge distributions as being continuous. The total charge on the surface of the sphere in Figure 1.1 is Q. The charge density on the surface of the sphere is

$$\frac{Q}{A} = \frac{Q}{4\pi r^2}. \tag{1.1}$$

We will use the convention that a line starts on a unit of positive charge and terminates on a unit of negative charge. This unit can be selected so that the graphical representation of the field is useful. If the total charge is doubled, then the number of lines is doubled. For representations in this book, no attempt will be made to relate the number of lines to any specific amount of charge. In general, we are interested in the shape of the field, areas of field concentration, and where the field lines terminate. For the ideal single sphere, the field lines are straight and extend from the surface to infinity.

In Figure 1.1, the force f on a small test charge q in the field of a charge Q located on a sphere is proportional to the product of the two charges and inverse to the square of radius r or

$$f = \frac{qQ}{4\pi\varepsilon_0 r^2} \tag{1.2}$$

The constant ε_0 is called the permittivity of free space. Equation 1.2 is known as Coulomb's law. The force per unit charge or f/q is a measure of the electric field intensity. The letter E is used for this measure. The force field around a group of charges is referred to as an E field. Mathematically, the E field around a charge Q is

$$\mathrm{E} = \frac{Q}{4\pi\varepsilon_0 r^2} \tag{1.3}$$

The E field falls off as the square of the distance r. In Figure 1.1, the force field intensity E decreases as the field lines diverge. The forces are greatest at the surface of the sphere. Note that the field lines do not enter the sphere. This is because there are no excess charges inside the conductor. The field lines must terminate on the sphere perpendicular to its surface. If there were a tangential component of force on the surface, the charges on the surface would be accelerated. If there were an absence of electrons on the surface, this absence of charge would also be accelerated. Remember the absence of negative charge can be considered the presence of a positive charge. For conductors, the mobility of a group of electrons is no different than the mobility of an absence of electrons. Except for the direction assigned to the force field, we will assume that positive and negative charges behave the same way. Figure 1.1 shows a sphere with a positive charge Q. If the charge were negative (the presence of electrons), the field lines would be shown with the arrows pointing inward.

The field lines in Figure 1.1 start at the surface of the sphere. If the charge Q were located at the center of the sphere and the sphere were removed, the field pattern at every initial value of r would be unchanged. A point charge Q implies an infinite charge density, which is impossible. Often it is mathematically convenient to consider the fields from point charges even though this cannot exist.

N.B.

The electric force field E is called a vector field as it has a magnitude and direction at every point in space.

N.B.

The field intensity is greatest where the lines are closest together.

1.5 THE DEFINITION OF VOLTAGE

A test charge q in the field of a charge Q experiences a force f given by Equation 1.2. The work required to move the test charge a small distance Δd is $f \cdot (\Delta d)$. The work to move it from infinity to a point r_1 is the integral of force times distance from infinity to r_1. If we follow one of the field lines, the force is always tangent to this line. The work is

$$W = \int_{\infty}^{r_1} f \ dr = -\frac{qQ}{4\pi\varepsilon_0 r_1}. \tag{1.4}$$

If we divide both sides of this equation by q, we obtain the work per unit charge. This term has the familiar name volts. In equation form, the voltage V is given by

$$\mathrm{V} = \frac{Q}{4\pi\varepsilon_0 r}. \tag{1.5}$$

DEFINITION

A voltage difference is the work required to move a unit charge between two points in space in an electric field.

In Equation 1.4, we can make the assumption that the voltage at infinity is zero. This allows us to assign a voltage to intermediate points in space. In a circuit, the work required to move a unit charge between two conducting surfaces is called a potential difference or a voltage difference. It is important to realize that potential differences do exist between points in space. Of course, it is difficult to place a voltmeter in space to get a measure of this voltage.

The voltage difference between two points in space is

$$V_2 - V_1 = \left(\frac{Q}{4\pi\varepsilon_0}\right)\left(\frac{1}{r_2} - \frac{1}{r_1}\right). \quad (1.6)$$

N.B.

A voltage difference cannot exist without the presence of an electric field.

In the presence of conductors, an electric field cannot exist without charges on the surface of these conductors. These charges are not apparent from a schematic diagram.

When a circuit is in operation, there are surface charges everywhere and there are voltage differences. These surface charges are the first charges that move when a request is made to move energy. In a dc circuit, electrons move in the entire conductor. A current is considered dc only after a period of time has elapsed and no further change in the flow pattern can be detected.

N.B.

Fields and charges are not shown on schematic diagrams.

1.6 EQUIPOTENTIAL SURFACES

As the word implies, an equipotential surface is a surface of equal voltage. No work is required to move a test charge on this surface. This surface can be in space or on a conductor. Figure 1.2 shows equipotential surfaces around the charged sphere in Figure 1.1. Note that these surfaces are also spheres and the E field lines are always perpendicular to these equipotential surfaces.

