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CHAPTER 1

BASIC CONCEPTS

1.1 INTRODUCTION

The twentieth century began with the electric power industry in its infancy; Thomas Edison and Nikola Tesla were locked in battle with Edison advocating direct current (dc) and Tesla alternating current (ac). The century ended with the electric power industry expanding rapidly from the traditional power generation, transmission, and utilization into propulsion of air, ground, and sea transportation. The advent of the computer and the silicon-controlled rectifier in the mid-1900s brought about an expansion of the power area to include the smart grid, microgrids, efficient and robust electric drives, more-electric aircraft, ships, and land vehicles. This growth is likely to continue into the foreseeable future.

Before the advent of the computer, engineers were essentially limited to steady-state analysis and therefore unable to conveniently deal with the analytical challenges of the expanding power industry. This chapter sets forth some of the basic concepts and analysis tools that are part of the present-day power and electric drives area. Although not inclusive, the material covered in this chapter is representative and common to all disciplines of the power area.

1.2 PHASOR ANALYSIS AND POWER CALCULATIONS

Since the early twentieth century, we have lived in an alternating current (ac) world. Thanks to George Westinghouse and Nikola Tesla, power systems are predominately ac; power is generated by large ac generators, transmitted by high voltage transmission lines, and transformed to a low voltage and distributed to homes and factories. The evolution of the ac power system brought about many engineering challenges and, as we look back, it is difficult to comprehend how these problems were solved without a computer. Even steady-state ac-circuit analysis posed a problem until the early 1900s when Charles Stienmetz, who was a less flamboyant colleague of Edison and Tesla, came up with the concept of what is now known as phasors. Some may consider the phasor a casualty of the computer age along with the slide rule. It is, however, still a very useful means for understanding and portraying the steady-state performance of electric machines, power systems, and electric drives. Moreover, the

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phasor concept provides a means of visualizing sinusoidal variations from different frames of reference and in Chapter 2 we will find that the voltage and current phasors combined with Tesla's rotating magnetic field provides a straightforward means of analyzing and portraying the steady-state operation of ac machines.

The phasor can be established by expressing a steady-state sinusoidal variable as

$$F_a(t) = F_p \cos \theta_{ef} \tag{1.2-1}$$

where the a subscript is used here to denote sinusoidal quantities. The sinusoidal variations are expressed as cosines, capital letters are used to denote steady-state quantities, and F_p is the peak value of the sinusoidal variation. Generally, F or f represents voltage (V or v) or current (I or i) in circuit analysis, but it could be any sinusoidal variable. For steady-state conditions, θ_{ef} may be written as

$$\theta_{ef}(t) = \omega_e t + \theta_{ef}(0) \tag{1.2-2}$$

where ω_e is the electrical angular velocity in radians/second (2π times the frequency) and $\theta_{ef}(0)$ is the time-zero position of the electrical variable. Substituting (1.2-2) into (1.2-1) yields

$$F_a(t) = F_p \cos[\omega_e t + \theta_{ef}(0)] \tag{1.2-3}$$

Now, Euler's Identity is

$$e^{j\alpha} = \cos\alpha + j\sin\alpha \tag{1.2-4}$$

and since we are expressing the sinusoidal variation as a cosine, (1.2-3) may be written as

$$F_a(t) = \mathrm{Re}\left\{F_p e^{j[\omega_e t + \theta_{ef}(0)]}\right\} \tag{1.2-5}$$

where Re is the shorthand for "real part of." Equations (1.2-3) and (1.2-5) are equivalent. Let us rewrite (1.2-5) as

$$F_a(t) = \mathrm{Re}\left\{F_p e^{j\theta_{ef}(0)} e^{j\omega_e t}\right\} \tag{1.2-6}$$

We need to take a moment to define what is referred to as the root-mean-square (rms) of a sinusoidal variation. In particular, the rms value is defined as

$$F = \left(\frac{1}{T}\int_0^T F_a^2(t)dt\right)^{\frac{1}{2}} \tag{1.2-7}$$

where F is the rms value of $F_a(t)$ and T is the period of the sinusoidal variation. It is left to the reader to show that the rms value of (1.2-3) is $F_p/\sqrt{2}$. Therefore, we can express (1.2-6) as

$$F_a(t) = \mathrm{Re}[\sqrt{2}Fe^{j\theta_{ef}(0)}e^{j\omega_e t}] \tag{1.2-8}$$

By definition, the phasor representing $F_a(t)$, which is denoted with a raised tilde, is

$$\tilde{F}_a = Fe^{j\theta_{ef}(0)} \tag{1.2-9}$$

which is a complex number. The reason for using the rms value as the magnitude of the phasor will be addressed later in this section. Equation (1.2-6) may now be written as

$$F_a(t) = \text{Re}\left[\sqrt{2}\tilde{F}_a e^{j\omega_e t}\right] \tag{1.2-10}$$

A shorthand notation for (1.2-9) is

$$\tilde{F}_a = F\,\underline{/\theta_{ef}(0)} \tag{1.2-11}$$

Equation (1.2-11) is commonly referred to as the *polar form* of the phasor. The *Cartesian* form is

$$\tilde{F}_a = F\cos\theta_{ef}(0) + jF\sin\theta_{ef}(0) \tag{1.2-12}$$

When using phasors to calculate steady-state voltages and currents, we think of the phasors as being stationary at $t = 0$; however, we know that a phasor is related to the instantaneous value of the sinusoidal quantity it represents. Let us take a moment to consider this aspect of the phasor and thereby, give some physical meaning to it. From (1.2-4), we realize that $e^{j\omega_e t}$ is a line of unity length rotating counterclockwise at an angular velocity of ω_e. Therefore, backing up for a minute

$$\sqrt{2}\,\tilde{F}_a e^{j\omega_e t} = \sqrt{2}F e^{j\theta_{ef}(0)} e^{j\omega_e t} \tag{1.2-13}$$

is a line with a constant amplitude of $\sqrt{2}F$ rotating counterclockwise in the real-imaginary plane at an angular velocity of ω_e with a time-zero displacement from the positive real axis of $\theta_{ef}(0)$. Since $\sqrt{2}F$ is the peak value of the sinusoidal variation, the instantaneous value of $F_a(t)$ expressed as a cosine is the real part of (1.2-13). In other words, the real projection of the phasor $\tilde{F}_a$ rotating counterclockwise at ω_e is the instantaneous value of $F_a(t)/\sqrt{2}$. Thus, with $\theta_{ef}(0) = 0$ in (1.2-3)

$$F_a(t) = \sqrt{2}F\cos\omega_e t \tag{1.2-14}$$

the phasor representing (1.2-14) is

$$\tilde{F}_a = Fe^{j0} = F\,\underline{/0^\circ} = F + j0 \tag{1.2-15}$$

For

$$\begin{aligned} F_a(t) &= \sqrt{2}F\sin\omega_e t \\ &= \sqrt{2}F\cos(\omega_e t - 90^\circ) \end{aligned} \tag{1.2-16}$$

the phasor is

$$\tilde{F}_a = Fe^{-j\pi/2} = F\,\underline{/-90^\circ} = 0 - jF \tag{1.2-17}$$

We will use degrees and radians interchangeably when expressing phasors. Although there are several ways to arrive at (1.2-17) from (1.2-16), it is helpful to ask yourself where the rotating phasor must be positioned at time-zero so that, when it rotates counterclockwise at ω_e, its real projection is $(1/\sqrt{2})F_p \sin\omega_e t$. It follows that a phasor of amplitude F positioned at 90° represents $-\sqrt{2}F\sin\omega_e t$.

In other words, we are viewing a sinusoidal variation as the real projection in the real-imaginary plane of a rotating line equal in magnitude to the positive peak value ($\sqrt{2}F$) of the variation and rotating at the electrical angular velocity of the sinusoidal variation. Since we are dealing with a steady-state variation, we can stop the rotation at any time and view it as a fixed line, but knowing full well that it, in fact, represents a sinusoidal variation and to represent the sinusoidal variation we must rotate it counterclockwise at ω_e and take the real projection. Please understand that if we ran at ω_e in unison with the rotating $\sqrt{2}F$ line it would appear as a constant to us; therefore, in viewing a sinusoidal variation in this manner it would appear to us as a constant. This is no different than stopping the phasor at some arbitrary time-zero; but realizing that it actually represents a sinusoidal variation. We will talk more about this important aspect as we go along; in particular, see Example 1A.

In order to show the facility of the phasor in the analysis of steady-state performance of ac circuits and devices, it is useful to consider a series circuit consisting of a resistance, an inductance L, and a capacitance C. Thus, using uppercase letters to indicate steady-state variables

$$V_a = RI_a + L\frac{dI_a}{dt} + \frac{1}{C}\int I_a dt \tag{1.2-18}$$

Throughout the text, we will use either R or r to represent resistance. For steady-state operation, let

$$V_a = \sqrt{2}V\cos[\omega_e t + \theta_{ev}(0)] \tag{1.2-19}$$

$$I_a = \sqrt{2}I\cos[\omega_e t + \theta_{ei}(0)] \tag{1.2-20}$$

where we have dropped the functional notation and the subscript a helps to distinguish the instantaneous value from the rms value of the steady-state variables. The steady-state voltage equation may be obtained by substituting (1.2-19) and (1.2-20) into (1.2-18), whereupon we can write

$$\begin{aligned}\sqrt{2}V\cos[\omega_e t + \theta_{ev}(0)] &= R\sqrt{2}I\cos\left[\omega_e t + \theta_{ei}(0)\right]\\ &\quad + \omega_e L\sqrt{2}I\cos\left[\omega_e t + \tfrac{1}{2}\pi + \theta_{ei}(0)\right]\\ &\quad + \frac{1}{\omega_e C}\sqrt{2}I\cos\left[\omega_e t - \tfrac{1}{2}\pi + \theta_{ei}(0)\right]\end{aligned} \tag{1.2-21}$$

The second term on the right-hand side of (1.2-21), which is $L\frac{dI_a}{dt}$, can be written

$$\omega_e L\sqrt{2}I\cos\left[\omega_e t + \tfrac{1}{2}\pi + \theta_{ei}(0)\right] = \omega_e L\mathrm{Re}\left[\sqrt{2}Ie^{j\frac{1}{2}\pi}e^{j\theta_{ei}(0)}e^{j\omega_e t}\right] \tag{1.2-22}$$

Since $\tilde{I}_a = Ie^{j\theta_{ei}(0)}$, from (1.2-21), we can write

$$L\frac{dI_a}{dt} = \omega_e Le^{j\frac{1}{2}\pi}\tilde{I}_a \tag{1.2-23}$$

Since $e^{j\frac{1}{2}\pi} = j$, (1.2-23) may be written

$$L\frac{dI_a}{dt} = j\omega_e L\tilde{I}_a \tag{1.2-24}$$

If we follow a similar procedure, we can show that

$$\frac{1}{C}\int I_a dt = -j\frac{1}{\omega_e C}\tilde{I}_a \tag{1.2-25}$$

Differentiation of a steady-state sinusoidal variable rotates the phasor counter-clockwise by $\frac{1}{2}\pi$ or j; integration rotates the phasor clockwise by $\frac{1}{2}\pi$ or $-j$.

The steady-state voltage equation given by (1.2-21) can now be written in phasor form as

$$\tilde{V}_a = \left[R + j(\omega_e L - \frac{1}{\omega_e C})\right]\tilde{I}_a \tag{1.2-26}$$

We can express (1.2-26) compactly as

$$\tilde{V}_a = Z\tilde{I}_a \tag{1.2-27}$$

where the impedance, Z, is a complex number; it is not a phasor. It may be expressed as

$$Z = R + j(X_L - X_C) \tag{1.2-28}$$

where $X_L = \omega_e L$ is the inductive reactance and $X_C = \frac{1}{\omega_e C}$ is the capacitive reactance. We should be careful here. Some prefer to write (1.2-28) as $R + jX$ where X is $X_L + X_C$ and let X_C be negative. This is essentially a matter of choice and does not change the end result. We will deal primarily with X_L and not X_C, therefore, this will have little impact on our work; nevertheless, since some authors will use a negative X_C we should make the reader aware of this difference.

It is appropriate to discuss the notation that will be used throughout the text. When an equation is written with the variables in lowercase letters it is valid for transient and steady state. If the variables are written with uppercase letters as in (1.2-18) the equation is a function of time and valid for instantaneous steady-state conditions. Equations (1.2-26) and (1.2-27) are phasor equations representing steady-state sinusoidal variables and are written in uppercase letters with an over tilde.

Power and Reactive Power

The instantaneous steady-state power is

$$\begin{aligned} P &= V_a I_a \\ &= \sqrt{2}V\cos\left[\omega_e t + \theta_{ev}(0)\right]\sqrt{2}I\cos\left[\omega_e t + \theta_{ei}(0)\right] \end{aligned} \tag{1.2-29}$$

where V and I are rms values. After some manipulation, we can write (1.2-29) as

$$P = VI\cos[\theta_{ev}(0) - \theta_{ei}(0)] + VI\cos[2\omega_e t + \theta_{ev}(0) + \theta_{ei}(0)] \tag{1.2-30}$$

The instantaneous steady-state power given by (1.2-30) varies about an average value at a frequency of $2\omega_e$. That is, the second term of (1.2-30) has a zero average value and the average power P_{ave} may be written

$$P_{\text{ave}} = |\tilde{V}_a|\,|\tilde{I}_a| \cos[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1.2-31}$$

where $|\tilde{V}_a|$ and $|\tilde{I}_a|$ are V and I, respectively which are the magnitudes of the phasors (rms value); $\theta_{ev}(0) - \theta_{ei}(0)$ is referred to as the *power factor angle* φ_{pf}, and $\cos[\theta_{ev}(0) - \theta_{ei}(0)]$ is the *power factor*. Power is in watts. If current is assumed positive in the direction of voltage drop then (1.2-31) is positive if power is consumed and negative if power is generated. It is interesting to point out that in going from (1.2-29) to (1.2-30), the coefficient of the two right-hand terms is $\frac{1}{2}(\sqrt{2}V\sqrt{2}I)$ or one-half the product of the peak values of the sinusoidal variables. Therefore, it was considered more convenient to use the rms values for the phasors, whereupon average steady-state power could be calculated by the product of the magnitude of the voltage and current phasors as given by (1.2-31).

