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Introduction

1.1 Motivation and Challenges

The current book investigates emerging applications of multiagent cooperative control. It is motivated by the ubiquity of networked systems and the need to control their behaviors for real-world applications. We first review collective behaviors and then introduce major technical challenges in cooperative control.

1.1.1 From Collective Behaviors to Cooperative Control

Collective behaviors are observed in natural systems. Groups of ants create colony architectures that no single ant intends. Populations of neurons create structured thought, permanent memories, and adaptive responses that no neuron can comprehend by itself. In the study of collective behaviors, usually some types of agent-based-models are expressed with mathematical and computational formalisms, and the descriptive model is capable of quantitative and objective predictions of the system under consideration. The descriptive equations of fish schools and other animal aggregations were proposed in Ref. [1] in the 1950s, and it is more than three decades later that renewed mainstream attention has been received in a range of fields—including computer graphics, physics, robotics, and controls. A distributed behavior model, which is based on the individual agent's motion, is built by Reynolds [2] and computer simulations are done therein for flock-like group motion. Individual-based models and simulation of collective behaviors are also addressed in Ref. [3] with discussions of collective effects of group characteristics. Simulated robots are used in Ref. [4] to simulate collective behaviors where different types of group motions are displayed. While the aforementioned work is mainly on descriptive models and simulated behaviors, controlling the movement of a group using simulated robots with dynamic motion is addressed in Ref. [5]. Collective behaviors such as seen in herds of animals and biological aggregations are also referred to as swarming in the literature. Models of swarming are discussed in Refs. [6, 7], where attraction–repulsion interactions

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are included in the system's dynamics. Stability analysis of swarms is given in Refs. [8, 9] based on certain artificial interaction forces. The research has progressed rapidly in recent years from modeling and simulation of specific examples toward a more fundamental explanation applicable to a wide range of systems with collective behaviors.

In physics, the phenomenon of collective synchronization, in which coupled oscillators lock to a common frequency, was studied in the early work [10, 11]. In the 1970s, Kuramoto proposed a tractable model (referred to as the Kuramoto model) for oscillator synchronization [12, 13]. A related problem, the collective motion and phase transition of particle systems, is considered from the perspective of analogies to biologically motivated interactions in Refs. [14, 15] where simulated behaviors are presented. The studied models are capable of explaining certain observed behaviors in biological systems, including collective motion (rotation and flocking) of bacteria, networks of pacemaker cells in the heart, circadian pacemaker cells in the nucleus of the brain, metabolic synchrony in yeast cell suspensions, and physical systems such as arrays of lasers and microwave oscillators. Despite 40 years having elapsed since Kuramoto proposed his important model, there remain important theoretical aspects of the collective motion that are not yet understood; see Ref. [16] for a review on the topic.

More recently, the phase transition behavior described by Vicsek and coauthors [15] was revisited and theoretically explained by Jadbabaie et al. [17]. Their work is significant since it provides a graph theory–based framework to analyze a group of networking systems. Since then, coordination of mobile agents has received considerable attention. The consensus problem, which considers the agreement upon certain quantities of interest, was posed and studied by Olfati-Saber and Murray [18]. Here, the network topology was explicitly configured and the relationship between this topology and the system convergence was addressed using graph theory–based methods. The problem is further studied in Refs. [19–22], and necessary and sufficient conditions are given for a networked system to achieve consensus with the switching topology (see a survey [19]). Subsequent studies extended the principles of cooperative control to applications related to vehicle systems, for example, in Refs. [23–29]. Since then, cooperative control has gone through periods of rapid development [30–33].

1.1.2 Challenges

Despite rapid development, the field of cooperative control is far from mature. Major technical challenges arise from system dynamics and network complexity, which include the following:

- *Nonlinear agent dynamics*: Most agent systems are nonlinear dynamic systems. For example, cooperating robot vehicles, such as ground, aerial, and underwater vehicles, are *nonlinear* dynamic systems: the states of

the system vary in time in complicated ways. Most existing cooperative control based on graph theory methods assumes single integrator or simple linear dynamics, which is not adequate for real-world applications where the performance of the designed control system can deviate greatly from the performance suggested by these simplified system models. Design of cooperative control for nonlinear systems is not a trivial task. There is no general framework available for nonlinear cooperative control.

- *Nonlinear agent interactions*: In many natural systems, the adhesive and repulsive forces among agents are nonlinear. For example, the repulsive force between two agents may need to become very large (and approach infinity) when they are very close in order to avoid collisions. Similarly, when the distance between them is greater than a threshold, the repulsive force may either become very small or vanish. Most existing cooperative control framework addresses linear agent interactions, while many real-world multiagent systems, such as nanoscale particle systems, have complicated nonlinear interactions, for example, Morse-type interactions. New methodologies are called upon to solve cooperative control problems to support systems with more general (nonlinear) agent interactions.
- *Robustness*: Due to uncertainties in agent dynamics, communication links, and operating environments, robustness has to be considered for a successful system design. For example, uncertainties in the communication links of cognitive radio networks (CRNs) include time delay of information exchange and time-varying and/or switching of the network connectivity. Such uncertainties can lead to unexpected or perhaps unstable behaviors. Robustness consideration has been discussed for the basic consensus problem in existing work, but general robust cooperative control for complicated real-world systems has not been adequately addressed. The ultimate goal is that, under a well-designed control scheme, the closed-loop networked system will be tolerant of and robust to network and environment disturbances.
- *Diversity of real-world problems and application domains*: Networked systems are becoming increasingly ubiquitous. Depending on the domain of applications, the control objectives and constraints are inherently different. For example, control of nanoscale particle systems has strict confinement constraints, the system is not readily accessible, and not all particles can be targeted or controller individually. In addition feedback control is very difficult to implement since the characteristic time is usually shorter than that of the available control devices. Although cooperative control has provided analysis methods and synthesis tools that were successfully applied to real-world systems such as autonomous vehicle systems at the macro- and microscales, cooperative control in other application domains such as the nanoscale systems has not been fully explored yet. New real-world problems and application domains pose new challenges for nonlinear cooperative control design.