N.B.

Conducting surfaces are equipotential surfaces regardless of their shape. This assumes that the surface charges are not in motion.

In practice, a conducting surface is an equipotential surface even when the charge distribution is not uniform. This is true even when there are

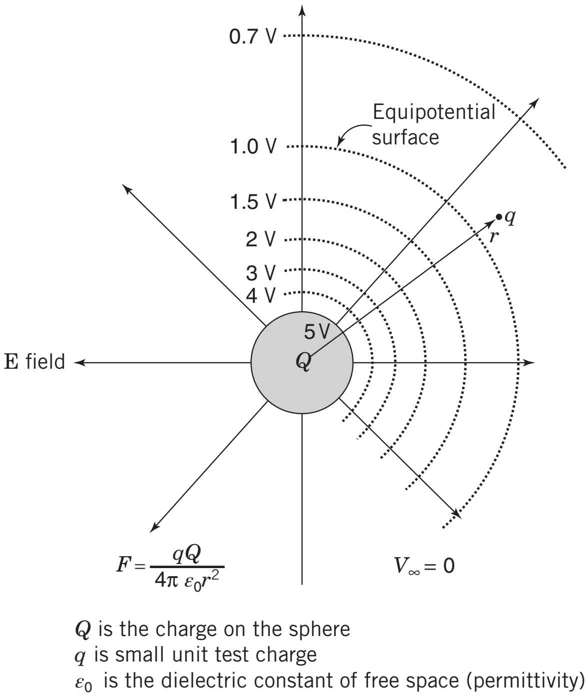


Figure 1.2 Equipotential surfaces around a charged sphere

areas of positive and negative charges on the same conductor. We will assume it takes no work to move charges on the surface of a conductor. If work were required, there would be a tangential electric field and this means that free charges would have to be in motion.

1.7 THE FORCE FIELD OR E FIELD BETWEEN TWO CONDUCTING PLATES

Consider two conducting plates separated by a distance h. On the top plate there is a charge $+Q$ and on the bottom plate there is a charge $-Q$. This configuration is shown in Figure 1.3.

If we ignore edge effects, the force field can be represented by equally spaced straight lines that run from the top plate to the bottom plate.[2] In this configuration, the net charge in the system is zero. There is no loss in generality if we assume that all of the field lines stay in the volume between the two plates. Since the lines do not diverge, the force on a test charge q is constant everywhere in the space between the plates. In other

[2] Edge effects can be ignored when the spacing between conductors is very small.

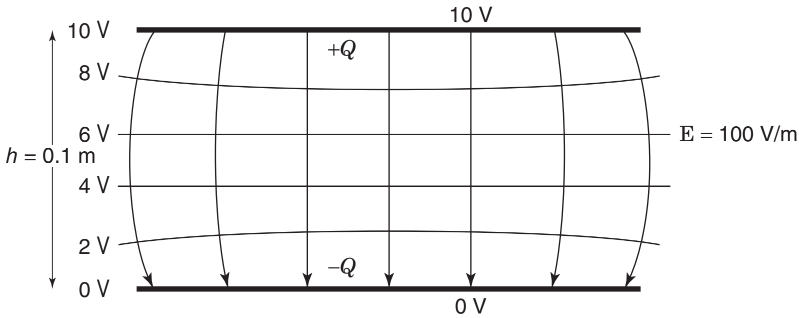


Figure 1.3 The force field between two conducting plates with equal and opposite charges and a spacing distance *h*

words, the electric field intensity E is a constant between the plates. If the charge density on the plates is made equal to the charge density of the sphere in Figure 1.1, the force field between the plates will have the same intensity as the force field at the surface of the sphere. The work required to move a unit charge between the plates will then be force times distance or

$$W = \frac{Qh}{4\pi\varepsilon_0 r^2} = \mathrm{E}h. \tag{1.7}$$

This work W is the potential difference between the plates. If the bottom plate is assumed to be at 0 V, the upper plate will be at a voltage $\mathrm{V} = \mathrm{E}h$. Note that the E field has units of volts per meter. If the voltage between the two planes of Figure 1.2 is 5 V and the spacing is 1 cm, the E field intensity between the plates is 500 V/m.

1.8 ELECTRIC FIELD PATTERNS

Figure 1.4 shows a printed circuit trace over a conducting surface. Such a surface is often called a ground plane. There are many types of ground planes and we will talk about this later. The spacing between these two conductors might be as small as 0.005 in. or 1.3×10^{-4} m. A typical logic voltage might be 5 V. The E field intensity under the trace would be 38,000 V/m, a very surprising figure.

The E field lines terminate and concentrate on the surfaces between the trace and the ground plane. Field lines terminate on charges. Remember this is a static situation. Note that surface charge distributions are not considered in circuit theory. The path taken by the charges to achieve this distribution is also not considered.

