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WHAT IS FUZZY MODELING

1.1 INDETERMINACY IN HUMAN LIFE

Fuzzy modeling is a group of special mathematical methods that make it possible to include in the model imprecise or vaguely formulated expert information that is often characterized using natural language. The developed models (we call them fuzzy models) are very successful because they provide solution in situations when traditional mathematical models fail—either due to their non-adequacy, or due to their inability to utilize the full available information.

Note that the idea to include imprecise information in our models contradicts to what has always been required: as high precision as possible. There is, however, a good reason for doing it, namely, we face a discrepancy between relevance and precision. The so-called *principle of incompatibility* formulated by L. A. Zadeh in [149] says the following:

> As a complexity of system increases, our ability to make absolute, precise, and significant statements about the system's behavior diminishes. At some moment, there will be trade-off between precision and relevance. Increase in precision can be gained only through decrease in relevance; increase in relevance can be gained only through the decrease in precision.

For example, from the description of an enterprise in several sentences, we may learn about its main activity, size, total number of its employees, its business successes, and problems. But we will know nothing about individual people, specific machines, and their parts. To describe everything in detail, we would need much

Insight into Fuzzy Modeling, First Edition. Vilém Novák, Irina Perfilieva, and Antonín Dvořák.

Companion Website: www.wiley.com/go/novak/fuzzy/modeling

more sentences, numbers, tables, and so on. But then the amount of information exponentially increases. We would thus learn more, but any detail would concern only a small part of the enterprise. The requirement to describe the whole enterprise in full detail would lead to a big pile of thick books that, however, nobody would be able to read. And if yes, to understand the content, he/she would need natural language, which means that he/she would have to return to imprecise characterization. Otherwise, he/she would be lost in the abundance of irrelevant details.

We can see that to express relevant information, we need natural language. This is the only and very accomplished tool that enable us to work effectively with vague concepts.

Is full precision achievable? We argue that full precision is only our illusion and is not achievable, even in principle. Otherwise, we could obtain the same result independently on the chosen precision. But this is, in general, impossible. For example, let us compare two containers according to their volume. If their volume is absolutely the same, then we obtain the same number independently if we measure in m^3, mm^3, or in arbitrary fractions such as billionths, quadrillionths, 10^{-120} of m^3, and so on. But this is impossible because at the level of atoms or even elementary particles, we would not be able to distinguish which of the latter belongs to the body of the container and which does not. We conclude that the struggle for limit precision brings us to contradiction.

Let us emphasize that vagueness is inseparable feature of the semantics of natural language. We argue that it is not its weakness but its strength. Natural language is used in almost any human activity. For example, if we want to learn driving a car, we need a teacher who explains us—in natural language—what should we do, for example, “slow down a little”, “now accelerate but not too much”, and so on. Though such commands are vague, they are sufficient for us to be able to learn driving.

The main theories applied in fuzzy modeling are (mathematical) fuzzy logic and the fuzzy set theory. When facing vagueness, we may ask why we speak about fuzzy sets and fuzzy logic and do not consider techniques of probability and statistics?

The probability theory provides a mathematical model of uncertainty that is met when considering an event that has not yet occurred and we do not know whether it will indeed occur or not. Such an event can be, for example, a result of an experiment we are going to realize. Uncertainty is thus a lack of information about occurrence of some event.[1]

The basic concept in probability theory is a *probability distribution*. This gives us information about occurrence of events from more to less likely ones. Further important concept is *independence of events*. If they are independent, then the probability of their simultaneous occurrence is equal to *product* of their respective probabilities.

On the other hand, let us consider, for example, a cupboard full of red dresses. Then to answer whether the given dress is “red” requires to characterize *truth* of the statement “the color of the given dress is red”. This cannot be probability because to be red color is a property, not an occurring event. Moreover, the class of all wave lengths representing red color cannot be a set because we are not able to specify precisely the borderline between “redness” and “non-redness”.

[1]This holds also in the case when we know that some event has occurred but we do not know which one.