1.2 Background and Related Work

The book addresses real-world applications of cooperative control in three application domains: networked communication systems, cooperating multirobotic systems, and multiagent physics systems. In this section, we provide background and related work for each of the application domains.

1.2.1 Networked Communication Systems

Part I studies distributed consensus for networked communication systems. In particular, after presenting average consensus for quantized communication, two emerging applications of distributed consensus in CRNs will be discussed, which include distributed spectrum sensing and radio environment mapping (REM).

CRNs are an innovative approach to wireless engineering in which radios are designed with an unprecedented level of intelligence and agility. This advanced technology enables radio devices to use spectrum (i.e., radio frequencies) in entirely new and sophisticated ways. Cognitive radios have the ability to monitor, sense, and detect the conditions of their operating environment and also dynamically reconfigure their own characteristics to best match those conditions [34].

1.2.1.1 Cooperative Spectrum Sensing

Due to rapidly growing demands of emerging wireless services and new mobile applications for anytime and anywhere connectivity in our daily lives, we expect to face a shortage of wireless spectrum. However, this spectrum-shortage problem is reported to be rooted in the conventional static spectrum-allocation policy where only licensed devices can operate on a designated spectrum band. For example, according to the report from the Shared Spectrum Company, only an average of 5.2% of wireless spectrum under 3 GHz was actively used, indicating that a large fraction of spectrum bands were unutilized or underutilized at any given location and time.

CRNs have emerged as an enabling technology to mitigate the spectrum-scarcity problem. In CRNs, unlicensed (or secondary) devices/users can opportunistically access temporarily available licensed spectrum bands, that is, spectrum bands not being used by the primary users. As a first step toward realization of the new concept of opportunistic spectrum access, the Federal Communications Commission (FCC) has approved the operation of unlicensed cognitive radio (CR) devices in ultra high frequency (UHF) bands (a.k.a. TV white spaces). The first standardization effort based on this CR technology, that is, the IEEE 802.22 wireless regional area networks, is also in its final stage of development [35, 36]. Thus, the openness of the lower-layer protocol stacks in CR and their subsequent ability to adapt their waveforms make them an appealing solution to dynamic spectrum access and alleviate the spectrum-scarcity problem.

Accurate and robust spectrum sensing is essential to spectral efficiency in CRNs. Conventional centralized cooperative spectrum sensing requires that the entire received data be gathered at one place, which may be difficult due to communication constraints [37]. In particular, the multi-hop communication channel requirements of the relay-assisted sensing may bring extra power cost and the sensing data quality may degrade during the multi-hop communication paths. Since future CRNs will consist of heterogeneous devices such as smartphones, tablets, and laptops moving with the collective behaviors of people, consensus-based distributed spectrum sensing [38, 39] reveals great potential for future development of distributed CRNs due to one-hop communication, self-organization, and scalable network structure. Chapter 4 presents a new weighted average consensus approach for distributed spectrum sensing.

1.2.1.2 Radio Environment Mapping

REM is first proposed in Ref. [40]. REM information to the CRNs is like the GPS traffic density map for car drivers. Car drivers with traffic density information can choose better routes to avoid traffic jam. CRNs with REM information will improve the utilization of the dynamic spectrum resources. The REM covers a wide range of functions as an integrated database that provides multidomain environmental information and prior knowledge for CRs, such as the geographical features, available services and networks, locations and activities of neighboring radios, and so on. Among those, one of the most fundamental features is the heat map estimation and tracking, such as power spectral density map estimation [41] or, as an alternative, the channel gain estimation [42] and tracking [43].

There are mainly two different types of methodologies for REM. The traditional method is to detect the existence of signal sources, estimate their number, location and parameters, and then estimate the radio effect they induce in their space (i.e., the field) based on signal propagation models. The second method is referred to as direct methods for field estimation, which is to estimate the field without resorting to source identification. In the book, we adopt the second approach to construct REM efficiently in real time. Most existing work on REM uses centralized methods, where a central data collection and processing machine is available to generate the global radio map. Similar to the centralized spectrum sensing problem, those methods suffer the dependency of reporting channels, bandwidth constraints, and scalability issues [44]. There has been limited work on distributed solutions to the REM problem without a central station. In Chapter 5, we will provide new distributed consensus filter-based methods for distributed cooperative estimation of REM.

1.2.2 Cooperating Autonomous Mobile Robots

Part II considers cooperative control of distributed multirobotic systems. Using multirobotic systems rather than a single robot can have several advantages. For

example, collectives of simple robots may be simpler in physical design than a large, complex robot, providing opportunities for systems that are more economical, more scalable, and less susceptible to overall failure. Also, through distributed sensing and action, multirobotic systems have the ability to solve problems that are inherently distributed in space, time, or functionality. Perhaps most important, technologies have advanced to the point where mobile, autonomous robot collectives are technically feasible at reasonable prices. A collection of autonomous robots is described as a swarm [45], a colony [46, 47], a collective [48], or robots exhibiting cooperative behaviors [49]. Most work in cooperative mobile robotics began after the introduction of the new robotics paradigm of behavior-based control [50, 51], which is rooted in biological inspirations. Researchers found it instructive to examine the social characteristics of insects and animals, and to apply the findings to the design of multirobot systems [52]. Interested readers are referred to Refs. [53–55] for reviews on multirobot systems.

