

CHAPTER

1

SYMMETRICAL COMPONENTS USING MATRIX METHODS

THE METHOD of symmetrical components was originally proposed by Fortescue in 1918 [1]. We study three-phase balanced systems, by considering these as single-phase system. The current or voltage vectors in a three-phase balanced system are all displaced by 120 electrical degrees from each other. The fundamental texts on electrical circuits [2] derive the equations governing the behavior of three-phase balanced systems. This simplicity of representing a three-phase as a single-phase system is lost for unbalanced systems. The method of symmetrical components has been an important tool for the study of unbalanced three-phase systems, unsymmetrical short-circuit currents, models of rotating machines and transmission lines, etc.

There have been two approaches for the study of symmetrical components:

- A physical description, without going into much mathematical matrix algebra equations.
- A mathematical approach using matrix theory.

This book will cover each of these two approaches to provide a comprehensive understanding. The mathematical approach is adopted in this chapter followed by Chapter 2, which provides some practical concepts and physical significance of symmetrical components. Some publications on symmetrical components are in References [3–6].

It can be mentioned that in the modern age of digital computers, the long-hand calculations using symmetrical components is outdated. See an example of short-circuit calculations in Chapter 6, which is a tedious and lengthy hand calculation for a simple system consisting of five components connected to two buses. In practical power systems, the number of buses can exceed 1000. Today, the systems are modeled with raw input data, and the programs will calculate the sequence components and apply these to derive a result demanded by the problem. Yet, an understanding of symmetrical components is necessary to understand the results from system simulation programs. The reader should understand the limitations of simulation models which are discussed in the chapters to follow.

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1.1 TRANSFORMATIONS

Symmetrical component method is a *transform*. There are number of transformations in electrical engineering, for example, Laplace transform, Fast Fourier transform, Park's transform, Clarke component transform, and the like. There are three steps that are applicable in any transform for the solution of a problem:

- The parameters of the original problem are transformed by the application of the transform to entirely new parameters.
- The solution with the altered parameters is arrived at. The fundamental concept is that the transformed parameters are much easier to solve than the parameters of the original problem.
- Inverse transform is applied to the solved parameters to get to the solution of the original problem.

1.2 CHARACTERISTIC ROOTS, EIGENVALUES, AND EIGENVECTORS

The matrix theory can be applied to understand some fundamental aspects of symmetrical components. The reader must have some knowledge of the matrices as applied to electrical engineering [7, 8], though enough material is provided for continuity of reading.

1.2.1 Definitions

1.2.1.1 Characteristic Matrix

For a square matrix $\bar{A}$, the matrix formed as $|\bar{A} - \lambda\bar{I}|$ is called the *characteristic matrix*. Here λ is a scalar and $\bar{I}$ is a unity matrix.

1.2.1.2 Characteristic Polynomial

The determinant $|\bar{A} - \lambda\bar{I}|$ when expanded gives a polynomial is called the *characteristic polynomial* of matrix $\bar{A}$.

1.2.1.3 Characteristic Equation

The equation $|\bar{A} - \lambda\bar{I}| = 0$ is called the *characteristic equation* of matrix $\bar{A}$.

1.2.1.4 Eigenvalues

The roots of the characteristic equation are called the *characteristic roots* or *eigenvalues*.

1.2.1.5 Eigenvectors, Characteristic Vectors

Each characteristic root λ has a corresponding non-zero vector $\bar{x}$ that satisfies the equation

$$|\bar{A} - \lambda\bar{I}| \bar{x} = 0 \quad (1.1)$$

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The non-zero vector $\bar{x}$ is called the *characteristic vector* or *eigenvector*.