The reactive power is defined as

$$Q = |\tilde{V}_a|\,|\tilde{I}_a| \sin[\theta_{ev}(0) - \theta_{ei}(0)] \tag{1.2-32}$$

The unit of Q is var (volt-ampere reactive). An inductance is said to absorb reactive power where the current lags the voltage by 90° and Q is positive and supplied by a capacitor where the current leads the voltage by 90° and Q is negative. Actually, Q is a measure of the interchange of energy stored in the electric (capacitor) and magnetic (inductor) fields. However, unlike instantaneous real power, the average value of instantaneous reactive power is zero. We will talk more about reactive power when we get to Chapter 5.

Example 1A. Phasor analysis. The parameters of a series RLC circuit are $R = 6\ \Omega$, $L = 20\,\text{mH}$, $C = 1 \times 10^3\ \mu\text{F}$. The 60 Hz applied voltage is $V_a = 155.6 \cos \omega_e t$. Calculate $\tilde{I}_a$, P_{ave}, Q and draw the phasor diagram and the sinusoidal variations as viewed running at counterclockwise with the phasor representing maximum V_a ($\sqrt{2}V_s\underline{/0°}$). From the expression of V_a

$$\tilde{V}_a = 110\underline{/0°}\ \text{V} \tag{1A-1}$$

Now, $\omega_e = 2\pi f = 2\pi \times 60 = 377\,\text{rad/s}$ and

$$\begin{aligned} Z &= R + j(X_L - X_C) \\ &= R + j\left(\omega_e L - \frac{1}{\omega_e C}\right) \\ &= 6 + j\left(377 \times 20 \times 10^{-3} - \frac{1}{377 \times 1 \times 10^{-3}}\right) = 7.73\underline{/39.1°}\ \Omega \end{aligned} \tag{1A-2}$$

$$\tilde{I}_a = \frac{\tilde{V}_a}{Z} = \frac{110\underline{/0°}}{7.73\underline{/39.1°}} = 14.2\underline{/-39.1°}\ \text{A} \tag{1A-3}$$

$$P_{\text{ave}} = |\tilde{V}_a|\,|\tilde{I}_a| \cos \phi_{pf} \tag{1A-4}$$

where

$$\begin{aligned}\phi_{pf} &= \theta_{ev}(0) - \theta_{ei}(0)\\ &= 0 - (-39.1°) = 39.1° \qquad (1A\text{-}5)\\ P_{\text{ave}} &= 110 \times 14.2 \cos 39.1°\\ &= 1212.2\,\text{W} \qquad (1A\text{-}6)\\ Q &= \left|\tilde{V}_a\right| \left|\tilde{I}_a\right| \sin \phi_{pf}\\ &= 110 \times 14.2 \sin 39.1° = 985.1 \text{ vars} \qquad (1A\text{-}7)\end{aligned}$$

The phasor diagram is shown in Fig. 1A-1.

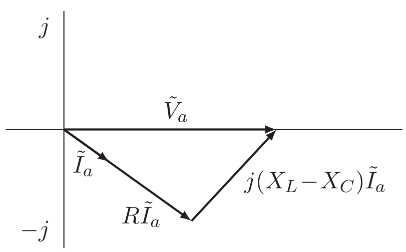

Figure 1A-1 Phasor diagram.

Fig. 1A-2 shows the "stationary" waveforms of the voltages viewed with 360° vision while running at ω_e with the counterclockwise rotating phasor representing V_a. The current I_a is not shown; however, $\sqrt{2}\left|\tilde{I}_a\right|$ would be in phase with the voltage $R\sqrt{2}\left|\tilde{I}_a\right|$. Also, the type of diagram shown in Fig. 1A-2, is not convenient to portray that the real part of the rotating phasor is the instantaneous value of the sinusoidal variation. Rotating the phasor diagram given in Fig. 1A-1 best illustrates that feature of a phasor.

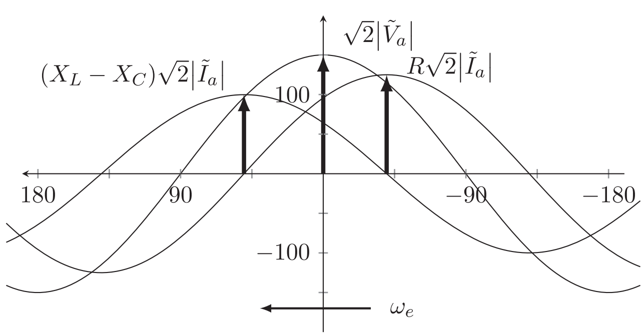

Figure 1A-2 Voltage waveforms of *RLC* circuit viewed with 360° vision while rotating with phasors at ω_e counterclockwise.

SP1.2-1. Express the instantaneous steady-state power for Example 1A. [Substitute into (1.2-30)]

SP1.2-2. Redraw the phasor diagram shown in Fig. 1A-1 showing $jX_L\tilde{I}_a$ and $-jX_C\tilde{I}_a$ as individual voltages. [Show $jX_L\tilde{I}_a$ and then from the terminus of $jX_L\tilde{I}_a$ show $-jX_C\tilde{I}_a$]

SP1.2-3. We know that $P_{\text{ave}} = |\tilde{I}|^2 R$, does Q equal $|\tilde{I}_a|^2 X_L - |\tilde{I}_a|^2 X_C$? [Yes]

SP1.2-4. If $\tilde{V} = 1\underline{/0^\circ}$ V and $\tilde{I} = 1\underline{/180^\circ}$ A in the direction of the voltage drop, calculate Z and P_{ave}. Is power generated or consumed? [$(-1 + j0)$ ohms, 1 watt, generated]

SP1.2-5. Express the instantaneous power for 60 Hz voltage, $\tilde{V}_a = 1\underline{/0^\circ}$ V, applied to a resistive circuit, $\tilde{I}_a = 1\underline{/0^\circ}$ A. [$1 + \cos 754t$]

SP1.2-6. Repeat SP1.2-5 for (a) an inductance, $\tilde{I}_a = I_L\underline{/-90^\circ}$ A and (b) a capacitance, $\tilde{I}_a = I_C\underline{/90^\circ}$ A. [(a) $I_L\cos(754t - 90^\circ)$, (b) $I_C\cos(754t + 90^\circ)$]

1.3 ELEMENTARY MAGNETIC CIRCUITS

Electric machines and transformers, which are the backbone of the power industry, are electromagnetic systems. Therefore, magnetic circuits and magnetic coupling play a major role in power and drives systems and it is necessary to establish the principles of magnetic systems sufficiently to convey the basic operation of the electromagnetic devices considered in later chapters. We will attempt to do this without becoming too involved.

An elementary magnetic circuit is shown in Fig. 1.3-1. It consists of a ferromagnetic member (core) with a coil of wire of N turns wound on it and an air gap of length x. The ferromagnetic member could be iron, nickel, cobalt, or steel, for example. The voltage equation of the electric circuit may be written

$$v = ri + e \tag{1.3-1}$$

where r is the total resistance of the circuit, v is the source voltage, e is the voltage induced in the coil according to Faraday's Law, and i is the current flowing in the circuit. The current flowing through the coil causes a magnetomotive force (mmf), which produces flux in the magnetic circuit denoted as Φ_m and Φ_l in Fig. 1.3-1, much as an electromotive force (emf) or source voltage produces current in an electric circuit.

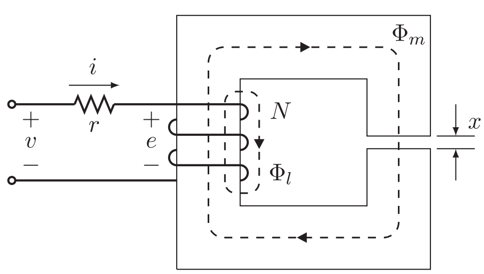

Figure 1.3-1 Elementary magnetic circuit with one air gap.

There are arrows associated with the dashed lines representing the flux paths in Fig. 1.3-1. These arrows indicate the assumed positive direction of flux which is determined from the assumed positive direction of current by the so called "right-hand" rule. If you grasp the coil with your right hand with your fingers in the assumed direction of positive current flow around the coil, your thumb will point in the direction of positive flux. Or, imagine grasping a turn of the coil with thumb in the assumed direction of positive current. If you ungrasp your fingers they will point in the direction of positive flux.

The total flux, Φ, that travels through (links) all of the turns N is

$$\Phi = \Phi_l + \Phi_m \tag{1.3-2}$$

where Φ_l is the equivalent flux that links all the turns of the coil but does not traverse the ferromagnetic member and Φ_m is referred to as the magnetizing flux that transverses the ferromagnetic member and links all of the turns of the coil. The leakage flux Φ_l is shown by one streamline in Fig. 1.3-1, it represents the aggregate of the flux that occurs around each wire of the coil, traveling partially in the ferromagnetic material and partially in air.

The concept of magnetic poles can be used to advantage to explain the operation of electromechanical devices. The poles can be established by considering the magnetic circuit shown in Fig. 1.3-1. In particular, to locate the north and south poles of an electromechanical device for the assumed positive direction of coil current, place yourself on the member that has the coil (windings). Use the right-hand rule to establish the direction of positive flux Φ_m for the assumed positive direction of coil current; where positive flux issues from the member with the coil, into the air is the assumed north pole. The south pole is where the positive flux returns from the air to the member with the coil. In the case of the magnetic circuit shown in Fig. 1.3-1, with the assumed direction of positive current a north pole exists over the upper face of the air gap and a south pole over the lower face.

A magnetically linear circuit behaves much as a resistive electric circuit. According to Ohm's Law, the current i in a resistive circuit is equal to the applied electromotive force (emf) or applied voltage divided by the resistance r. In the case of a magnetic circuit, the total flux Φ is equal to the magnetomotive force (mmf), which is in ampere turns Ni, divided by the equivalent reluctance $\mathcal{R}$ of the magnetic circuit. Thus, in (1.3-2)

$$\Phi_l = \frac{Ni}{\mathcal{R}_l} \tag{1.3-3}$$

$$\Phi_m = \frac{Ni}{\mathcal{R}_i + \mathcal{R}_g} \tag{1.3-4}$$

where $\mathcal{R}_l$ is the reluctance of the leakage flux path, $\mathcal{R}_i$ is the reluctance of the ferromagnetic member, and $\mathcal{R}_g$ is the reluctance of the air gap. Although the leakage reluctance is generally determined by test or an involved calculation, $\mathcal{R}_i$ and $\mathcal{R}_g$ may be calculated from

$$\mathcal{R} = \frac{\ell}{\mu_r \mu_0 A} \tag{1.3-5}$$

where ℓ is the length of the flux path, A is the cross sectional area of the flux path, μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ Wb/A · m or H/m), and μ_r is the permeability relative to free space. The unit of reluctance is (henry^{-1}) or H^{-1}. In the case of the air gap the relative permeability is considered to be unity ($\mu_{rg} \approx 1$), for the ferromagnetic member typical μ_{ri} values vary from 500 to 4000 depending upon the type of ferromagnetic material. It follows that the reluctance of the ferromagnetic member is much less than the reluctance of either the air gap or the part of the leakage path that is in air. In fact, the reluctance of the ferromagnetic member is generally neglected when an air gap is present as in the case of an electric machine. The magnetic equivalent circuit shown in Fig. 1.3-2 may be helpful to visualize the flux paths of the magnetic system shown in Fig. 1.3-1.

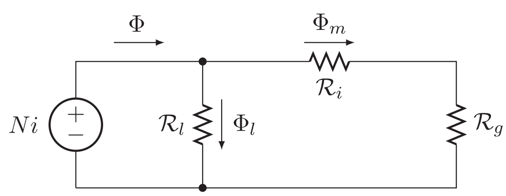

Figure 1.3-2 Magnetic equivalent circuit for magnetic system shown in Fig. 1.3-1.

Before proceeding, there are a couple of things that we should talk about. We have arrived at (1.3-3) and (1.3-4) by exploiting the similarities between a resistive electric circuit and a linear magnetic circuit. Although this approach is straight forward and easy to follow, let us take a minute to apply basic laws of magnetically linear systems that we learned in early physics courses to justify (1.3-3) and (1.3-4). Ampere's Law states that the line integral of the field intensity (field strength), H, about a closed path is equal to the net current enclosed within the closed path of integration. Now, for a two dimensional magnetically linear system

$$B = \mu H \tag{1.3-6}$$

where B is the flux density, μ is the permeability, and H is the field strength. If we assume the flux, Φ, is uniform over the cross-sectional surface area of the magnetic path then

$$B = \frac{\Phi}{A} \tag{1.3-7}$$

where A is the cross-sectional area. Thus, H may be expressed

$$H = \frac{\Phi}{A\mu} \tag{1.3-8}$$

Now, if H is integrated over the closed path that encloses the total turns of the coil and assuming H is the same over this path, then the magnetomotive force, mmf, or Ni is

$$Ni = \int_0^{\ell} \frac{\Phi}{A\mu} d\xi \tag{1.3-9}$$

and

$$Ni = \frac{\Phi}{A\mu}\ell \tag{1.3-10}$$

Thus Φ may be expressed

$$\begin{aligned}\Phi &= \frac{Ni}{\frac{\ell}{A\mu_r\mu_0}} \\ &= \frac{Ni}{\mathcal{R}}\end{aligned} \tag{1.3-11}$$

where we have substituted $\mu_r\mu_0$ for μ. We now understand the difficulty in determining the reluctance to the leakage flux, however, the reluctance of the iron and air gap can be calculated quite accurately for a magnetically linear system.