Early work in the field of distributed robotics demonstrated the use of the simple local control rules of various biological societies—particularly ants, bees, and birds—to the development of similar behaviors in cooperating robot systems [56, 57]. Such systems can be used in applications including search and rescue [58], satellite clustering [59, 60], formation flight [61], formation flying of spacecraft [62, 63], platoon of underwater vehicle [64], cooperative hunting [65], and mobile sensor network [66]. The control strategies for cooperative robots can be organized into three different approaches: behavior-based approaches, virtual structure approaches, and leader-following approaches.

Behavior-based approaches include the work of Parker [67] and of Balch and Arkin [68]. Parker [67] proposed a software architecture for fault-tolerant multirobot cooperation, which incorporates the use of mathematically modeled motivations within each robot to achieve adaptive action selection. Formation keeping is studied within behavior-based framework using motor schemas by Balch and Arkin [68]. The virtual structure approach was proposed and applied to formations of mobile robots by Lewis and Tan [69]; here, the idea is to force an ensemble of robots to behave as if they were particles embedded in a rigid structure. Virtual structure–based methods are also used by Beard and coauthors in [63] for spacecraft formation control, and by Egerstedt and Hu [70]. Leader-following strategies are reported in Refs. [71–73] and the references therein. In comparing the approaches, coordination is achieved through different types of shared knowledge. In the behavioral-based approach, shared knowledge of the relative configuration states are used to achieve coordination. In the virtual structure approach, coordination is achieved through shared knowledge of the states of the virtual structure. In the leader-following approach, coordination is achieved through shared knowledge of the leader's states.

More recently, a graph theorem–based approach has been suggested and its applications to formation control, rendezvousing, and flocking are studied. Studies using the "consensus" concept includes work in Refs. [23, 74, 75], where stability property of a group of agents is connected to the information flow structure characterized by a communication graph. Similar results are presented independently in Refs. [20, 26], where the dynamics of a unicycle robot is considered. Results in Refs. [76, 77] present general formation control frameworks that apply to robot vehicles with high-order linear dynamics. Various results on cooperative control using graph and system theory–based methods can be found in Refs. [29, 66, 78, 79]. Complementary to graph theory–based methods, Refs. [22a and 27], apply matrix theory–based methods and present cooperative and formation control results for general dynamic vehicle systems in their linear canonical forms.

Despite extensive efforts from the robotics and controls communities to develop distributed control methods solving formation control and other aforementioned multirobot cooperation tasks, new emerging applications call for more advanced robotics control techniques. One of such demands is to respond to the recent Deepwater Horizon oil spill and use ocean robots to detect, monitor, and track the propagation of oil plumes. In Part II, we present new cooperative control methods for multirobot systems to conduct cooperative tasks including source seeking and plume tracking.

1.2.3 Nanoscale Systems and Laser Synchronization

Part III addresses distribute control of multiagent physics systems, including coupled nanoparticle arrays and coupled laser arrays, both of which present unique challenges that have not be addressed before in nonlinear cooperative control.

1.2.3.1 Control of Nanoscale Systems

The integration of physics principles of macroscopic mechanical systems with control mechanisms and control principles has a rich history, enabling new applications and establishing new research directions. Recent technological advances have allowed the scaling of basic mechanical structures to ever smaller dimensions, including the microelectromechanical system (MEMS) technologies that have evolved over the past few decades and the more recent further scaling of mechanical structures to the nanorange. Early work on MEMS devices quickly demonstrated that the physical behaviors of the microscale elements were quite different from those familiar in macroscale components. This led to an intense period of research and experimentation, seeking to establish the physics principles associated with the microscale mechanical elements and to achieve means of controlling the motion of those elements. These studies led to technologies capable of creating microstructures with predicable and controllable behaviors suitable for practical applications.

More recently, technologies have moved into the nanoscale regime, with basic electronic, optoelectronic, sensor, mechanical, and other components enabling many new applications emerging. At these even smaller scales, nanodevices and nanostructures exhibit behaviors different from those seen at the microscale. The behavior of mechanical nanostructures reflects the approaching of structures with a number of atoms, with physics principles migrating into quantum and interacting atom regimes. Ultimately, technologies for these nanostructures will advance to the point where major applications will become routine. However, the application of these nanomechanical structures will require an integration of the new physics principles for their behavior with new control theoretical principles appropriate for the new behaviors exhibited by nanostructures. Similar to the case of MEMS, progress will require an integration of new principles drawing upon emerging experimental results on nanostructures. The availability of new tools and the development of new techniques in handling nanoparticles give unprecedented opportunity for theoretical breakthrough toward controllable nanoscale systems. In particular, scanning probes can control and visualize molecular motion that was not possible before, and nanofabrication technology can now make nanoscale features envisioned for the on-chip infrastructure.

Control at the nanoscale presents many challenges. Due to strict confinement and additional constraints, nanosystems are not readily accessible, and not all particles can be targeted or controlled individually. Also, the system dynamics is highly nonlinear; feedback control may be difficult to implement since the characteristic time may be shorter than that of the available control devices. In addition, particle interaction is intrinsic for nanoscale systems due to the interparticle potentials of various physical origins (e.g., the van del Waals' interactions). Although nonlinear control theory has provided analysis methods and synthesis tools that were successfully applied to dynamic systems at the macro- and microscales, control of nanoscale systems from the perspective of nonlinear control theory has not been adequately addressed yet. Chapter 8 presents control of coupled nanoparticles that slide on a surface, and the control goal is to achieve smooth sliding, thereby reducing friction.

