1

Introduction

1.1 Background

1.1.1 Networked Multi-agent Systems

Most large-scale systems in nature and human societies, such as biological neural networks, ecosystems, metabolic pathways, the Internet, the WWW, and electrical power grids can be described by networks with nodes representing individuals in the system and edges representing the connections between them. Recently, the study of various complex networks and systems has attracted increasing attention from researchers in various fields of physics, mathematics, engineering, biology, and sociology alike [8, 35, 62, 90, 117, 118, 119, 123, 142].

In the early 1960s, Erdös and Rényi (ER) proposed a random-graph model, which laid a solid foundation for modern network theory [35]. In a random network, each pair of nodes is connected with a certain probability. In order to describe a transition from a regular network to a random network, Watts and Strogatz (WS) proposed an interesting small-world network model [123]. Then, Newman and Watts (NW) modified the original WS model to generate another version of the small-world model [80]. Meanwhile, Barabási and Albert (BA) proposed a scale-free network model, in which the degree distribution of the nodes follows a power-law form [8]. Since then, small-world and scale-free networks have been extensively investigated worldwide.

Cooperative and collective behaviors in networks of multiple autonomous agents have also received considerable attention in recent years due to the growing interest in understanding the amazing animal group behaviors, such as flocking and swarming, and also due to their emerging broad applications in sensor networks, UAV (unmanned air vehicles) formations, and robotic teams. To coordinate with other agents in a network, every agent needs to share information with its adjacent peers so that all can agree on a common goal of interest, such as the value of some measurement in a sensor network, the heading in a UAV formation, or the target position of a robotic team.

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Recently, some progress has been made in analyzing cooperative control for collective behaviors in dynamical multi-agent systems, for which some closely related focal topics are synchronization [90, 117, 118, 142], consensus [15, 57, 77, 81, 98, 99, 101, 115], swarming [44, 45], and flocking [82]. More details can be found in survey papers [4, 17, 84, 120].

1.1.2 Collective Behaviors and Cooperative Control in Multi-agent Systems

Synchronization is a typical collective behavior in nature. Since the pioneering work of Pecora and Carroll [90], chaos control and synchronization have received a great deal of attention due to their potential applications in secure communications, chemical reactions, biological systems, and so on [143, 145]. Typically, there are large numbers of nodes in real-world complex networks. In recent years, a lot of work has been devoted to the study of synchronization in various large-scale complex networks [14, 70, 117, 118, 142]. In [117, 118], local synchronization was investigated by the transverse stability to the synchronization manifold, where synchronization was discussed on small-world and scale-free networks. In [132, 134], a distance from the collective states to the synchronization manifold was defined, and based on this, some results were obtained for global synchronization of coupled systems [14, 70]. A general criterion was derived in [142], where the network sizes can be extended to be much larger than those given in [14, 70]. However, it is still very difficult to ensure global synchronization in general large-scale networks due to the computational complexity. Recently, global pinning synchronization for a class of complex networks with switched topologies was addressed in [130] by using tools from stability analysis of switched systems.

The consensus problem has a long history in the field of computer science especially for distributed computing [74]. The idea of consensus was originated from statistical consensus theory by DeGroot [28], which was revisited two decades later for pattern recognition using multi-sensors [10]. Usually, it refers to the problem of how to reach agreement among a group of autonomous agents in a dynamically changing environment [99]. One of the main challenges in solving such a consensus problem is that an agreement has to be reached by all agents in the whole dynamic network while the information of each agent is shared only locally. Various models have been used to study the consensus problem. Vicsek et al. studied a discrete-time system that models a group of autonomous agents moving in the plane with the same speed but different headings [115]. It was shown, through simulation, that using a distributed averaging rule, agents could eventually move in the same direction without centralized coordination. Vicsek's model by nature is a simplified version of the model proposed earlier by Reynolds [101]. Analysis on Vicsek's model, or its continuous-time version, shows that the connectivity of the time-varying graph that describes the neighboring relationships within the group is key in achieving consensus [15, 57, 81, 77, 98]. In particular, in [81], Olfati-Saber and Murray established the relationship between the algebraic connectivity (also called the Fiedler eigenvalue [37]) and the speed of convergence when the underlying directed graph is balanced. A broader class of directed graphs that may lead to reaching consensus are those that contain spanning trees [98], which are also called rooted graphs [15].