Some properties of the eigenvalues are:

- Any square matrix $\bar{A}$ and its transpose $\bar{A}'$ have the same eigenvalues.
- The sum of the eigenvalues of a matrix is equal to the trace of the matrix (the sum of the elements on the principal diagonal is called the trace of the matrix).
- The product of the eigenvalues of the matrix is equal to the determinant of the matrix. If

$$\lambda_1, \lambda_2, \dots, \lambda_n$$

are the eigenvalues of $\bar{A}$, then the eigenvalues of

$$\begin{aligned} k\bar{A} &\text{ are } k\lambda_1, k\lambda_2, \dots, k\lambda_n \\ \bar{A}^m &\text{ are } \lambda_1^m, \lambda_2^m, \dots, \lambda_n^m \\ \bar{A}^{-1} &\text{ are } 1/\lambda_1, 1/\lambda_2, \dots, 1/\lambda_n \end{aligned} \quad (1.2)$$

- Zero is a characteristic root of a matrix, only if the matrix is singular.
- The characteristic roots of a triangular matrix are diagonal elements of the matrix.
- The characteristics roots of a Hermitian matrix are all real.
- The characteristic roots of a real symmetric matrix are all real, as the real symmetric matrix will be Hermitian. A square matrix $\bar{A}$ is called a Hermitian matrix if every i - j th element of the matrix is equal to the conjugate complex j - i th element, that is, the matrix

$$\begin{vmatrix} 1 & 2+j3 & 3+j \\ 2-j3 & 2 & 1-j2 \\ 3-j & 1+j2 & 5 \end{vmatrix}$$

is a Hermitian matrix.

Example 1.1 Find eigenvalues and eigenvectors of matrix

$$\bar{A} = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & 2 & 0 \end{vmatrix}$$

Write the characteristic equation

$$\bar{A} = \begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & 2 & 0-\lambda \end{vmatrix} = 0$$

Its solution can be shown to be

$$(\lambda + 3)(\lambda + 3)(\lambda - 5) = 0$$

Therefore, the eigenvalues are

$$\lambda = -3, -3, 5$$

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When $\lambda = -3$,

$$\begin{vmatrix} -2+3 & 2 & -3 \\ 2 & 1+3 & -6 \\ -1 & 2 & 3 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

By manipulation of rows, this can be written as

$$\begin{vmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

Therefore,

$$x_1 + 2x_2 - 3x_3 = 0 \quad (1.3)$$

Assume that

$$\begin{aligned} x_1 &= k_1 \\ x_2 &= k_2 \end{aligned}$$

Then,

$$x_3 = \frac{1}{3}(k_1 + 2k_2)$$

The eigenvectors are

$$\begin{vmatrix} k_1 \\ k_2 \\ 1/3(k_1 + 2k_2) \end{vmatrix} \quad \begin{vmatrix} k_1 \\ k_2 \\ 1/3(k_1 + 2k_2) \end{vmatrix}$$

Similarly for $\lambda = 5$,

$$\begin{vmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & 2 & -5 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

This can be reduced to

$$\begin{vmatrix} -1 & -2 & -5 \\ 0 & 16 & 32 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_1 \\ x_2 \\ x_3 \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

$$\begin{aligned} -x_1 - 2x_2 - 5x_3 &= 0 \\ 16x_2 + 32x_3 &= 0 \end{aligned} \quad (1.4)$$

There are two equations and three unknowns. Consider $x_3 = 1$. Then,

$$\begin{aligned} -x_1 - 2x_2 &= 5 \\ 16x_2 &= -32 \end{aligned}$$

Then, eigenvector for $\lambda = 5$ is

$$\begin{vmatrix} -1 \\ -2 \\ 1 \end{vmatrix}$$

Concept 1.1 *The eigenvectors are not unique. Equations (1.3) and (1.4) can be satisfied with other assumed numbers.*

1.3 DIAGONALIZATION OF A MATRIX

If a square matrix $\bar{A}$ of $n \times n$ has n linearly independent eigenvectors, then a matrix $\bar{P}$ can be found so that

$$\bar{P}^{-1}\bar{A}\bar{P} \quad (1.5)$$

is a diagonal matrix.

The matrix $\bar{P}$ is found by grouping the eigenvectors of $\bar{A}$ into a square matrix, that is, $\bar{P}$ has eigenvalues of $\bar{A}$ as its diagonal elements.