1.2.3.2 Laser Synchronization

For many energy-related applications, it is important to concentrate significant amount of energy in a tiny spot. Lasers are naturally one of the more popular light concentration devices and, consequently, are used as a source for directed energy, space and fiber communication, welding, cutting, fusion, and so on. Lasers are also used for clean energy production and can completely operate on solar energy [80–83].

While for some applications a single high beam quality laser can provide sufficient intensity, for others it is imperative to combine lasers into arrays. A coherent beam combination from N lasers on an array will result in total

output intensity that scales as the square number of lasers (N^2). However, a coherent combination requires phase synchronization in lasers and does not naturally occur in laser arrays.

While Kuramoto model describes synchronization behaviors of coupled *phase* oscillators, *coupled laser arrays* have a highly nonlinear model and represent a complex system with both technological [84] and theoretical (Refs. [85–89]) importance. Synchronized or phase-locked state, where all the lasers oscillate at a common frequency *and* with fixed phase relationships, is sought from an applied perspective, because such coherent arrays generate much greater power than a single laser. From a theoretical perspective, laser arrays provide a prime example of a system of coupled limit-cycle oscillators, which connects to explorations of pattern formation and many other topics throughout physics, chemistry, biology, and engineering [90, 91].

Most existing theoretical work on laser arrays assumes identical lasers [92–95]. But real devices are nonidentical and have different intrinsic frequencies [93]. Previous work has studied the system's collective behaviors depending on the parameters of the coupling strength, the pump strength, and the width of the distribution of natural frequencies Refs. [85, 86, 88, 96–98]. For intermediate coupling, the dynamics becomes more complicated. In fact, numerical simulations reveal various unsteady collective states between incoherence and phase locking [96]. Global coupling governed by a complex coupling parameter was investigated for a large system of nonidentical lasers in Ref. [98]. So far, there is no systematic analysis of the complex behaviors exhibited by coupled semiconductor lasers with nonidentical parameters, especially the effects of dynamic coupling and different topologies of the network (neither nearest neighbor nor global) are unknown. In Chapter 9, we present new synchronization results for coupled semiconductor laser arrays using recent distributed cooperative control tools; synchronization conditions are characterized with rigorous mathematical proof.

1.3 Overview of the Book

The rest of the book is organized as follows. Chapter 2 reviews fundamental consensus and consensus filter techniques. The remaining chapters are organized in three parts, each of which represents a specific application domain of distributed cooperative control. Part I considers applications in networked communication systems, and includes three chapters to study average consensus in quantized communication, cooperative spectrum sensing, and distributed REM, respectively. Part II deals with distributed multirobotic systems, and includes two chapters to investigate source seeking and dynamic plume tracking by cooperating mobile robots, respectively. Part III presents distributed cooperative control for multiagent physics systems,

and includes two chapters to discuss friction control of nanoparticle arrays and synchronizing coupled laser arrays, respectively. We highlight the contribution of each chapter in the following text.

Chapter 2 introduces the concepts of distributed consensus and consensus filters, which are the cornerstones of modern cooperative control techniques. Brief literature review on both distributed consensus and consensus filters are given. Then, following the introduction of graph theory preliminaries, basic distributed consensus protocols are given in both continuous-time and discrete-time formulations. Distributed consensus filters are also reviewed in both continuous-time and discrete-time formulations for the proportional-integral (PI) average consensus filters, which has better convergence performances than other early consensus filter protocols. It is not our intention to provide a comprehensive review on this rich topic. This chapter presents necessary background for proceeding to the following chapters on applications.

Chapter 3 studies average consensus for directed graphs with quantized communication under fixed and switching topologies. In the presence of quantization errors, conventional consensus algorithms fail to converge and may suffer from an unbounded asymptotic mean square error. Robust consensus algorithms are developed in the chapter to reduce the effect of quantization. Specifically, a robust weighting matrix design is introduced that uses the H_∞ performance index to measure the sensitivity from the quantization error to the consensus deviation. Linear matrix inequalities are used as design tools. The mean-square deviation is proven to converge, and its upper bound is explicitly given in the case of fixed topology with probabilistic quantization. Numerical results demonstrate the effectiveness of this method.

Chapter 4 discusses distributed spectrum sensing in CRNs. Existing distributed consensus-based fusion algorithms only ensure equal gain combining of local measurements, whose performance may be incomparable to various centralized soft combining schemes. Motivated by this fact, practical channel conditions and link failures are considered in the chapter, and new weighted soft measurement combining technique is developed without a centralized fusion center. Following the measurement by its energy detector, each secondary user exchanges its own measurement statistics with its local one-hop neighbors, and chooses the information exchanging rate according to the measurement channel condition, for example, the signal-to-noise ratio. Convergence of the new consensus algorithm is rigorously proved, and it is shown that all secondary users hold the same global decision statistics from the weighted soft measurement combining throughout the network. The chapter also provides distributed optimal weight design under uncorrelated measurement channels. The convergence rate of the consensus iteration is given under the assumption that each communication link has an independent probability to fail, and the upper bound of the iteration number of the ϵ-convergence is explicitly given as a function of system parameters. Simulation results show

significant improvement of the sensing performance compared to existing consensus-based approaches, and the performance of the distributed weighted design is comparable to the centralized weighted combining scheme.