It is interesting to observe that Vicsek's model is similar to a class of models discussed in synchronization of complex networks [14, 70, 117, 118, 134, 142]. In 1998, Pecora and Carroll made use of a master stability function to study the synchronization of coupled complex networks [90]. To date, stability and synchronization of small-world and scale-free networks have been investigated extensively using this master stability function method.

In the literature, most work on the consensus problem considered the case where agents are governed by first-order dynamics [11, 57, 72, 81, 98, 114, 134, 141, 142, 151]. Meanwhile, there is a growing interest in consensus algorithms where all agents are governed by second-order dynamics [50, 51, 82, 93, 95, 97, 146]. More precisely, second-order consensus refers to the problem of reaching an agreement among a group of autonomous agents governed by second-order dynamics. A detailed analysis of second-order consensus algorithms is a key step to bring more realistic dynamics into the model of each individual agent based on the general framework of multi-agent systems, thus it can help control engineers to implement distributed cooperative control strategies for networked multi-agent systems. It has been shown that, in sharp contrast to the first-order consensus problem, consensus may fail to be achieved for agents with second-order dynamics even if the network topology has a directed spanning tree [97].

On the other hand, time delay is ubiquitous in biological, physical, chemical, and electrical systems [11, 114]. In biological and communication networks, time delays are usually inevitable due to the possibly slow process of interactions among agents. It has been observed from numerical experiments that consensus algorithms without considering time delays may lead to unexpected instability. In [11, 114], some sufficient conditions were derived for first-order consensus in delayed multi-agent systems.

Very recently, some higher-order consensus algorithms in cooperative control of multi-agent systems were studied, such as in [100] based on the results derived in [97]. However, only third-order consensus was discussed in detail therein. In this book, a general higher-order consensus protocol is designed and analyzed based on the transverse stability to the consensus manifold, which originates from the study of synchronization in complex networks [117]. A detailed analysis of the higher-order consensus algorithms is a prerequisite to introducing more realistic dynamics into the model of each individual agent.

As validated by biological field studies and engineering robotic experiments, swarm cohesion can be achieved in a distributed fashion despite the fact that each agent may only have local information about its nearest neighbors. An in-depth understanding of the principles behind swarming behaviors will help engineers to develop distributed cooperative control strategies and algorithms for networked dynamical systems, such as formations of UAVs, autonomous robotic teams, and mobile sensors networks. Synchronous distributed coordination rules for swarming groups in one- or two-dimensional spaces were studied in [58], where convergence and stability analysis were given. In [44, 45], stability properties of a continuous-time model for swarm aggregation in the n-dimensional space was discussed, and an asymptotic bound for the spatial size of the swarm was computed using the parameters of the swarm model.

In [101], three heuristic rules were suggested by Reynolds to animate flocking behavior: (1) velocity consensus, (2) center cohesion, and (3) collision avoidance. In order to embody the three Reynolds' rules, Tanner et al. designed flocking algorithms in [110, 111], where a collective potential function and a velocity consensus term were introduced. Later, in [82], Olfati-Saber proposed a general framework to investigate distributed flocking algorithms where, in particular, three algorithms were developed for free and constrained flocking. In [110, 111], it was pointed out that due to the time-varying network topology, the set of differential equations describing the flocking behavior of a multi-agent dynamical system is in general nonsmooth; therefore, several techniques from nonsmooth analysis, such as Filippov solutions [38], generalized gradient [25], and differential inclusion [87], were applied for analysis.

1.1.3 Network Control in Multi-agent Systems

In the case where the whole network cannot synchronize by itself, some controllers may be designed and applied to force the network to synchronize. However, it is literally impossible to add controllers to all nodes. To reduce the number of controlled nodes, some local feedback injections may be applied only to a fraction of network nodes, which is known as pinning control [23, 119, 151]. In [47], pinning control of spatiotemporal chaos, and later in [89], global and local control of spatiotemporal chaos in coupled map lattices, were discussed. Recently, in [119], both specific and random pinning schemes were studied, where specific pinning to the nodes with large degrees is shown to require a smaller number of controlled nodes than the random pinning scheme, but the former requires more information about the network than the latter.