1.4 SIMILARITY TRANSFORMATION

The transformation of matrix $\bar{A}$ into $\bar{P}^{-1}\bar{A}\bar{P}$ is called a *similarity transformation*. Diagonalization is a special case of similarity transformation.

Example 1.2 Continuing with Example 1.1, in Equation (1.4), by assuming that $x_3 = 1$, and solving, one eigenvector is

$$(-1, -2, 1)^t$$

Similarly in Equation (1.3), choose arbitrarily $x_1 = 2$, $x_2 = -1$, then

$$(2, -1, 0)^t$$

is an eigenvector. Similarly,

$$(3, 0, 1)^t$$

can be the third eigenvector. A matrix formed of these vectors is

$$\bar{P} = \begin{vmatrix} -1 & 2 & 3 \\ -2 & -1 & 0 \\ 1 & 0 & 1 \end{vmatrix}$$

and the diagonalization is obtained by application of Equation (1.5):

$$\bar{P}^{-1}\bar{A}\bar{P} = \begin{vmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{vmatrix}$$

This contains the eigenvalues as the diagonal elements.

Now choose some other eigenvectors and form a new matrix, say

$$\bar{P} = \begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{vmatrix}$$

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Again with these values, $\bar{P}^{-1}A\bar{P}$ is

$$\begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{vmatrix}^{-1} \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix} \begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{vmatrix}$$

This is the same result as before.

Concept 1.2 *The sets of equations are an uncoupled system, which can be obtained with different eigenvectors.*

1.5 DECOUPLING A THREE-PHASE SYMMETRICAL SYSTEM

Let us decouple a three-phase transmission line section, where each phase has a mutual coupling with respect to ground. This is shown in Figure 1.1a. An impedance matrix of the three-phase transmission line can be written as

$$\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \quad (1.6)$$

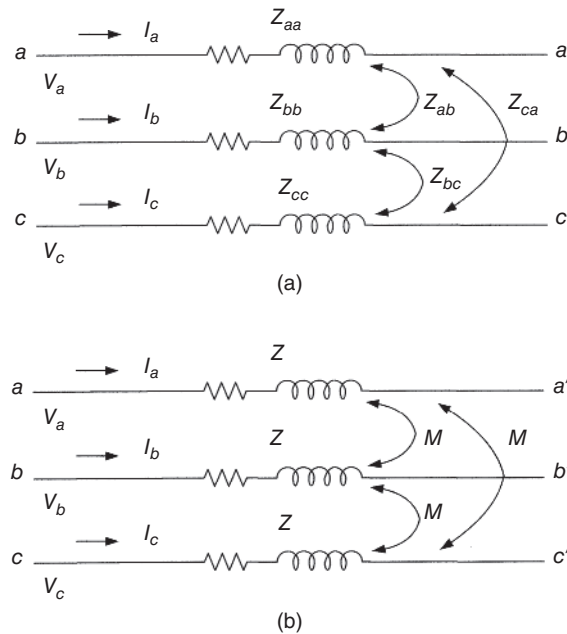


Figure 1.1 (a) An unbalanced three-phase section of a transmission line; (b) a balanced section.

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where Z_{aa} , Z_{bb} , and Z_{cc} are the self-impedances of the phases a , b , and c ; Z_{ab} is the mutual impedance between phases a and b , and Z_{ba} is the mutual impedance between phases b and a .

Assume that the line is *perfectly symmetrical*. This means all the mutual impedances, $Z_{ab} = Z_{ba} = M$, and all the self-impedances, $Z_{aa} = Z_{bb} = Z_{cc} = Z$, are equal (Figure 1.1b). This reduces the impedance matrix to

$$\begin{vmatrix} Z & M & M \\ M & Z & M \\ M & M & Z \end{vmatrix} \quad (1.7)$$

It is required to decouple this system using symmetrical components. First, find the eigenvalues:

$$\begin{vmatrix} Z - \lambda & M & M \\ M & Z - \lambda & M \\ M & M & Z - \lambda \end{vmatrix} = 0 \quad (1.8)$$

