1

Reliability Concepts

1.1 Definitions of Reliability

It is difficult to define reliability precisely because this term evokes many different meanings in different contexts. In the field of reliability engineering, we primarily deal with engineered devices and systems. Single-word descriptions may depict one or two aspects of reliability in an engineering application context, but they are inadequate for a technical definition of *engineering reliability*. So, how do engineers and technical experts define *reliability*?

- Radio Electronics Television Manufacturers Association (1955) – "Reliability is the probability of a device performing its purpose adequately for the period of time intended under the operating conditions encountered."
- ASQ (2020) – "Reliability is defined as the probability that a product, system, or service will perform its intended function adequately for a specified period of time, or will operate in a defined environment without failure."
- Meeker and Escobar (1998a) – "Reliability is often defined as the probability that a system, vehicle, machine, device, and so on will perform its intended function under operating conditions, for a specified period of time."
- Condra (2001a) – "Reliability is quality over time."
- Yang (2007) - "Reliability is defined as the probability that a product performs its intended function without failure under specified conditions for a specified period of time."

There are some variations in the aforementioned definitions, but they all either explicitly or implicitly state the following characteristics of reliability:

- Reliability is a probabilistic measure – the probability of a functioning product, service, or system.
- Reliability is a function of time – the probability function of successfully performing tasks, as designed, over time.
- Reliability is defined under specified or intended operating conditions.

We define a function, $S(t)$, to be the survival, or reliability, function, which is the probability of the product, service, or system being successfully operated under its normal operating condition at time t; in other words, the unit survived past time t.

Design of Experiments for Reliability Achievement, First Edition.
Steven E. Rigdon, Rong Pan, Douglas C. Montgomery, and Laura J. Freeman.

Companion website: www.wiley.com/go/rigdon/designexperiments

1.2 Concepts for Lifetimes

When an item fails, the "fix" sometimes involves making a repair to bring it back to a working condition. Another possibility is to discard the item and replace it with a working item. In general, the more complex a system is, the more likely we are to repair it, and the simpler it is the more likely we are to scrap it and replace it with a new item. For example, if the starter on our automobile fails, we would probably take out the old starter and replace it with a new one. In a case like this, the automobile is a *repairable* system, but the starter is *nonrepairable* since our fix has been to replace it entirely.

Since complex systems, which are usually repairable, are made up of component parts that are nonrepairable, we will focus in this book on nonrepairable items. If these nonrepairable items are designed and built to have high reliability, then the system should be reliable as well. For nonrepairable systems we are interested in studying the distribution of the time to the first (and only) failure, or more generally, the effect of predictor variables on this lifetime. This lifetime need not be measured in calendar time; it could be measured in operating time (for an item that is switched on and off periodically), miles driven (for a motor vehicle like a car or truck), copies made (for a copier or printer), or cycles (for an industrial machine). For nonrepairable systems, we study the occurrence of events in time, such as failures (and subsequent repairs) or recurrence of a disease or its symptoms. See Rigdon and Basu (2000) for a treatment of repairable systems.

The lifetime T of a unit is a random variable that necessarily takes on nonnegative values. Usually, but not always, we think of T as a continuous random variable taking on values in the interval $[0, \infty)$. There are various forms that the distribution may take, many of which, including the exponential, Weibull and gamma, are presented in detail Chapter 2. Here we present the fundamental ideas and terms for continuous random variables.

Definition 1.1 *The probability density function (PDF) of a continuous random variable T is a function $f(t)$ with the property that*

$$P(a < T < b) = \int_a^b f(t)\ dt.$$

Thus, probabilities for a continuous random variable are found as areas under the PDF. (See Figure 1.1a.) Note that a and b can be $-\infty$ or ∞. Since $\int_a^a f(t)\ dt = 0$, the probability that $T = a$, that is, the probability that T *equals* a particular value a, is equal to zero. This also implies that

$$P(a < T < b) = P(a < T \leq b) = P(a \leq T < b) = P(a \leq T \leq b) = \int_a^b f(t)\ dt.$$

See Figure 1.1b.

Since the probability is 1 that T is between $-\infty$ and ∞, we have the property that

$$\int_{-\infty}^{\infty} f(t)\ dt = 1. \tag{1.1}$$

See Figure 1.1c. Also, since all probabilities must be nonnegative, the PDF $f(t)$ must satisfy

$$f(t) \geq 0, \quad \text{all } t. \tag{1.2}$$

The results in (1.1) and (1.2) are the fundamental properties for a PDF.