Recently, hybrid systems, namely complex systems with both continuous-time and discrete-time event dynamics, have been extensively investigated in the literature, for example continuous-time systems with impulsive responses, sampled data, quantization, to name just a few. Some real-world applications can be modeled by continuous-time systems together with some discrete-time events, such as an A/C unit containing some discrete modes with on or off states, changing the temperature continuously over time. In practice, it is quite difficult to measure the continuous information transmission due to the unreliability of information channels, the capability of transmission bandwidths of networks, etc. Thus, it is more practical to apply sampled-data control, which has been widely studied recently and applied in many real-world systems such as radar tracking systems, power grids, and temperature control. It has been found that sampled-data control has many good properties such as robustness and low cost. Recently, many results have been established from investigating the second-order consensus in multi-agent systems with sampled data. For example, some conditions were derived for multi-agent systems with sampled control by using zero-order holds or direct discretization [18, 41, 66, 163]. On the other hand, consensus of continuous-time multi-agent systems with time-varying topologies and sampled-data control was discussed in [42], and communication delays were considered in multi-agent systems with sampled-data control in [43, 162].

It should be noted that most of the results on consensus problems in multi-agent systems are derived based on a common assumption that the information is transmitted continuously among the agents, that is, each agent shares information with its neighbors without any communication constraints. However, this may not be the case in reality. In some cases, the mobile agents can only communicate with their neighbors at some disconnected time intervals. In order to describe such multi-agent systems more appropriately, a second-order consensus protocol based on globally synchronous intermittent local information feedback was proposed in [124, 129, 126] to guarantee the states of agents converging to consensus asymptotically.

In the literature, many derived conditions for ensuring network synchronization were only sufficient but not necessary, thus are somewhat conservative. Lately, a lot of work has been devoted to using adaptive strategies to adjust network parameters so as to derive better conditions for reaching network synchronization, which employed some existing results from adaptive synchronization in nonlinear systems [139, 155]. For example, in [23, 151, 165, 166], adaptive laws were applied to the control gains from the leader to the followers, and in [23, 151] centralized adaptive schemes were designed on the network coupling strengths. However, there is not much work on how to update the coupling weights of the network for reaching synchronization. In addition, in order to reach consensus or synchronization, some additional global conditions in terms of the spectrum of the Laplacian matrix or its eigenvalues must be satisfied even if the multi-agent system is linear, which actually did not take full advantage of the powerful distributed protocol technology. For example, in [50, 95, 146, 149], in order to reach second-order consensus, the Laplacian matrix or its eigenvalues must be known a priori. To overcome the disadvantage for checking the global information under the local distributed protocol, fully distributed adaptive control in multi-agent systems was investigated recently, which will be introduced in this book.

1.1.4 Distributed Consensus Filtering in Sensor Networks

Sensor networks have attracted increasing attention due to their wide applications in robotics, surveillance and environment monitoring, information collection, wireless communication networks, and so on. A sensor network consists of a large number of sensor nodes distributing over a spatial region. Each sensor has some levels of communication, intelligence for signal processing, and data fusion, which build up a sensing network. Due to the limited energy, computational ability, and communication capability, typically a large number of sensor nodes are used in a wider region so as to achieve higher accuracy of estimating the quantities of interest. Each sensor node is equipped with a microelectronic device having limited power source, which might not be able to transmit messages over a large sensor network. In order to save energy, a natural way is to carry out data fusion to reduce the communication overhead. Therefore, distributed estimation and tracking is one of the most important problems in large-scale sensor networks today. From a network-theoretic perspective, a large-scale sensor network can be viewed as a complex network or a multi-agent system with each node representing a sensor and each edge carrying the information exchange between two sensors. It would be interesting to see how synchronization of complex networks [70, 117, 118, 142] and consensus of multi-agent systems [57, 81, 98] can be used in distributed consensus filtering design. In a complex network, each node communicates with its neighboring nodes to exchange information, so that all the states could finally reach the synchronization or consensus manifold. Therefore, it is quite natural to use the synchronization fundamentals of complex networks and consensus in multi-agent systems as the theoretic basis for distributed consensus filtering design. Practically, it is very difficult to observe all the states of the target, so pinning observers may be designed in the case where the informed sensors can only measure partial states of the target. This notion will also be introduced in the present book.

1.2 Organization

This book focuses on distributed cooperative control in multi-agent systems, which includes complex dynamics, hybrid control, adaptive control, distributed filtering, etc. The contents of the book are summarized as follows.

In Chapter 2, first-order consensus for cooperative agents with nonlinear dynamics in a directed network is discussed. Both local and global consensus are defined and investigated. Techniques for studying synchronization in such complex networks are exploited to establish various sufficient conditions for reaching consensus. The local consensus problem is first studied by combining tools of complex analysis, local consensus manifold approach, and Lyapunov methods. A generalized algebraic connectivity is then derived for studying the global consensus problem in strongly connected networks and also in a broad class of networks containing spanning trees, for which ideas from algebraic graph theory, matrix theory, and Lyapunov methods are utilized. The concept of a virtual network, which has the same spectrum as the original one, is formulated to simplify the analysis.