The eigenvalues are

$$\begin{aligned} \lambda &= Z + 2M \\ &= Z - M \\ &= Z - M \end{aligned} \quad (1.9)$$

The eigenvectors can be found by making $\lambda = Z + 2M$ and then $Z - M$. Substituting $\lambda = Z + 2M$,

$$\begin{vmatrix} Z - (Z + 2M) & M & M \\ M & Z - (Z + 2M) & M \\ M & M & Z - (Z + 2M) \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} = 0 \quad (1.10)$$

By manipulation of the rows, this can be reduced to

$$\begin{vmatrix} -2 & 1 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} X_1 \\ X_2 \\ X_3 \end{vmatrix} = 0 \quad (1.11)$$

This means that

$$\begin{aligned} -2X_1 + X_2 + X_3 &= 0 \\ -X_2 + X_3 &= 0 \\ X_3 &= 0 \end{aligned}$$

This gives $X_1 = X_2 = X_3 = \text{any arbitrary constant } k$. Thus, one of the eigenvectors of the impedance matrix is

$$\begin{vmatrix} k \\ k \\ k \end{vmatrix} \quad (1.12)$$

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It can be called the zero sequence eigenvector of the symmetrical component transformation matrix and can be written as

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad (1.13)$$

Similarly for $\lambda = Z - M$,

$$\begin{bmatrix} Z - (Z - M) & M & M \\ M & Z - (Z - M) & M \\ M & M & Z - (Z - M) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 0 \quad (1.14)$$

which gives

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = 0 \quad (1.15)$$

$$\begin{aligned} X_1 + X_2 + X_3 &= 0 \\ 0X_2 + 0X_3 &= 0 \\ 0X_3 &= 0 \end{aligned}$$

This gives the general relation $X_1 + X_2 + X_3 = 0$. Any choice of X_1, X_2, X_3 , which satisfies this relation is a solution vector. Some choices are shown below:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 \\ a^2 \\ a \end{bmatrix}, \begin{bmatrix} 1 \\ a \\ a^2 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{3}/2 \\ -\sqrt{3}/2 \end{bmatrix}, \begin{bmatrix} 1 \\ -1/2 \\ -1/2 \end{bmatrix}, \text{ and so on,} \quad (1.16)$$

where a is a unit vector operator, which rotates by 120 degrees in the counterclockwise direction, that is,

$$\begin{aligned} a &= -0.5 + j0.866 \\ a^2 &= -0.5 - j0.866 \\ a^3 &= 1 \\ 1 + a + a^2 &= 0 \end{aligned} \quad (1.17)$$

Concept 1.3 Equation (1.16) is an important result and shows that, for perfectly symmetrical systems, multiple eigenvectors can be constructed for the transformation.

1.6 SYMMETRICAL COMPONENT TRANSFORMATION

The voltage drop across a symmetrical section of a three-phase line can be written as

$$\begin{bmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \end{bmatrix} = \begin{bmatrix} Z & M & M \\ M & Z & M \\ M & M & Z \end{bmatrix} \begin{bmatrix} \Delta I_a \\ \Delta I_b \\ \Delta I_c \end{bmatrix} \quad (1.18)$$

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Based on discussions in Section 1.5, the symmetrical component transformation eigenvectors are chosen as

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \quad (1.19)$$

A symmetrical component transformation matrix can, therefore, be written as

$$\bar{T}_s = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \quad (1.20)$$

Its inverse is

$$\bar{T}_s^{-1} = \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \quad (1.21)$$

For the transformation of currents, we can write

$$\begin{vmatrix} I_a \\ I_b \\ I_c \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} I_0 \\ I_1 \\ I_2 \end{vmatrix} \quad (1.22)$$