Chapter 5 presents distributed estimation and tracking for REM. Compared to existing REM using centralized methods, a distributed solution eliminating the central station is provided for map construction. Based on the random field model of the REM with shadow fading effects, consensus-based Kalman filter design is adopted to estimate and track the temporal dynamic REM variation. The unknown parameters of REM temporal dynamics are estimated by a distributed expectation maximization algorithm that is incorporated with Kalman filtering. The proposed approach features distributed Kalman filtering with unknown system dynamics and achieves dynamic REM recovery without localizing the transmitter. Simulation results show satisfactory performances of the proposed method where spatial correlated shadowing effects are successfully recovered.

Chapter 6 considers the problem of source seeking using a group of mobile robots equipped with sensors for source concentration measurement. In the formulation, the robot team cooperatively estimates the gradient of the source field, moves to the source by tracing the gradient-ascending direction, and keeps a predefined formation in movement. Two control algorithms are presented in the chapter with all-to-all and limited communications, respectively. For the case of all-to-all communication, rigorous analytic analysis proves that the formation center of the robots converges to the source in the presence of estimation errors with a bounded error, the upper bound of which is explicitly given. In the case of limited communication where centralized quantities are not available, distributed consensus filters are used to distributively estimate the centralized quantities, and then embedded in the distributed control laws. Numerical simulations are given to validate the effectiveness of the proposed approaches. Experimental results on the E-puck robot platform demonstrate satisfactory performances in a light source-seeking scenario.

Chapter 7 presents robotic tracking of dynamic plume front modeled by the advection–diffusion equation. Different from existing work purely relying on gradient measurement, the transport model of pollution source is explicitly considered in tracking control design. The problem using a single robot is first studied and solved in an estimation and control framework. It is then extended to the multirobot case in a nearest-neighbor communication topology, where the robots form formation while patrolling along the plume front. The distributed control is scalable to a large number of robots. Simulation results show satisfactory performances of the proposed method.

Chapter 8 studies sliding friction of a nanoparticle array. While the problem is approached by chemical means traditionally, a recent approach has received increasing attention to control the system mechanically to tune frictional responses. In the chapter, feedback control laws is explicitly designed for a one-dimensional particle array sliding on a surface subject to friction. The

Frenkel–Kontorova model describing the dynamics is a nonlinear interconnected system and the accessible control elements are average quantities only. Local stability of equilibrium points of the uncontrolled system is proved in the presence of linear and nonlinear particle interactions, respectively. A tracking control problem is then formulated, whose control objective is for the average system to reach a designated targeted velocity using accessible elements. Sufficient stabilization conditions are explicitly derived for the closed-loop error systems using the Lyapunov theory–based methods. Simulation results show satisfactory performances. The results can be applied to other physical systems whose dynamics is described by the Frenkel–Kontorova model.

Chapter 9 considers synchronization of coupled semiconductor lasers modeled by coupled Lang and Kobayashi equations. Decoupled laser stability is first analyzed, and synchronization conditions of coupled laser dynamics is then characterized. It is rigorously proven that the coupled system locally synchronizes to a limit cycle under the coupling topology of an undirected connected graph with equal in-degrees. Graph and systems theory is used in synchronization analysis. The results not only contribute to analytic understanding of semiconductor lasers but also advance cooperative control by providing a real-world system of coupled limit-cycle oscillators.

References

1 Breder, C.M. (1954) Equations descriptive of fish schools and other animal aggregations. *Ecology*, **35**, 361–370.
2 Reynolds, C.W. (1987) Flocks, herds, and schools: A distributed behavioral model. *Computer Graphics*, **21** (4), 25–34.
3 Kunz, H. and Hemelrijk, C.K. (2003) Artificial fish schools: Collective effects of school size, body size, and body form. *Artificial Life*, **9**, 237–253.
4 Baldassarre, G., Nolfi, S., and Parisi, D. (2003) Evolving mobile robots able to display collective behaviors. *Artificial Life*, **9**, 255–267.
5 Brogan, D. and Hodgins, J.K. (1997) Group behaviors for systems with significant dynamics. *Autonomous Robots*, **4**, 137–153.
6 Mogilner, A. and Edelstein-Keshet, L. (1999) A non-local model for a swarm. *Journal of Mathematical Biology*, **38**, 534–570.
7 Topaz, C.M. and Bertozzi, A.L. (2004) Swarming patterns in a two-dimensional kinematic model for biological groups. *SIAM Journal of Applied Mathematics*, **65** (1), 152–174.
8 Gazi, V. and Passino, K.M. (2003) Stability analysis of swarms. *IEEE Transactions on Automatic Control*, **48** (4), 692–696.
9 Gazi, V. and Passino, K. (2004) Stability analysis of social foraging swarms. *IEEE Transactions on Systems, Man, and Cybernetics–Part B: Cybernetics*, **34** (1), 539–557.