In Chapter 3, some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems are presented. Here, theoretical analysis is carried out for the case where each agent with second-order dynamics is governed by the position and velocity terms and the asymptotic velocity is constant. A necessary and sufficient condition is given to ensure second-order consensus and it is found that both the real and imaginary parts of the eigenvalues of the Laplacian matrix of the corresponding network play key roles in reaching consensus. Based on this result, a second-order consensus algorithm is derived for the multi-agent system with communication delays. A necessary and sufficient condition is established, which shows that consensus can be achieved in a multi-agent system whose network topology contains a directed spanning tree if and only if the time delay is less than a critical value. Then, the second-order consensus problem is extended to multi-agent systems with nonlinear dynamics and directed topologies where the final asymptotic velocity is time-varying. Some sufficient conditions are derived for reaching second-order consensus in multi-agent systems with nonlinear dynamics based on algebraic graph theory, matrix theory, and the Lyapunov control approach.

Next, some general higher-order distributed consensus protocols in multi-agent dynamical systems are designed in Chapter 4. The notion of network synchronization is first introduced, with some necessary and sufficient conditions derived for higher-order consensus. It is found that consensus can be reached if and only if all subsystems are asymptotically stable. Based on this result, consensus regions are characterized. It is proved that for the mth-order consensus, there are at most $\left\lfloor \frac{m+1}{2} \right\rfloor$ disconnected stable and unstable consensus regions. It is shown that consensus can be achieved if and only if all the nonzero eigenvalues of the Laplacian matrix lie in the stable consensus regions. Moreover, since the ratio of the largest to the smallest nonzero eigenvalues of the Laplacian matrix plays a key role in reaching consensus, a scheme for choosing the coupling strength is derived, which determines the eigen-ratio. Furthermore, a leader-follower control problem with full-state or partial-state observations in multi-agent dynamical systems is considered respectively, which reveals that the agents with very small degrees must be informed.

In Chapter 5, the stability of a continuous-time swarm model with nonlinear profiles is investigated. It is shown that, under some mild conditions, all agents in a swarm can reach cohesion within a finite time, where some upper bounds for the cohesion are derived in terms of the parameters of the swarm model. The results are then generalized by allowing stochastic noise and switching between nonlinear profiles. Furthermore, swarm models with changing communication topologies and unbounded repulsive interactions among agents are studied by nonsmooth analysis, where the sensing range of each agent is limited and the possibility of collision among nearby agents can be high.

In Chapter 6, using tools from algebraic graph theory and nonsmooth analysis in combination with the ideas of collective potential functions, velocity consensus and navigation feedback, a distributed leader-follower flocking algorithm for multi-agent dynamical systems with time-varying velocities is developed, where each agent is governed by second-order dynamics. The distributed leader-follower flocking algorithm deals with the case where the group has one virtual leader with time-varying velocity. For each agent, this algorithm consists of four terms: the first term is the self nonlinear dynamics, which determines the final time-varying velocity; the second term is determined by the gradient of the collective potential between this agent and all of its neighbors; the third term is the velocity consensus term; the fourth term is the navigation feedback term of the leader. To avoid an unpractical assumption that the informed agents sense all the states of the leader, the new distributed algorithm is developed by making use of observer-based pinning navigation feedback. In this case, each informed agent only requires partial information about the leader, yet the velocity of the whole group can still converge to that of the leader; furthermore, the centroid of those informed agents, having the leader's position information, follows the trajectory of the leader asymptotically.

