Chapter 1

The Mathematical Toolbox: A Summary

1.2 LINEAR FUNCTIONS

An expression such as $Y = b_0 + b_1X$ represents a *linear equation (function)*, where b_0 is the Y-intercept (it gives the value of Y when $X = 0$) and $b_1 = \Delta Y/\Delta X$ is the *slope* (often referred to as *rise/run*). Here Y is the *dependent variable* and X is the *independent variable*. Note that both b_0 and b_1 are constants.

1.3.1 Solving Two Simultaneous Linear Equations

At times you will need to obtain a solution to a set of simultaneous linear equations, that is, to a set of equations of the general form:

$$aX + bY = e, \tag{1.1}$$

$$cX + dY = f. \tag{1.2}$$

A system such as this is said to be *consistent* if it has at least one solution. Moreover, if $ad - cb \neq 0$, then this equation system is consistent. For instance, the equation system

$$X - Y = 6 \tag{1.3}$$

$$3X - 2Y = 4 \tag{1.4}$$

is consistent since $(1)\,(-2) - (3)\,(-1) = 1 \neq 0$. In fact, to obtain the (unique) solution, we can multiply Equation (1.3) by -3 so as to obtain $-3X + 3Y = -18$, and then add this multiple to Equation (1.4) to get $Y = -14$. If we then substitute $Y = -14$ into Equation (1.3), we obtain $X = -8$. How do we know that we have generated the correct solution to this equation system?

Solutions Manual to Accompany Introduction to Quantitative Methods in Business: With Applications Using Microsoft® Office Excel®, First Edition. Bharat Kolluri, Michael J. Panik, and Rao N. Singamsetti.

Answer: Substitute $X = -8$ and $Y = -14$ back into, say, Equation (1.4) and show that equality holds.

It is easily demonstrated that the equation system

$$4X_1 + 2X_2 = 3,$$

$$16X_1 + 8X_2 = 12,$$

is *inconsistent* or *dependent* in that it has no solution. Here $(4)(8) - (16)(2) = 0$. Clearly, these two equations represent parallel lines—they do not intersect.

1.4 SUMMATION NOTATION

The operation of addition of a set of n values is readily carried out by using the "sigma" notation. In this regard, the left-hand side of the expression

$$\sum_{i=1}^{n} X_i = X_1 + X_2 + \cdots + X_n$$

reads: "the sum of all values X_i as i goes from 1 to n." The right-hand side shows that the operation of addition has been executed. Some useful summation rules are as follows:

Rule 1: $\sum_{i=1}^{n} (X_i \pm Y_i) = \sum_{i=1}^{n} X_i \pm \sum_{i=1}^{n} Y_i$.
Rule 2: $\sum_{i=1}^{n} cX_i = c\sum_{i=1}^{n} X_i$, where c is a constant.
Rule 3: $\sum_{i=1}^{n} c = nc$, where c is a constant.

Note also that

$$\sum_{i=1}^{n} X_i^2 \neq \left(\sum_{i=1}^{n} X_i\right)^2,$$

$$\sum_{i=1}^{n} X_i Y_i \neq \left(\sum_{i=1}^{n} X_i\right)\left(\sum_{i=1}^{n} Y_i\right),$$

if $\overline{X} = \sum_{i=1}^{n} X_i/n$ is the sample mean, then

$$\sum_{i=1}^{n} \left(X_i - \overline{X}\right) = 0.$$

The *Pearson sample correlation coefficient* can be written as either

$$r = \frac{\sum_{i=1}^{n} \left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sqrt{\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 \sum_{i=1}^{n} \left(Y_i - \overline{Y}\right)^2}} \quad \text{(long formula)},$$

where $\overline{X} = \sum_{i=1}^{n} X_i/n$ and $\overline{Y} = \sum_{i=1}^{n} Y_i/n$ are the sample means of X and Y, respectively; or as

$$r = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\left(\sum X_i^2 - \left[\left(\sum X_i\right)^2/n\right]\right)\left(\sum Y_i^2 - \left[\left(\sum Y_i\right)^2/n\right]\right)\right]^{1/2}} \quad \text{(short formula)}.$$

Given a collection of data points (X_i, Y_i), $i = 1, \ldots, n$, we can fit a linear equation of the form $Y = a + bX$ through them, where a is the Y-intercept and b is the slope. Here,

$$b = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2} \quad \text{(long formula)}$$

or

$$b = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\sum X_i^2 - \frac{(\sum X_i)^2}{n}\right]}$$

and $a = \overline{Y} - b\overline{X}$.

1.5 SETS

We know that a *set* is a collection or grouping of items (called *elements*) without regard to structure or order and that a set is usually defined by listing its elements. A set containing no elements is called the *null set* or the *empty set* and is denoted as ϕ. A set containing all elements under a particular discussion or in a given context is termed the *universal set* and is denoted as U. Given a set A, its complement, denoted $\overline{A}$, is the set containing all the elements within U that lie outside of A.

The *intersection* of two sets A and B is the set of elements common to both A and B. It is denoted as $A \cap B$. The *union* of two sets A and B contains the elements in A, or in B, or in both A and B. It is denoted as $A \cup B$. A moments reflection reveals the following:

$$\overline{\phi} = U$$
$$\overline{U} = \phi$$
$$A \cup \phi = A$$
$$A \cap \phi = \phi$$
$$A \cup U = U$$
$$A \cap U = A$$
$$A \cup \overline{A} = U$$
$$A \cap \overline{A} = \phi$$

1.6 FUNCTIONS AND GRAPHS

A *function f* is a rule or law of correspondence that associates with each value of a variable x a unique value of a variable y. Here y is termed the *image* of x under rule f. This "rule" is written as $y = f(x)$ and is read: "y is a function of x." Think

x	$y = f(x) = x^2$
0	0
1	1
2	4
3	9
4	16

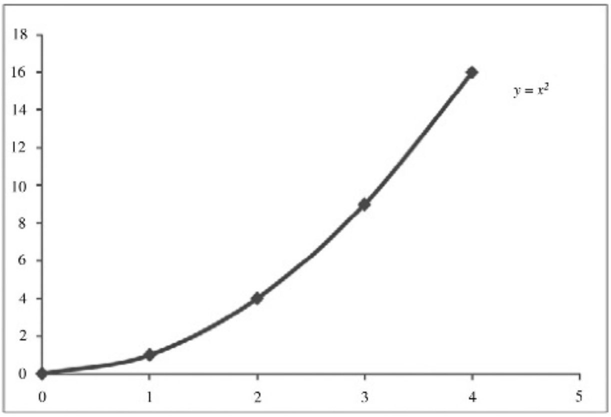


Figure 1.1 The graph of the function $y = f(x) = x^2$.

of a function as a recipe for getting unique y values from x values. Here x is termed the *independent variable* and y is called the *dependent variable*. The set of all admissible x values is called the *domain* of the function; the collection of y values, which are the image of at least one x, is termed the *range* of the function. For instance, a function may appear as $y = f(x) = x^2 + 2$. What is the rule that is operative here? The rule is: Select a value of x, multiply it by itself, and then add 2 to the result to get y. So, if $x = 2$, the image of $x = 2$ via rule f is

$$y = (2)^2 + 2 = 6.$$

It is important to note that for each x value there is one and only one resulting value of y. However, a y value may be the image of more than one x value. However, if $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$, then f is said to be a *one-to-one* function.

A useful device for illustrating a function is its *graph*. Here y is plotted on the vertical axis and x is plotted on the horizontal axis. Then, via the rule f, a set of x values are chosen and their corresponding y values are determined. These ordered (x, y) combinations are then connected to form the graph of the function in the x–y-plane (see Figure 1.1).

Given a particular graph, how can we determine whether or not it represents a function?

Answer: We can employ the *vertical line test*: For a given value of x, find y by drawing a vertical line at x. If the line cuts the graph at only a single value of y, then the graph indeed represents a function.