We now see that (1.3-3) is obtained by applying Ampere's Law about the leakage flux, Φ_ℓ, path and (1.3-4) is obtained by applying Ampere's Law about the magnetizing flux, Φ_m, path. In regard to (1.3-4), since for a magnetically linear system the reluctance of an air gap is much larger than the reluctance of the iron, the mmf drop across the iron is generally neglected and (1.3-4) would become

$$\Phi_m = \frac{Ni}{\mathcal{R}_g} \tag{1.3-12}$$

where the total mmf is dropped across the air gap which is commonly referred to as the air-gap mmf.

Let us modify Fig. 1.3-1 by cutting a second identical air gap as shown in Fig. 1.3-3. We will assume that the positions of the iron members are fixed. If now we neglect the reluctance of the iron, then one-half of the mmf is dropped across each air gap that is, each air-gap mmf is $\frac{Ni}{2}$.

The total flux linking all the turns of the coil may be expressed from (1.3-2) as

$$\begin{aligned}\lambda &= N\Phi \\ &= N(\Phi_l + \Phi_m)\end{aligned} \tag{1.3-13}$$

Substituting (1.3-3) and (1.3-4) into (1.3-13) yields the flux linkage as

$$\lambda = \left(\frac{N^2}{\mathcal{R}_l} + \frac{N^2}{\mathcal{R}_i + \mathcal{R}_g}\right) i \tag{1.3-14}$$

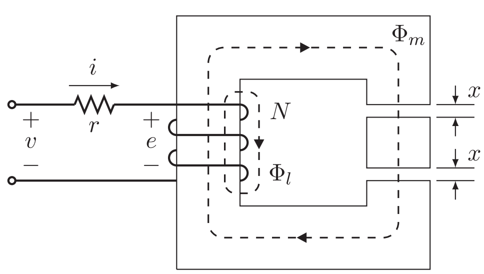

Figure 1.3-3 Elementary magnetic circuit with two air gaps.

It is now time to get back to the electric circuit. Faraday's Law tells us that the voltage induced in the coil, e in Fig. 1.3-1, may be expressed

$$e = \frac{d\lambda}{dt} \tag{1.3-15}$$

Note that the coil resistance has been lumped outside the coil in the circuit shown in Fig. 1.3-1. If we now substitute (1.3-14) into (1.3-15) and then substitute the result into (1.3-1) for e we obtain

$$v = ri + \frac{d}{dt}\left[\left(\frac{N^2}{\mathcal{R}_l} + \frac{N^2}{\mathcal{R}_i + 2\mathcal{R}_g}\right) i\right] \tag{1.3-16}$$

where $\mathcal{R}_g$ is the reluctance of one of the two identical air gaps. Since we are dealing exclusively with a magnetically linear system, the reluctances are assumed to be constant, therefore (1.3-16) becomes

$$v = ri + L\frac{di}{dt} \tag{1.3-17}$$

where L is the self-inductance which is

$$\begin{aligned} L &= \frac{N^2}{\mathcal{R}_l} + \frac{N^2}{\mathcal{R}_i + 2\mathcal{R}_g} \\ &= L_l + L_m \end{aligned} \tag{1.3-18}$$

where L_l is the leakage inductance and L_m is the magnetizing inductance. In particular,

$$L_l = \frac{N^2}{\mathcal{R}_l} \tag{1.3-19}$$

$$L_m = \frac{N^2}{\mathcal{R}_i + 2\mathcal{R}_g} \tag{1.3-20}$$

Since inductance (unit of H) is turns (no unit) squared divided by reluctance, the unit of reluctance is H^{-1}. Moreover, the use of an inductance in the analysis signifies that a magnetically linear system is being assumed.

Let us rewrite (1.3-14) as

$$\lambda = Li \tag{1.3-21}$$

Here we see a linear relationship between flux linkages and current. The λ versus i relationship shown in Fig. 1.3-4 is often referred to as the magnetization curve.

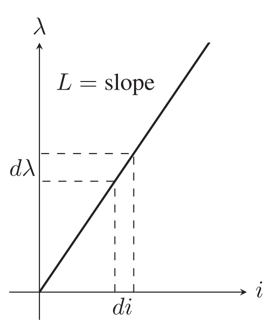

Figure 1.3-4 λi characteristic of a magnetically linear system.

Field Energy and Coenergy

Since power is the rate of energy transfer, the energy entering the magnetic field from the electrical system (Fig. 1.3-1) may be expressed

$$W_e = \int ei\,dt \tag{1.3-22}$$

As previously mentioned, the resistance of the coil is lumped external to the coil; therefore, W_e is the energy transferred only to the field of the magnetic circuit from the electrical system. Thus, the energy stored in the field which is referred to as the field energy is

$$\begin{aligned} W_f &= W_e \\ &= \int i\frac{d\lambda}{dt}dt = \int i\,d\lambda \\ &= \frac{1}{2}Li^2 \end{aligned} \tag{1.3-23}$$

Therefore, the energy stored in the field (W_f) for a given λ or i is the area to the left of the magnetization curve shown in Fig. 1.3-4. Also, since, for the magnetic system shown in Fig. 1.3-1, $L_\ell \ll L_m$ and since $\mathcal{R}_i \ll \mathcal{R}_g$, most of the field energy is stored in the air gap.

The area to the right of the magnetization curve is referred to as the coenergy; it is expressed as

$$W_c = \int \lambda\,di \tag{1.3-24}$$

It has no physical meaning; however, it is often used as a convenience in expressing the force and torque, since it is easier to work with λ in terms of i rather than i in terms of λ. Also, we see from Fig. 1.3-4, that

$$\lambda i = W_f + W_c \tag{1.3-25}$$

Clearly, $W_f = W_c$ for a magnetically linear system.

We will not take saturation into account in our analysis in this text. To do so would greatly complicate our work compared to treating all electromechanical devices as magnetically linear and little is sacrificed in portraying their salient operating characteristics. Nevertheless, magnetically nonlinear systems should be mentioned in passing. In this regard, Fig. 1.3-5 is a plot of the flux linkage λ versus the current i for a magnetically nonlinear system. The actual characteristic is dependent on the type of ferromagnetic material. The knee of the curve occurs due to the saturation of the ferromagnetic member. We will assume that the λ versus i characteristic is a single-valued function in that there is only one value of λ for a given value of i, whereupon hysteresis is neglected. When a ferromagnetic material is subjected to a magnetic field its magnetic characteristics tend to aid in producing flux. This continues at a near linear rate until this rate begins to decrease, causing the knee of the magnetization curve. As the strength of the applied field continues to increase

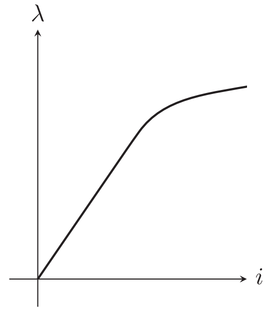

Figure 1.3-5 λ versus i for a magnetically nonlinear magnetic system.

(by increasing i), the rate will continue to decrease and ultimately the magnetic characteristic of the ferromagnetic material approaches that of air. It is clear that after the knee of the magnetization curve, the field energy W_f is no longer equal to the coenergy W_c.

Example 1B. Inductance calculations of a magnetic circuit. The parameters of the magnetic circuit shown in Fig. 1.3-1 are: $r = 1\,\Omega$, $N = 100$ turns, $v = 10$ V, $\ell_i = 40$ cm, $x = 3$ mm, $A_i = A_g = 40\,\text{cm}^2$, $\mu_0 = 4\pi \times 10^{-7}$ H/m, $\mu_{ri} = 1000$. Determine (a) the steady-state total flux Φ, assume $\mathcal{R}_l = 6.77 \times 10^7\text{H}^{-1}$; (b) the leakage inductance L_l, the magnetizing inductance L_m, and the self-inductance L; (c) express the energy from the source to the system following a step applied voltage.

(a) It would be helpful to refer to Fig. 1.3-2. Therein,

$$\mathcal{R}_i = \frac{\ell_i}{\mu_{ri}\mu_0 A_i} = \frac{40 \times 10^{-2}}{(1000)(4\pi \times 10^{-7})(4 \times 10^{-4})} = 7.95 \times 10^5\ \text{H}^{-1} \quad \text{(1B-1)}$$

$$\mathcal{R}_g = \frac{x}{\mu_{rg}\mu_0 A_g} = \frac{3 \times 10^{-3}}{(1)(4\pi \times 10^{-7})(4 \times 10^{-4})} = 5.97 \times 10^6\ \text{H}^{-1} \quad \text{(1B-2)}$$

$$\Phi_m = \frac{Ni}{\mathcal{R}_i + \mathcal{R}_g} = \frac{100\left(\frac{10}{1}\right)}{7.95 \times 10^5 + 5.97 \times 10^6} = 1.48 \times 10^{-4}\ \text{Wb} \quad \text{(1B-3)}$$

$$\Phi_l = \frac{Ni}{\mathcal{R}_l} = \frac{100\left(\frac{10}{1}\right)}{6.77 \times 10^7} = 1.48 \times 10^{-5}\ \text{Wb} \quad \text{(1B-4)}$$

$$\Phi = \Phi_l + \Phi_m = 1.48 \times 10^{-5} + 1.48 \times 10^{-4} = 1.63 \times 10^{-4}\ \text{Wb} \quad \text{(1B-5)}$$

(b)

$$L_l = \frac{N^2}{\mathcal{R}_l} = \frac{(100)^2}{6.77 \times 10^7} = 0.148\,\text{mH} \tag{1B-6}$$

$$L_m = \frac{N^2}{\mathcal{R}_m} = \frac{(100)^2}{(7.96 \times 10^5) + (5.97 \times 10^5)} = 1.48\,\text{mH} \tag{1B-7}$$

$$L = L_l + L_m = (0.148 \times 10^{-3}) + (1.48 \times 10^{-3}) = 1.63\,\text{mH} \tag{1B-8}$$

(c) The expression for the total energy from the source, W_E, is

$$W_E = \int v i dt \tag{1B-9}$$

From (1.3-1), which can now be written

$$v = ri + L\frac{di}{dt} \tag{1B-10}$$

Substituting (1B-10) into (1B-9) yields

$$\begin{aligned} W_E &= \int ri^2 dt + \int L\frac{di}{dt} i dt \\ &= r\int i^2 dt + \int L\, i di \\ &= r\int i^2 dt + \frac{1}{2}Li^2 \\ &= r\int i^2 dt + \frac{1}{2}(L_\ell + L_m)i^2 \end{aligned} \tag{1B-11}$$

The first term on the right side of (1B-11) is the energy dissipated in the resistor while the second term is the energy stored in the inductance (magnetic field).

Since the expression for the current after the step voltage is applied is

$$i = \frac{v}{r}(1 - e^{-t/\tau}) \tag{1B-12}$$

where τ is the time constant L/r, the energy supplied by the source may be expressed

$$\begin{aligned} W_E &= r\int \left[\frac{v}{r}(1 - e^{-t/\tau})\right]^2 dt + \frac{1}{2}Li^2 \\ &= r\left(\frac{v}{r}\right)^2 \int (1 - 2e^{-t/\tau} + e^{-2t/\tau})dt + \frac{1}{2}Li^2 \\ &= \frac{v^2}{r}\left[t + 2\tau e^{-t/\tau} - \frac{\tau}{2}e^{-2t/\tau}\right] \\ &\quad + \frac{1}{2}L\left(\frac{v}{r}\right)^2(1 - e^{-t/\tau})^2 \end{aligned} \tag{1B-13}$$

For $t \gg \tau$,

$$W_E \cong \frac{v^2}{r}t + \frac{1}{2}L\left(\frac{v}{r}\right)^2 \tag{1B-14}$$

It is interesting to note that the energy stored in the inductance converges to a constant; however, the resistor continues to dissipate energy at a rate of v^2/r.

SP1.3-1. Determine two changes in the magnetic circuit shown in Example 1B that would result in interchanging the position of the poles. [Reverse the direction of positive current or reverse the sense of the winding.]

SP1.3-2. Express (a) W_f and (b) W_c if $L = 1$ H and the current is $I = I_p \cos \omega_e t$. (a) $\left[\frac{1}{2} I_p^2 \left(\frac{1}{2} + \frac{1}{2} \cos 2\omega_e t\right)\right]$, (b) [same].

SP1.3-3. Draw the λ versus i plot for Example 1B. Indicate the change in the slope as the length of the air gap increases. [slope decreases]

SP1.3-4. Consider Fig. 1.3-1. Assume the reluctance of the iron may be neglected. Express the energy stored in the air gap in terms of its air-gap mmf (mmf_{ag}) and its reluctance $\mathcal{R}_g$. $\left[\frac{(\text{mmf}_{ag})^2}{2\mathcal{R}_g}\right]$

SP1.3-5. Consider Fig. 1.3-3. Neglect the reluctance of the iron and express the energy stored in each air gap in terms of its air-gap mmf. Let mmf_{ag} and $\mathcal{R}_g$ be for one air gap. $\left[\frac{2(\text{mmf}_{ag})^2}{\mathcal{R}_g}\right]$

1.4 STATIONARY COUPLED CIRCUITS

Faraday's Law tells us that a voltage is induced in an electric circuit due to a change of flux linkage. In the case of stationary circuits, such as the transformer, a change of flux linkage occurs due to time varying currents. Therefore, in an ac power system we can change the level of the voltage and current. That is, power can be generated at convenient voltage and current levels and transmitted at a high voltage with a low current to minimize transmission losses and then lower the voltage to a safe level at the point of usage.