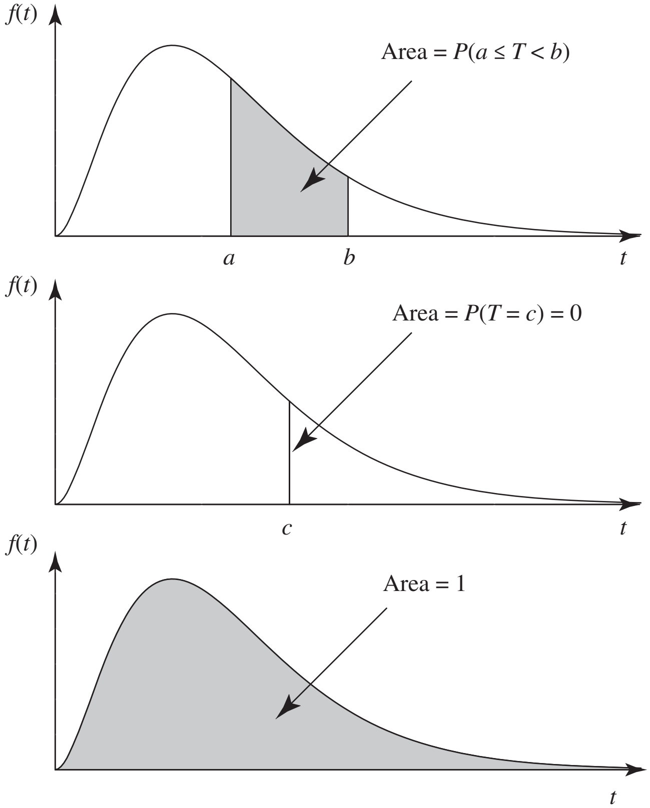

Figure 1.1 Properties of the PDF for a lifetime distribution.

The development earlier makes no assumption about the possible values that the random variable T can take on. For lifetimes, which must be nonnegative, we have $f(t) = 0$ for $t < 0$. Thus, for lifetimes, the PDF must satisfy

$$\begin{aligned} f(t) &\geq 0, \quad \text{all } t, \\ f(t) &= 0, \quad t < 0, \\ \int_0^\infty f(t)\ dt &= 1. \end{aligned}$$

The set of values for which the PDF of the random variable T is positive is called the *support* of T. The support for a lifetime distribution is $[0, \infty)$, although for some distributions we exclude the possibility of $t = 0$.

Note that the PDF does not give probabilities directly; for example, $f(4)$ does not give the probability that $T = 4$. Rather, as an approximation we can write

$$P(4 < T < 4 + \Delta t) = \int_4^{4+\Delta t} f(t)\ dt \approx \Delta t\, f(4).$$

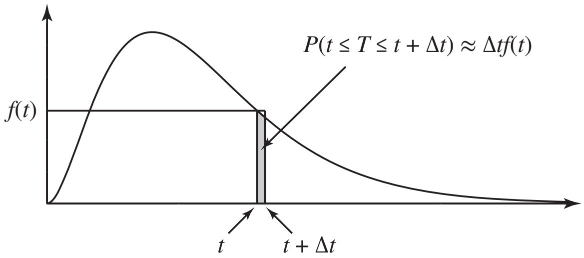

Figure 1.2 Approximation $P(t \le T \le t + \Delta t) \approx \Delta t f(t)$.

(See Figure 1.2.) Thus, the PDF can be interpreted as

$$\Delta t\, f(t) \approx P(t < T < t + \Delta t), \tag{1.3}$$

or equivalently

$$f(t) \approx \frac{P(t < T < t + \Delta t)}{\Delta t}.$$

To be precise, the PDF is equal to the limit of the right side earlier as $\Delta t \to 0^+$:

$$f(t) = \lim_{\Delta t \to 0^+} \frac{P(t < T < t + \Delta t)}{\Delta t}.$$

Definition 1.2 *The cumulative distribution function (CDF) of the random variable T is defined as*

$$F(t) = P(T \le t) = \int_{-\infty}^{t} f(x)\ dx, \tag{1.4}$$

where $f(x)$ is the PDF for T.