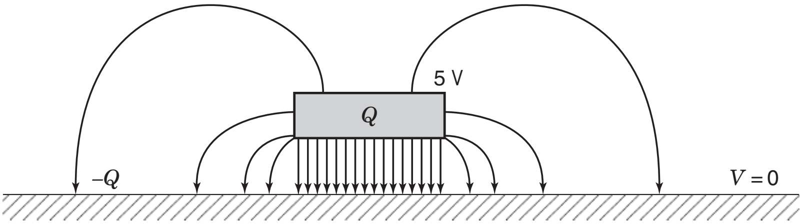


Figure 1.4 The electric field pattern of a circuit trace over a ground plane

FACTS

1. The charge distribution on the surface of conductors is usually not uniform.
2. There is no potential gradient along the surface of the ground plane as the charges are not moving.
3. The charges concentrate at the surfaces between the circuit trace and the ground plane.
4. There is some E field above the circuit trace.
5. Surface charges concentrate on the sharp edges of the circuit trace.
6. If the voltage were to increase slowly, new charges must be supplied to the conductors.

Consider the field pattern when there are two traces over a ground plane. This pattern is shown in Figure 1.5. If the voltages are of opposite polarity, the charge distribution on the ground plane will reverse

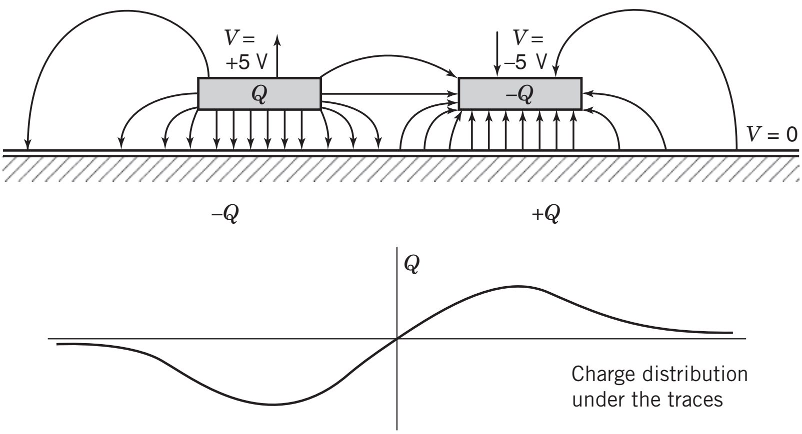


Figure 1.5 The electric field pattern around two traces over a ground plane

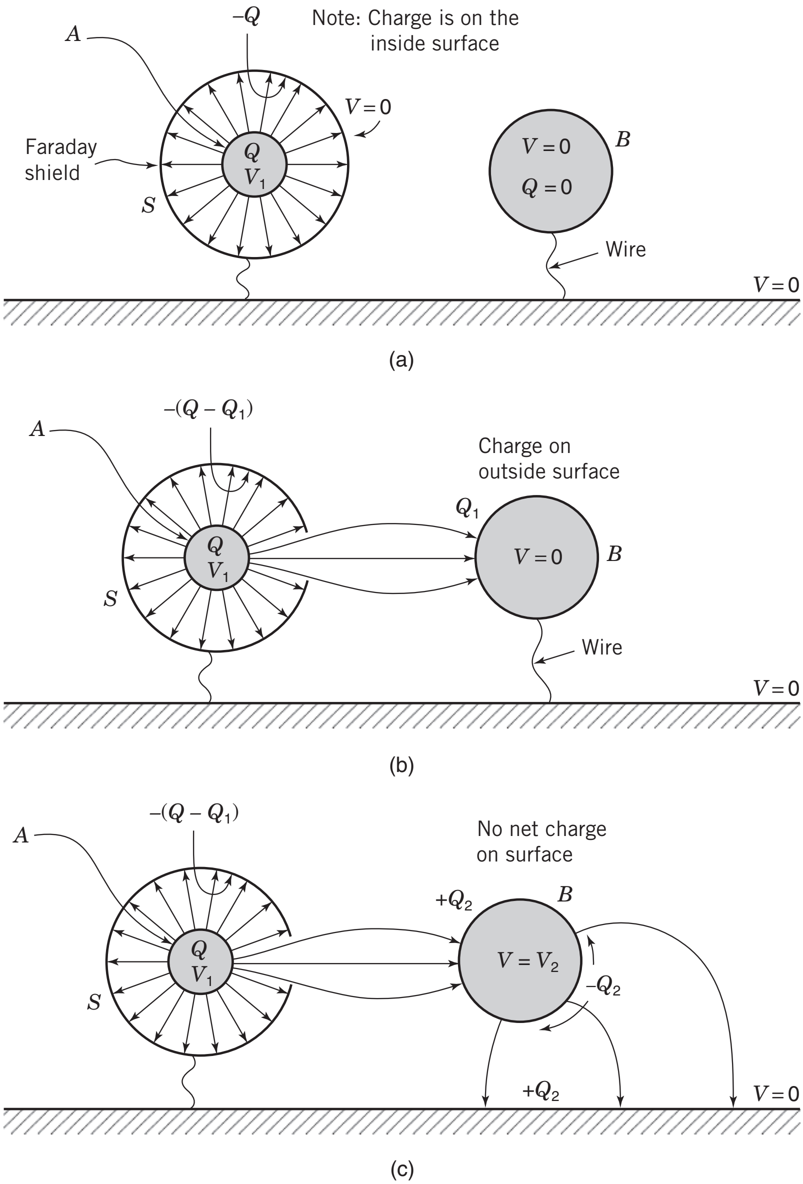


Figure 1.6 Field configurations around a shielded conductor

polarity under the traces. Again, the ground plane is an equipotential surface.

Consider the field pattern around a section of shielded cable as in Figure 1.6a.

In Figure 1.6a, the shield S fully encloses the center conductor A and no electric field escapes. In Figure 1.6b and c, there is a hole in the shield that allows some of the electric field to terminate on conductor B. The field lines that terminate on conductor B imply a charge distribution on conductor B. In Figure 1.6c, conductor B floats in space. Note there is still a charge distribution on this conductor, but the net charge on the conductor is zero. This floating conductor would not be at zero potential, but it would still be an equipotential surface. The grounded conductor by definition would be at zero potential over its entire surface even though there is a net charge on its surface.

If the voltage on the shielded conductor in Figure 1.6b is slowly changed, the field intensity changes everywhere. The amount of charge on conductor B in Figure 1.6b must now change. This change in charge must flow in the connection to the ground plane. The charge on conductor B is called an induced charge. The flow of charge is called a current. Any current that flows to conductor B is called an induced current. In the case of Figure 1.6c, the charge density on the surface of the floating conductor must change. This means that induced currents must flow locally on this conductor. Note there is no path for new charges to reach the isolated conductor. There simply needs to be a change in the electric field in the space around the conductor for there to be a change in the charge distribution.

N.B.

Surface currents can flow on a conductor that is floating, that is, not connected to any other circuit.