1.7 WORKING WITH FUNCTIONS

At times it is important to be able to graph a linear function in a quick and efficient fashion. To this end, let us consider the following approach:

Intercept method: Since two points are enough to draw a unique straight line, find both the *horizontal intercept* (set $y = 0$ and solve for the x-intercept) and the *vertical intercept* (set $x = 0$ and solve for the y-intercept) and connect the resulting two points, which are called *basic points*. For instance, suppose $y = 2x - 6$. Then,

a. if $y = 0$, then $2x - 6 = 0$ or $x = 3$; and
b. if $x = 0$, then $y = -6$.

Hence, the two basic points are (3, 0) and (0, −6). These points are then connected to obtain the desired graph.

An alternative method for graphing a function (which is useful for both linear and nonlinear equations) is the point method:

Point method: Given an admissible set of x values, we use the rule f to get the corresponding y values and then we connect each of the resulting ordered pairs (x, y) (see Figure 1.1).

1.8 DIFFERENTIATION AND INTEGRATION

We may think of a function as being *continuous* if its graph is without breaks. Additionally, the *derivative of a function $y = f(x)$ with respect to x* is defined as

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = f'(x).$$

Here dy/dx is the instantaneous rate of change in y per unit change in x as Δx gets smaller and smaller. Note that if f has a derivative at a point $x = a$, then it is also continuous at $x = a$. However, if f is continuous at a point $x = a$, it does not follow that f has a derivative there.

The derivative $f'(x) = dy/dx$ is also interpreted as the slope of the function $y = f(x)$ at a given point, that is, for $x = a$, $f'(a)$ is the slope of f at this x value. Additionally, dy/dx depicts the *marginal change in y with respect to x.*

Rules of Differentiation:

1. *Power rule:* If $y = f(x) = x^n$, then $dy/dx = nx^{n-1}$. (If $y = x^4$, $dy/dx = 4x^3$.)
2. The derivative of a constant is zero.
3. *Coefficient rule:* If $y = f(x) = kg(x)$, then $dy/dx = k(dg/dx)$. (If $y = 6x^2$, then $dy/dx = 12x$. Here, $k = 6$ and $g(x) = x^2$.)
4. *Sum (difference) rule:* If $y = f(x) = g(x) \pm h(x)$, then $dy/dx = dg/dx \pm dh/dx$. (If $y = 2x^2 - 6x^3$, then $dy/dx = 4x - 18x^2$.)
5. *Product rule:* If $y = f(x) = g(x) \times h(x)$, then $dy/dx = (dg/dx) \times h(x) + (dh/dx) \times g(x)$. (If $y = (3x - 2)(2x^2 + 8)$, then $dy/dx = (3)(2x^2 + 8) + (3x - 2)(4x)$.)

6. *Quotient rule:* If $y = f(x) = g(x)/h(x),\ h(x) \neq 0$, then

$$\frac{dy}{dx} = \frac{h(x) \times (dg/dx) - g(x) \times (dh/dx)}{(h(x))^2}.$$

$$\left(\text{If } y = \frac{10x - x^2}{2x^3 + 6}, \text{ then } \frac{dy}{dx} = \frac{(2x^3 + 6)(10 - 2x) - (10x - x^2)(6x^2)}{(2x^3 + 6)^2}.\right)$$

7. Derivatives of exponential and logarithmic functions:

a. If $y = e^x$, $dy/dx = e^x$.

b. If $y = e^{kx}$, $dy/dx = ke^{kx}$, where k is a constant.

c. If $y = \ln x$, $dy/dx = \frac{1}{x}$.

Higher Order Derivatives:

1. Given that the *first derivative of* f *with respect to* x appears as $f'(x) = dy/dx$, the *second derivative of* f *with respect to* x is written as $f''(x) = d^2y/dx^2$ and is calculated as the derivative of the first derivative.

(If $y = 3x^3 + 6x^2 - 2$, $f'(x) = 9x^2 + 12x$, and $f''(x) = 18x + 12$.)

Geometrically, the second derivative of f is the rate of change of the slope of f at a specific point on the graph of f. For instance, at a point $x = a$, if $f''(a) < 0$, then f is said to be *concave downward at* $x = a$; and if $f''(a) > 0$, then f is termed *concave upward of* $x = a$.

2. The *third derivative of* f *with respect to* x is the derivative of the second derivative. (Given $f''(x) = 18x + 12$ above, $f'''(x) = 18$.)

3. In general, the *nth order derivative of f with respect to x* is the derivative of the $(n - 1)$th order derivative, for example, the sixth derivative of f is the derivative of the fifth derivative of f.

There are two distinct ways to view the concept of an integral. On the one hand, we have the *definite integral*—an integral that is the limit of an addition process and can be thought of as an area under a given curve. On the other hand, we have the *indefinite integral*—an integral determined as the result of reversing the process of differentiation.

1. *Definite integral:* Given the derivative $dy/dx = f(x)$, $a < x < b$, we need to find $y = F(x)$. In this regard, the function $y = F(x)$ is a solution to $dy/dx = f(x)$ if $F(x)$ is differentiable over $a < x < b$ and $dF(x)/dx = f(x)$. Hence, F(x) is the *integral of f(x) with respect to x.* So, if $y = F(x)$ is a particular solution of $dy/dx = f(x)$, then "all" solutions can be written as $y = F(x) + c$, where c is an arbitrary constant. This is indicated by the expression $\int f(x)\,dx = F(x) + c$.

2. *Definite integral:* Suppose we wish to determine the area under the continuous curve $y = f(x)$ and above the x-axis for $a \le x \le b$. This area, called the *definite integral of f (x)*, is the limiting sum of a set of rectangles as the number of rectangles increases without bound and is denoted as

$$\int_a^b f(x)dx.$$

This integral is determined via the *fundamental theorem of integral calculus*: If $f(x)$ is continuous on $a \le x \le b$ and F is any differentiable function such that $F'(x) = f(x),\ a \le x \le b$, then

$$\int_a^b f(x)dx = F(x)]_a^b = F(b) - F(a).$$

Rules of Integration:

1. *Power rule:*

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad n \neq -1$$

if $n = -1$,

$$\int x^{-1} dx = \ln x + c.$$

(Here, $\int x^3 dx = \frac{1}{4}x^4 + c$.)

2. The integral of zero is a constant or $\int 0dx = c$.

3. The integral of a constant times a function is the constant times the integral of the function or $\int kf(x)dx = k\int f(x)dx$, where k is a constant.
(Here, $\int 3x^5 dx = 3\int x^5 dx = 3\left(\frac{1}{6}x^6\right) + c$.)

4. Integral of a sum or difference rule:

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx.$$

Here,

$$\int \left(8x^2 + 3x - 6\right)dx = 8\int x^2 dx + 3\int xdx$$

$$-6\int dx = 8\left(\frac{1}{3}x^3\right) + 3\left(\frac{1}{2}x^2\right) - 6x + c.$$

5. *Integration by parts:* $\int udv = uv - \int vdu$.

Find $\int x(x+3)^3 dx$. Set $u = x$ and $dv = (x+3)^3 dx$. Then, $du = 1$ and $v = \int (x+3)^3 dx = \frac{1}{4}(x+3)^4$. Hence, from $\int u dv = uv - \int v du$,

$$\begin{aligned}\int x(x+3)^3 dx &= x\left(\frac{1}{4}(x+3)^4\right) - \frac{1}{4}\int (x+3)^4 dx \\ &= \frac{1}{4}x(x+3)^4 - \frac{1}{4}\left(\frac{1}{5}\right)(x+3)^5 + c.\end{aligned}$$

6. Integral of an exponential or logarithmic function:

a. $\int e^x dx = e^x + c$.

b. $\int e^{kx} dx = \frac{1}{k} e^{kx} + c$, where k is a constant.

c. $\int \ln x \, dx = x(\ln x - 1) + c$.