Note that we have changed the variable of integration from t to x, in order to avoid confusion with the upper limit on the integral. For a lifetime distribution with support $(0, \infty)$, we have the result

$$F(t) = P(T \le t) = P(0 \le T \le t) = \int_{0}^{t} f(x)\ dx. \tag{1.5}$$

As $t \to \infty$ on the right side earlier, the integral goes to $\int_0^\infty f(x)\ dx$, which equals 1. Also, since the probability of having a negative lifetime is 0, the CDF must be zero for all $t < 0$. Finally, since the CDF "accumulates" probability up to and including t, increasing t can only increase (or hold constant) the CDF. Thus, for a lifetime distribution, the CDF must satisfy

$$F(t) = 0, \quad \text{for } t \le 0,$$

$$\lim_{t \to \infty} F(t) = 1,$$

$$F(t) \ \text{ is nondecreasing.}$$

Equation (1.4) shows how to get the CDF given the PDF. A formula for the reverse (getting the PDF from the CDF) can be obtained by differentiating both sides of (1.4) with respect to t and applying the fundamental theorem of calculus:

$$F'(t) = \frac{d}{dt} \int_{-\infty}^{t} f(x)\ dx = f(t). \tag{1.6}$$

In other words, the PDF is the derivative of the CDF.

Definition 1.3 *The survival function, or reliability function, is defined to be*

$$S(t) = P(T > t) = \int_t^\infty f(x)\ dx.$$

In other words, $S(t)$ is the probability that an item survives past time t, while $F(t)$ is the probability that it fails at or before time t (that is, that it *doesn't* survive past time t). Thus, $S(t)$ and $F(t)$ are related by

$$S(t) = 1 - F(t).$$

One of the most important concepts in lifetime analysis is the hazard function.

Definition 1.4 *The hazard function is*

$$h(t) = \lim_{\Delta t \to 0^+} \frac{P(t \leq T \leq t + \Delta t | T > t)}{\Delta t} \tag{1.7}$$

As an approximation, we can write

$$\Delta t\ h(t) \approx P(t \leq T \leq t + \Delta t | T > t) \tag{1.8}$$

analogous to (1.3).

The probability in the definition of the hazard is a *conditional* probability; it is conditioned on survival to the beginning of the interval. This is a natural quantity to consider because it makes intuitive sense to talk about the failure probability of an item that is still working. It is conceptually more difficult to talk about the probability of an item failing if the item might or might not be working. If we replace the conditional probability in the definition of the hazard function with an unconditional probability, we get

$$\lim_{\Delta t \to 0^+} \frac{P(t \leq T \leq t + \Delta t)}{\Delta t} \tag{1.9}$$

which is equal to the PDF $f(t)$. Thus, the PDF is the (limit of) the probability of failing in a small interval when viewed *before* testing begins. The hazard is the (limit of) the probability of failing in a small interval for a unit that is known to be working.

The hazard function can be written as

$$\begin{aligned} h(t) &= \lim_{\Delta t \to 0^+} \frac{P(t < T \leq t + \Delta t | T > t)}{\Delta t} \\ &= \lim_{\Delta t \to 0^+} \frac{P(t < T \leq t + \Delta t \wedge T > t)}{\Delta t P(T > t)} \\ &= \lim_{\Delta t \to 0^+} \frac{P(t < T \leq t + \Delta t) / \Delta t}{P(T > t)} \\ &= \frac{f(t)}{S(t)}. \end{aligned} \tag{1.10}$$

Indeed, many books *define* the hazard function in this way. We choose to define the hazard as the limit of a conditional probability because this intuitive concept is helpful for understanding the failure mechanism.

To illustrate the difference between hazard and density, consider a discrete case, say, where items are placed on test and are observed every 1000 hours. Let h_i denote the probability that an item fails in the ith interval $(1000(i-1), 1000i]$. Suppose first that $h_i = \frac{1}{10}$, so that there is a probability of $\frac{1}{10} = 0.1$ that a working unit will fail in any time interval. Thus, at the end of a time interval when we inspect those units still operating, we would expect that about one-tenth of them would fail. We could naturally ask the question "What is the probability that a unit fails in the ith interval?" This is different from the question "What is the probability that a unit that is currently operating fails in the ith interval?" The difference is that the latter is a conditional probability (conditioned on the unit still operating), whereas the former is an unconditional probability. The answer to the latter question is: the hazard h_i. To get at the answer to the latter question, we can observe that