In Chapter 7, based on full sampled-data information, a distributed linear consensus protocol with second-order dynamics is first designed, where both sampled position and velocity data are utilized. A necessary and sufficient condition based on the sampling period, coupling gains, and spectrum of the Laplacian matrix, is established for reaching consensus of the system. It is found that second-order consensus in such a multi-agent system can be achieved by appropriately choosing the sampling period determined by a third-order polynomial function. Interestingly, second-order consensus cannot be reached for a sufficiently large sampling period while it can be reached for a sufficiently small one under some conditions. Then, the coupling gains are designed under the given network structure and the sampling period. Furthermore, the consensus regions are characterized based on the spectrum of the Laplacian matrix. On the other hand, second-order consensus in delayed undirected networks with sampled position and velocity data is discussed. A necessary and sufficient condition is also given, by which appropriate sampling periods can be chosen to achieve consensus in multi-agent systems. Then, a distributed linear consensus protocol for second-order dynamics is designed, where both the current and some sampled past position data are utilized. In particular, consensus regions are characterized. In such cases, it may even be possible to find some disconnected stable consensus regions determined by choosing appropriate sampling periods. Furthermore, the problem of consensus in directed networks of multiple agents with intrinsic nonlinear dynamics and sampled-data information is discussed. A new protocol is deduced from a class of continuous-time linear consensus protocols by implementing data-sampling technique and a zero-order hold circuit. On the basis of a delayed-input approach, the sampled-data multi-agent system is converted to an equivalent nonlinear system with a time-varying delay. Theoretical analysis on this time-delayed system shows that consensus with asymptotic time-varying velocities in a strongly connected network can be achieved over some suitable sampled-data intervals. A multi-step procedure is further presented to estimate the upper bound of the maximal allowable sampling intervals. The results are then extended to a network topology with a directed spanning tree. For the case of the topology without a directed spanning tree, it is shown that the new protocol can still guarantee the system to achieve consensus by appropriately informing a fraction of agents.

In Chapter 8, the problem of second-order consensus is investigated for a class of multi-agent systems with a fixed directed topology and some communication constraints, where each agent is assumed to share information only with its neighbors on some disconnected time intervals. A consensus protocol is designed based on synchronous intermittent local information, to coordinate the states of agents to converge to second-order consensus under a fixed strongly connected topology, which is then extended to the case where the communication topology contains a directed spanning tree. By using tools from algebraic graph theory and Lyapunov control methods, it is proved that second-order consensus can be reached if the general algebraic connectivity of the communication topology is larger than a threshold value, and the mobile agents communicate with their neighbors frequently enough as the network evolves. Furthermore, consensus of second-order multi-agent systems with nonlinear dynamics and intermittent communication is investigated.

In Chapter 9, distributed adaptive control in multi-agent systems is studied. First, distributed adaptive control of synchronization in complex networks is discussed. An effective distributed adaptive strategy to tune the coupling weights of a network is designed, based on local information of the node dynamics. The analysis is then extended to the case where only a small fraction of coupling weights are adjusted. A general criterion is derived and it is found that synchronization can be reached if the subgraph consisting of the edges and nodes corresponding to the updated coupling weights contains a spanning tree. Then, pinning control in complex networks is investigated. The design of distributed control gains for consensus in multi-agent systems with second-order nonlinear dynamics is discussed. First, an effective distributed adaptive gain-design strategy is proposed, based only on local information of the network structure. Then, a leader-follower consensus problem in multi-agent systems with updated control gains is studied. A distributed adaptive law is derived for each follower, based on local information of its neighboring agents and the leader if this follower is an informed agent. Finally, a distributed leader-follower consensus problem in multi-agent systems with unknown nonlinear dynamics is investigated by combining the variable structure approach and the adaptive method.

In Chapter 10, some applications of collective behaviors in multi-agent systems are studied. A new filtering problem for sensor networks is investigated. A new type of distributed consensus filters is designed, where each sensor can communicate with the neighboring sensors, and filtering can be performed in a distributed way. In the pinning control approach, only a small fraction of sensors need to measure the target information, with which the whole network can be controlled. Furthermore, pinning observers are designed in the case that the sensors can only observe partial target information.

Finally, in Chapter 11, delay-induced consensus and quasi-consensus protocols in multi-agent dynamical systems are designed, where both the current and delayed position information are utilized. Time delays, in a common perspective, can induce periodic oscillations or even chaos in dynamical systems. However, it is found that consensus and quasi-consensus in a multi-agent system cannot be reached without the delayed position information under the given protocol while they can be achieved with a relatively small time delay by appropriately choosing the coupling strengths. A necessary and sufficient condition for reaching consensus in time-delayed multi-agent dynamical systems is established. It is shown that consensus and quasi-consensus can be achieved if and only if the time delay is bounded by some critical values which depend on the coupling strengths and the largest eigenvalue of the Laplacian matrix of the network. The motivation for studying quasi-consensus is revealed, and a potential relationship between the second-order multi-agent system with delayed positive feedback and the first-order system with distributed-delay control input is discussed.

Finally in Chapter 12, conclusions are drown and future research outlook is presented with some brief discussions.