SOLUTIONS TO ODD-NUMBERED EXERCISES

Solve the following:

1. $\frac{1}{3}X + 7 = 51$

3. $2.3X - 8.9 = -4.3$

SOLUTION:

1. $\frac{1}{3}X = 51 - 7$

$\frac{1}{3}X = 44$

$X = 3 \times 44 = 132$

SOLUTION:

3. $2.3X = -4.3 + 8.9$

$2.3X = 4.6$

$X = \frac{4.6}{2.3} = 2$

Solve the following sets of simultaneous equations:

5. $2X + 3Y = 18$
$X + 3Y = 15$

7. $2X_1 - X_2 = 3$
$7X_1 + 2X_2 = 27$

SOLUTION:

5. Designate the first equation as (1) and the second equation as (2)

$$2X + 3Y = 18 \tag{1}$$

$$X + 3Y = 15 \tag{2}$$

Subtracting (2) from (1), we get

$$X = 3$$

Substitute the value of X in any of the above equations, for example,

$$\begin{aligned} X + 3Y &= 15 \\ 3Y = 15 - 3 &= 12 \\ Y &= \frac{12}{3} = 4 \end{aligned}$$

Check of (1):

$$2(3) + 3(4) = 6 + 12 = 18$$

Check of (2):

$$3 + 3(4) = 3 + 12 = 15$$

SOLUTION:

7. Designate the first equation as (1) and the second equation as (2):

$$2X_1 - X_2 = 3 \quad (1)$$

$$7X_1 + 2X_2 = 27 \quad (2)$$

Multiply (1) by 2 and call the resulting equation (3). Then, adding equations (3) and (2), we get

$$7X_1 + 2X_2 = 27 \quad (2)$$

$$\underline{4X_1 - 2X_2 = 6} \quad (3)$$

$$11X_1 = 33$$

$$X_1 = \frac{33}{11} = 3$$

Now substitute the value of X_1 into equation (1) to get

$$2(3) - X_2 = 3$$

$$-X_2 = 3 - 6 = -3, \; X_2 = 3$$

Check of (1):

$$2(3) - 3 = 6 - 3 = 3$$

Check of (2):

$$7(3) + 2(3) = 21 + 6 = 27$$

Suppose that there are six observations for the variables X and Y as given below:

$$X_1 = 2, \; X_2 = 1, \; X_3 = 3, \quad X_4 = -5, \; X_5 = 1, \; X_6 = -2$$

$$Y_1 = 4, \; Y_2 = 0, \; Y_3 = -1, \; Y_4 = 2, \quad Y_5 = 7, \; Y_6 = -3$$

For Exercises 9, 11, 13, 15, 17, and 19, compute the following:

9. $\sum_{i=1}^{6} X_i$

11. $\sum_{i=1}^{6} X_i^2$

13. $\sum_{i=1}^{6} X_i Y_i$

15. $\sum_{i=1}^{6}(X_i - Y_i)$

17. $\sum_{i=1}^{6} cX_i, \quad \text{where } c = -1$

19. $\sum_{i=1}^{6} (X_i - Y_i)^2$

$$= \sum_{i=1}^{6} X_i^2 - \sum_{i=1}^{6} 2X_i Y_i + \sum_{i=1}^{6} Y_i^2$$

SOLUTION:

9. $\sum_{i=1}^{6} X_i = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$

$$= 2 + 1 + 3 + (-5) + 1 + (-2) = 7 - 7 = 0$$

SOLUTION:

11. $\sum_{i=1}^{6} X_i^2 = X_1^2 + X_2^2 + X_3^2 + X_4^2 + X_5^2 + X_6^2$

$= (2)^2 + (1)^2 + (3)^2 + (-5)^2 + (1)^2 + (-2)^2$

$= 4 + 1 + 9 + 25 + 1 + 4 = 44$

SOLUTION:

13. $\sum_{i=1}^{6} X_i Y_i = X_1 Y_1 + X_2 Y_2 + X_3 Y_3 + X_4 Y_4 + X_5 Y_5 + X_6 Y_6$

$= (2)(4) + (1)(0) + (3)(-1) + (-5)(2) + (1)(7) + (-2)(-3)$

$= 8 + 0 - 3 - 10 + 7 + 6 = 8$

SOLUTION:

15. $= (X_1 - Y_1) + (X_2 - Y_2) + (X_3 - Y_3) + (X_4 - Y_4) + (X_5 - Y_5) + (X_6 - Y_6)$

$= (2 - 4) + (1 - 0) + (3 + 1) + (-5 - 2) + (1 - 7) + (-2 + 3)$

$= -2 + 1 + 4 - 7 - 6 + 1 = -9$

SOLUTION:

17. $c\sum_{i=1}^{6} X_i = -1 \times 0 = 0$ since $c = -1$ and $\sum_{i=1}^{6} X_i = 0$

SOLUTION:

19. The given expression can be written as

$$\sum_{i=1}^{6} X_i^2 - 2\sum_{i=1}^{6} X_i Y_i + \sum_{i=1}^{6} Y_i^2$$

We already know that

$$\sum_{i=1}^{6} X_i^2 = 44$$

$$\sum_{i=1}^{6} X_i Y_i = 8$$

$$\sum_{i=1}^{6} Y_i^2 = 79$$

Substituting these values in the above equation yields

$$44 - 2 \times 8 + 79 = 44 - 16 + 79 = 107$$

21. Write set notation for the set of odd whole numbers greater than 0.

SOLUTION:

21. $\{1,3,5,7,9, \ldots\}$

Determine whether true or false in Exercises 23 and 25.

23. $2 \in \{-3, 2, 4, 5, 7\}$

25. $\{0, 1, 7\} \subset \{0, 1, 8, 13\}$

SOLUTION:

23. True

SOLUTION:

25. False, because set $\{0,1,7\}$ is not a subset of the $\{0, 1, 8, 13\}$.

27. Find $\{3,5,9\} \cup \{1,6,7\}$

SOLUTION:

27. $\{1, 3, 5, 6, 7, 9\}$

29. Using the information
$U = \{$3 red marbles, 7 blue marbles, 8 white marbles$\}$ and $A=\{$8 white marbles$\}$, find $A \cup \overline{A}$, $A \cap \overline{A}$, $\overline{U}$, and $\overline{\phi}$.

SOLUTION:

29. $A \cup \overline{A} = \{8\text{ white marbles}, 3\text{ red marbles}, 7\text{ blue marbles}\}$

$$A \cap \overline{A} = \phi$$

$$\overline{U} = \phi$$

$$\overline{\phi} = U$$

Graph the following functions:

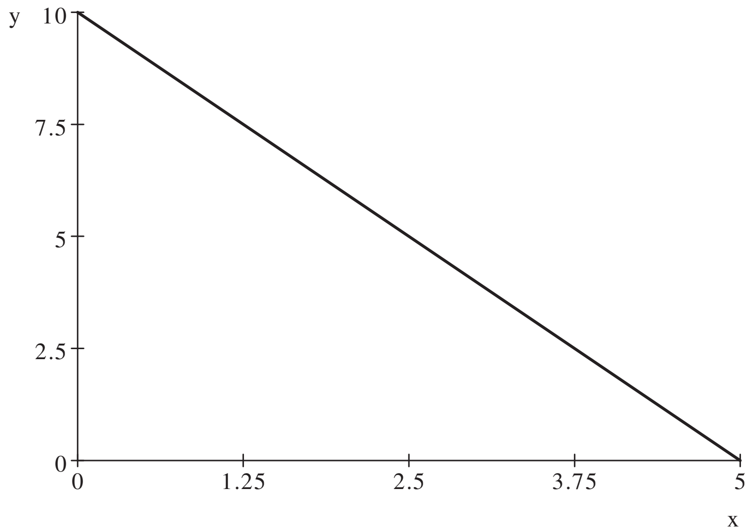
31. $2x + y - 10 = 0$

33. $y - 3\sqrt{x} = 5$

SOLUTION:

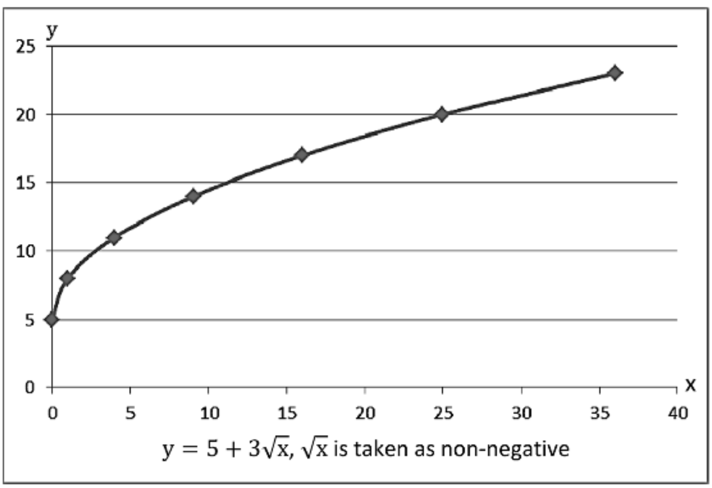
31. We can write this equation in standard form as $y = 10 - 2x$; prepare a table of some arbitrary x values and the corresponding y values and plot the points on a graph and connect them to get a straight line as shown below:

x	y
0	10
1	8
2	6
5	0



SOLUTION:

33.



x	y
0	5
1	8
4	11
9	14
16	17
25	20
36	23

Find dy/dx for the following functions given in Exercises 35, 37, 39, 41, and 43.

35. $y = 1/x^3, \quad x \neq 0$. Evaluate dy/dx at $x = 1$.

37. $y = (1/2)$

39. $y = (1/5)x^5$

41. $y = -0.5x + 3$

43. $y = 5x^3 + 4x^2 + 3x + 2$

SOLUTION:

35. $\frac{dy}{dx} = \frac{d\left(x^{-3}\right)}{dx} = -3x^{-3-1} = -3x^{-4} = -3\left(\frac{1}{X^4}\right)$. At $x = 1$, $\frac{dy}{dx} = -3$

SOLUTION:

37. $\frac{dy}{dx} = 0$

SOLUTION:

39. $\frac{dy}{dx} = \frac{1}{5}\frac{d(x^5)}{dx} = \frac{1}{5} \times 5x^4 = x^4$

SOLUTION:

41. $\frac{dy}{dx} = -0.5\frac{d(x)}{dx} + 0 = -0.5$

SOLUTION:

43. $\frac{dy}{dx} = 5\frac{d(x^3)}{dx} + 4\frac{d(x^2)}{dx} + 3\frac{d(x)}{dx} + 0$

$= 5 \times 3x^2 + 4 \times 2x + 3 = 15x^2 + 8x + 3$

45. **(a)** If $f(x) = 3x - 6$, $g(x) = 4x^2 + x$, and $y = f(x)/g(x)$, find $\frac{dy}{dx}$.

(b) If $f(x) = 5x^3 - 10$, $g(x) = -x^2 + x$, and $y = \frac{f(x)}{g(x)}$, find $\frac{dy}{dx}$.

SOLUTION:

45. **(a)** $\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{\left(4x^2 + x\right)(3) - (3x - 6)(8x + 1)}{(4x^2 + x)^2}$

$= \frac{12x^2 + 24x + 6}{16x^4 + 8x^3 + x^2}$

(b) $\frac{dy}{dx} = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2} = \frac{\left(-x^2 + x\right)\left(15x^2\right) - \left(5x^3 - 10\right)(-2x + 1)}{(-x^2 + x)^2}$

$= \frac{-5x^4 + 10x^3 - 20x + 10}{-x^4 - 2x^3 + x^2}$

47. $\int \frac{1}{x^2} dx$

SOLUTION:

47. $\int x^{-2} dx = \frac{1}{(-2+1)} x^{-2+1} + c$, where c is a constant

$= -1x^{-1} + c = \frac{-1}{x} + c$

49. $\int\left(3x^2 - 4x + 8\right)dx$

SOLUTION:

49. $\displaystyle\int 3x^2dx - \int 4xdx + \int 8dx = \frac{3x^{2+1}}{(2+1)} + c_1 - \frac{4x^{1+1}}{(1+1)} + c_2 + 8x + c_3$

$= x^3 - 2x^2 + 8x + c,$ where $c = c_1 + c_2 + c_3$ ($c_1, c_2,$ and c_3 are constants)

51. $\displaystyle\int_a^b \frac{1}{(b-a)}dx$

SOLUTION:

51. $\displaystyle\left[\frac{1}{(b-a)}x\right]_a^b = \frac{b}{b-a} - \frac{a}{b-a} = \frac{b-a}{b-a} = 1$

53. Given

$X_1 = -2,\ X_2 = -3,\ X_3 = 0,\ X_4 = 5,\ X_5 = 9$

$Y_1 = 5,\quad Y_2 = 9,\quad Y_3 = 6,\ Y_4 = 8,\ Y_5 = 7$

(a) Evaluate the Pearson correlation coefficient

$$r = \sum_{i=1}^{5} \frac{\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{\sqrt{\sum_{i=1}^{5}\left(X_i - \overline{X}\right)^2 \sum_{i=1}^{5}\left(Y_i - \overline{Y}\right)^2}}, \qquad (-1 \le r \le 1)$$

where $\overline{X} = \sum X_i/5$ and $\overline{Y} = \sum Y_i/5$. How is r interpreted?

(b) Also, re-evaluate r using the following "short formula":

$$r = \frac{\sum X_iY_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\left(\sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n}\right)\left(\sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n}\right)\right]^{\frac{1}{2}}}$$

(*Hint:* To facilitate your calculations, construct a table.)

SOLUTION:

53. (a)

Serial No.	X_i	Y_i	$\left(X_i - \overline{X}\right)$	$\left(Y_i - \overline{Y}\right)$	$\left(X_i - \overline{X}\right) \times \left(Y_i - \overline{Y}\right)$	$(X_i - \overline{X})^2$	$(Y_i - \overline{Y})^2$
1	−2	5	−3.8	−2	7.6	14.44	4
2	−3	9	−4.8	2	−9.6	23.04	4
3	0	6	−1.8	−1	1.8	3.24	1
4	5	8	3.2	1	3.2	10.24	1
5	9	7	7.2	0	0	51.84	0
Sum	9	35	0	0	3	102.8	10
Mean	1.8	7					

Then,

$$r = \frac{\sum_i^5 (X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^5 (X_i - \overline{X})^2 \sum_{i=1}^5 (Y_i - \overline{Y})^2}}$$
$$= \frac{3}{\sqrt{(102.8)(10)}} = 0.093567$$

(b)

Serial No.	X_i	Y_i	$X_i Y_i$	X_i^2	Y_i^2
1	−2	5	−10	4	25
2	−3	9	−27	9	81
3	0	6	0	0	36
4	5	8	40	25	64
5	9	7	63	81	49
Sum	9	35	66	119	255

Using the short formula:

$$r = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\left(\sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n}\right)\left(\sum Y_i^2 - \frac{\left(\sum Y_i\right)^2}{n}\right)\right]^{\frac{1}{2}}}$$
$$= \frac{66 - \frac{(9)(35)}{5}}{\sqrt{\left(119 - \frac{(9)^2}{5}\right)\left(255 - \frac{(35)^2}{5}\right)}}$$
$$= \frac{3}{32.0624} = 0.093567$$

Interpretation: In general, if r > 0, X and Y move together; and if r < 0, X and Y move in opposite directions.

More specifically, if

(i) $r = -1$, we have perfect negative linear association between X and Y;
(ii) $r = 0$, no linear association between X and Y;
(iii) $r = 1$, we have perfect positive linear association between X and Y.