$$\begin{aligned}
p_1 &= P\left(\text{failure in } (0, 1000]\right) = \frac{1}{10} \\
p_2 &= P\left(\text{failure in } (1000, 2000]\right) \\
&= P\left(\text{no failure in } (0, 1000]\right) P\left(\text{failure in } (1000, 2000] | \text{no failure in } (0, 1000]\right) \\
&= \frac{9}{10}\frac{1}{10} \\
p_3 &= P\left(\text{failure in } (2000, 3000]\right) \\
&= P\left(\text{no failure in } (0, 1000]\right) \\
&\quad \times P\left(\text{no failure in } (1000, 2000] | \text{no failure in } (0, 1000]\right) \\
&\quad \times P\left(\text{failure in } (2000, 3000] | \text{no failure in } (0, 2000]\right) \\
&= \frac{9}{10}\frac{9}{10}\frac{1}{10},
\end{aligned}$$

and in general,

$$\begin{aligned}
p_i &= P\left(\text{failure in } (1000\,(i-1)\,, 1000i]\right) \\
&= P\left(\text{no failure in } (0, 1000]\right) \\
&\quad \times P\left(\text{no failure in } (1000, 2000] | \text{no failure in } (0, 1000]\right) \\
&\quad \times \cdots \\
&\quad \times P\left(\text{failure in } (1000\,(i-1)\,, 1000i] | \text{no failure in } (0, 1000\,(i-1)]\right) \\
&= \left(\frac{9}{10}\right)^{i-1}\frac{1}{10}, \quad i = 1, 2, \ldots
\end{aligned}$$

This is, of course, the geometric distribution. Plots of h_i and p_i for the case of a constant (discrete) hazard are shown in Figure 1.3.

Suppose now that the probability mass function (rather than the hazard function) is constant, with $p_i = 0.1$. Since $\sum p_i = 1$, we conclude that $p_i = 0.1$ for $i = 1, 2, \ldots, 10$. The hazard is then

$$\begin{aligned}
h_i &= P(\text{failure in interval } i | \text{survival to beginning of interval } i) \\
&= \frac{P\left(\text{failure in interval } i\right)}{P\left(\text{survival to beginning of interval } i\right)} \\
&= \frac{P\left(\text{failure in interval } i\right)}{1 - P\left(\text{death before beginning of interval } i\right)} \\
&= \frac{1/10}{1-(i-1)/10} \\
&= \frac{1}{10-i+1}.
\end{aligned}$$

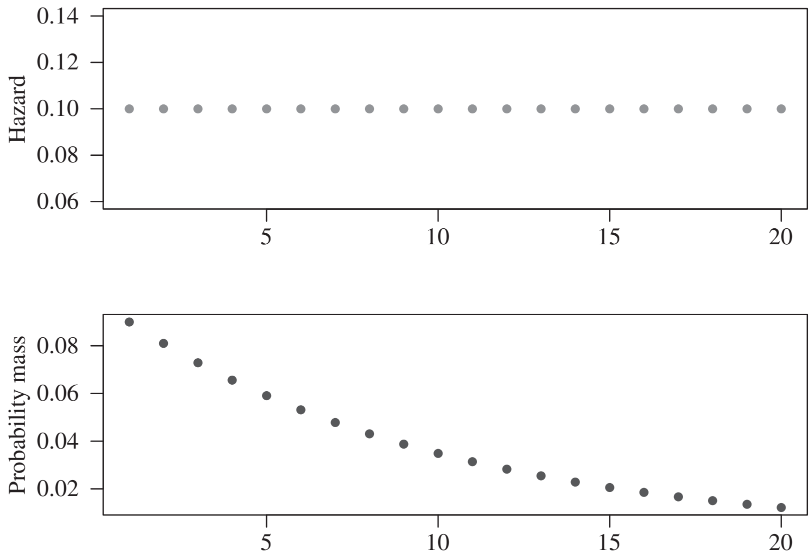

Figure 1.3 Hazard and probability mass function for the case of constant hazard.

Thus, $h_1 = \frac{1}{10}, h_2 = \frac{1}{9}, \dots, h_{10} = \frac{1}{1} = 1$. The last number, $h_{10} = 1$, may seem a little surprising, but if $p_i = 0$ for $i > 10$, and if an item hasn't failed up through interval $i = 9$, then it *must* fail at time $i = 10$. The hazard and probability mass for the constant probability case are shown in Figure 1.4.