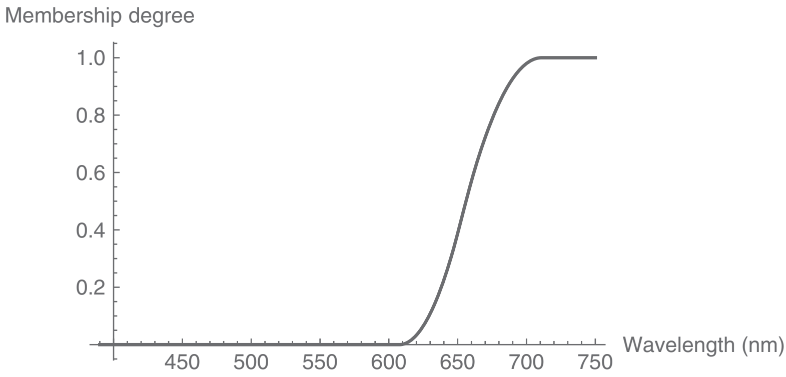

Figure 1.1 Fuzzy set modeling the meaning of "red color".

We can model the meaning of "red" using the concept of a fuzzy set. A fuzzy set A is a function

$$A : U \longrightarrow [0, 1],$$

where U is a set called *universe*. Each element $x \in U$ is assigned a membership degree $A(x) \in [0, 1]$ which is a *truth value*[2] of the proposition saying that $x \in A$. The value $A(x) = 1$ means that x *belongs* to A ($x \in A$). The value $A(x) = 0$ means that x *does not belong* to A ($x \notin A$). All other values mean only partial belonging to the fuzzy set A. To stress that A is a fuzzy set on U, we often write $A \underset{\sim}{\subset} U$.

If now we want to model what does it mean "red", we first define the universe of wave lengths that cover visible spectrum of light. People are able to see wave lengths from the interval $[390, 750]$ nm. Then "red" can be modeled by a fuzzy set $R \underset{\sim}{\subset} [390, 750]$ depicted in Figure 1.1. This means that light of wave length shorter than 600 nm is not red at all. Then the degree of "redness" increases with the increase in wavelengths up to full redness.

Of course, one may ask what is the probability of taking a red dress out of the cupboard. In this case, we face a combination of uncertainty and vagueness because the considered event is vaguely specified. We can thus summarize that there is a general concept called *indeterminacy*.[3] It has at least two distinguished facets: *vagueness* and *uncertainty*. Vagueness can be mathematically modeled using the fuzzy set theory, while uncertainty is mathematically modeled using the probability theory.[4] Of course, in reality, we often face both these facets together. For example, we can ask: "What is the probability that a tall man will come to our party?"[5]

Let us emphasize that indeterminacy cannot be removed. On one hand, it turns out that laws of nature inherently include uncertainty and it is not possible even in

[2]The interval $[0, 1]$ of truth values can be replaced by any other ordered set having the greatest and the smallest elements.

[3]One can also find this concept under the name "uncertainty in the wider sense".

[4]Note that there are also other theories of uncertainty, for example, possibility or belief theory.

[5]We can paraphrase this discussion as follows: the probability theory gives us an answer to the question "what event will occur?" and the fuzzy set theory gives us an answer to the question "what is the occurring event?"

principle to know all aspects causing occurrence of some event. On the other hand, vagueness is related to our way of regarding the world around us and its properties.

We argue that the presence of vagueness is the only way to familiarize with a new situation, or to communicate. Imagine, for example, that when parking a car, we would have at disposal instructions such as "turn the steering wheel by $19°25'32''$ to the left and move by 368.1256 mm back". Following such instructions would require great effort to make sufficiently precise measurements and to move accordingly. However, this would, in fact, be wasting of time because in practice, we do not need so precise parking position. It is sufficient to follow only vague instructions such as "turn the steering wheel a little to the left and move slightly back". Finally, note that we always face imprecision even when high precision is required, for example, when programming precise manipulating robots; the difference is only in the considered scale, that is, "small" could mean, for example, values around 1.3 mm or less.

1.2 FUZZY MODELING: WITH AND WITHOUT WORDS

The attempt to utilize the imprecise information in mathematical models led to the development of fuzzy modeling techniques. Recall that mathematical models manipulate with variables. In traditional models, values of the considered variable are taken from some set of numbers called a universe. Traditional mathematical models manipulate directly with its elements. In a fuzzy model, however, variables may represent fuzzy subsets of the universe. Hence, fuzzy models require partitioning of the universe into parts, for which it is specific that they need not be precisely formed and can overlap.

One of the very important modeling methods is *cluster analysis*. Its idea is the following: for a given set V of some elements, find its partition into c sets of subsets $R_j \subset V$, $j = 1, \ldots, c$, called *clusters*, in such a way that if two objects x, y belong to the same cluster R_j, then they are similar, while if they belong to different clusters, then they are not similar. For example, sizes of shoes represent subsets of lengths of human feet; the length of feet of people having, for example, size 6 is between 241 and 250 mm, for size 7 it is between 251 and 259 mm, and so on.