The electric field in Figure 1.6a is totally contained. The internal field can change and there are no induced currents on nearby conductors. This containment of the electric field is called electric shielding. The outer conductor S is often called a Faraday shield. A conductor with an outer conducting sheath is called a shielded cable. Later we will discuss a shielded conductor called coax.

N.B.

The electric field lines in Figure 1.6 terminate on the inside surface of the cable. If the voltage changes slowly, the resulting change in field causes current flow on the inside surface of the shield. Ideally, this field does not penetrate into this shield and get to the outside surface.

N.B.

Shielding has nothing to do with external connections to the shield conductor. If the electric field is contained, the shield is effective. The shield need not be at "ground potential" to be effective.

1.9 THE ENERGY STORED IN AN ELECTRIC FIELD

It takes work to move a charge in an electric field. In Figure 1.3, the work required to move a unit charge between the plates is the voltage difference between the plates. As more charge is moved across the space, the voltage between the plates increases. The work done on the charges is stored as potential energy. Where is this energy stored? Since it is not stored inside the conductors or on their surfaces, the only place that is left is in the space between the plates. The same problem exists with gravity. When a weight is lifted in a gravitational field, the added energy is stored in this field not in the mass.

The force on a small increment of charge dq is $\mathrm{E}\,dq$, where E is the force field. Assume the top plate has a charge q and the bottom plate has a charge $-q$. The E field from Equation 1.3 is $q/\varepsilon_0 A$.

The work dW required to move an increment of charge dq across the distance h is

$$dW = f\,h\,dq = \left(\frac{q}{\varepsilon_0 A}\right) h\,dq. \tag{1.8}$$

By integrating Equation 1.8 over charge from 0 to Q, the total work W required to move a charge Q is

$$W = \int_0^Q \frac{q\,dq}{\varepsilon_0 A} = \frac{Q^2 h}{2\varepsilon_0 A}. \tag{1.9}$$

Since $\mathrm{E} = Q/(\varepsilon_0 A)$, the work W can be written as

$$W = \frac{\mathrm{E}^2\,\varepsilon_0 A h}{2} = \frac{1}{2}\mathrm{E}^2 \boldsymbol{V} \varepsilon_0 \tag{1.10}$$

where $\boldsymbol{V}$ is the volume of the space between the two conductors.

N.B.

Every volume of space that contains an E field stores electric field energy.

> **N.B.**
>
> Every circuit with voltages stores electric field energy.

Somehow space has a quality like a spring that can be used to store potential energy. The conductors seem to provide a handle for holding on to this spring. We cannot see it or feel it, yet we can do work with the energy that is stored in space. Without the restraint of these conductors, the field energy would have to leave the area at the speed of light. The correct word here is radiation.