The cumulative hazard is defined to be the accumulated area under the hazard function. To be precise, the cumulative hazard is defined to be

$$H(t) = \int_0^t h(u)\, du. \tag{1.11}$$

Any one of the PDF, CDF, survival function, hazard, or cumulative hazard function is enough to determine the lifetime distribution. In other words, knowing any one of these can get you all of the others. For example, Eq. (1.5) shows how you can get the CDF if you know the PDF. If we know the hazard function $h(t)$, we can use the relationship

$$h(t) = \frac{f(t)}{S(t)} = -\frac{S'(t)}{S(t)}$$

to find $S(t)$. To see this, notice that this is a simple first-order linear differential equation with initial condition $S(0) = 1$, which can be solved by integrating both sides from $u = 0$ to $u = t$. This yields

$$\int_0^t h(u)\, du = \int_0^t -\frac{S'(u)}{S(u)}\, du = -\log S(t) + \log S(0) = -\log S(t) \tag{1.12}$$

from which we obtain the relationship

$$S(t) = \exp\left(-\int_0^t h(u)\, du\right). \tag{1.13}$$

We leave the other relationships as an exercise. Table 1.1 shows most of the relationships.

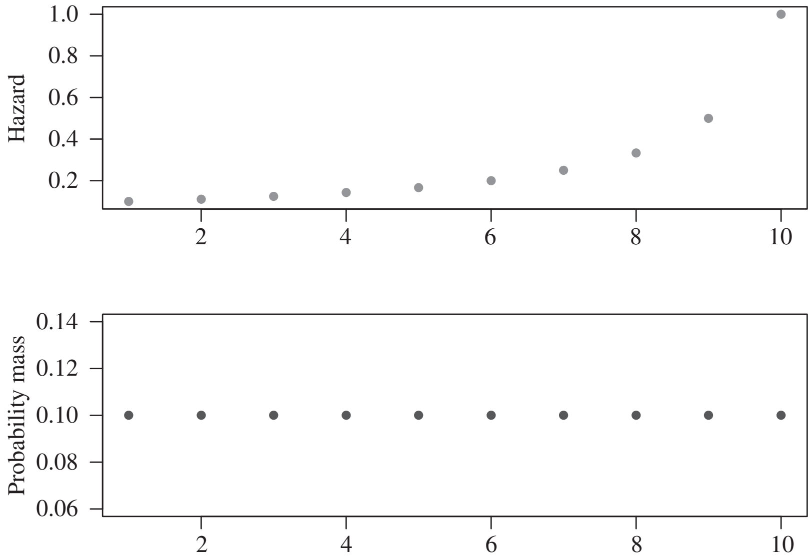

Figure 1.4 Hazard and probability mass function for the case of constant probability mass.

1.3 Censoring

A life testing experiment, where many units are operating simultaneously, may be terminated before all of the units have failed. This is a typical scenario, because many units will be highly reliable and will not fail during a test. This leads to the concept of censoring, where we cannot observe the lifetime of an item because of the design of the life testing experiment.

Definition 1.5 *When we cannot observe the exact failure time of an item, but rather we can only observe an* interval *in which the failure was observed, we say that the observation is* ***censored****. This interval could be an unbounded interval, such as* (c, ∞) *or a bounded interval such as* $(0, a)$*.*

The most common type of censoring occurs when the life test is stopped before all items have failed. Let τ denote the censoring time, that is, the time of termination of the test. In this case, we would know the exact failure times of all items that failed before time τ, but for those still operating at the end of the experiment, we know only that the failure would occur past time τ. For those item still operating, we would only know that the failure time was in the interval (τ, ∞). Actually, the censoring time need not be the same for all items. For example, if the items were placed into service at different times, then the censoring times would be different even if the test was terminated at the same (calendar) time. This type of lifetime censoring, where we observe a survival event (where we know that the failure must occur in the interval (τ, ∞)), is called **right censoring**.

Table 1.1 General relationships between PDF, CDF, hazard function, and cumulative hazard function.