55. Using the data and results in Exercise 53 above,

(a) Compute the slope value, $b = \left[\sum_{i=1}^5 (X_i - \overline{X})(Y_i - \overline{Y})\right] / \left[\sum_{i=1}^5 (X_i - \overline{X})^2\right]$, and the intercept value, $a = \overline{Y} - b\overline{X}$, of the linear equation $Y = a + bX$. Use this estimated equation to predict the value of Y when $X = 11$.

(b) Also, reevaluate the slope value b using the following "short formula":

$$b = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\sum X_i^2 - \frac{\left(\sum X_i\right)^2}{n}\right]}$$

SOLUTION:

55. (a)

Serial No.	X_i	Y_i	$(X_i - \overline{X})$	$(Y_i - \overline{Y})$	$(X_i - \overline{X}) \times (Y_i - \overline{Y})$	$(X_i - \overline{X})^2$	$(Y_i - \overline{Y})^2$
1	−2	5	−3.8	−2	7.6	14.44	4
2	−3	9	−4.8	2	−9.6	23.04	4
3	0	6	−1.8	−1	1.8	3.24	1
4	5	8	3.2	1	3.2	10.24	1
5	9	7	7.2	0	0	51.84	0
Sum	9	35	0	0	3	102.8	10
Mean	1.8	7					

Using the values from the table above, we find

$$b = \frac{\sum_{i=1}^{5}(X_i - \overline{X})(Y_i - \overline{Y})}{\sum_{i=1}^{5}(X_i - \overline{X})^2} = \frac{3}{102.8} = 0.029183$$

and

$$\begin{aligned} a &= \overline{y} - b\overline{x} \\ &= 7 - 0.029183 \times 1.8 = 6.947471. \text{ Therefore,} \\ Y &= 6.947471 + 0.029183(11) = 7.268484. \end{aligned}$$

(b)

Serial No.	X_i	Y_i	X_iY_i	X_i^2	Y_i^2
1	−2	5	−10	4	25
2	−3	9	−27	9	81
3	0	6	0	0	36
4	5	8	40	25	64
5	9	7	63	81	49
Sum	9	35	66	119	255

Using the values from the table above, we find

$$\begin{aligned} b &= \frac{\sum X_iY_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\sum X_i^2 - \frac{(\sum X_i)^2}{n}\right]} \\ &= \frac{66 - \left(\frac{9 \times 35}{5}\right)}{\left(119 - \frac{9^2}{5}\right)} = 0.029183 \end{aligned}$$

57. Find the derivative of $y = f(x)g(x)$, where

(a) $f(x) = 9x + 7$, $g(x) = 2x^3 + x$

(b) $f(x) = 4x^2 - 3x$, $g(x) = x^7 - 7$

SOLUTION:

57. **(a)** $$\frac{dy}{dx} = 9\left(2x^3 + x\right) + (9x + 7)\left(6x^2\right)$$

$$= 18x^3 + 9x + 54x^3 + 42x^2$$

$$= 72x^3 + 42x^2 + 9x$$

(b) $$\frac{dy}{dx} = (8x - 3)\left(x^7 - 7\right) + \left(4x^2 - 3x\right)\left(7x^6\right)$$

$$= 8x^8 - 56x - 3x^7 + 21 + 28x^8 - 21x^7$$

$$= 36x^8 - 24x^7 - 56x + 21$$

59. Find the derivative of $y = \frac{f(x)}{g(x)}$, where

(a) $f(x) = x^2 - 9$, $g(x) = x + 3$

(b) $f(x) = \frac{1}{x} + x$, $x \neq 0$, $g(x) = 2x^3 + 3$

SOLUTION:

59. **(a)** $$\frac{dy}{dx} = \frac{(x + 3)(2x) - \left(x^2 - 9\right)(1)}{(x + 3)^2}$$

$$= \frac{2x^2 + 6x - x^2 + 9}{x^2 + 6x + 9}$$

(b) $$\frac{dy}{dx} = \frac{\left(2x^3 + 3\right)\left(-(1/x^2) + 1\right) - ((1/x) + x)\left(6x^2\right)}{(2x^3 + 3)^2}$$

$$= \frac{-2x + 2x^3 - (3/x^2) + 3 - 6x - 6x^3}{4x^6 + 12x^3 + 9}$$

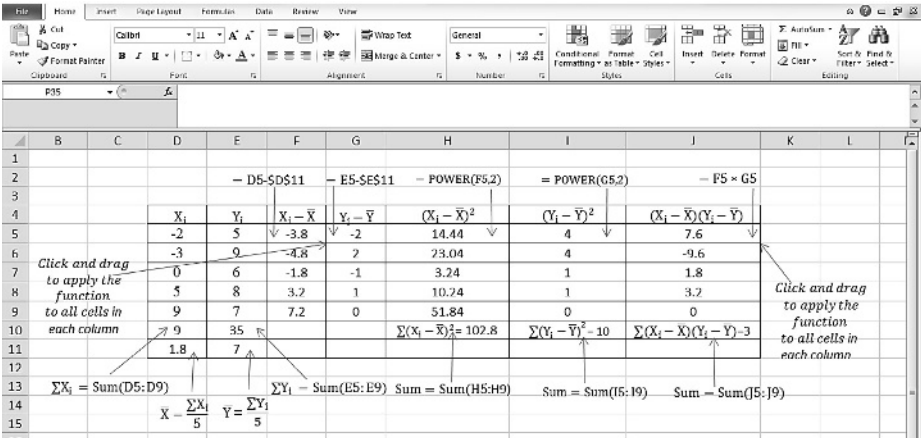
$$= \frac{-4x^3 - 3x^{-2} - 8x + 3}{4x^6 + 12x^3 + 9}$$

EXCEL APPLICATIONS

61. Answer Exercise 53, both parts (a) and (b), by formulating appropriate Excel spread sheets.

SOLUTION:

(a)

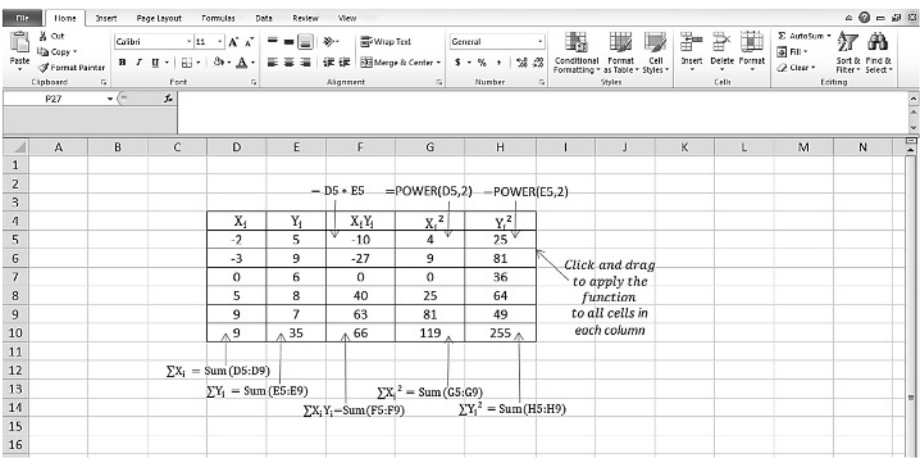


X_i	Y_i	$X_i-\bar{X}$	$Y_i-\bar{Y}$	$(X_i-\bar{X})^2$	$(Y_i-\bar{Y})^2$	$(X_i-\bar{X})(Y_i-\bar{Y})$
-2	5	-3.8	-2	14.44	4	7.6
-3	9	-4.8	2	23.04	4	-9.6
0	6	-1.8	-1	3.24	1	1.8
5	8	3.2	1	10.24	1	3.2
9	7	7.2	0	51.84	0	0
9	35			$\sum(X_i-\bar{X})^2_i$ = 102.8	$\sum(Y_i-\bar{Y})^2$ = 10	$\sum(X_i-\bar{X})(Y_i-\bar{Y})$=3
1.8	7					

$$r = \sum_i^5 \frac{(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^5 (X_i - \bar{X})^2 \sum_{i=1}^5 (Y_i - \bar{Y})^2}}$$

$$= \frac{3}{\sqrt{(102.8)(10)}} = 0.093567$$

(b)