In practical circuits, the electric field patterns are complex and the intensity of the field varies over space. To calculate the total stored energy, space can be divided into small volumes of near-constant field intensity. The important fact to remember is that the energy stored per unit volume is proportional to the square of the field intensity. In many practical problems, the region of high field intensity is all that is important. Remember this is where most of the energy is stored.

The potential assigned to a point in space is a number not a vector. Some surface in the system must be assigned to the zero of potential. The potential at every other point in the system is the work required to move a test charge against the forces created by every element of charge in the system. This voltage value is a summation based on Equation 1.5. The electric field vector can be determined by locating the direction in which the voltage change is maximum. Stated mathematically, the gradient of the voltage is the electric field intensity.

The electric fields we will consider in circuits are located in the spaces between conductors. These fields terminate on charges distributed on the conducting surfaces. There can be no electric field in the conductors or there would be current flow. This means there is essentially no field energy stored in the conductors that is available to us to do work. This supports the theme of this book that the spaces between conductors carry the fields that represent signal, energy flow, as well as interference.

1.10 DIELECTRICS

We next consider the effect dielectric materials have on electric fields. Typical dielectrics are rubber, silk, Mylar®, polycarbonate, BST, epoxy, air, and nylon. Up until now, we have considered the electric field in air (technically in a vacuum). Consider the two plates in Figure 1.3. If the space between the plates is filled by an insulating dielectric, it takes less work to move a charge Q from one plate to the other. This means the force field inside the insulator is reduced. This reduction factor ε_R is known as the relative dielectric constant. The force field between the planes in the dielectric medium is given by

$$E = \frac{Q}{A\varepsilon_0\varepsilon_R}. \tag{1.11}$$

If the space is first filled with air, then a voltage V results in a charge Q. When a dielectric material is inserted between the plates, the voltage must drop to V/ε_R. If the voltage is again increased to V, the amount of charge on the surface would increase by the factor ε_R. This factor is called the relative dielectric constant. This factor is exactly 1 in free space.

N.B.

The relative dielectric constant of air is 1.0006.

N.B.

Dielectric materials are used in capacitors to increase the charge stored per unit voltage.

1.11 THE D FIELD

It is convenient to discuss two measures of the electric field. The voltage between two points defines the E field intensity. A second field measure called the D field relates directly to charges. In a vacuum, the E field and D field patterns are exactly the same. In a region where there are dielectrics, the E field intensity changes at every dielectric interface. The D field starts and stops on charges but does not change intensity at a charge-free boundary. In Figure 1.1, if the charge Q were located in a dielectric medium, the E field would be reduced by a factor equal to the relative dielectric constant. The new E field is then given by Equation 1.12.

$$E = \frac{Q}{4\pi\varepsilon_0\varepsilon_R r^2}. \tag{1.12}$$

The energy stored in the field of this charge is inversely proportional to the relative dielectric constant. Figure 1.7 shows the field pattern

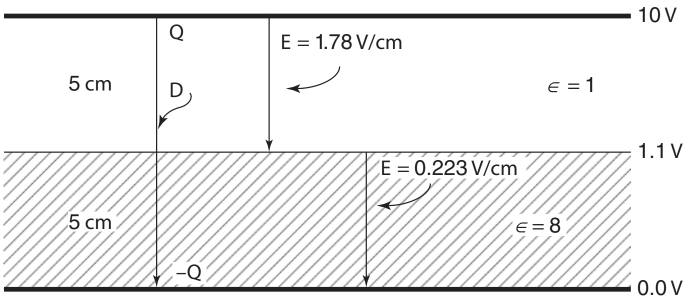


Figure 1.7 The electric field pattern in the presence of a dielectric

between two planes, where half of the space has a dielectric constant of 8. If the total spacing is 10 cm and the dielectric material has a dielectric constant 8, the E field pattern must adjust so the total voltage difference is 10 V. The voltages in terms of the E field are $(E/8) \cdot 5\ \text{cm} + (E/1) \cdot 5\ \text{cm} = 10\,\text{V}$. E in the open space is obviously 8.9 V/cm and inside the dielectric it is 1.1 V/cm. The voltages are 8.9 V across the air space and 1.1 V across the dielectric. The electric field intensity in air is now 8.9 V/5 cm = 1.78 V/cm. Before the dielectric was inserted, the field intensity in air was 1 V/cm. This means that the added dielectric has increased the charge Q on the plates by 78%. Note that the majority of energy is stored in the air space not in the dielectric. In Figure 1.7, the D field is continuous from the top to bottom plates. If E equals D/ε_0 in the air space, then in the dielectric

$$E = \frac{D}{\varepsilon_R \varepsilon_0}. \tag{1.13}$$

N.B.

Electric field energy is stored in the E field.