	PDF $f(t)$	CDF $F(t)$	Hazard $h(t)$	Cumulative hazard $H(t)$
$f(t)$	—	$F(t) = \int_0^t f(u)\, du$	$h(t) = \frac{f(t)}{\int_t^\infty f(u)\, du}$	$H(t) = \exp\left(\frac{f(t)}{\int_t^\infty f(u)\, du}\right)$
$F(t)$	$f(t) = F'(t)$	—	$h(t) = \frac{F'(t)}{1 - F(t)}$	$H(t) = -\log(1 - F(t))$
$h(t)$	$f(t) = h(t)\exp\left(-\int_0^t h(u)\, du\right)$	$F(t) = 1 - \exp\left(-\int_0^t h(u)\, du\right)$	—	$H(t) = \int_0^t h(u)\, du$
$H(t)$	$f(t) = H'(t)\exp(-H(t))$	$F(t) = 1 - \exp(-H(t))$	$h(t) = H'(t)$	—

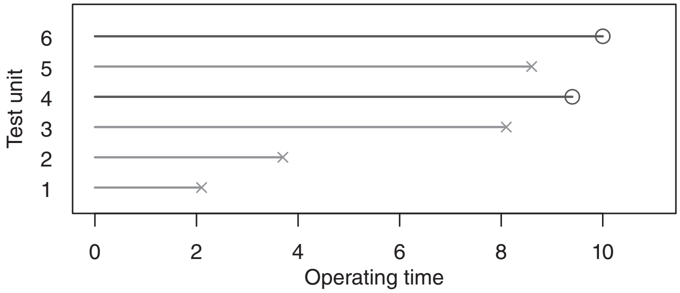

Figure 1.5 Times to failure for six items. The X indicates a failure time and the O indicates a censored time.

Figure 1.5 shows an illustration of right censoring. In this situation six items were placed on test at the same time. Items 1, 2, 3, and 5 failed during the test at times 2.1, 3.7, 8.1, and 8.6, respectively. The other two units were still operating at times 9.4 and 10.0. The failure times are denoted by an X and the censoring times are denoted by an O. The solid lines cover the times for which it is known that the items were operating.

It sometimes happens that observation of an item may begin well after the it was placed into service. This can occur when items are observed in the field. In some instances, it may be the case that the item is observed to have already failed when it is first inspected. For example, suppose an item is placed into service and then not inspected until an age of 100 days. If it was observed to be in a failed condition at that time, then we know only that it failed before time $t = 100$. In other words, the failure must have occurred sometime in the interval $(0, 100)$, but the exact failure time is unknown. This is called **left censoring**, because the failure is known to have occurred before time 100. Figure 1.6 illustrates the concept of left (and right) censoring. Here, items 1, 2, and 4 were placed on test and the failure times were observed to be 24, 37, and 77, respectively. Items 3 and 6 were observed at times 10 and 28, respectively, to be in a failed condition. A dashed line is used to indicate the possible times of actual failure. In general, we use a solid line to indicate times when the units were known to be operating. Finally, items 5 and 7 were still operating at time 99 when the test was terminated. Thus, items 3 and 6 were left censored and items 5 and 7 were right censored.

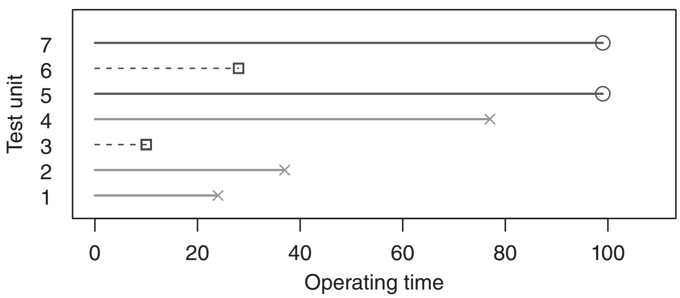

Figure 1.6 Times to failure for seven items. The X indicates a failure event and the O indicates a right censoring event, whereby the failure occurred to the right of the of the O. The dashed line that ends with a □ indicates a left censoring event, whereby the failure occurred to the left of the □.