Flux linkages of an electric circuit can also be changed, and thus voltages induced, due to relative motion between magnetic fields. This allows power to be generated at the source where mechanical motion is converted to electric power (generator action). At the load, electric power is often converted back to mechanical motion (motor action). The work of Faraday and later the inventions and development of the ac power system by Tesla and Westinghouse doomed Edison's dc power system in the very early twentieth century. We will start our analytical journey into the world of Faraday and Tesla with the transformer.

Magnetically Linear Transformer

Two coupled circuits are shown in Fig. 1.4-1. Actually, all we have done is "closed up" the air gap of Fig. 1.3-1 and added a second coil. About all we have to concern ourselves with is how to handle the coupling between coils since in the previous

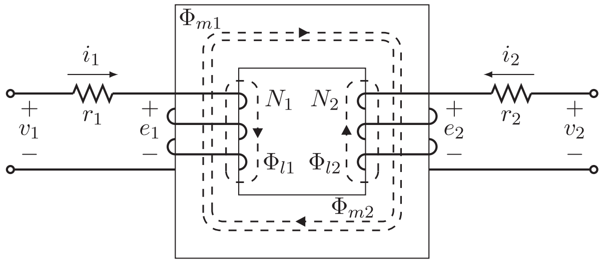

Figure 1.4-1 Magnetically coupled circuits.

section we became familiar with leakage flux and magnetizing flux. The flux linkages are

$$\lambda_1 = N_1\Phi_1 \tag{1.4-1}$$
$$\lambda_2 = N_2\Phi_2 \tag{1.4-2}$$

where Φ_1 and Φ_2 are the total flux linking N_1 and N_2, respectively. From Fig. 1.4-1

$$\begin{aligned}\Phi_1 &= \Phi_{l1} + \Phi_{m1} + \Phi_{m2}\\ &= \frac{N_1 i_1}{\mathcal{R}_{l1}} + \frac{N_1 i_1}{\mathcal{R}_m} + \frac{N_2 i_2}{\mathcal{R}_m}\end{aligned} \tag{1.4-3}$$
$$\begin{aligned}\Phi_2 &= \Phi_{l2} + \Phi_{m2} + \Phi_{m1}\\ &= \frac{N_2 i_2}{\mathcal{R}_{l2}} + \frac{N_2 i_2}{\mathcal{R}_m} + \frac{N_1 i_1}{\mathcal{R}_m}\end{aligned} \tag{1.4-4}$$

Mutual coupling occurs since Φ_{m2}, which is the flux created by current flowing in coil N_2, links coil N_1. Likewise, Φ_{m1}, which is the flux created by i_1 current flowing in coil N_1, links coil N_2.

Substituting (1.4-3) into (1.4-1) and (1.4-4) into (1.4-2) yields

$$\lambda_1 = \frac{N_1^2}{\mathcal{R}_{l1}}i_1 + \frac{N_1^2}{\mathcal{R}_m}i_1 + \frac{N_1N_2}{\mathcal{R}_m}i_2 \tag{1.4-5}$$
$$\lambda_2 = \frac{N_2^2}{\mathcal{R}_{l2}}i_2 + \frac{N_2^2}{\mathcal{R}_m}i_2 + \frac{N_2N_1}{\mathcal{R}_m}i_1 \tag{1.4-6}$$

The coefficients of the first two terms of (1.4-5) and (1.4-6), respectively, are the self-inductances L_{11} and L_{22}, that is

$$L_{11} = \frac{N_1^2}{\mathcal{R}_{l1}} + \frac{N_1^2}{\mathcal{R}_m} = L_{l1} + L_{m1} \tag{1.4-7}$$
$$L_{22} = \frac{N_2^2}{\mathcal{R}_{l2}} + \frac{N_2^2}{\mathcal{R}_m} = L_{l2} + L_{m2} \tag{1.4-8}$$

The coefficient of the last term of (1.4-5) and (1.4-6) is the mutual inductance

$$L_{12} = L_{21} = \frac{N_1 N_2}{\mathcal{R}_m} \tag{1.4-9}$$

We can now write λ_1 and λ_2 as

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \tag{1.4-10}$$

$$\lambda_2 = L_{22} i_2 + L_{21} i_1 \tag{1.4-11}$$

It is important to note that L_{m1}, L_{m2}, and L_{12} have a common term $\mathcal{R}_m$ which is the reluctance of the mutual path around the ferromagnetic core. This fact will allow us to derive a very useful equivalent circuit; however, before getting started on that, let us go back to (1.4-5) and (1.4-6). The mutual inductance can be positive or negative in these equations depending upon the relative directions of Φ_{m1} and Φ_{m2}. If the assumed positive directions of the currents or the sense of the winding of the coil causes Φ_{m1} and Φ_{m2} to aid each other, L_{12} (L_{21}) would be positive as in the case here. If they oppose L_{12} (L_{21})would be negative.

Now, λ_1 may be written

$$\begin{aligned} \lambda_1 &= L_{l1} i_1 + L_{m1} i_1 + L_{12} i_2 \\ &= L_{l1} i_1 + L_{m1}\left(i_1 + \frac{N_2}{N_1} i_2\right) \end{aligned} \tag{1.4-12}$$

Here we multiplied L_{12}, given by (1.4-9), by $\frac{N_1}{N_2}$ whereupon

$$\begin{aligned} \frac{N_1}{N_2} L_{12} &= \left(\frac{N_1}{N_2}\right) \frac{N_1 N_2}{\mathcal{R}_m} \\ &= L_{m1} \end{aligned} \tag{1.4-13}$$

thus,

$$L_{12} = \frac{N_2}{N_1} L_{m1} \tag{1.4-14}$$

From (1.4-12) we see there is a common inductance L_{m1} that carries the currents i_1 and $\frac{N_2}{N_1} i_2$. We have referred $\frac{N_2}{N_1} i_2$ to the winding with N_1 turns; if we would have expressed λ_2 rather than λ_1 we could have referred i_1 to the winding with N_2 turns by $\frac{N_1}{N_2} i_1$. Let us stay with $\frac{N_2}{N_1} i_2$, that is generally written, for compactness, as

$$i_2' = \frac{N_2}{N_1} i_2 \tag{1.4-15}$$

or

$$N_1 i_2' = N_2 i_2 \tag{1.4-16}$$

From (1.4-16), i_2' flowing in N_1 produces the same mmf as i_2 flowing in N_2.

To maintain the same power

$$v_2' i_2' = v_2 i_2 \tag{1.4-17}$$

Thus

$$v_2' = \frac{N_1}{N_2} v_2 \tag{1.4-18}$$

Since λ_2 has units of volt · sec, the same turns ratio used for voltages can be applied to it; thus,

$$\lambda_2' = \frac{N_1}{N_2} \lambda_2 \tag{1.4-19}$$

There is considerable algebraic manipulation to get to the equivalent circuit. We will only set forth the steps. First, let us write the voltage equations in terms of 1- and 2-variables.

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} \tag{1.4-20}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} \tag{1.4-21}$$

where r_1 and r_2 are the resistances of winding 1 and winding 2, respectively. Since we are referring the 2-variables to the 1-winding, (1.4-20) remains unchanged in form; however, (1.4-21) becomes

$$v_2' = r_2' i_2' + \frac{d\lambda_2'}{dt} \tag{1.4-22}$$

The flux linkage equations become

$$\lambda_1 = L_{l1} i_1 + L_{m1}(i_1 + i_2') \tag{1.4-23}$$

$$\lambda_2' = L_{l2}' i_2' + L_{m1}(i_1 + i_2') \tag{1.4-24}$$

where

$$L_{l2}' = \left(\frac{N_1}{N_2}\right)^2 L_{l2} \tag{1.4-25}$$

$$r_2' = \left(\frac{N_1}{N_2}\right)^2 r_2 \tag{1.4-26}$$

The equivalent circuit is shown in Fig. 1.4-2. This circuit is valid for all modes of operation, transient and steady state; however, it is valid only for a magnetically linear system.

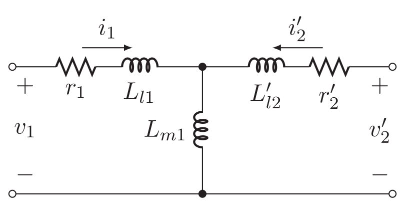

Figure 1.4-2 Transformer equivalent *T* circuit with winding 1 selected as reference winding.

Field Energy

Before leaving our work with transformers, let us derive an expression for the energy stored in the field W_f. In the case of a single coil the field energy was $\frac{1}{2}Li^2$. Here we have two coils with mutual coupling between them. In this case, the energy from the electrical system is

$$W_e = \int e_1 i_1 dt + e_2 i_2 dt \tag{1.4-27}$$

Since r_1 and r_2 are external to the respective coils and using unreferred variables

$$e_1 = \frac{d\lambda_1}{dt} \tag{1.4-28}$$

$$e_2 = \frac{d\lambda_2}{dt} \tag{1.4-29}$$

Substituting (1.4-28) and (1.4-29) into (1.4-27) yields

$$W_e = \int i_1 d\lambda_1 + i_2 d\lambda_2 \tag{1.4-30}$$

From (1.4-10) and (1.4-11)

$$d\lambda_1 = L_{11} di_1 + L_{12} di_2 \tag{1.4-31}$$

$$d\lambda_2 = L_{22} di_2 + L_{12} di_1 \tag{1.4-32}$$

Substituting (1.4-31) and (1.4-32) into (1.4-30) and since $W_f = W_e$ we have

$$W_f = \int [i_1(L_{11} di_1 + L_{12} di_2) + i_2(L_{22} di_2 + L_{12} di_1)] \tag{1.4-33}$$

For a magnetically linear system these integrals can be calculated in two steps; first, let the current in winding 1 be the variable of integration and vary it from zero to i_1 while holding i_2 and di_2 at zero. Next, hold the current in winding 1 at $i_1(di_1 = 0)$ and let the current in winding 2 be the variable of integration varying from zero to i_2. Performing the first step we have

$$W_{f(1)} = \int_0^{i_1} L_{11}\xi d\xi = \frac{1}{2}L_{11}i_1^2 \tag{1.4-34}$$

For the second step

$$\begin{aligned} W_{f(2)} &= \int_0^{i_2} L_{12} i_1 d\xi + L_{22}\xi d\xi \\ &= L_{12} i_1 i_2 + \frac{1}{2}L_{22}i_2^2 \end{aligned} \tag{1.4-35}$$

Thus

$$W_f = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1 i_2 + \frac{1}{2}L_{22}i_2^2 \tag{1.4-36}$$

The energy stored in the coupling field is W_f given by (1.4-36) less the energy stored in the leakage inductances. In a problem at the end of the chapter you are asked to express (1.4-36) in terms of i_1 and i_2' and to identify the energy stored in the coupling field.

The pattern is clear, the field energy may be expressed for multiple electrical inputs (coils) as $\frac{1}{2}$ the self-inductance of each coil times the square of the current flowing in the coil and the mutual inductance between each set of coils times the product of the current flowing in the mutually coupled coils. For three coupled coils

$$W_f = \tfrac{1}{2}L_{11}i_1^2 + \tfrac{1}{2}L_{22}i_2^2 + \frac{1}{2}L_{33}i_3^2 + L_{12}i_1i_2 + L_{13}i_1i_3 + L_{23}i_2i_3 \qquad (1.4\text{-}37)$$

Now, for multiple electrical inputs (windings), (1.3-25) becomes

$$\sum_{j=1}^{J} \lambda_j i_j = W_f + W_c \qquad (1.4\text{-}38)$$

where j is the winding. Thus,

$$W_c = \sum_{j=1}^{J} \lambda_j i_j - W_f \qquad (1.4\text{-}39)$$

In SP1.4-3, you are asked to prove that W_c is W_f using (1.4-37) and (1.4-39).

Example 1C. Parameters of the transformer equivalent circuit. It is instructive to illustrate the method of deriving an equivalent T circuit from open- and short-circuit measurements of the transformer. When winding 2 of the two-winding transformer shown in Fig. 1.4-2 is open-circuited and a voltage of 110 V (rms) at 60 Hz is applied to winding 1, the average power supplied to winding 1 is 6.66 W. The measured current in winding 1 is 1.05 A (rms). Next, with winding 2 short-circuited, the current flowing in winding 1 is 2 A when the applied voltage is 30 V at 60 Hz. The average input power is 44 W. If we assume $L_{l1} = L'_{l2}$, an approximate equivalent T circuit can be determined from these measurements with winding 1 selected as the reference winding.

The average power supplied to winding 1 may be expressed from (1.2-31) as

$$P_1 = |\tilde{V}_1|\,|\tilde{I}_1| \cos\varphi_{pf} \qquad (1C\text{-}1)$$

where

$$\varphi_{pf} = \theta_{ev}(0) - \theta_{ei}(0) \qquad (1C\text{-}2)$$

Here, $\tilde{V}_1$ and $\tilde{I}_1$ are phasors with the positive direction of $\tilde{I}_1$ taken in the direction of voltage drop, and $\theta_{ev}(0)$ and $\theta_{ei}(0)$ are the phase angles of $\tilde{V}_1$ and $\tilde{I}_1$, respectively. Solving for φ_{pf} during the open-circuit test, we have

$$\varphi_{pf} = \cos^{-1}\frac{P_1}{|\tilde{V}_1|\,|\tilde{I}_1|} = \cos^{-1}\frac{6.66}{(110)(1.05)} = 86.7° \qquad (1C\text{-}3)$$

Although $\varphi_{pf} = -86.7°$ is also a legitimate solution of (1C-3), the positive value is taken since $\tilde{V}_1$ leads $\tilde{I}_1$ in an inductive circuit. With winding 2 open-circuited, the input impedance of winding 1 is

$$Z = \frac{\tilde{V}_1}{\tilde{I}_1} = r_1 + j(X_{l1} + X_{m1}) \tag{1C-4}$$

With $\tilde{V}_1$ as the reference phasor, $\tilde{V}_1 = 110\underline{/0°}$, $\tilde{I}_1 = 1.05\underline{/-86.7°}$. Thus,

$$r_1 + j(X_{l1} + X_{m1}) = \frac{110\underline{/0°}}{1.05\underline{/-86.7°}} = 6 + j104.6\,\Omega \tag{1C-5}$$

From (1C-5), $r_1 = 6\,\Omega$. We also see from (1C-5) that $X_{l1} + X_{m1} = 104.6\,\Omega$.