10 Wiener, N. (1958) *Nonlinear Problems in Random Theory*, MIT Press, Cambridge, MA.
11 Winfree, A.T. (1967) Biological rhythms and the behavior of populations of coupled oscillators. *Journal of Theoretical Biology*, **16** (1), 15–42.
12 Kuramoto, Y. (1975) Self-entrainment of a population of coupled non-linear oscillators, in *International Symposium on Mathematical Problems in Theoretical Physics*, Lecture Notes in Physics, (ed. H. Arakai), Springer, New York, p. 420.
13 Kuramoto, Y. and Nishikawa, I. (1989) Onset of collective rhythms in large populations of coupled oscillators, in *Cooperative Dynamics in Complex Physical Systems* (ed. H. Takayama), Springer, Berlin.
14 Shimoyama, N., Sugawara, K., Mizuguchi, T., Hayakawa, Y., and Sano, M. (1996) Collective motion in a system of motile elements. *Physical Review Letters*, **76** (20) 3870–3873.
15 Vicsek, T., Czirok, A., Ben-Jacob, E., Cohen, I., and Shochet, O. (1995) Novel type of phase transition in a system of self-driven particles. *Physical Review Letters*, **75** (6), 1226–1229.
16 Strogatz, S.H. (2000) From Kuramoto to Crawford: Exploring the onset of synchronization in populations of coupled oscillators. *Physica D*, **143**, 1–20.
17 Jadbabaie, A., Lin, J., and Morse, A. (2003) Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Transactions on Automatic Control*, **48** (6), 988–1001.
18 Olfati-Saber, R. and Murray, R.M. (2004) Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, **49** (9), 1520–1533.
19 Ren, W., Beard, R.W., and Atkins, E.M. (2005) A survey of consensus problems in multi-agent coordination, in *Proceedings of American Control Conference*, Portland, OR, pp. 1859–1864.
20 Lin, Z., Francis, B., and Maggiore, M. (2004) Local control strategies for groups of mobile autonomous agents. *IEEE Transactions on Automatic Control*, **49** (4), 622–629.
21 Moreau, L. (2005) Stability of multi-agent systems with time-dependent communication links. *IEEE Transactions on Automatic Control*, **50**, 169–182.
22 Qu, Z., Wang, J., and Hull, R.A. (2008) Cooperative control of dynamical systems with application to autonomous vehicles. *IEEE Transactions on Automatic Control*, **53** (4), 894–911.
23 Fax, J.A. and Murray, R.M. (2004) Information flow and cooperative control of vehicle formation. *IEEE Transactions on Automatic Control*, **49** (9), 1465–1476.
24 Khatir, M.E. and Davison, E.J. (2004) Cooperative control of large systems, in *Cooperative Control, Lecture Notes in Control and Information Sciences*,

vol. 309 (eds V. Kumar, N.E. Leonard, and A.S. Moore), Springer-Verlag, Berlin/Heidelberg, pp. 119–136.
25 Ren, W. and Beard, R.W. (2005) Consensus seeking in multiagent systems under dynamically changing interaction topologies. *IEEE Transactions on Automatic Control*, **50** (5), 655–661.
26 Lin, Z., Francis, B., and Maggiore, M. (2005) Necessary and sufficient graphical conditions for formation control of unicycles. *IEEE Transactions on Automatic Control*, **50** (1), 121–127.
27 Qu, Z., Wang, J., and Hull, R. (2005) Leadless cooperative formation control of autonomous mobile robots under limited communication range constraints, in *Cooperative Control and Optimization*, Gainesville, FL.
28 Lafferriere, G., Caughman, J., and Williams, A. (2004) Graph theoretic methods in the stability of veicle formations, in *Proceedings of the American Control Conference*, Boston, pp. 3729–3734.
29 Tanner, H.G., Jadbabaie, A., and Pappas, G.J. (2004) Flocking in teams of nonholonomic agents, in *Cooperative Control*, *Lecture Notes in Control and Information Sciences*, vol. 309 (eds V. Kumar, N.E. Leonard, and A.S. Moore), Springer-Verlag, Berlin/Heidelberg, pp. 229–239.
30 Qu, Z. (2009) *Cooperative Control of Dynamical Systems: Applications to Autonomous Vehicles*, Springer-Verlag, London.
31 Bai, H. and Arcak, M. (2011) *Cooperative Control Design: A Systematic, Passivity-Based Approach*, Springer, New York.
32 Ren, W. and Cao, Y. (2011) *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*, Springer-Verlag, London.
33 Lewis, F.L., Zhang, H., Hengster-Movric, K., and Das, A. (2014) *Cooperative Control of Multi-agent Systems: Optimal and Adaptive Design Approaches*, Springer, London.
34 Mitola, J. (2000) *An integrated agent architecture for software defined radio*, Ph.D. thesis, The Royal Institute of Technology, KTH.
35 Stevenson, C., Chouinard, G., Lei, Z., Hu, W., Shellhammer, S., and Caldwell, W. (2009) IEEE 802.22: The first cognitive radio wireless regional area network standard. *Communications Magazine, IEEE*, **47** (1), 130–138.
36 IEEE 802.22, Working Group on Wireless Regional Area Networks (WRANs). Http://grouper.ieee.org/groups/802/22/ (accessed September 21, 2016).
37 Mishra, S., Sahai, A., and Brodersen, R. (Istanbul, 2006) Cooperative sensing among cognitive radios, in *IEEE International Conference on Communications*, vol. 4, pp. 1658–1663.
38 Yildiz, M.E., Aysal, T.C., and Barner, K.E. (2009) In-network cooperative spectrum sensing, in *EURASIP European Signal Processing Conference*, Glasgow, Scotland.
39 Li, Z., Yu, F.R., and Huang, M. (2010) A distributed consensus-based cooperative spectrum-sensing scheme in cognitive radio. *IEEE Transaction on Vehicular Technology*, **59** (1), 383–393.

40 Zhao, Y. (2007) *Enabling cognitive radios through radio environment maps*, Ph.D. thesis, Virginia Polytechnic Institute and State University.

41 Bazerque, J. and Giannakis, G. (2010) Distributed spectrum sensing for cognitive radio networks by exploiting sparsity. *IEEE Transaction on Signal Processing*, **58** (3), 1847–1862.

42 Kim, S.J., Dall'Anese, E., and Giannakis, G. (2011) Cooperative spectrum sensing for cognitive radios using Kriged Kalman filtering. *IEEE Journal of Selected Topics in Signal Processing*, **5** (1), 24–36.