X_i	Y_i	X_iY_i	X_i^2	Y_i^2
-2	5	-10	4	25
-3	9	-27	9	81
0	6	0	0	36
5	8	40	25	64
9	7	63	81	49
9	35	66	119	255

$$r = \frac{\sum X_i Y_i - \frac{\sum X_i \sum Y_i}{n}}{\left[\left(\sum X_i^2 - \frac{(\sum X_i)^2}{n}\right)\left(\sum Y_i^2 - \frac{(\sum Y_i)^2}{n}\right)\right]^{\frac{1}{2}}}$$

$$= \frac{66 - \frac{(9)(35)}{5}}{\sqrt{\left(119 - \frac{(9)^2}{5}\right)\left(255 - \frac{(35)^2}{5}\right)}}$$

$$= \frac{3}{32.0624} = 0.093567$$

63. Solve the simultaneous equations given in Exercises 4, 6, and 8 (a) using the Solver Add-in in Excel.

Note: Exercises 4, 6, and 8 (a) are typed here as they are even numbers in the textbook.

Solve the following sets of simultaneous equations:

4. $7X + 3Y = 14$
$X - Y \quad = 12$

6. $5X - 7Y = 28$
$2X + 5Y = 19$

8. **(a)** $3P + Q = 26$
$P - 5Q = 2$

(b) $-6X + 3Y = 0$
$3X + Y = 0$

(c) $P + 2Q = 10$
$4P + 8Q = 20$

SOLUTION:

63. Solver solution for Exercise 4.

Let $7X + 3Y = 14$ be equation (1) and $X - Y = 12$ be equation (2).

INSTRUCTIONS:

A. Excel spread sheet set up of the problem:

Enter data in Excel spread sheet.

- **a.** Type “Variables” in A1, X in B1, and Y in C1. Type “Trial values” in A2,1 in B2, and 1 in C2.
- **b.** Type “equation (1)” in A3 and enter the coefficient, 7, of the variable X in B3 and 3 of the variable Y in C3. In the cell D3 enter the Array formula “=Sumproduct (highlight the trial values 1 and 1, highlight 7 and 3)” and hit Enter. You will see 10 in D3, which is the left-hand side value of the equation (1) with the given trial values. In the cell E3 type “equals” (not the symbol = since Excel expects a formula to follow = symbol) and enter 14 in cell F3, which is the right-hand side value of equation (1).

c. Repeat the above step after typing equation (2) in cell A4. This will result in 0 in D4, the word "equals" in E4 and 12 in F4.
d. Click any empty cell away from the setup table, which is as follows:

	A	B	C	D	E	F
1	Variables	X	Y			
2	Trial values	1	1			
3	Equation 1	7	3	10	equals	14
4	Equation 2	1	-1	0	equals	12
5						
6						
7						

B. Using Solver Add-In*

Click on "DATA" in the menu bar and then click on "Solver" in the extreme right side of the ribbon at the top of your screen to get Solver parameters dialog box.

a. In the Solver parameters dialog box, at "set objective", select Target Cell D3, where 10 is obtained by formula.
b. At "to" select the button "value of" and enter 14 in white box.
c. Under "By Changing Variable Cells' click in the white box and highlight both the trial value cells at the same time.
d. Click inside the lower bigger rectangular box and click on "Add" on the right-hand side to get "Add Constraint" dialog box.
e. Under Cell Reference: Highlight Cell, D4 where "0" is displayed by the formula.
f. Change the middle symbol to "=".
g. Constraint: Highlight Cell F4 where "12" is typed.
h. Click "OK" since you don't have a third constraint equation.
i. Uncheck "Make Unconstrained Variables Non-Negative" box and select solving method as "Simplex LP."
j. Click on "Solve" to see a circle marked "Keep Solver Solution" in the Solver Results dialog box.
k. Click OK to see the solution as in the table below:

	A	B	C	D	E	F
1	Variables	X	Y			
2	Trial values	5	-7			
3	Equation 1	7	3	14	equals	14
4	Equation 2	1	-1	12	equals	12
5						
6						
7						

Note: The trial values of variables X and Y are changed by solver program to $X = 5$ in cell B2 and $Y = -7$ in cell C2, which satisfy both the equations. Therefore, the final solution of the two equations is $X = 5$ and $Y = -7$. It may be observed that you obtained the same solution by the elimination method in Exercise 4 above.

* If you don't find "solver" when you click on "Data" in the menu bar in Microsoft Excel (Office 2007 and onward), then

1. Click **the MICROSOFT ICON or FILE** (at the top left of your screen).
2. Click **Excel Options** (at the bottom of the menu).
3. Click **ADD-INS.**
4. Click **GO** (at the bottom of the window).
5. Check the box with "Solver Add-in."
6. Click on **OK** and see **"Solver"** at the top right of your screen.

63. Solver Solution for Exercise 6

Let $5X - 7Y = 28$ be equation (1) and $2X + 5Y = 19$ be equation (2).

INSTRUCTIONS:

A. Excel spreadsheet setup of the problem:
Enter data in Excel spreadsheet.

a. Type "Variables" in A1, X in B1, and Y in C1. Type "Trial values" in A2, 1 in B2, and 1 in C2.

b. Type "equation (1) " in A3 and enter the coefficient, 5, of the variable X in B3 and −7 of the variable Y in C3. In the cell D3, enter the Array formula "=Sumproduct(highlight the trial values 1 and 1, highlight 5 and −7)" and hit Enter. You will see −2 in D3, which is the left-hand side value of the equation (1) with the given trial values. In the cell E3 type "equals" (not the symbol = since Excel expects a formula to follow = symbol) and enter 28 in cell F3, which is the right-hand side value of equation (1).

c. Repeat the above step after typing equation (2) in cell A4. This will result in 7 in D4, the word "equals" in E4 and 19 in F4.

d. Click any empty cell away from the setup table, which is as follows:

	A	B	C	D	E	F	G
1	Variables	X	Y				
2	Trial values	1	1				
3	Equation 1	5	-7	-2	equals	28	
4	Equation 2	2	5	7	equals	19	
5							
6							
7							

B. Using Solver Add-In*
Click on "DATA" in the menu bar and then click on "Solver" on the extreme right side of the ribbon at the top of your screen to get Solver parameters dialog box.

a. In the Solver parameters dialog box, at "set objective," select Target Cell D3, where −2 is obtained by formula.

b. At "to" select the button "value of " and enter 28 in white box.

c. Under "By Changing Variable Cells" click in the white box and highlight both the trial value cells at the same time.

d. Click inside the lower bigger rectangular box and click on "Add" on the right-hand side to get "Add Constraint" dialog box.

e. Under Cell Reference: Highlight Cell, D4 where "7" is displayed by the formula.
f. Change the middle symbol to "=".
g. Constraint: Highlight Cell F4 where "19" is typed.
h. Click "OK" since you don't have a third constraint equation.
i. Uncheck "Make Unconstrained Variables Non-Negative" box and select solving method as "Simplex LP."
j. Click on "Solve" to see a circle marked "Keep Solver Solution" in the Solver Results dialog box.
k. Click OK to see the solution as in the table below:

	A	B	C	D	E	F
1	Variables	X	Y			
2	Trial values	7	1			
3	Equation 1	5	-7	28	equals	28
4	Equation 2	2	5	19	equals	19
5						
6						
7						
8						

Note: The trial values of variables X and Y are changed by solver program to $X=7$ in cell B2 and $Y=1$ in cell C2, which satisfy both the equations. Therefore, the final solution of the two equations is $X=7$ and $Y=1$. It may be observed that you obtained the same solution by the elimination method in Exercise 6 above.