For the short-circuit test, we will assume that $\tilde{I}_1 = -\tilde{I}'_2$ since transformers are designed so that at rated frequency $X_{m1} \gg |r'_2 + jX'_{l2}|$. Hence, using (1C-1) again,

$$\varphi_{pf} = \cos^{-1}\frac{44}{(30)(2)} = 42.8° \tag{1C-6}$$

In this case, the input impedance is $Z = (r_1 + r'_2) + j(X_{l1} + X'_{l2})$. This may be determined as

$$Z = \frac{30\underline{/0°}}{2\underline{/-42.8°}} = 11 + j10.2\,\Omega \tag{1C-7}$$

Hence, $r'_2 = 11 - r_1 = 5\,\Omega$ and, since it is assumed that $X_{l1} = X'_{l2}$, both are $10.2/2 = 5.1\,\Omega$. Therefore, $X_{m1} = 104.6 - 5.1 = 99.5\,\Omega$. In summary, $r_1 = 6\,\Omega$, $L_{l1} = 13.5$ mH, $L_{m1} = 263.9$ mH, $r'_2 = 5\,\Omega$, $L'_{l2} = 13.5$ mH. It is left to the reader to verify the conversion from X's to L's.

SP1.4-1. Consider the transformer and parameters calculated in Example 1C. Winding 2 is short-circuited and 12 V (dc) is applied to winding 1. Calculate the steady-state values of i_1 and i_2. Repeat with winding 2 open-circuited. [$I_1 = 2$ A and $I_2 = 0$ in both cases]

SP1.4-2. Calculate the no-load current (winding 2 open-circuited) for the transformer given in Example 1C if $V_1 = \sqrt{2}10\cos 100t$. [$\tilde{I}_1 = 0.35\underline{/-77.8°}$ A]

SP1.4-3. Determine W_c from (1.4-37) and (1.4-39). Show that $W_c = W_f$.

SP1.4-4. Use the equivalent T circuit given in Fig. 1.4-2 to identify the energy stored in the coupling field. [$\frac{1}{2}L_{m1}(i_1^2 + i'^2_2) + L_{m1}i_1 i'_2$]

1.5 COUPLED CIRCUITS IN RELATIVE MOTION

In the previous section, we considered the "transformer" coupling that exists between two stationary windings. An induced voltage due to a time varying current (transformer action) is only "half" of Faraday's Law. In the case of electric machines, there can be windings on the stationary member (stator) and windings on the rotating member (rotor). Transformer coupling may exist between only stator windings or between only rotor windings. Transformer coupling may also exist between the stator and rotor windings depending upon their relative positions. In addition, there is a voltage induced in the circuits due to the rate of change of the relative position of the stator and rotor windings; commonly referred to as a speed voltage. In this section, we will take our first look at the magnetic coupling when windings are in relative motion and what Faraday's Law tells us about the voltages that can be induced.

The rotational device shown in Fig. 1.5-1 will be used to illustrate windings in relative motion. This device consists of two coils each containing several turns of a conductor. Winding 1 has N_1 turns and it is on the stationary member (stator); winding 2 has N_2 turns and it is on the rotating member (rotor). The ⊗ indicates that the assumed direction of positive current flow in the conductors is into the paper, where ⊙ indicates positive current flow in the conductors is out of the paper. In a practical device, the turns of a winding are distributed over an arc (often 30° to 60° of the stator or rotor; however, in this introductory consideration, it is sufficient to assume, for now, that the turns are concentrated in one position, as shown in Fig. 1.5-1. Also, the length of the air gap between the stator and rotor is shown exaggerated relative to the actual inside diameter of the stator.

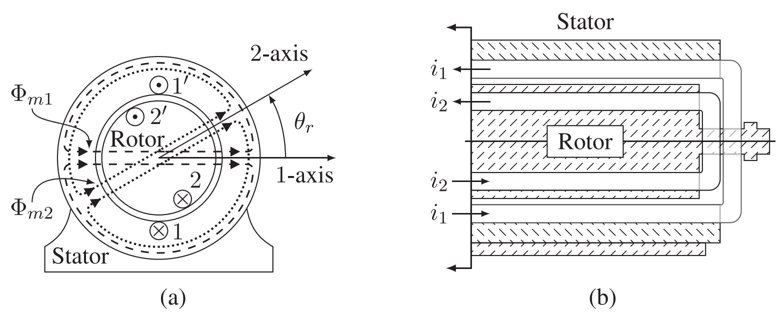

Figure 1.5-1 Elementary rotational electromechanical device: (a) end view; (b) cross-sectional view.

The voltage equations may be written as

$$v_1 = r_1 i_1 + \frac{d\lambda_1}{dt} \tag{1.5-1}$$

$$v_2 = r_2 i_2 + \frac{d\lambda_2}{dt} \tag{1.5-2}$$

where r_1 and r_2 are the resistances of windings 1 and 2, respectively. The magnetic system is assumed to be linear; therefore, the flux linkages may be expressed as

$$\lambda_1 = L_{11} i_1 + L_{12} i_2 \tag{1.5-3}$$

$$\lambda_2 = L_{21} i_1 + L_{22} i_2 \tag{1.5-4}$$

Since the air gap is uniform, the self-inductances L_{11} and L_{22} are constants and may be expressed in terms of leakage and magnetizing inductances as

$$\begin{aligned} L_{11} &= L_{l1} + L_{m1} \\ &= \frac{N_1^2}{\mathcal{R}_{l1}} + \frac{N_1^2}{\mathcal{R}_m} \end{aligned} \tag{1.5-5}$$

$$\begin{aligned} L_{22} &= L_{l2} + L_{m2} \\ &= \frac{N_2^2}{\mathcal{R}_{l2}} + \frac{N_2^2}{\mathcal{R}_m} \end{aligned} \tag{1.5-6}$$

where $\mathcal{R}_m$ is the reluctance of the complete magnetic path of Φ_{m1} and Φ_{m2}, which is through the rotor and stator iron and twice across the air gap $(\mathcal{R}_i + 2\mathcal{R}_g)$. Clearly, $\mathcal{R}_m$ is the same for the magnetic system established by either winding 1 or 2.

Take a moment to note the designation of axis 1 and axis 2 in Fig. 1.5-1. These axes denote the positive direction of the respective magnetic systems with the assumed positive direction of current flow in the windings (right-hand rule). Now let us consider L_{12}. (Is it clear that $L_{12} = L_{21}$?) When θ_r is zero, the coupling between windings 1 and 2 is maximum. In particular, with $\theta_r = 0$ the magnetic system of winding 1 aids that of winding 2 with positive currents assumed. Hence, the mutual inductance is positive and

$$L_{12}(0) = \frac{N_1 N_2}{\mathcal{R}_m} \tag{1.5-7}$$

When $\theta_r = \frac{1}{2}\pi$, the windings are orthogonal. The mutual coupling is zero. Hence,

$$L_{12}\left(\frac{1}{2}\pi\right) = 0 \tag{1.5-8}$$

When $\theta_r = \pi$, $L_{12}(\pi)$ is the negative of (1.5-7) and at $\theta_r = \frac{3}{2}\pi$, L_{12} is zero; the same as (1.5-8). Let us make it as simple as possible by assuming that as a first approximation the mutual inductance may be written

$$L_{12}(\theta_r) = L_{sr}\cos\theta_r \tag{1.5-9}$$

where L_{sr} is the amplitude of the sinusoidal mutual inductance between the stator and rotor windings as given by (1.5-7).

The flux linkages are given by (1.5-3) and (1.5-4), let us rewrite these equations indicating the functionality

$$\lambda_1(i_1, i_2, \theta_r) = L_{11}i_1 + L_{12}(\theta_r)i_2 \tag{1.5-10}$$

$$\lambda_2(i_1, i_2, \theta_r) = L_{12}(\theta_r)i_1 + L_{22}i_2 \tag{1.5-11}$$

The total derivative of λ_1 becomes

$$\begin{aligned} \frac{d\lambda_1}{dt} &= \frac{\partial(L_{11}i_1)}{\partial i_1}\frac{di_1}{dt} + \frac{\partial(L_{11}i_1)}{\partial i_2}\frac{di_2}{dt} + \frac{\partial(L_{11}i_1)}{\partial\theta_r}\frac{d\theta_r}{dt} \\ &\quad + \frac{\partial(L_{12}i_2)}{\partial i_1}\frac{di_1}{dt} + \frac{\partial(L_{12}i_2)}{\partial i_2}\frac{di_2}{dt} + \frac{\partial(L_{12}i_2)}{\partial\theta_r}\frac{d\theta_r}{dt} \end{aligned} \tag{1.5-12}$$

Only the first, fifth, and sixth terms on the right-hand side of (1.5-12) are nonzero, thus v_1 can be written

$$v_1 = r_1 i_1 + L_{11}\frac{di_1}{dt} + L_{sr}\cos\theta_r \frac{di_2}{dt} - i_2 L_{sr}\sin\theta_r \frac{d\theta_r}{dt} \tag{1.5-13}$$

Following the same procedure, we can express v_2 as

$$v_2 = r_2 i_2 + L_{22}\frac{di_2}{dt} + L_{sr}\cos\theta_r \frac{di_1}{dt} - i_1 L_{sr}\sin\theta_r \frac{d\theta_r}{dt} \tag{1.5-14}$$

Faraday's Law has given us the last three terms of (1.5-13) and (1.5-14). The first of these terms is due to the change of current flowing in the winding; this term would exist independent of the other winding. The second term is due to the "transformer" coupling from one winding to the other. The degree and sense of the coupling would depend upon the relative position or displacement between the windings. The last of these three terms is the voltage induced due to the rate of change of the displacement between windings; commonly referred to as the "speed voltage."

Field Energy

Regardless, if the electromechanical device is magnetically linear or nonlinear, the λi relationship is a single-valued function with the assumptions we have made. Thus, the energy stored in the field is single-valued state function, meaning that for a given i_1, i_2, and θ_r (or x in case of translational motion) there is only one value of W_f. Therefore, even though (1.4-33) was derived for magnetically linear, stationary coupled circuits it is also valid for magnetically linear coupled circuits in relative motion. Thus, from (1.4-36)

$$W_f = \frac{1}{2}L_{11}i_1^2 + i_1 i_2 L_{sr}\cos\theta_r + \frac{1}{2}L_{22}i_2^2 \tag{1.5-15}$$

The assumption of a magnetically linear system may not be as limiting as it might first appear. In many cases, most of the mmf is dropped across the air gap of the electromechanical device and therefore most of the field energy W_f is stored in the air gap and air is a magnetically linear, conservative medium.

Example 1D. Inductances of windings in relative motion. Consider the windings in relative motion shown in Fig. 1D-1. The outer stator housing has been omitted from that shown in Fig. 1.5-1. We will often omit this housing for convenience. Winding 3 has been added to the rotor. Windings 2 and 3 are identical and displaced by $\frac{\pi}{2}$ relative to each other. Express L_{11}, L_{22}, L_{33}, L_{12}, L_{13}, L_{23}, and v_1.

We have L_{11} and L_{22} from (1.5-5) and (1.5-6), respectively. It follows that

$$L_{33} = \frac{N_3^2}{\mathcal{R}_{l3}} + \frac{N_3^2}{\mathcal{R}_m} \tag{1D-1}$$

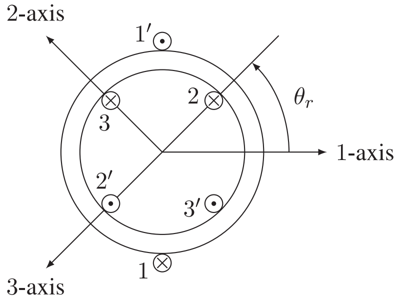

Figure 1D-1 Windings in relative motion (stator housing omitted).

Since windings 2 and 3 are identical $N_2 = N_3$ thus $L_{22} = L_{33}$. In the case of L_{12} and L_{13}

$$L_{12} = -\frac{N_1 N_2}{\mathcal{R}_m} \sin\theta_r \tag{1D-2}$$

$$L_{13} = -\frac{N_1 N_3}{\mathcal{R}_m} \cos\theta_r \tag{1D-3}$$

Since windings 2 and 3 are orthogonal, $L_{23} = 0$. The voltage v_1 may be written by using (1.5-13) or (1.5-14) as a guide.

$$\begin{aligned} v_1 = r_1 i_1 + L_{11}\frac{di_1}{dt} - L_{sr}\sin\theta_r\frac{di_2}{dt} - L_{sr}\cos\theta_r\frac{di_3}{dt} \\ - i_2 L_{sr}\cos\theta_r\frac{d\theta_r}{dt} + i_3 L_{sr}\sin\theta_r\frac{d\theta_r}{dt} \end{aligned} \tag{1D-4}$$

where

$$L_{sr} = \frac{N_1 N_2}{\mathcal{R}_m} \tag{1D-5}$$

and

$$N_2 = N_3 \tag{1D-6}$$

SP1.5-1. Consider Fig. 1.5-1 $I_1 = 1$ A, $L_{sr} = 0.1$ H, $L_{11} = 10L_{sr}, \omega_r = 100$ rad/s, $\theta_r(0) = 0$, and winding 2 is open circuited. Express V_2. [$V_2 = -10\sin 100t$]

SP1.5-2. Calculate the field energy for SP1.5-1. [$\frac{1}{2}$ J]

SP1.5-3. Repeat SP1.5-1 if $I_1 = 1\cos 100t$. [$V_2 = -10\sin 200t$]

SP1.5-4. Can we use phasors to solve steady-state versions of (1.5-13) or (1.5-14)? [Only if a single frequency is present.]