43 Dall'Anese, E., Kim, S.J., and Giannakis, G. (2011) Channel gain map tracking via distributed Kriging. *IEEE Transactions on Vehicular Technology*, **60** (3), 1205–1211.

44 Akyildiz, I.F., Lo, B.F., and Balakrishnan, R. (2011) Cooperative spectrum sensing in cognitive radio networks: A survey. *Physical Communication*, **4**, 40–62.

45 Beni, G. and Wang, J. (1989) Swarm intelligence in cellular robotic systems, in *Proceedings of the NATO Advanced Workshop on Robotics and Biological Systems*, Tuscany, Italy.

46 Dorigo, M., Maniezzo, V., and Colorni, A. (1996) The ant system: Optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics*, **26**, 1–13.

47 Agah, A. and Bekey, G. (1997) Phylogenetic and ontogenetic learning in a colony of interacting robots. *Autonomous Robots*, **4**, 85–100.

48 Kube, C.R. and Zhang, H. (1993) Collective robotics: From social insects to robots. *Adaptive Behavior*, **2** (2), 189–219.

49 Parker, L.E. (1993) Designing control laws for cooperative agent teams, in *Proceedings of IEEE International Conference on Robotics and Automation*, Atlanta, GA, pp. 582–587.

50 Arkin, R. (1990) Integrating behavioral, perceptual, and world knowledge in reactive navigation. *Robotics and Autonomous Systems*, **6**, 105–122.

51 Brooks, R.A. (1986) A robust layered control system for a mobile robot. *IEEE Transactions on Robotics and Automation*, **2** (1), 14–23.

52 Reif, J.H. and Wang, H. (1999) Social potential fields: A distributed behavioral control for autonomous robots. *Robotics and Autonomous Systems*, **27**, 171–194.

53 Arai, T., Pagello, E., and Parker, L.E. (2002) Editorial: Advances in multi-robot systems. *IEEE Transactions on Robotics and Automation*, **18** (5), 655–661.

54 Balch, T. and Parker, L.E. (2002) *Robot Teams: From Polymorphism to Diversity*, A K Peters, Natick, MA.

55 Cao, Y.U., Fukunaga, A.S., and Kahng, A.B. (1997) Cooperative mobile robotics: Antecedents and directions. *Autonomous Robotics*, **4** (1), 7–27.

56 Mataric, M.J. (1992) Designing emergent behaviors: From local interactions to collective intelligence, in *Proceedings of the Second International*

Conference on Simulation of Adaptive Behavior (eds J. Meyer, H. Roitblat, and S. Wilson), MIT Press, Honolulu, Hawaii, pp. 432–441.

57 Drogoul, A. and Ferber, J. (1992) From Tom Thumb to the Dockers: Some experiments with foraging robots, in *Proceedings of the Second International Conference on Simulation of Adaptive Behavior* (eds J. Meyer, H. Roitblat, and S. Wilson), MIT Press, Honolulu, Hawaii, pp. 451–459.

58 Murphy, R. (2000) Biomimetic search for urban search and rescue, in *Proceedings of the 2000 IEEE/RSJ International Conference on Intelligent Robots and Systems*, Takamatsu, Japan, pp. 2073–2078.

59 Kang, W. and Yeh, H.H. (2002) Co-ordinated attitude control of multi-satellite systems. *International Journal of Robust and Nonlinear Control*, **12**, 185–205.

60 McInnes, C.R. (1995) Autonomous ring formation for a planar constellation of satellites. *AIAA Journal of Guidance, Control, and Dynamics*, **18** (5), 1215–1217.

61 Giulietti, F., Pollini, L., and Innocenti, M. (2000) Autonomous formation flight. *IEEE Control Systems Magazine*, **20**, 34–44.

62 Mesbahi, M. and Hadaegh, F.Y. (2001) Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching. *AIAA Journal of Guidance, Control, and Dynamics*, **24**, 369–377.

63 Beard, R.W., Lawton, J., and Hadaegh, F.Y. (2001) A coordination architecture for spacecraft formation control. *IEEE Transactions on Control Systems Technology*, **9** (6), 777–790.

64 Stilwell, D.J. and Bishop, B.E. (2000) Platoons of underwater vehicles. *IEEE Control Systems Magazine*, **20** (6), 45–52.

65 Yamaguchi, H. (2003) A distributed motion coordination strategy for multiple nonholonomic mobile robots in cooperative hunting operations. *Robotics and Autonomous Systems*, **43**, 257–282.

66 Ogren, P., Fiorelli, E., and Leonard, N.E. (2004) Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment. *IEEE Transactions on Automatic Control*, **40** (8), 1292–1302.

67 Parker, L.E. (1998) ALLIANCE: An architecture for fault tolerant multi-robot cooperation. *IEEE Transactions on Robotics and Automation*, **14**, 220–240.

68 Balch, T. and Arkin, R.C. (1998) Behavior-based formation control for multirobot teams. *IEEE Transactions on Robotics and Automation*, **14** (6), 926–939.

69 Lewis, M.A. and Tan, K.H. (1997) High precision formation control of mobile robots using virtual structures. *Autonomous Robots*, **4**, 387–403.

70 Egerstedt, M. and Hu, X. (2001) Formation constrained multi-agent control. *IEEE Transactions on Robotics and Automation*, **17** (6), 947–951.

71 Wang, P.K.C. (1991) Navigation strategies for multiple autonomous mobile robots moving in formation. *Journal Robotic Systems*, **8** (2), 177–195.