Or in abbreviated form, we can write

$$\bar{I}_{abc} = \bar{T}_s \bar{I}_{012} \quad (1.23)$$

where $\bar{I}_{abc}$ are the original currents in phases a , b , and c , which are transformed into zero sequence, positive sequence, and negative sequence currents, $\bar{I}_{012}$. The original phasors are subscripted abc and the sequence components are subscripted 012. Similarly, for transformation of voltages,

$$\begin{vmatrix} \Delta V_a \\ \Delta V_b \\ \Delta V_c \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \begin{vmatrix} \Delta V_0 \\ \Delta V_1 \\ \Delta V_2 \end{vmatrix} \quad (1.24)$$

Or in the abbreviated form

$$\Delta \bar{V}_{abc} = \bar{T}_s \Delta \bar{V}_{012} \quad (1.25)$$

Or in general

$$\bar{V}_{abc} = \bar{T}_s \bar{V}_{012} \quad (1.26)$$

where $\bar{V}_{abc}$ are the original voltages in phases a , b , and c , which are transformed into zero sequence, positive sequence, and negative sequence voltages, $\bar{V}_{012}$. The original phasors are subscripted abc and the sequence components are subscripted 012.

Conversely,

$$\bar{I}_{012} = \bar{T}_s^{-1} \bar{I}_{abc}, \quad \bar{V}_{012} = \bar{T}_s^{-1} \bar{V}_{abc} \quad (1.27)$$

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The transformation of impedance is not straightforward and is derived as follows:

$$\begin{aligned}\bar{V}_{abc} &= \bar{Z}_{abc} \bar{I}_{abc} \\ \bar{T}_s \bar{V}_{012} &= \bar{Z}_{abc} \bar{T}_s \bar{I}_{012} \\ \bar{V}_{012} &= \bar{T}_s^{-1} \bar{Z}_{abc} \bar{T}_s \bar{I}_{012} = \bar{Z}_{012} \bar{I}_{012}\end{aligned}\quad (1.28)$$

Therefore,

$$\bar{Z}_{012} = \bar{T}_s^{-1} \bar{Z}_{abc} \bar{T}_s \quad (1.29)$$

$$\bar{Z}_{abc} = \bar{T}_s \bar{Z}_{012} \bar{T}_s^{-1} \quad (1.30)$$

Applying the impedance transformation to the original impedance matrix of the three-phase symmetrical transmission line in Equation (1.7), the transformed matrix is

$$\begin{aligned}\bar{Z}_{012} &= \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} Z & M & M \\ M & Z & M \\ M & M & Z \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \\ &= \begin{vmatrix} Z + 2M & 0 & 0 \\ 0 & Z - M & 0 \\ 0 & 0 & Z - M \end{vmatrix}\end{aligned}\quad (1.31)$$

The original three-phase coupled system has been decoupled through symmetrical component transformation. It is diagonal, and all off-diagonal terms are zero, meaning that there is no coupling between the sequence components. Decoupled positive, negative, and zero sequence networks are shown in Figure 1.2. The transformations described above are called symmetrical component transformations.

1.7 DECOUPLING A THREE-PHASE UNSYMMETRICAL SYSTEM

Now consider that the original three-phase system is not completely balanced. *Ignoring the mutual impedances* in Equation (1.6), let us assume unequal phase impedances, Z_1 , Z_2 , and Z_3 , that is, the impedance matrix is

$$\bar{Z}_{abc} = \begin{vmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_3 \end{vmatrix} \quad (1.32)$$

The symmetrical component transformation is

$$\begin{aligned}\bar{Z}_{012} &= \frac{1}{3} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} Z_1 & 0 & 0 \\ 0 & Z_2 & 0 \\ 0 & 0 & Z_3 \end{vmatrix} \begin{vmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{vmatrix} \\ &= \frac{1}{3} \begin{vmatrix} Z_1 + Z_2 + Z_3 & Z_1 + a^2 Z_2 + a Z_3 & Z_1 + a Z_2 + Z_3 \\ Z_1 + a Z_2 + a Z_3 & Z_1 + Z_2 + Z_3 & Z_1 + a^2 Z_2 + a Z_3 \\ Z_1 + a^2 Z_2 + a Z_3 & Z_1 + a Z_2 + a Z_3 & Z_1 + Z_2 + Z_3 \end{vmatrix}\end{aligned}\quad (1.33)$$