1.6 ELECTROMAGNETIC FORCE AND TORQUE

Our goal in this section is to derive an expression for the electromagnetic torque developed in rotational systems or force in translational systems. Although there are several approaches that could be used for this derivation, with the background that we have established, the "energy balance" approach is perhaps most convenient [2]. This method is quite direct and results in an easy to use expression for evaluating torque or force in magnetically linear electromagnetic systems.

Electromechanical devices consist of an electrical system, a coupling field, and a mechanical system. The electrical and the mechanical systems either supply or absorb energy by way of the coupling field. The coupling field can either be electromagnetic, which will be our focus, or electrostatic. The coupling field enables the electrical and mechanical systems to interact thereby providing a means of transferring energy between the two systems while at the same time providing a means of storing energy. A block diagram depicting this interaction and possible directions of energy flow is given in Fig. 1.6-1.

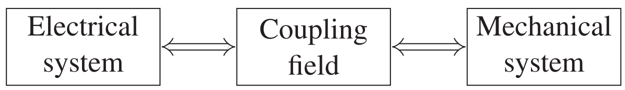

Figure 1.6-1 Block diagram of possible energy interchange in an elementary electromechanical system.

The energy balance is shown in Fig. 1.6-2. W_E and W_M are positive for energy being supplied to the electromechanical device from an external electrical system (W_E) and mechanical system (W_M). Likewise W_e and W_m are positive when supplying energy to the coupling field in the electrical form (W_e) and mechanical form (W_m).

In Fig. 1.6-2, the subscripts E and e pertain to the electrical system; M and m to the mechanical; and f to the field. Also, S and L indicate energy stored and energy lost, respectively. The energy W_{eL} is the energy lost due to i^2r; W_{eS} is the energy stored in a field external to the coupling field. Some authors consider energy stored in the field of the leakage inductances as W_{eS} while others will include the leakage flux in the coupling field. This is irrelevant since the energy stored in the field of the leakage inductance is generally not a function of the mechanical motion of the electromechanical device.

The energy W_{mL} represents energy lost due to mechanical friction and windage losses. W_{mS} is the energy stored as kinetic energy in the rotating mass (rotor) or as potential energy in a spring.

The energy W_e is the energy coming into the coupling field from the electrical system. The energy W_m is the energy coming into the coupling field from the

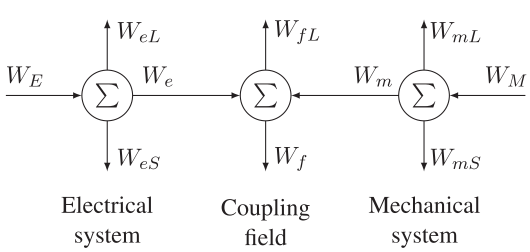

Figure 1.6-2 Energy balance.

mechanical system. The energy W_{fL} is the energy lost due to hysteresis loss and circulating currents induced in the mechanical components of the coupling field. Electric machines are designed to minimize these losses making the λi relationship approach a single-valued function as shown in Fig. 1.3-3. We will neglect W_{fL}; whereupon, we can express the energy balance of the coupling field as

$$W_f = W_e + W_m \tag{1.6-1}$$

where W_f is the energy stored in the coupling field [1].

Fig. 1.5-1a is shown again in Fig. 1.6-3 wherein the flux streamlines and the outside housing have been omitted from the cross-sectional view similarly in Fig. 1D-1. Also electromagnetic torque T_e, the load torque T_L, and the angular velocity of the rotor ω_r are indicated. Note that T_e is positive in the direction of positive angular displacement of the rotor θ_r while the load torque T_L is positive in the direction of negative θ_r.

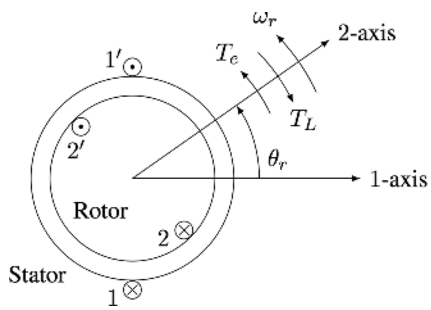

Figure 1.6-3 Elementary rotational electromechanical device.

The torque relationship between the electromechanical device and the mechanical system may be written

$$T_e = J\frac{d\omega_r}{dt} + B_m\omega_r + T_L \tag{1.6-2}$$

which is the rotational form of Newton's Second Law. Here, J is the inertia of the rotor and any tightly mechanically coupled rotating mass, B_m is the mechanical damping which is generally small and often neglected. Note that positive T_e acts to increase ω_r whereas a positive load torque T_L would act to retard ω_r. Therefore, with T_e positive for motor action, then positive W_m entering the coupling field as shown in Fig. 1.6-2 and as expressed in (1.6-1), would be

$$W_m = -\int T_e d\theta_r \tag{1.6-3}$$

The energy entering the coupling field from the two electrical sources shown in Fig. 1.6-3 W_e is

$$W_e = \int e_1 i_1 dt + e_2 i_2 dt \tag{1.6-4}$$

where

$$e_1 = \frac{d\lambda_1}{dt} \tag{1.6-5}$$

$$e_2 = \frac{d\lambda_2}{dt} \tag{1.6-6}$$

and e_1 and e_2 are $v_1 - r_1 i_1$ and $v_2 - r_2 i_2$, respectively. Substituting (1.6-5) and (1.6-6) into (1.6-4) yields

$$W_e = \int i_1 d\lambda_1 + i_2 d\lambda_2 \tag{1.6-7}$$

For J electrical inputs (1.6-7) may be expressed

$$W_e = \int \sum_{j=1}^{J} i_j d\lambda_j \tag{1.6-8}$$

where j is an index (1, 2, 3, … , J) not to be confused with the imaginary part of a complex number or with inertia or Joules. Thus, (1.6-1) may now be written

$$W_f = \int \sum_{j=1}^{J} i_j d\lambda_j - \int T_e d\theta_r \tag{1.6-9}$$

In differential form, which we will use extensively, (1.6-9) can be written

$$T_e d\theta_r = \sum_{j=1}^{J} i_j d\lambda_j - dW_f \tag{1.6-10}$$

Equation (1.3-25) may be written for multiple electrical inputs as

$$\sum_{j=1}^{J} \lambda_j i_j = W_c + W_f \tag{1.6-11}$$

where W_c is the coenergy. If we take the derivative of (1.6-11) we have

$$\sum_{j=1}^{J} \lambda_j di_j + \sum_{j=1}^{J} i_j d\lambda_j = dW_c + dW_f \tag{1.6-12}$$

Solving (1.6-12) for $\sum_{j=1}^{J} i_j d\lambda_j$ and substituting the result into (1.6-10) yields

$$T_e d\theta_r = -\sum_{j=1}^{J} \lambda_j di_j + dW_c \tag{1.6-13}$$

Although (1.6-10) and (1.6-13) are valid for magnetically linear and nonlinear systems, we will consider only magnetically linear systems. Therefore, it is convenient to express the flux linkages in terms of currents and either θ_r for rotational systems or x for translational systems. Hence, we will choose currents and θ_r or x

as independent variables. In this case, (1.6-13) is most convenient for obtaining an expression for T_e which can be written

$$T_e(\mathbf{i}, \theta_r)d\theta_r = -\sum_{j=1}^{J} \lambda_j(\mathbf{i}, \theta_r)di_j + dW_c(\mathbf{i}, \theta_r) \tag{1.6-14}$$

for compactness,

$$\mathbf{i} = (i_1, i_2, i_3, \ldots, i_J) \tag{1.6-15}$$

Equation (1.6-14) may now be written

$$T_e(\mathbf{i}, \theta_r)d\theta_r = -\sum_{j=1}^{J} \lambda_j(\mathbf{i}, \theta_r)di_j + \sum_{j=1}^{J} \frac{\partial W_c}{\partial i_j}(\mathbf{i}, \theta_r)di_j + \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r}d\theta_r \tag{1.6-16}$$

Equating coefficients of $d\theta_r$

$$T_e(\mathbf{i}, \theta_r) = \frac{\partial W_c}{\partial \theta_r} \tag{1.6-17}$$

In case of a translational system

$$f_e\,(\mathbf{i}, x) = \frac{\partial W_c}{\partial x} \tag{1.6-18}$$

where f_e is the electromagnetic force and x is the displacement. Since we will consider only magnetically linear systems $W_c = W_f$.

Example 1E. Force between pole faces of air gap. Determine the force that exists between the pole faces of the elementary magnetic circuit given in Fig. 1.3-1 and the parameters given in Example 1B. In order to make use of (1.6-18) to calculate the electromagnetic force f_e we must allow what is referred to as a virtual displacement. That is, we will assume that the air-gap length x increases in the positive direction by dx; therefore, according to (1.6-18) f_e will be positive if it acts to lengthen the air gap. If f_e is negative, it acts to shorten the air gap.

From (1.3-11) the self-inductance is

$$L = L_l + L_m \tag{1E-1}$$

where L_l is the leakage inductance and L_m is the magnetizing inductance. In particular,

$$L = \frac{N^2}{\mathcal{R}_l} + \frac{N^2}{\mathcal{R}_i + \mathcal{R}_g} \tag{1E-2}$$

The coenergy is

$$W_c = \frac{1}{2}(L_l + L_m)i^2 \tag{1E-3}$$

In order to determine f_e, we must take the partial derivative of W_c with respect to x. Since L_l is not a function of x, we only need to concern ourselves with L_m. We can write L_m from (1B-1) and (1B-2) as

$$L_m = \frac{N^2}{\mathcal{R}_i + \mathcal{R}_g} = \frac{N^2}{\dfrac{\ell_i}{\mu_{ri}\mu_0 A_i} + \dfrac{x}{\mu_0 A_g}} \tag{1E-4}$$

which can be written

$$L_m(x) = \frac{k_0}{k_1 + k_2 x} = \frac{\frac{k_0}{k_2}}{\frac{k_1}{k_2} + x} \tag{1E-5}$$

where

$$k_0 = N^2 = (1 \times 10^2)^2 = 1 \times 10^4 \text{ turns} \tag{1E-6}$$

$$k_1 = \frac{\ell_i}{\mu_{ri}\mu_0 A_i} = \frac{4 \times 10^{-1}}{(1 \times 10^3)(4\pi \times 10^{-7})(4 \times 10^{-4})} = 7.96 \times 10^5 \text{ H}^{-1} \tag{1E-7}$$

$$k_2 = \frac{1}{\mu_0 A_g} = \frac{1}{(4\pi \times 10^{-7})(4 \times 10^{-4})} = 1.99 \times 10^9 \text{ m}^{-1}\text{H}^{-1} \tag{1E-8}$$

Therefore

$$W_c = \frac{\frac{1}{2}\frac{k_0}{k_2} i^2}{\frac{k_1}{k_2} + x} \tag{1E-9}$$

Now

$$f_e(i,x) = \frac{\partial W_c}{\partial x} = -\frac{\frac{1}{2}\left(\frac{k_0}{k_2} i^2\right)}{\left(\frac{k_1}{k_2} + x\right)^2} = -\frac{\frac{1}{2}(5.025 \times 10^{-6} \times 10^2)}{(3.995 \times 10^{-4} + 3 \times 10^{-3})^2} = -21.74\,\text{N (an attractive force)} \tag{1E-10}$$

A force is established in a magnetic system to minimize the reluctance.

SP1.6-1. Neglect the reluctance of the ferromagnetic core in Example 1E and calculate f_e. [−27.92 N]

SP1.6-2. In a rotational system, the self-inductance of stator winding is $L_{11} = k_0 + k_1 \cos 2\theta_r$. There is only one stator winding and no rotor winding. Express T_e, which is referred to as the reluctance torque, if $I_1 = \cos \omega_e t$. [$-k_1 I_1^2 \sin 2\theta_r$]

SP1.6-3. Rearrange the magnetic circuit in Fig. 1.3-1 to produce a repelling force between pole faces. [The force will always be in the direction to minimize the reluctance of the magnetic system.]

1.7 ELEMENTARY ELECTROMECHANICAL DEVICE

It is instructive to derive the expression for the torque of the elementary electromagnetic device shown in Fig. 1.6-3. This will allow us to describe the steady-state torque-angle characteristics of this elementary device with constant currents and provides a means of determining the regions of stable operation of electric machines. Figure 1.6-3 is repeated here for convenience as Fig. 1.7-1. From (1.6-17),

$$T_e(\mathbf{i}, \theta_r) = \frac{\partial W_c(\mathbf{i}, \theta_r)}{\partial \theta_r} \tag{1.7-1}$$

where the coenergy W_c is equal to the field energy W_f since the system is assumed to be magnetically linear. Thus,

$$W_c(i_1, i_2, \theta_r) = \frac{1}{2}L_{11}i_1^2 + L_{12}i_1 i_2 + \frac{1}{2}L_{22}i_2^2 \tag{1.7-2}$$

where

$$L_{11} = L_{l1} + L_{m1} \tag{1.7-3}$$

$$L_{22} = L_{l2} + L_{m2} \tag{1.7-4}$$

$$L_{12} = L_{sr} \cos \theta_r \tag{1.7-5}$$

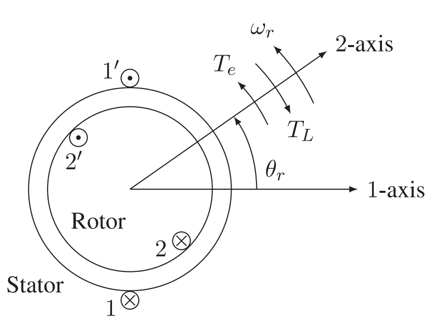

Figure 1.7-1 Elementary rotational electromechanical device.