72 Desai, J.P., Ostrowski, J.P., and Kumar, V. (2001) Modeling and control of formations of nonholonomic mobile robots. *IEEE Transactions on Robotics and Automation*, **17**, 905–908.

73 Tanner, H., Pappas, G., and Kumar, V. (2004) Leader-to-formation stability. *IEEE Transactions on Robotics and Automation*, **20** (3), 443–455.

74 Olfati-Saber, R. (2006) Flocking for multi-agent dynamic systems: Algorithms and theory. *IEEE Transactions on Automatic Control*, **51** (3), 401–420.

75 Ren, W., Beard, R.W., and McLain, T.W. (2004) Coordination variable and consensus building in multiple vehicle systems, in *Cooperative Control*, *Springer-Verlag Series : Lecture Notes in Control and Information Sciences*, vol. 309 (eds V. Kumar, N.E. Leonard, and A.S. Moore), Springer-Verlag, Berlin/Heidelberg, pp. 171–188.

76 Dong, W. and Guo, Y. (2005) Formation control of nonholonomic mobile robots using graph theoretical methods, in *Cooperative Control and Optimization*, Gainesville, FL.

77 Lafferriere, G., Williams, A., Caughman, J., and Veerman, J. (2005) Decentralized control of vehicle formations. *Systems and Control Letters*, **54**, 899–910.

78 Leonard, N.E. and Fiorelli, E. (2001) Virtual leaders, artificial potentials and coordinated control of groups, in *Proceedings of IEEE Conference on Decision and Control*, Orlando, FL, pp. 2968–2973.

79 Lin, J., Morese, A.S., and Anderson, B.D.O. (2003) The multi-agent rendezvous problem, in *Proceedings of IEEE Conference on Decision and Control*, Maui, Hawaiian, pp. 1508–1513.

80 Yabe, T., Bagheri, B., Ohkubo, T., Uchida, S., Yoshida, K., Funatsu, T., Oishi, T., Daito, K., Ishioka, M., Yasunaga, N., Sato, Y., Baasandash, C., Okamoto, Y., and Yanagitani, K. (2008) 100 W-class solar pumped laser for sustainable magnesium-hydrogen energy cycle. *Journal of Applied Physics*, **104** (8), 083104.

81 International Energy Agency, Key World Energy Statistics, 2007, http://www.coprocem.com/documents/key_stats_2007.pdf (accessed September 21, 2016).

82 Ohkubo, T. (2009) Solar-pumped 80 W laser irradiated by a fresnel lens. *Optics Letters*, **34** (2), 175–177.

83 Sakurai, Y., Yabe, T., Ikuta, K., Sato, Y., Uchida, S., and Matsunaga, E. (2008) Basic characterization of the mg combustion engine for a renewable energy cycle using solar-pumped laser. *The Review of Laser Engineering Supplemental Volume*, **36**, 1157–1160.

84 Botez, D. and Scifres, D.R. (1994) *Diode Laser Arrays*, Cambridge University Press, Cambridge, UK.

85 Braiman, Y., Kennedy, T.A.B., Wiesenfeld, K., and Khibnik, A. (1995) Entrainment of solid-state laser arrays. *Physical Review A*, **52** (2), 1500–1506.

86 Fabiny, L., Colet, P., Roy, R., and Lenstra, D. (1993) Coherence and phase dynamics of spatially coupled solid-state lasers. *Physical Review A*, **47** (5), 4287–4296.

87 Garcia-Ojalvo, J., Casademont, J., Torrent, M.C., Mirasso, C.R., and Sancho, J.M. (1999) Coherence and synchronization in diode-laser arrays with delayed global coupling. *International Journal of Bifurcation and Chaos*, **9** (11), 2225–2229.

88 Hohl, A., Gavrielides, A., Erneux, T., and Kovanis, V. (1997) Localized synchronization in two nonidentical coupled lasers. *International Journal of Bifurcation and Chaos*, **23** (25), 4745–4748.

89 Hohl, A., Gavrielides, A., Erneux, T., and Kovanis, V. (1999) Quasiperiodic synchronization for two delay-coupled semiconductor lasers. *Physical Review A*, **59** (5), 3941–3949.

90 Winfree, A.T. (1980) *Geometry of Biological Time*, Springer-Verlag, New York.

91 Kuramoto, Y. (1984) *Chemical Oscillations, Waves and Turbulence*, Springer-Verlag, Berlin.

92 Li, R. and Erneux, T. (1992) Preferential instability in arrays of coupled lasers. *Physical Review A*, **46** (7), 4252–4260.

93 Silber, M., Fabiny, L., and Wiesenfeld, K. (1993) Stability results for in-phase and splay-phase states of solid-state laser arrays. *Journal of the Optical Society of America. B*, **10** (6), 1121–1129.

94 Wang, S.S. and Winful, H.G. (1988) Dynamics of phase-locked semiconductor laser arrays. *Applied Physics Letters*, **52** (21), 1774–1776.

95 Winful, H.G. and Wang, S.S. (1988) Stability of phase locking in semiconductor laser arrays. *Applied Physics Letters*, **53** (20), 1894–1896.

96 Oliva, R.A. and Strogatz, S.H. (2001) Dynamics of a large array of globally coupled lasers with distributed frequencies. *International Journal of Bifurcation and Chaos*, **11** (9), 2359–2374.

97 Zehnle, V. (2000) Theoretical model for coupled solid-state lasers. *Physical Review A*, **63** (03), 033 814.

98 Jiang, Z.P. and McCall, M. (1993) Numerical simulation of a large number of coupled lasers. *Journal of the Optical Society of America. B*, **10** (1), 155–163.