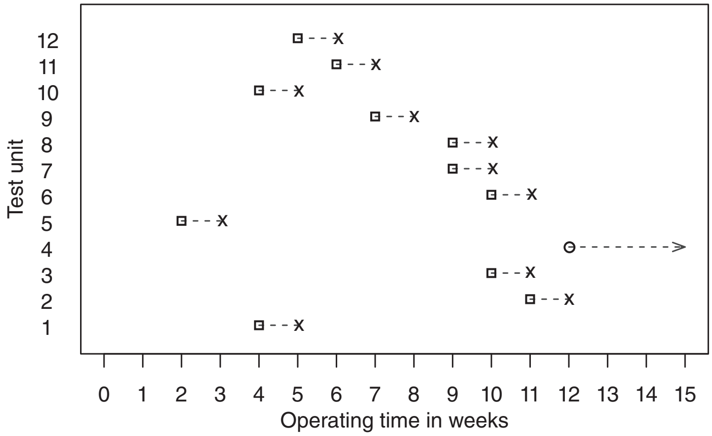

Figure 1.7 Intervals containing the failures for items that are inspected only once a week. The dashed lines indicate possible failure times. One item, the fourth one, was right censored at time $t = 12$. All other observations were interval censored. The □ on the left indicates that this is the left endpoint of the censoring interval. The X on the right indicates that the item had failed before the end of the interval.

Another type of censoring occurs when items are inspected only periodically and observed to be either operating or failed. For example, the items may be inspected once every week. If an item was observed at time $t = 2$ to be operating and at time $t = 3$ to be failed, then it is known that the failure must have occurred in the interval (2,3]. This is called **interval censoring**. The concept is illustrated in Figure 1.7.

Two special kinds of right censoring are called Type I and Type II censoring. For both we assume that all items were placed on test at the same time. Under **Type I** censoring, we terminate the test at a predetermined time T, whereas under **Type II** censoring, we terminate the test after a predetermined number r of failures. Thus, under Type I censoring, the testing time is fixed, but the number of failures is random. Conversely, for Type II censoring the number of failures is fixed, but the testing time is random. For planning reliability experiments it is important to note that time based censoring can result in a failed experiment if the time allowed is not sufficient for achieving the required number of failures for estimating the reliability distribution. Alternatively, Type II censoring guarantees a certain number of failures, but can lead to long experiment timelines.

Often, the censoring mechanism does not fit either of these categories. For example, items are placed on test at random points in time, and some are removed from test for reasons unrelated to the life test. This situation is called *arbitrary censoring*. We hypothesize two random variables T_i and C_i, where T_i is the failure time and C_i is the censoring time. We observe only the first of these to occur, that is, $U_i = \min(T_i, C_i)$. We define the indicator variable δ_i to be 1 if $T_i = \min(T_i, C_i)$ and 0 if $C_i = \min(T_i, C_i)$. Thus, $\delta_i = 1$ if we observe a failure, and $\delta = 0$ if the failure observation is right censored. In most reliability data sets, we will observe the event times $u_1, u_2, \ldots, u_n$ (i.e. the times of either failure or censoring) along with the corresponding indicators $\delta_1, \delta_2, \ldots, \delta_n$. These two sets of variables completely describe the observed failure process.

When the systems under test are repairable, we usually observe multiple failures per unit. These can be represented in a graph, similar to those shown earlier, but with multiple failures per line. In Figure 1.8, we see two repairable systems whose failure times are marked with an X on the time axis. System 1

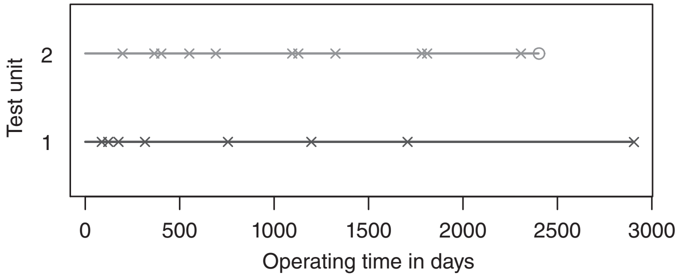

Figure 1.8 Failure times for two repairable systems. For the first system (bottom), the period of observation ended at the time of the last failure. For the second system, the period of observation ended *after* the last failure; this is indicated by the O at the end of the line segment.

(on the bottom part of the figure) is **failure truncated**, that is, the termination of the test coincided with the time of the last failure. System 2 (top) is **time truncated**, that is, the test terminated after the last failure. The fact that system 2 continued to operate failure free from time 1955 to time 2400 is a piece of information that cannot be ignored.