In high-voltage transformers, an oil dielectric is often used to reduce the E field around conductors. This reduction in the E field reduces the chance of arcing. The oil also helps to conduct heat away from the windings.

1.12 CAPACITANCE

The ratio of charge to voltage is capacitance C. The unit of capacitance is the farad F. A capacitor of 1 F stores one coulomb of charge Q for a voltage of 1 V. In Figure 1.1, the voltage on the surface of the sphere is associated with a stored charge Q. The voltage V is $Q/4\pi r\varepsilon_0$. The ratio $Q/$V equals $4\pi r\varepsilon_0$. If the sphere is located in a dielectric medium, the voltage V is reduced by ε_R and the ratio $Q/$V is $4\pi r\varepsilon_0\varepsilon_R$. The capacitance of a conducting sphere in a dielectric medium is

$$C = 4\pi r\varepsilon_R\varepsilon_0. \tag{1.14}$$

For the parallel planes in Figure 1.3, the voltage between the conducting plants is the E field times the spacing h. The voltage from Equation 1.11 is V = $Qh/(\varepsilon_0 A)$. If there is a dielectric present, the ratio $Q/$V is

$$C = \frac{\varepsilon_0\varepsilon_R A}{h}. \tag{1.15}$$

Capacitance is a function of conductor geometry. So far, we have discussed two simple geometries: the sphere and parallel conducting planes. In most practical circuits, the geometries are complex and the capacitances are not simply calculated. It is important to recognize that capacitance is controlled by three factors. It is proportional to surface area, inversely proportional to the spacing between surfaces, and proportional to the relative dielectric constant.

N.B.

Capacitance is a geometric concept. All conductor geometries can store some electric field energy; therefore, they all have capacitance.

The idea of capacitance can be extended into free space. Consider a cube in space oriented so that electric field lines are perpendicular to two of the cube's faces. If there were equal and opposite charge distributions on the two opposite faces of the cube, the field lines would be no different. The voltage between the faces is the E field times the distance across the cube. Since we have an equivalent surface charge and a potential difference, the ratio is capacitance.

N.B.

Free space has the ability to store electric field energy. A volume of space has a capacitance. An electric field cannot use this space to store static energy unless there are nearby conductors to hang on.

The factor ε_0 is called the permittivity of free space and it is equal to 8.85×10^{-12} F/m. Consider the capacitance of a printed circuit trace over a ground plane. If the spacing h is 5 mm and the trace is 10 mm wide by 10 cm long, the trace area is 100 mm^2. The value of A/h is 20 mm or 20×10^{-3} m. If the relative dielectric constant is 10, then the capacitance between the trace and the ground plane is equal to $(A/h)\varepsilon_R\varepsilon_0 = 177 \times 10^{-12}$ F or 177 pF.

It is interesting to calculate the capacitance of the earth as a conductor. The radius of the earth is 6.6×10^6 m. Using Equation 1.14, the capacitance is 711 μF.

1.13 MUTUAL CAPACITANCE

A mutual capacitance is often referred to as a leakage capacitance or parasitic capacitance. The electric field pattern in most practical circuits is complex. The voltage on any one conductor implies a self-charge and induced charges on all the other conductors. For small component geometries, a large percentage of the field energy may be parasitic in nature.

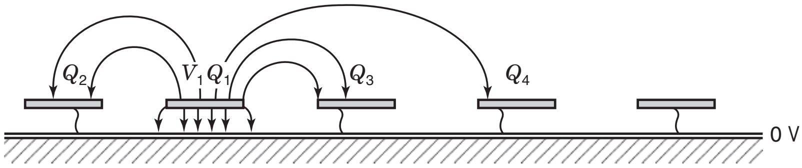
The ratio of charge to voltage on any one conductor is called a self-capacitance. Examples of self-capacitance are shown in Figures 1.1 and 1.4. The ratio of the charge induced on a second conductor to voltage on a first conductor is called a mutual capacitance. An example of a mutual capacitance is shown in Figure 1.6b. A measure of this capacitance requires a test voltage be placed on one conductor and all other conductors must be at zero potential. The mutual capacitance C_{12} is the ratio of charge induced on conductor 2 for a voltage on conductor 1. It turns out that $C_{12} = C_{21}$.[3]

All mutual capacitance values are negative as the induced charge for a positive voltage is always negative. A simple geometry showing a few mutual capacitances is shown in Figure 1.8.

A voltage V_1 is placed on trace 1 and traces 2, 3, and 4 are at 0 V. The ground plane is also at 0 V. The capacitances C_{11}, C_{12}, C_{13}, and C_{14} are the ratios V_1/Q_1, V_1/Q_2, V_1/Q_3, and V_1/Q_4, respectively. Mutual capacitance C_{32} would be the ratio V_3/Q_2.