The torque becomes

$$T_e(i_1, i_2, \theta_r) = -i_1 i_2 L_{sr} \sin\theta_r \tag{1.7-6}$$

Although (1.7-6) is valid regardless of the form of i_1 and i_2, let i_1 and i_2 be positive constants. For the positive direction of current shown, the torque would then be of the form

$$T_e = -K \sin\theta_r \tag{1.7-7}$$

where K is a positive constant equal to $i_1 i_2 L_{sr}$. Positive θ_r in Fig. 1.7-1 is counterclockwise; we will let positive θ_r be from right to left in Fig. 1.7-2.

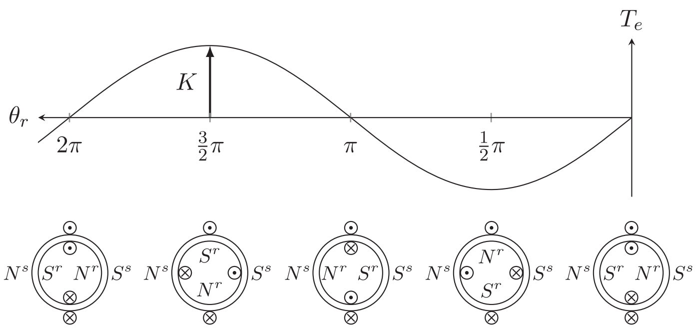

Figure 1.7-2 Electromagnetic torque versus angular displacement with constant winding currents.

Although the device being considered is not practical, Fig. 1.7-2, which depicts the torque and poles versus θ_r, allows us to describe some important aspects of electromechanical devices. Depending on the type of electric machine the steady-state torque is often plotted versus displacement (angle) or speed. Stable operating regions can be determined from these plots which are similar to Fig. 1.7-2. Remember a positive torque T_e will cause a positive movement in θ_r, a negative torque will cause a decrease in θ_r. If, for example, we hold the rotor at exactly $\theta_r = \pi$, the torque T_e on the rotor would be zero; however, what happens when we let loose of the rotor? Well, any slight movement in θ_r due to a disturbance would cause a positive T_e if θ_r increases slightly or a negative T_e if θ_r decreases slightly. Therefore, regardless of the direction of the initial movement, a torque would occur to move the rotor away from $\theta_r = \pi$; thus at this position the rotor is in unstable equilibrium. If, however, we position the rotor at either $\theta_r = 0$ or 2π and release the rotor it will remain in that position. That is, if a slight disturbance should cause θ_r to increase, the resulting negative increase in T_e would cause θ_r to decrease back to the zero T_e position ($\theta_r = 0$ or 2π). It follows that if the disturbance should cause a decrease in θ_r, a positive increase in T_e from zero would cause θ_r to increase back to its original position. With the positive directions of currents and θ_r that we have assumed and with the assumption of constant currents, stable operation occurs over the negative slope portion of the torque versus θ_r plot. Therefore, in Fig. 1.7-2 positive θ_r is from right to left, therefore stable operation occurs from $0 \le \theta_r < \frac{\pi}{2}$ and $\frac{3}{2}\pi < \theta_r \le 2\pi$. It is important to emphasize that if the currents change with a change of displacement or speed, then the region of stable operation may occur with the relative displacement between the stator and rotor poles greater than $\frac{\pi}{2}$. We will see this in the case of the electric machine considered in Chapters 2 and 4.

At $\theta_r = \frac{3}{2}\pi$ the torque is maximum positive and equal to

$$T_e = K = I_1 I_2 L_{sr} \tag{1.7-8}$$

which is motor action since positive torque is in the direction of counterclockwise rotation. This is the position of maximum torque per ampere.

Example 1F. Inductance equations with clockwise rotation. In Fig. 1F-1, θ_r is positive in the clockwise direction. The peak amplitude of the mutual inductance is L_{sr}. Express (a) the mutual inductance L_{ab} and (b) the electromagnetic torque T_e.

(a) The maximum negative coupling occurs when $\theta_r = -\frac{\pi}{6}$. Therefore,

$$L_{ab} = -L_{sr}\cos\left(\theta_r + \frac{\pi}{6}\right) \tag{1F-1}$$

(b) The coenergy or field energy is

$$W_c = \frac{1}{2}L_{aa}i_a^2 + L_{ab}i_a i_b + \frac{1}{2}L_{bb}i_b^2 \tag{1F-2}$$

Since the self-inductances L_{aa} and L_{bb} are constant, the torque is

$$\begin{aligned} T_e &= \frac{\partial W_c(i_a, i_b, \theta_r)}{\partial \theta_r} \\ &= L_{sr}\sin\left(\theta_r + \frac{\pi}{6}\right) i_a i_b \end{aligned} \tag{1F-3}$$

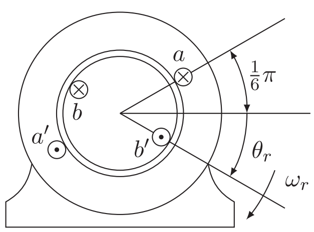

Figure 1F-1 A two-winding device with clockwise rotation.

SP1.7-1. In the system shown in Fig. 1.7-1, $L_{sr} = 0.1$ H, $i_1 = 2$ A, and $i_2 = 10$ A. (a) A torque of 1 N · m is applied in the clockwise direction. Calculate the steady-state value of θ_r. (b) Repeat for an applied torque of 2 N · m. [(a) $\theta_r = -30°$; (b) unstable]

SP1.7-2. Reverse the direction of i_1 in Fig. 1.7-1 and determine the range of stable operation for constant, positive currents. [$\frac{1}{2}\pi < \theta_r < \frac{3}{2}\pi$]

SP1.7-3. In Fig. 1.7-1, winding 1 is moved so that ⊗ is at three o'clock and ⊙ at nine. Express T_e. [$T_e = i_1 i_2 L_{sr}\cos\theta_r$]

SP1.7-4. The currents i_1 and i_2 are positive constants in the device shown in Fig. 1.7-1. (a) An external torque is applied to increase θ_r to 45° and then released. Assume damping exists. What is the final position of the rotor? Repeat for θ_r increased to (b) 90°, (c) 120°, (d) 180°, and (e) 210°. [(a) $\theta_r = 0$; (b) 0; (c) 0; (d) 0 or 2π; (e) 2π]

1.8 TWO- AND THREE-PHASE SYSTEMS

High voltage transmission, most inverter-supplied electric drives, and the alternator of your car are examples of three-phase systems. Although two-phase systems are not common, a two-phase system is far less involved when it comes to machine analysis than its three-phase big sister. Fortunately, once the derivations have been set forth for a two-phase machine the extension to a three-phase machine is straightforward and easily achieved. This section is devoted to the introduction of two- and three-phase systems.

By definition, a two-phase set of variables is balanced if the variables are equal-amplitude sinusoidal quantities in time quadrature (90° out of time phase). A three-phase set of variables is balanced if the sinusoidal variables are equal-amplitude quantities that are 120° out of time phase with each other.

Two-Phase Systems

In the broadest sense of the above definition, two-phase balanced sets may be expressed as

$$f_a(t) = \pm f(t)\cos\theta_{ef} \tag{1.8-1}$$

$$f_b(t) = \pm f(t)\sin\theta_{ef} \tag{1.8-2}$$

where

$$\theta_{ef}(t) = \int_0^t \omega_e(\xi)\,d\xi + \theta_{ef}(0) \tag{1.8-3}$$

In (1.8-1) and (1.8-2), $f(t)$ can represent voltage, current, or flux linkage. The amplitude may be any function of time; however, for steady-state balanced conditions $f(t)$ is a constant. In (1.8-3), ω_e is the electrical angular velocity and ξ is a dummy variable of integration. Equations (1.8-1) and (1.8-2) express four balanced two-phase sets. Like signs of (1.8-1) and (1.8-2) define balanced sets where $f_a(t)$ leads $f_b(t)$ by 90°, an *ab* sequence; for unlike signs $f_a(t)$ lags $f_b(t)$ by 90°, a *ba* sequence.

For steady-state balanced conditions ω_e is constant and (1.8-3) becomes

$$\theta_{ef}(t) = \omega_e t + \theta_{ef}(0) \tag{1.8-4}$$

Whereupon (1.8-1) and (1.8-2) are written as

$$F_a(t) = \pm\sqrt{2}F\cos\left[\omega_e t + \theta_{ef}(0)\right] \tag{1.8-5}$$

$$F_b(t) = \pm\sqrt{2}F\sin\left[\omega_e t + \theta_{ef}(0)\right] \tag{1.8-6}$$

For like signs of (1.8-5) and (1.8-6) $F_a(t)$ leads $F_b(t)$ by 90°; thus $\tilde{F}_a = j\tilde{F}_b$. For unlike signs $F_a(t)$ lags $F_b(t)$; thus $\tilde{F}_a = -j\tilde{F}_b$.

Most large horsepower electric machines are three phase and smaller household machines are single phase. Therefore, it can be argued that there is no need to treat two-phase devices. This is an understandable position; however, we will find that it is convenient to first analyze a two-phase device and then extend this to a three-phase device. This turns out to be much less involved and more instructive than to start the analysis with the three-phase device. Although this is perhaps not evident at this juncture, it will become very evident as we get into the later chapters.

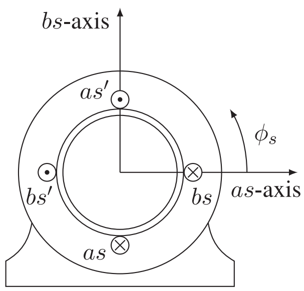

Figure 1.8-1 Elementary two-pole two-phase concentrated stator windings.

It is helpful to take a brief look at the stator winding arrangement of the two-phase machine, shown in Fig. 1.8-1. The windings are assumed to be concentrated as in Fig. 1.5-1. The displacement around the stator is denoted ϕ_s. The two windings are displaced 90° degrees from each other. The voltage equations may be written (dropping the functional notation)

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt} \tag{1.8-7}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt} \tag{1.8-8}$$

In matrix form

$$\mathbf{v}_{abs} = \mathbf{r}_s \mathbf{i}_{abs} + p\boldsymbol{\lambda}_{abs} \tag{1.8-9}$$

The stator windings are identical, and the air gap is uniform. The magnetic axes are orthogonal, (thus $L_{asbs} = 0$) and the flux linkage equations may be written

$$\begin{aligned} \lambda_{as} &= L_{asas} i_{as} \\ &= (L_{ls} + L_{ms}) i_{as} \end{aligned} \tag{1.8-10}$$

$$\begin{aligned} \lambda_{bs} &= L_{bsbs} i_{bs} \\ &= (L_{ls} + L_{ms}) i_{bs} \end{aligned} \tag{1.8-11}$$

where L_{ls} is the leakage inductance and L_{ms} is the magnetizing inductance of the stator windings. In matrix form

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \end{bmatrix} = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \end{bmatrix} \tag{1.8-12}$$

or

$$\boldsymbol{\lambda}_{abs} = \mathbf{L}_s \mathbf{i}_{abs} \tag{1.8-13}$$

where

$$\mathbf{L}_s = \begin{bmatrix} L_{ss} & 0 \\ 0 & L_{ss} \end{bmatrix} \tag{1.8-14}$$

and

$$L_{ss} = L_{ls} + L_{ms} \tag{1.8-15}$$

An important feature of multiphase systems is that the instantaneous power is constant for balanced operation. You are asked to show this in SP1.8-2. Recall that in a single-phase system the instantaneous power has an average value and a double frequency component.

Three-Phase Systems

A three-phase balanced set may be expressed as

$$f_a(t) = f(t)\cos\theta_{ef} \tag{1.8-16}$$

$$f_b(t) = f(t)\cos\left(\theta_{ef} - \frac{2}{3}\pi\right) \tag{1.8-17}$$

$$f_c(t) = f(t)\cos\left(\theta_{ef} + \frac{2}{3}\pi\right) \tag{1.8-18}$$

This set is referred to as an *abc* sequence, since $f_a(t)$ leads $f_b(t)$ by 120° and $f_b(t)$ leads $f_c(t)$ by 120°. An *acb* sequence is

$$f_a(t) = f(t)\cos\theta_{ef} \tag{1.8-19}$$

$$f_b(t) = f(t)\cos\left(\theta_{ef} + \frac{2}{3}\pi\right) \tag{1.8-20}$$

$$f_c(t) = f(t)\cos\left(\theta_{ef} - \frac{2}{3}\pi\right) \tag{1.8-21}$$

For steady-state balanced conditions, the *abc* sequence may be written

$$F_a(t) = \sqrt{2}F\cos[\omega_e t + \theta_{ef}(0)] \tag{1.8-22}$$

$$F_b(t) = \sqrt{2}F\cos\left[\omega_e t - \frac{2}{3}\pi + \theta_{ef}(0)\right] \tag{1.8-23}$$

$$F_c(t) = \sqrt{2}F\cos\left[\omega_e t + \frac{2}{3}\pi + \theta_{ef}(0)\right] \tag{1.8-24}$$

With $\tilde{F}_a = F\underline{/\theta_{ef}(0)}$, $\tilde{F}_b = F\underline{/-\frac{2}{3}\pi + \theta_{ef}(0)}$, and $\tilde{F}_c = F\underline{/+\frac{2}{3}\pi + \theta_{ef}(0)}$. For an *acb* sequence $\tilde{F}_b$ and $\tilde{F}_c$ are interchanged.