* If you don't find "solver" when you click on "Data" in the menu bar in Microsoft Excel (Office 2007 and onward), then

1. Click *the MICROSOFT ICON* or *FILE* (at the top left of your screen).
2. Click *Excel Options* (at the bottom of the menu).
3. Click *ADD-INS.*
4. Click *GO* (at the bottom of the window).
5. Check the box with "Solver Add-in."
6. Click on *OK* and see *"Solver"* at the top right of your screen.

63. Solver Solution for Exercise 8 (a)
Let $3P+Q=26$ be equation (1) and $P-5Q=2$ be equation (2).

INSTRUCTIONS:

A. Excel spreadsheet setup of the problem:
Enter data in Excel spreadsheet.

a. Type "Variables" in A1, P in B1, and Q in C1. Type "Trial values" in A2, 1in B2, and 1 in C2.
b. Type "equation (1)" in A3 and enter the coefficient, 3 of the variable P in B3, and 1 of the variable Q in C3. In the cell D3, enter the Array formula "=Sumproduct (highlight the trial values 1 and 1, highlight 3 and 1)" and hit Enter. You will see 4 in D3, which is the left-hand side value of the equation (1) with the given trial values. In the cell E3 type "equals" (not the symbol = since Excel expects a formula to follow = symbol) and enter 26 in cell F3, which is the right-hand side value of equation (1).

c. Repeat the above step after typing equation (2) in cell A4. This will result in −4 in D4, the word "equals" in E4 and 2 in F4.

d. Click any empty cell away from the setup table, which is as follows:

	A	B	C	D	E	F
1	Variables	P	Q			
2	Trial values	1	1			
3	Equation 1	3	1	4	equals	26
4	Equation 2	1	-5	-4	equals	2
5						
6						

B. Using Solver Add-In*

Click on "DATA" in the menu bar and then click on "Solver" in the extreme right side of the ribbon at the top of your screen to get Solver parameters dialog box.

a. In the Solver parameters dialog box, at "set objective," select Target Cell D3, where 4 is obtained by formula.

b. At "to" select the button "value of" and enter 26 in white box.

c. Under "By Changing Variable Cells" click in the white box and highlight both the trial value cells at the same time.

d. Click inside the lower bigger rectangular box and click on "Add" on the right-hand side to get "Add Constraint" dialog box.

e. Under Cell Reference: Highlight Cell, D4 where "−4" is displayed by the formula.

f. Change the middle symbol to "=".

g. Constraint: Highlight Cell F4 where "2" is typed.

h. Click "OK" since you don't have a third constraint equation.

i. Uncheck "Make Unconstrained Variables Non-Negative" box and select solving method as "Simplex LP."

j. Click on "Solve" to see a circle marked "Keep Solver Solution" in the Solver Results dialog box.

k. Click OK to see the solution as in the table below:

	A	B	C	D	E	F
1	Variables	P	Q			
2	Trial values	8.25	1.25			
3	Equation 1	3	1	26	equals	26
4	Equation 2	1	-5	2	equals	2
5						

Note: The trial values of variables P and Q are changed by solver program to $P = 8.25$ in cell B2 and $Q = 1.25$ in cell C2, which satisfy both the equations. Therefore, the final solution of the two equations is $P = 8.25$ and $Q = 1.25$. It may be observed that you obtained the same solution by the elimination method in Exercise 8 (a) above.

* If you don't find "solver" when you click on "Data" in the menu bar in Microsoft Excel (Office 2007 and onward), then

1. Click *the MICROSOFT ICON* or *FILE* (at the top left of your screen).
2. Click *Excel Options* (at the bottom of the menu).
3. Click *ADD-INS.*

4. Click *GO* (at the bottom of the window).
5. Check the box with "Solver Add-in."
6. Click on *OK* and see "Solver" at the top right of your screen.

APPENDIX A EXERCISES

Simplify the following:

A.1 $(18 \div 12) \times 8^2 - 88$

A.3 $[(4)^2 \times 4] \times 16 + 10$

A.5 $X^2YXY^3X^2Y^2$

A.7 $\left(\dfrac{X^3}{-64Y^6}\right)^{1/3}$

A.9 $\left[3\sqrt{\dfrac{1}{27}}\right]^2$

A.11 $0.30 + 1.645\sqrt{\dfrac{0.3(1-0.3)}{100}}$

A.13 $\{(0.3)xyz\}^0$

A.15 $\dfrac{(0.3)^4}{(0.2)^3} \times 10^3$

A.17 $\left(\dfrac{1.24(0.25)}{0.62}\right)^{1/2}$

A.19 $x \cdot x \cdot y^2$

A.21 $(x^4)^4$

A.23 $(4 - 2^2)^0$

A.25 $\left(\dfrac{x^3}{y^4}\right)^5$

A.27 $\left(\dfrac{x^4}{x^3}\right)^5$

A.29 $\dfrac{12^3}{12^0}$

A.31 $\left(\dfrac{9}{16}\right)^{1/2}$

A.33 $27^{4/3}$

A.35 $\dfrac{x^8/y^4}{8/y^2}$

A.37 $\left(\dfrac{1}{3}\right)^3$

SOLUTION:

A1. $\frac{18}{12} \times 64 - 88 \quad = \frac{3}{2} \times 64 - 88 = 96 - 88 = 8$

SOLUTION:

A.3 $[16 \times 4] \times 16 + 10 = 64 \times 16 + 10 = 1024 + 10 = 1034$

SOLUTION:

A.5 $x^{2+1+2}y^{2+3+1} = x^5y^6$

SOLUTION:

A.7 $\left(\frac{x^{3/3}}{-4^{3/3}y^{6/3}}\right) = \frac{x}{-4y^2}$

SOLUTION:

A.9 $\left[\frac{1}{3^{3/3}}\right]^2 = \frac{1}{9}$

SOLUTION:

A.11 $0.30 + 1.645\sqrt{\frac{0.3 \times 0.7}{100}}$

$$0.30 + 1.645\sqrt{\frac{0.21}{10^2}} = 0.30 + 1.645\sqrt{0.0021} = 0.30 + 0.075 = 0.375$$

SOLUTION:

A.13 $(0.3)^0 \times (xyz)^0 = 1 \times 1 = 1$

SOLUTION:

A.15 $\frac{0.3 \times 0.3 \times 0.3 \times 0.3}{0.2 \times 0.2 \times 0.2} \times 10^3 = \frac{0.0081}{0.008} \times 1000 = 1012.5$

SOLUTION:

A.17 $\left(\frac{0.31}{0.62}\right)^{1/2} = \sqrt[2]{0.5} = 0.7071$

SOLUTION:

A.19 $x^{1+1}y^2 = x^2y^2$

SOLUTION:

A.21 $x^4 \times x^4 \times x^4 \times x^4 = x^{4+4+4+4} = x^{16}$

SOLUTION:

A.23 $(4-4)^0 = 0^0 = \text{undefined}$

SOLUTION:

A.25 $\dfrac{x^{15}}{y^{20}} = x^{15} \times y^{-20}$

SOLUTION:

A.27 $\left(\dfrac{x^{4\times5}}{x^{3\times5}}\right) = \dfrac{x^{20}}{x^{15}} = x^{20-15} = x^5$

SOLUTION:

A.29 $12^{3-0} = 12^3 = 1728$

SOLUTION:

A.31 $\dfrac{\sqrt[2]{9}}{\sqrt[2]{16}} = \dfrac{3}{4}$

SOLUTION:

A.33 $3^{3\times(4/3)} = 3^4 = 81$

SOLUTION:

A.35 $\dfrac{x^8}{y^4} \times \dfrac{y^2}{8} = \dfrac{x^8}{y^{4-2} \times 8} = \dfrac{x^8}{8y^2}$

SOLUTION:

A.37 $\dfrac{1}{3} \times \dfrac{1}{3} \times \dfrac{1}{3} = \dfrac{1}{27}$

SOLUTION:

A.39 Evaluate $y = \left(\dfrac{1}{x}\right)^{-(4/6)}$ at $x = 27$

$$Y = \left(\frac{1}{27}\right)^{-(4/6)} = \left(\frac{1}{(3)^{3(-4/6)}}\right) = \frac{1}{3^{-2}} = 3^2 = 9$$

Factorize the following:

A.41 $x^2 - 7x + 12$

A.43 $x^2 + x - 12$

A.45 $5x^2 + 7x + 2$

A.47 $x^2 + 5x + 6$

SOLUTION:

A.41 $x^2 - 3x - 4x + 12 = x(x-3) - 4(x-3) = (x-3)(x-4)$

SOLUTION:

A.43 $x^2 + 4x - 3x - 12 = x(x+4) - 3(x+4) = (x+4)(x-3)$

SOLUTION:

A.45 $5x^2 + 5x + 2x + 2 = 5x(x + 1) + 2(x + 1) = (5x + 2)(x + 1)$

SOLUTION:

A.47 $x^2 + 2x + 3x + 6 = x(x + 2) + 3(x + 2) = (x + 3)(x + 2)$

Evaluate the following expressions:

A.49 $\frac{7}{8} - \frac{3}{24}$

A.51 $\frac{1}{2} - \frac{1}{4}$

SOLUTION:

A.49 $\text{LCM} = 24 = 2 \times 2 \times 2 \times 3$

$$\frac{21}{24} - \frac{3}{24} = \frac{21 - 3}{24} = \frac{18}{24} = \frac{3}{4}$$

SOLUTION:

A.51 $\text{LCM} = 4 = 2 \times 2$

$$\frac{2}{4} - \frac{1}{4} = \frac{2 - 1}{4} = \frac{1}{4}$$

Solve the following:

A.53 $\frac{(1/3) - (1/5)}{(2/3) - (1/6)}$

A.55 $\frac{3}{5} \text{x} \frac{-2}{7}$

A.57 $\frac{-2}{3} + \frac{3}{5}$

A.59 $\frac{0}{5} + \frac{5}{6}$

SOLUTION:

A.53 $\text{LCM 1} = 5 \times 3 = 15$

$\text{LCM 2} = 2 \times 3 = 6$

$$\frac{(5/15) - (3/15)}{(4/6) - (1/6)} = \frac{(5 - 3)/15}{(4 - 1)/6} = \frac{2/15}{3/6} = \frac{2}{15} \times \frac{6}{3} = \frac{4}{15}$$

SOLUTION:

A.55 $\frac{3 \times -2}{5 \times 7} = \frac{-6}{35}$

SOLUTION:

A.57 $\frac{-2}{3} \times \frac{5}{3} = \frac{-2 \times 5}{3 \times 3} = \frac{-10}{9}$

SOLUTION:

A.59 $\left(0 \times \frac{6}{5}\right) = 0$

Simplify the following:

A.61 $2\frac{3}{5} + \frac{1}{3} - \frac{2}{7}$

A.63 $\left(2\frac{3}{4}\right)\left(4\frac{1}{2}\right)$

A.65 $\left(2\frac{3}{4}\right) / \left(4\frac{1}{3}\right)$

SOLUTION:

A.61 $\frac{13}{5} + \frac{1}{3} - \frac{2}{7}$

LCM $= 3 \times 5 \times 7$

$$\frac{273}{105} + \frac{35}{105} - \frac{30}{105} = \frac{273 + 35 - 30}{105} = \frac{278}{105}$$

SOLUTION:

A.63 $\left(\frac{11}{4}\right)\left(\frac{9}{2}\right) = \frac{99}{8}$

SOLUTION:

A.65 $\frac{11}{4} \times \frac{3}{13} = \frac{33}{52}$

Show the position of each digit in the following:

A.67 0.873

SOLUTION:

A.67 $\frac{8}{10} + \frac{7}{100} + \frac{3}{1000} = 0.8 + 0.07 + 0.003$

Find:

A.69 $1.273 + 3.2$

A.71 $0.34 - 0.07 + 1.7$

SOLUTION:

A.69 4.473

SOLUTION:

A.71 $0.34 + 1.7 - 0.07 = 1.97$

Evaluate:

A.73 $0.1 \times 0.01 \times 0.005$

A.75 $\dfrac{(1.5)(-0.6)-(2.5)(1.7)}{20}$

A.77 $\dfrac{0.4}{0.04} - \dfrac{6(3.6)}{0.072}$

SOLUTION:

A.73 0.000005

SOLUTION:

A.75 $\dfrac{-0.9-4.25}{20} = -0.2575$

SOLUTION:

A.77 $10 - \dfrac{21.6}{0.072} = 10 - 300 = -290$

Round-off the resulting decimal to the given place:

A.79 $\dfrac{5}{7}$, to the nearest hundredths

A.81 $\dfrac{22}{7}$, to the nearest ten thousandth

SOLUTION:

A.79 0.71

SOLUTION:

A.81 3.1429

A.83 In a business school, the ratio of female to male undergraduate students is 1 to 4. After 200 female students are admitted, the ratio of female to male students became $2/3$. Find the total number of students after admitting 200 female students in the school.

SOLUTION:

A.83 Let x denote number of female students to start with.
Then,

$$\frac{x+200}{4x} = \frac{2}{3}$$
$$3x + 600 = 8x$$
$$5x = 600$$
$$x = \frac{600}{5}$$
$$x = 120$$

So, male students to start with are 480. After 200 female students joined, the total number of female students is $120 + 200 = 320$.

Therefore, the total number of students after the 200 female students are admitted is $320 + 480 = 800$.

A.85 Mr. John Smith receives \$1400.00 as salary every week after taxes. If he is in the 30% tax bracket, what is his salary per week before taxes?

SOLUTION:

A.85 Let's call his salary before Taxes "S."

$$100\% - 30\% = 70\%$$

70% of Mr. John Smith's salary before taxes (S) is equal to 1400.00.

$$\frac{70}{100} \times S = 1400, \qquad S = \frac{1400 \times 100}{70} = 2000 \text{ (which is in dollars)}$$

$$S = \$2000.00$$

A.87 Fill in the blanks:

Row	Fraction	Decimal	%, Percentage
1	---------	0.02	--------
2	---------	0.125	---------
3	----------	3.750	---------
4	$\frac{1}{16}$	----------	----------
5	$\frac{1}{8}$	----------	-----------
6	$\frac{3}{2}$	-----------	-----------
7	-----------	-----------	30
8	-----------	-----------	50
9	-----------	-----------	10
10	-----------	-----------	200
11	$\left(\frac{1}{4}\right)^2$	------------	-----------
12	$\left(\frac{1}{10}\right)^5$	------------	-----------

SOLUTION:

A.87

Row	Fraction	Decimal	%, Percentage
1	$\frac{1}{50}$	0.02	2
2	$\frac{1}{8}$	0.125	12.5
3	$\frac{15}{4}$	3.750	375
4	$\frac{1}{16}$	0.0625	6.25
5	$\frac{1}{8}$	0.125	12.5
6	$\frac{3}{2}$	1.5	150
7	$\frac{3}{10}$	0.3	30
8	$\frac{1}{2}$	0.5	50
9	$\frac{1}{10}$	0.1	10
10	2	2.0	200
11	$\left(\frac{1}{4}\right)^2$	0.0625	6.25
12	$\left(\frac{1}{10}\right)^5$	0.00001	0.001

A.89 *The Wall Street Journal*, Friday, May 25, 2012, reported that the 52-week Crude Oil high and low prices per barrel were, respectively, \$109.77 and \$75.67. Find the percentage decrease from high price to low price.

SOLUTION:

A.89 Percentage change is

$$\frac{109.77 - 75.67}{109.77} \times 100 = \frac{34.1}{109.77} \times 100\% = 0.3106 \times 100\% = 31.06\%$$