Mutual capacitances are a function of circuit geometry. These capacitances often limit or determine circuit performance. In an integrated

[3] Measuring a small mutual capacitance can be difficult. One method of making a measure is to use a sinusoidal voltage at about 10 kHz and observe the current flow in a 10 k-ohm series resistor. Leakage capacitances as low as 0.1 pF can be measured this way. This capacitance measurement requires very careful shielding of the driving voltage.



$$C_{11} = \frac{V_1}{Q_1} \qquad C_{12} = \frac{V_1}{Q_2} \qquad C_{13} = \frac{V_1}{Q_3}$$

Figure 1.8 The mutual capacitances between several traces on a ground plane

circuit amplifier, mutual capacitances are an integral part of the design. They may define circuit bandwidth and circuit stability.

1.14 DISPLACEMENT CURRENT

Figure 1.3 shows two conducting plates. If a charge Q is placed on the top plate, a charge $-Q$ must exist on the lower plate. The ratio of charge Q to the voltage on the top plate is the capacitance of this geometry. This geometry is typical of many small commercial capacitors.

If the charge stored on the capacitor plates increases linearly with time, the voltage difference V will also increase in a linear manner. A constant current source can serve to provide this increasing charge. The equivalent circuit is shown in Figure 1.9.

For convenience, we will use a standard circuit symbol for this capacitance and label it with the letter C. The current flow in this circuit can be looked at in two ways. First viewpoint: The electrons flow on to the plates of the capacitor, but they do not flow through the dielectric. Second viewpoint: In a loop analysis, the current flows through the

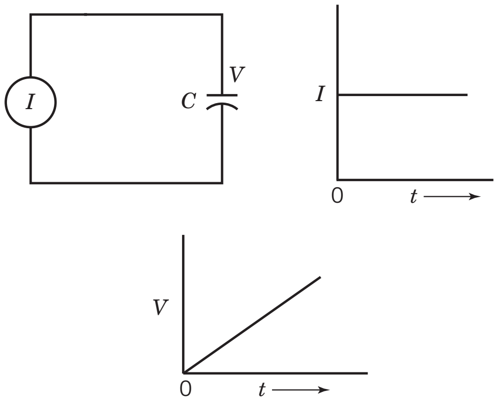


Figure 1.9 A capacitor driven from a constant current source

capacitor. Which viewpoint is correct? They both are correct providing we interpret the changing field in the dielectric correctly. As the charge Q accumulates on the plates, the D field in the dielectric also increases. This changing D field is equivalent to a current flow. We call a changing D field a displacement current. This statement is one of Maxwell's wave equations. This current has an associated magnetic field.

> **N.B.**
>
> A changing D field in space is equivalent to a displacement current flowing in space.

It is important to realize that we cannot have physical laws that work some of the time. Physical laws must work the same way all of the time at all frequencies. Circuit theory does not deal with the electric fields, but they are present if there are voltage differences. Later, we will discuss electromagnetic radiation. Field energy or radiation that leaves a circuit involves both an electric and a magnetic field. In a capacitor, the changing electric field is a current that has an associated magnetic field. As you will see in the next chapter, a changing magnetic field requires an electric field. In other words, these changing fields go hand in hand. In effect, energy can be carried across the capacitor plates as radiation and this requires the presence of both an electric and a magnetic field. Obviously, it is very cumbersome to view the operation of a capacitor in terms of this radiation.

1.15 ENERGY STORED IN A CAPACITOR

The energy stored in the field of a capacitor from Equation 1.8 is $\frac{1}{2} E^2 \varepsilon Ah$. If we substitute $E = \mathrm{V}/h$ and remember from Equation 1.4 that $E/\varepsilon A = Q$, then we can write the energy $\boldsymbol{E}$ as a function of charge and voltage.

$$\boldsymbol{E} = \frac{1}{2} Q\mathrm{V}. \tag{1.16}$$

We can use the ratio $C = Q/\mathrm{V}$ in Equation 1.13 to obtain two equivalent equations for energy storage.

$$\boldsymbol{E} = \frac{1}{2} C\mathrm{V}^2 \tag{1.17}$$

$$\boldsymbol{E} = \frac{1}{2} \frac{Q^2}{C}. \tag{1.18}$$

The energy that is used in fast digital circuitry must be made available near the point of demand. As you will see, it takes time to move energy over traces as multiple reflections are required. Providing local energy is the function of decoupling capacitors. This topic is discussed later in greater detail. The limitation that we will discuss is that capacitors are two-terminal devices. This means that it is impossible to take energy out and put it back in at the same time. The analogy with a water tank is appropriate, where separate input and output conduits are provided. In an oil tanker, one loading port is all that is required as loading and unloading takes place at different times. In fluid flow, a port can be a metal conduit. In electrical circuits a port usually requires two conductors. At microwave frequencies, a single metal conduit can serve as a port.