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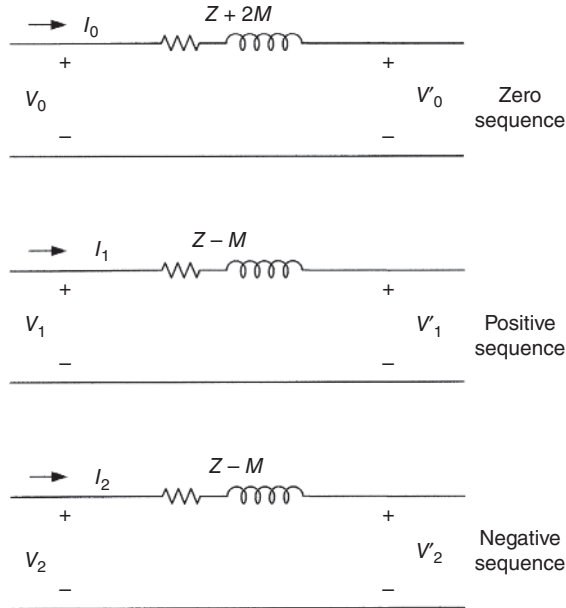


Figure 1.2 Decoupled system of Figure 1.1b.

The resulting matrix shows that the *original unbalanced system is not decoupled*. If we start with equal self-impedances and unequal mutual impedances or vice versa, the resulting matrix is nonsymmetrical. It is a minor problem today, as nonreciprocal networks can be easily handled on digital computers. Nevertheless, the main application of symmetrical components is for the study of unsymmetrical faults. Negative sequence relaying, stability calculations, and machine modeling are some other examples.

Concept 1.4 *It is assumed that the system is perfectly symmetrical before an unbalance condition occurs. The asymmetry occurs only at the fault point. The symmetrical portion of the network is considered to be isolated, to which an unbalanced condition is applied at the fault point. In other words, the unbalance part of the network can be thought to be connected to the balanced system at the point of fault. Practically, the power systems are not perfectly balanced and some asymmetry always exists. However, the error introduced by ignoring this asymmetry is, generally, small.*

1.8 CLARKE COMPONENT TRANSFORMATION

Review the concept outlined in Section 1.5, that *eigenvectors are not unique*. The Clarke component transformation is defined as

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ 1 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_0 \\ V_\alpha \\ V_\beta \end{bmatrix} \quad (1.34)$$

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Note that the eigenvalues satisfy the relations derived in Equation (1.16), and

$$\begin{bmatrix} V_0 \\ V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad (1.35)$$

Similar equations may be written for the current. Note that

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}, \begin{bmatrix} 0 \\ \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} \end{bmatrix} \quad (1.36)$$

are the eigenvectors of a perfectly symmetrical impedance.

The transformation matrices are

$$\bar{T}_c = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1/2 & \sqrt{3}/2 \\ 1 & 1/2 & \sqrt{3}/2 \end{bmatrix} \quad (1.37)$$

$$\bar{T}_c^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 2/3 & -1/3 & -1/3 \\ 0 & 1/\sqrt{3} & -1/\sqrt{3} \end{bmatrix} \quad (1.38)$$

and as before,

$$\bar{Z}_{0\alpha\beta} = \bar{T}_c^{-1} \bar{Z}_{abc} \bar{T}_c \quad (1.39)$$

$$\bar{Z}_{abc} = \bar{T}_c \bar{Z}_{0\alpha\beta} \bar{T}_c^{-1} \quad (1.40)$$

The Clarke component expression for a perfectly symmetrical system is

$$\begin{bmatrix} V_0 \\ V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} Z_{00} & 0 & 0 \\ 0 & Z_{\alpha\alpha} & 0 \\ 0 & 0 & Z_{\beta\beta} \end{bmatrix} \begin{bmatrix} I_0 \\ I_\alpha \\ I_\beta \end{bmatrix} \quad (1.41)$$

The same philosophy of transformation can also be applied to systems with two or more three-phase circuits in parallel. The instantaneous power theory and EMTP (Electromagnetic Transient Program) modeling of transmission lines, are based upon Clarke's transformation [9, 10].