The windings of a three-phase stator are shown in Fig. 1.8-2. The magnetic axes of the windings are displaced 120° and the windings are often "wye-connected" as

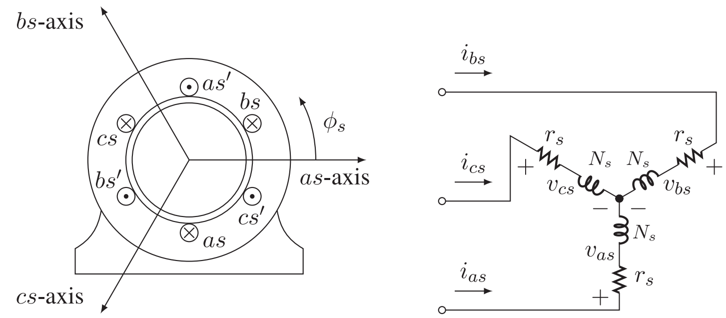

Figure 1.8-2 Elementary two-pole three-phase concentrated stator windings.

shown. We will talk more about three-phase connections later. The voltage equations may be written (again dropping the functional notation)

$$v_{as} = r_s i_{as} + \frac{d\lambda_{as}}{dt} \tag{1.8-25}$$

$$v_{bs} = r_s i_{bs} + \frac{d\lambda_{bs}}{dt} \tag{1.8-26}$$

$$v_{cs} = r_s i_{cs} + \frac{d\lambda_{cs}}{dt} \tag{1.8-27}$$

In matrix form

$$\mathbf{v}_{abcs} = \mathbf{r}_s \mathbf{i}_{abcs} + p\boldsymbol{\lambda}_{abcs} \tag{1.8-28}$$

Since the windings are displaced 120° from each other there is a mutual coupling between the stator windings. Recall that in Section 1.5 we approximated the mutual inductance between the two coils as $\cos\theta_r$. We can do something similar here. Let us assume that we can move to bs-winding clockwise through the iron until it is "on top" of the as-winding at $\phi_s = 0$. The coupling would be maximum positive. Now assume we can rotate the bs-winding counterclockwise back to $\phi_s = 120°$ where the mutual inductance between the as- and bs-windings would be

$$\begin{aligned} L_{asbs} &= L_{ms}\cos 120° \\ &= -\frac{1}{2}L_{ms} \end{aligned} \tag{1.8-29}$$

where L_{ms} is the magnetizing inductance of the stator windings. Following this same approach we can express the flux-linkage matrix as

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{ss} & -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & L_{ss} & -\frac{1}{2}L_{ms} \\ -\frac{1}{2}L_{ms} & -\frac{1}{2}L_{ms} & L_{ss} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \tag{1.8-30}$$

where

$$L_{ss} = L_{ls} + L_{ms} \tag{1.8-31}$$

Equation (1.8-30) may also be written as

$$\boldsymbol{\lambda}_{abcs} = \mathbf{L}_s \mathbf{i}_{abcs} \tag{1.8-32}$$

Example 1G. Voltage equations for a three-wire system. A three-phase stator similar to that given in Fig. 1.8-2 is connected to a three-phase source as shown in Fig. 1G-1. Assume the stator is symmetrical, that is, the windings have the same resistance and same number of turns and displaced 120°. The stator could be that of induction, synchronous, or brushless dc machines. The source voltages e_{ga}, e_{gb}, and e_{gc} may be of any form. Express v_{as}, v_{bs}, and v_{cs} in terms of e_{ga}, e_{gb}, and e_{gc}.

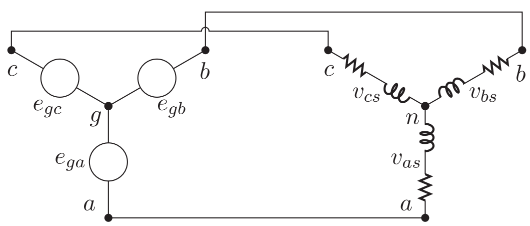

Figure 1G-1 Three-phase source connected to symmetrical stator windings.

From Fig. 1G-1 we can write

$$e_{ga} = v_{as} + v_{ng} \tag{1G-1}$$

$$e_{gb} = v_{bs} + v_{ng} \tag{1G-2}$$

$$e_{gc} = v_{cs} + v_{ng} \tag{1G-3}$$

where e_{ga} is a voltage rise and v_{as} and v_{ng} are voltage drops, etc. Adding (1G-1) through (1G-3) yields

$$e_{ga} + e_{gb} + e_{gc} = v_{as} + v_{bs} + v_{cs} + 3v_{ng} \tag{1G-4}$$

Let us look at $v_{as} + v_{bs} + v_{cs}$. From (1.8-25) through (1.8-27)

$$v_{as} + v_{bs} + v_{cs} = r_s(i_{as} + i_{bs} + i_{cs}) + p(\lambda_{as} + \lambda_{bs} + \lambda_{cs}) \tag{1G-5}$$

In a three-wire, wye-connected stator the sum of $i_{as} + i_{bs} + i_{cs}$ must be zero regardless of the form of the currents. Now from (1.8-30)

$$\lambda_{as} + \lambda_{bs} + \lambda_{cs} = L_{ss}(i_{as} + i_{bs} + i_{cs}) - L_{ms}(i_{as} + i_{bs} + i_{cs}) \tag{1G-6}$$

Thus,

$$v_{as} + v_{bs} + v_{cs} = 0 \tag{1G-7}$$

The question we have is will this be the case when we bring the rotor into play? We will find that for the electromechanical devices we will consider, it will be true. Substituting (1G-7) into (1G-4) yields

$$v_{ng} = \frac{1}{3}(e_{ga} + e_{gb} + e_{gc}) \tag{1G-8}$$

Going back to (1G-1) through (1G-3) we can write

$$v_{as} = e_{ga} - v_{ng}$$
$$= \frac{2}{3}e_{ga} - \frac{1}{3}(e_{gb} + e_{gc}) \tag{1G-9}$$
$$v_{bs} = e_{gb} - v_{ng}$$
$$= \frac{2}{3}e_{gb} - \frac{1}{3}(e_{gc} + e_{ga}) \tag{1G-10}$$
$$v_{cs} = e_{gc} - v_{ng}$$
$$= \frac{2}{3}e_{gc} - \frac{1}{3}(e_{ga} + e_{gb}) \tag{1G-11}$$

We will make use of these equations in Chapter 4 on electric drives.

SP1.8-1. In Fig. 1G-1, let $e_{ga} = 1$, $e_{gb} = 0$, $e_{gc} = \cos\omega_e t$. Determine v_{as}, v_{bs}, and v_{cs}. [$\frac{2}{3} - \frac{1}{3}\cos\omega_e t$; $-\frac{1}{3} - \frac{1}{3}\cos\omega_e t$; $\frac{2}{3}\cos\omega_e t - \frac{1}{3}$]. Note that $v_{as} + v_{bs} + v_{cs} = 0$.

SP1.8-2. In a two-phase system, let $V_a = \sqrt{2}V\cos[\omega_e t + \theta_{ev}(0)]$, $I_a = \sqrt{2}I\cos[\omega_e t + \theta_{ei}(0)]$, $V_b = \sqrt{2}V\sin[\omega_e t + \theta_{ev}(0)]$, and $I_b = \sqrt{2}I\sin[\omega_e t + \theta_{ei}(0)]$. Show that the total instantaneous power is $P = 2VI\cos[\theta_{ev}(0) - \theta_{ei}(0)]$.

REFERENCES

[1] P. C. Krause, O. Wasynczuk, and S. D. Pekarek, *Electromechanical Motion Devices*, 2nd edition, Wiley-IEEE Press, New York, 2012.
[2] D. C. White and H. H. Woodson, *Electromechanical Energy Conversion*, John Wiley and Sons, New York, 1959.

PROBLEMS

1. Using the same circuit parameters as given in Example 1A; $R = 6\,\Omega$, $L = 20\,\text{mH}$, and $C = 1 \times 10^3\,\mu\text{F}$, consider the circuit shown in Fig. 1.10-1. Let the 60 Hz source voltage be $\tilde{V}_a = 110\underline{/0^\circ}$. Determine all phasors and draw the phasor diagram.

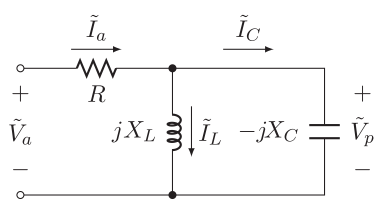

Figure 1.10-1 Series-parallel circuit.

2. For the magnetic system in Example 1B, let $V(t) = \sqrt{2}10\cos 377t$. Establish $I(t)$ and express $W_f(t)$.

3. During the short-circuit test of the transformer in Example 1C, which is a step-down distribution transformer, it was noticed that I_2 was 6 A. In re-doing the open-circuit test, the open-circuit current $|\tilde{I}_1|$ was not 1.05 A but 0.6 A. Upon checking the wiring, it was discovered that the lower terminal of N_1 was mistakenly connected to the top terminal of N_2 (See Fig. 1.4-1) and the 110 V source was connected between the top terminal of N_1 and the bottom terminal of N_2. Justify that $|\tilde{I}_1| = 0.6$ A. The equivalent "T" circuit will not yield the correct answer.

4. During the open-circuit test performed in Example 1C the rms voltage across the open-circuit 2-winding was 34.8 V. Determine X_{m2} ($\omega_e L_{m2}$).

5. Express the field energy W_f stored in the fields of the coils shown in Fig. 1.4-1 in terms of L_{l1}, L'_{l2}, L_{m1}, i_1, and i'_2. Identify the energy stored in the coupling field.

6. Winding 4 is added to the stator of the device given in Fig. 1D-1 as shown in Fig. 1.10-2. It is identical to winding 1 and displaced $\frac{\pi}{2}$ radians from it. Express L_{44}, L_{14}, L_{24}, L_{34}, and v_2. Let us mention in passing that the device in Fig. 1.10-2 has the winding configuration of an induction motor. There are many induction motors serving you in your home; furnace fan, garbage disposal, air conditioner, etc.

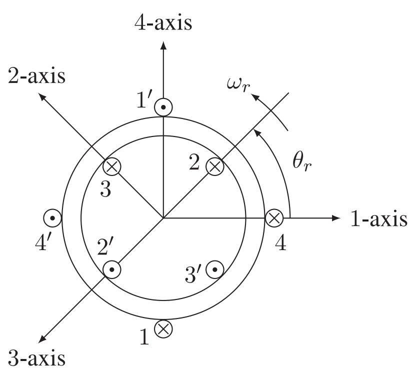

Figure 1.10-2 Two stator windings and two rotor windings.

7. Use the appropriate turns ratio to refer (1.5-1) through (1.5-4) to the 1-winding. Express v_1, v'_2, λ_1, and λ'_2 in terms of i_1, i'_2, L_{m1}, $L'_{\ell 2}$, $L_{\ell 1}$, r'_2, turns ratios, and θ_r.

8. Express the torque between windings 2 and 3 of Fig. 1D-1. Use the concept of virtual rotation with β, the angle between the positive magnetic axes of the windings.

9. Use the device depicted in Fig. 1.6-3 to show that the first two terms on the right-hand side of (1.6-16) sum to zero. That is, show that the coefficients of the di terms are equal.

10. Consider the device shown in Fig. 1F-1. (a) Identify the stable operating region for i_a and i_b positive constants. (b) Identify the stable region if i_b is a negative constant. (c) Reverse the direction of positive current in the rotor winding and identify the region of stable operation for constant positive currents.

11. Consider the device shown in Fig. 1.10-3. The 3-winding which is identical to the 1-winding has been added to the stator of Fig. 1.7-1. The device now has the winding configuration of an elementary synchronous machine. Express $T_e(i_1, i_2, i_3, \theta_r)$.

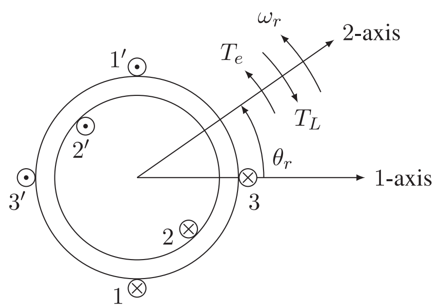

Figure 1.10-3 The stator windings and one rotor winding.

12. Consider Example 1G and Fig. 1G-1. The load is symmetrical and the source voltages are given in Fig. 1.10-4. (a) Plot v_{ng}, v_{as}, v_{bs}, and v_{cs}. (b) Connect n to g in Fig. 1G-1 and repeat part (a).

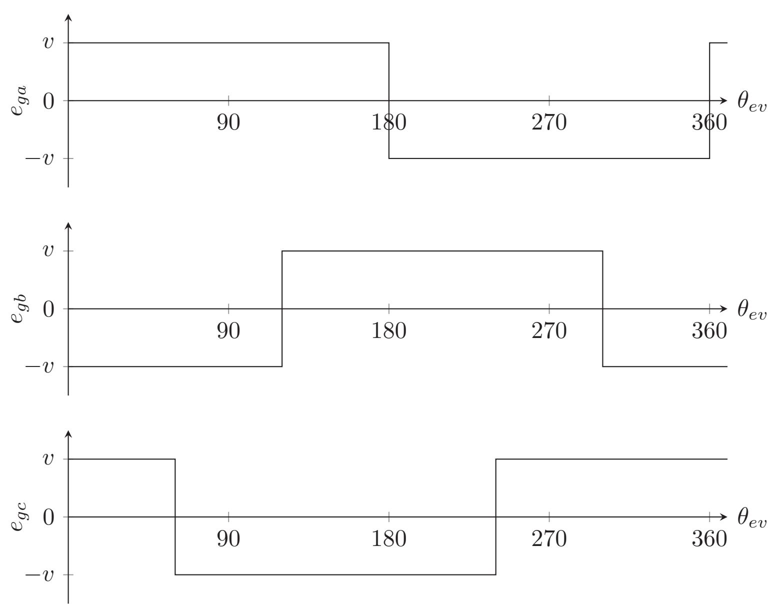

Figure 1.10-4 Waveforms of the source voltages of Fig. 1G-1.