The classical cluster analysis provides partitioning into disjoint clusters, that is, we require that

$$U = R_1 \cup \cdots \cup R_c \quad \text{and} \quad R_i \cap R_j = \emptyset \quad \text{if } i \neq j.$$

This is often not realistic, because, as everybody knows, people often fit to more than one size of shoes. To cope with problems like this, we need generalization of the classical cluster analysis to the fuzzy one where the crisp clusters are replaced by fuzzy (possibly overlapping) ones. The fuzzy cluster analysis is described in Chapter 6.

The most important tool in fuzzy modeling are *fuzzy IF-THEN rules*. These are special expressions, which characterize relations among parts of two or more universes. For example, let us consider an electric boiler and two universes: values of electric current (A) and temperature (°C). Then the following is a typical fuzzy IF-THEN rule:

$$\mathscr{R}: \ \mathsf{IF} \text{ electric current is very strong } \mathsf{THEN} \text{ temperature is high} \tag{1.1}$$

By this rule, part of the universe of values of electric current characterized by the expression "very strong" is related to the part of the universe of degrees characterized as "high temperature". In practice, we usually have more such rules at disposal. A set $\mathscr{R}_1, \ldots, \mathscr{R}_m$ of rules (1.1) is called a *linguistic description*.

The reader has certainly met rules of this form already, namely, in programming languages. These are, however, *crisp* rules that do not allow any imprecision. On the other hand, IF-THEN rules used by people are almost always vague. The reason is that they contain vague natural language expressions that are central for human thinking. In this book, we will describe the way how semantics of certain class of natural language expressions can be modeled mathematically. The possibility to grasp the meaning of IF-THEN rules in the form close to human thinking should be considered as a great scientific accomplishment.

We manipulate with fuzzy IF-THEN rules by means of a scheme that reminds classical modus ponens rule from logic. Therefore, it is called *generalized modus ponens*:

Condition:	IF error is *big* AND change of error is *small* THEN control action is *very big*
Observation:	error is *roughly big* AND change of error is *small*
Conclusion:	control action is *big*

This means that if we know both the condition and the observation, we can deduce what we should do. For example, we deduce whether we should brake or turn a regulator cock or do something else. The tools of fuzzy modeling described in this book enable us to transform a linguistic description into an algorithm whose result is an action.

It is surprising how strong is the application potential of fuzzy IF-THEN rules (which are, in fact, quite simple expressions). The applications started in process control, but it is possible to describe by means of them very wide class of decision-making problems carried out by people. It starts from common events (crossing the street, dressing) and leads to important decisions requiring expert knowledge, for example, in medicine, management, and technology.

The rules of the form (1.1) resemble sentences of natural language. In this book, we will describe two ways how such rules can be interpreted:

(i) relational interpretation,
(ii) linguistic interpretation.

In case (i), we take an IF-THEN rule $\mathscr{R}$ in (1.1) as a rough characterization of some dependence. The linguistic expressions occurring in the rule are, in fact, taken only as names (codes) of some fuzzy sets. The rule as well as the whole linguistic description are interpreted by a fuzzy relation, which is a fuzzy set in a Cartesian product of several universes. The role of natural language is here auxiliary and no model of its semantics is considered. On the other hand, the methods developed on the basis of this interpretation of fuzzy IF-THEN rules provide well-substantiated tools for approximation of continuous functions. The relational interpretation and elaboration of fuzzy IF-THEN rules are in detail described in Chapter 3.

In case (ii), we take an IF-THEN rule $\mathscr{R}$ in (1.1) as a conditional clause formulated in natural language and the linguistic description is construed as a special text characterizing the decision situation. This interpretation is related to a paradigm of *computing with words and perceptions*, which was proposed by Zadeh in [152] (cf. also [153]). This is a methodology, in which objects of computation are words and propositions drawn from a natural language. Several publications appeared since then (e.g., [144, 154], and other ones).

To stress that fuzzy IF-THEN rules are taken as conditional linguistic clauses, we will call them *fuzzy/linguistic* IF-THEN rules. The methods for their elaboration belong among mathematical tools developed in the so-called *fuzzy natural logic* (FNL). This is a class of mathematical theories with the goal to model natural human thinking that is characterized by the use of natural language. The central position in FNL is played by the theory of *evaluative linguistic expressions*, that is, the theory of the semantics of linguistic expressions such as "nice", "very deep", "more or less strong", and "extremely quick". Such expressions occur in the rules $\mathscr{R}$ in (1.1), which are then used by people in various situations when it is necessary to make a decision, to evaluate some product (e.g., very good, strong, not safe), and in various other occasions.