1.9 SIGNIFICANCE OF SELECTION OF EIGENVECTORS IN SYMMETRICAL COMPONENTS

The significance of symmetrical components will be illustrated with an example.

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Example 1.3 As the vector selection is arbitrary, design an entirely new transformation system and apply it to a transformation.

Let the eigenvectors be

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1/4 \\ -3/4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1/2 \\ -1/2 \end{bmatrix}$$

Then the transformation matrix is

$$\bar{T}_t = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1/4 & -1/2 \\ 1 & -3/4 & -1/2 \end{bmatrix} \quad (1.42)$$

Its inverse is

$$\bar{T}_t^{-1} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 2 & -2 \\ 2/3 & -2\frac{1}{3} & 1\frac{2}{3} \end{bmatrix} \quad (1.43)$$

Then,

$$\begin{aligned} \bar{Z}_{012} &= \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 2 & -2 \\ 2/3 & -2\frac{1}{3} & 1\frac{2}{3} \end{bmatrix} \begin{bmatrix} Z & M & M \\ M & Z & M \\ M & M & Z \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.25 & -0.5 \\ 1 & -0.75 & -0.5 \end{bmatrix} \\ &= \begin{bmatrix} Z+2M & 0 & 0 \\ 0 & Z-M & 0 \\ 0 & 0 & Z-M \end{bmatrix} \end{aligned} \quad (1.44)$$

This gives the same impedance transformation as with the symmetrical component vectors. Rather than being thrilled with this result let us examine further.

Let us denote the voltages in a balanced three-phase system as

$$\begin{aligned} V_a &= V \angle 0^\circ \\ V_b &= V \angle -120^\circ \\ V_c &= V \angle 120^\circ \end{aligned} \quad (1.45)$$

This is the standard representation in a three-phase system with counterclockwise rotation of the vectors. The “rotation” signifies the direction in which the vectors rotate. The “sequence” describes the order in which the three-phase vectors rotate, that is, it can be ABC, or ACB, see Chapter 2. Then, first apply symmetrical component transformation:

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \end{bmatrix} \begin{bmatrix} V \\ V(-0.5 - j0.866) \\ V(-0.5 + j0.866) \end{bmatrix} = \begin{bmatrix} 0 \\ V \\ 0 \end{bmatrix} \quad (1.46)$$

That is, zero and negative sequence components are zero. In the symmetrical components, the operator “ a ” rotates a unity vector by 120 degrees electrical in the counterclockwise direction. By international convention, this is the accepted rotation

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of the vectors in a three-phase system. Also in a balanced system, each current and voltage vector is displaced by 120 electrical degrees. Thus, for a balanced system these vectors can be called as positive sequence vectors (components), and the *negative sequence and zero sequence components should be zero*, as illustrated in Equation (1.46).

Now apply the transformation in Equation (1.43)

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0 & 2 & -2 \\ 2/3 & -2/3 & 1/3 \end{bmatrix} \begin{bmatrix} V \\ V(-0.5 - j0.866) \\ V(-0.5 + j0.866) \end{bmatrix} = \begin{bmatrix} 0 \\ -j3.5V \\ V(1 + j3.5) \end{bmatrix} \quad (1.47)$$

Meaningful interpretation cannot be applied to the end result of Equation (1.47). It flouts the accepted convention and phasors in a three-phase system. We question the results obtained in this equation, as a three-phase unbalanced system does not generate any negative or zero sequence components. For further simplified explanation, see Chapter 2.

Concept 1.5 *In symmetrical component transformation, the eigenvectors are selected to provide meaningful physical interpretation. The scientific and engineering logic of symmetrical components lies in selection of appropriate eigenvectors.*

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