Note the terminology used to describe the termination of a life test. Nonrepairable systems whose (one and only) failure occur in some interval (not at some point in time) are said to be censored. A repairable system whose period of observation ends at some point (either at a failure time or extends past the last observed failure) is said to be truncated. Since we deal almost exclusively with nonrepairable systems, we will refer to censored systems often.

Problems

1.1 Suppose that there are only two possible lifetimes for an item. The lifetime will be either 0 or 1, with equal probability. Let T denote the lifetime. Find
(a) the probability mass function
(b) $E(T)$
(c) the hazard function.

1.2 Rework Problem 1.1, but assume that the probabilities of being 0 and 1 are p and $1 - p$.

1.3 Suppose that the lifetime is a discrete distribution with equal mass at $t = 0,1, \dots, A$. Find
(a) the probability mass function
(b) $E(T)$
(c) the hazard function.

1.4 Repeat Problem 1.3, except assume that the probability that the lifetime T equals i is p_i for $i = 1,2, \dots, A$ where $p_0 + p_1 + \cdots + p_A = 1$.

1.5 Suppose that an item is observed each day at the same time, and it is observed whether the item is or isn't working. During each day there is a constant probability p that a working item will fail. The support for the lifetime T is therefore $1,2, \dots$. Find

(a) the probability mass function
(b) $E(T)$
(c) the hazard function.

1.6 Your town announces that it will test the new severe weather warning system at noon on some day next week (Sunday through Saturday). They will pick a day at random with equal probability to conduct the test.
(a) What is the probability that the test is conducted on Saturday?
(b) If it is Saturday morning and the test has not yet been conducted, what is the probability that it occurs on Saturday?
(c) If it is Friday morning and the test has not yet been conducted, what is the probability that the test occurs on Friday?

1.7 Suppose that the distribution of a lifetime T has PDF

$$f(x) = \begin{cases} \dfrac{1}{(x+1)^2} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Find
(a) the CDF
(b) $E(T)$
(c) the hazard function.

1.8 Suppose that the distribution of a lifetime T has PDF

$$f(x) = \begin{cases} \dfrac{1}{2}\, e^{-x/2} & \text{if } x \geq 0, \\ 0 & \text{if } x < 0. \end{cases}$$

Find
(a) the CDF
(b) $E(T)$
(c) the hazard function.

1.9 Suppose that the hazard function for a lifetime distribution is

$$h(t) = t, \qquad t \geq 0.$$

Find
(a) the PDF
(b) the CDF
(c) $E(T)$.

1.10 Suppose that the hazard function for a lifetime distribution is

$$h(t) = \sqrt{t}, \qquad t \geq 0.$$

Find
(a) the PDF
(b) the CDF
(c) $E(T)$.

1.11 Suppose that n items are put on test at the same time. Let $T_1, T_2, \dots, T_n$ denote the lifetimes, which we assume come from the distribution with the PDF given in Problem 1.7. The test will be terminated at time $t = 5$.
(a) What is the probability that one particular item will be censored?
(b) If we denote by X the number of items out of the n that fail before the censoring time $t = 5$, what is the distribution of X?
(c) Find $E(X)$.
(d) If we want the expected number of failed items to be at least 20, how large must n be?
(e) Describe the kind of censoring involved in this problem.

1.12 Repeat Problem 1.11 with the distribution from Problem 1.8.

1.13 Suppose that n items are put on test and then tested weekly, say, every Monday. Let T is the lifetime of these items, measured in hours. Describe the kind of censoring involved in this situation.

1.14 Suppose that $n = 10$ items are placed on test as described in the previous problem. The observed quantity X is the first week at which an item is not working. For example, if an item was working after week 3, but was not working when it was observed at week 4, then $x = 4$. The test is terminated after time 5 and the observations are

```
3   3   5   2   1   5+   3   1   4   5+
```

where a + indicates a right censored observation. Sketch a graph like the one in Figure 1.7 for this data.

1.15 In Eqs. (1.6), (1.5), (1.13), (1.12), and (1.11) we derived several of the relationships among the PDF, CDF, hazard, and cumulative hazard. For this problem, derive the other relationships:
(a) PDF → hazard
(b) hazard → PDF
(c) PDF → cumulative hazard
(d) cumulative hazard → PDF
(e) CDF → cumulative hazard
(f) cumulative hazard → CDF
(g) cumulative hazard → hazard.