As mentioned, the *linguistic description* is understood as a special text characterizing, for example, a sophisticated control strategy, decision-making, or behavior of a complex system. Applying a special reasoning method (called *perception-based logical deduction*), we can form models that effectively utilize expert knowledge and mimic the way how people behave when facing complicated decision situations. Interpretation and elaboration of fuzzy/linguistic IF-THEN rules, linguistic descriptions, and reasoning on the basis of them are described in detail in Chapter 5.

A special and very effective method of fuzzy modeling is *fuzzy transform* (*F-transform*). This method gains still more attention for its fascinating applications in diverse areas.

The basic concept of the F-transform is that of a *fuzzy partition* of an interval of real numbers $[a, b] \subset \mathbb{R}$. This is a finite set of fuzzy sets $A_k \subsetneq [a, b]$, $k = 0, \dots, n$ that fulfill special conditions. Using a fuzzy partition, a real continuous function $f : [a, b] \longrightarrow [c, d]$ is transformed into a finite vector $[F_0[f], \dots, [F_n[f]]$ of *components*. This procedure is called a *direct phase*. Then (possibly after some computations) we can transform the vector of components back to a space of continuous functions. The result is a function $\hat{f} : [a, b] \longrightarrow [c, d]$, which *approximates* the original function f. This procedure is called an *inverse phase*. Parameters of the F-transform can be set in such a way that $\hat{f}$ has desired interesting properties. This opens the door to various kinds of applications. In this book, we will describe applications of the F-transform in image processing and in analysis and forecasting of time series. It can also be applied in numerical solution of differential equations, signal processing, data mining, and elsewhere. The fuzzy transform is described in detail in Chapter 4.

There are many applications of fuzzy modeling. The most distinguished ones are in control—we speak about *fuzzy control*. There are several reasons for applying fuzzy control in practice. One of them is the relative ease of its design. For example, when Hubble telescope had to be repaired, the automatic arm that took the telescope in and out of the spaceship was controlled using fuzzy controller. Its development took about 2 weeks. The same control developed in parallel using classical proportional-integral-derivative (PID) controller was in 2 weeks far from being finished. In fact, the complexity of design of fuzzy controller depends very little on the complexity of the controlled process.

Surprisingly, there are even processes whose satisfactory automatic control can be realized using fuzzy controller only. A typical example is the control of purification process in the sewage treatment plant. One can hardly find a mathematical model on the basis of which classical control can be designed. Therefore, this facility is usually controlled by people who apply their practical experience. We can express the latter using rules such as (1.1) and, consequently, utilize it using fuzzy modeling techniques.

A very important property of fuzzy models is their *robustness*. This means that they are little sensitive to external disturbances. For example, one of the very successful applications of fuzzy control based on fuzzy/linguistic IF-THEN rules is control of five aluminum smelting furnaces in Al Invest company in a small village Břidličná in the Czech Republic. The control is subject to large disturbances caused by repeated opening, recharging, charging, and closing of the smelting furnace. However, the fuzzy control works without disruption already for many years (more details about this application are presented in Chapter 7). Thanks to all these properties, fuzzy control and other fuzzy modeling techniques have become standard tools used by companies in their production now. We can find fuzzy controllers in automatic washing machines, dishwashers, cars, and other products.

It should be emphasized that robustness is a typical feature not only of fuzzy control but also of applications of fuzzy modeling in general. For example, when applying fuzzy modeling methods to character recognition, we need only few patterns (see [90]), while in classical solutions, we need hundreds of them. In applications of the F-transform, robustness manifests itself on little dependence of the result on the choice of the initial conditions. For example, when solving differential equations, the result is similar to the classical regularization method. This means that the result is almost independent on the choice of initial values (cf. [57, 108, 117]).

The range of applications of fuzzy modeling methods is very wide because they enable us to work with imprecise information that is often available in the form of sentences of natural language only. Of course, this does not mean that classical methods should be abandoned. On the contrary, they should be combined with methods of fuzzy modeling in such a way that each of the methods is used in a situation when it can lead to the best result. Namely, classical methods are convenient if precise mathematical model of our problem can be found. On the other hand, if the available information is imprecise and we need robustness, then methods based on fuzzy modeling should be preferred.