1.16 FORCES IN THE ELECTRIC FIELD

Field energy exists in the space between individual electrons. In Figure 1.1, the charges moved apart on the surface of the sphere until they were uniformly spaced. In effect they arranged themselves to store the least amount of field energy. This is characteristic of nature in so many ways. In fact all static field configurations represent a minimum of field energy storage within the geometric constraints provided. For a set of charges on conductors, there is only one field configuration possible and that field configuration stores a minimum of field energy.

N.B.

If there is a way, nature will follow a path that will reduce the amount of potential energy stored in a system. The path taken is unique. The path can involve an oscillation and usually involves some dissipation.

If the plates of a capacitor are moved closer together, the capacitance increases. Equation 1.18 shows that if the charge Q is fixed, then a larger capacitance stores less energy. This means there is a force acting on the plates trying to reduce the spacing as this reduces the potential energy stored in the system. It helps to recognize that work equals force times distance. The derivative of work with respect to distance is simply force.

Acoustic tweeters work on this principle. Parallel metal plates can be made to move air by placing an audio voltage between the plates. Potential differences in the order of several hundred volts are required. The E field must be biased at dc so that there is no doubling of frequency.

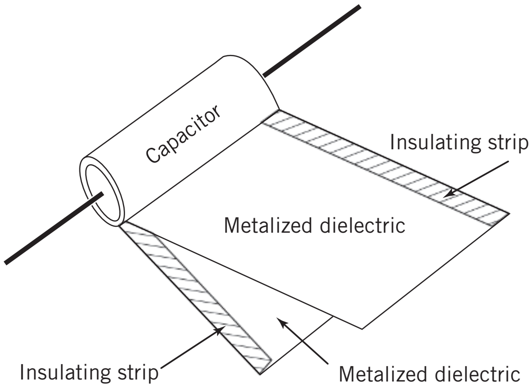


Figure 1.10 A typical wrap and foil capacitor

1.17 CAPACITORS

Capacitors are components that store electric field energy. A circuit engineer has a wide choice of component types and values to choose from. There are many construction styles, types of dielectric and voltage ratings. Capacitance values can range over 12 orders of magnitude from a few picofarads to 1 or 2 farads. Typically, electrolytic capacitors in the range 100 μF are used to store field energy for general circuit use. Capacitors in the range 0.001–0.1 μF are used on digital circuit boards for supplying local energy. These surface-mounted capacitors are often made using a ceramic dielectric called BST (barium–strontium–titanate). This material can have a relative dielectric constant as high as 10,000. In applications requiring stability over time or temperature, capacitor dielectrics can be mica or nylon. A capacitor of the wrap and foil type is shown in Figure 1.10.

Connections are made to the foil on the opposite ends of the cylinder. In many capacitor types, conductive material is vacuum deposited on a dielectric. An example of this technique is metalized Mylar®. Conductor surfaces can be made irregular to increase the effective surface area that increases the capacitance. Later, more will be said about the role capacitors play in digital circuits operating at high clock rates.

1.18 DIELECTRIC ABSORPTION

Dielectrics have a characteristic where a small charge is absorbed when an electric field is applied. In a capacitor, these trapped charges do not immediately return to the circuit when the electric field is removed. In time, these trapped electrons are released. The result can be a voltage on the terminals of the capacitor or a small current flow. This property is called dielectric absorption. This phenomenon can lead to misinformation in some measurements. Mica and nylon have a very low dielectric absorption.

The author once used a glass-encased 100-MΩ resistor as a feedback element in an amplifier. In production, the frequency response of the instrument was distorted. It was traced to a label that had been affixed to the glass. The gum backing on the label modified the parasitic capacitance associated with the resistor. Trapped charges in the dielectric were causing a small delayed current to flow in the feedback circuit.

1.19 RESISTANCE OF PLANE CONDUCTORS

There are places where large conducting surface areas must be considered. The largest is the surface of the earth or ocean, where fields cause surface currents. Sheet metal on the sides of a building provides another large surface. Compared to traces, the ground or power planes used in a circuit board design are large surface areas.

When a current flows uniformly in a conductor, the resistance is the resistivity ρ times the path length l divided by the cross-sectional area or

$$R = \frac{\rho l}{A}. \tag{1.19}$$

For a square of material where A is equal to $l \cdot t$, the resistance is

$$R = \frac{\rho}{t}. \tag{1.20}$$

Note the resistance is determined by the thickness and not the size of the square. The ohms-per-square for 1 mm thick copper at dc is 172 μΩ. For plane conductors that are not square, the resistance can be found by dividing the area into a number of squares and then using a series–parallel calculation. The resistance rises with frequency as magnetic effects limit the depth of current penetration. See the discussion of skin effect in Section 3.23.

When current does not use an entire conductor, the effective resistance rises. This is the situation at a point contact such as at a lightning strike or at a fault contact. For a copper bus bar, the contact at the ends must involve the entire surface or the end resistances will be high. In high-current applications, this can lead to overheating.