1

Reliability Assessment

Reliability is a critical attribute for the modern technological components and systems. Uncertainty exists on the failure occurrence of a component or system, and proper mathematical methods are developed and applied to quantify such uncertainty. The ultimate goal of reliability engineering is to quantitatively assess the probability of failure of the target component or system [1]. In general, reliability assessment can be carried out by both parametric or nonparametric techniques. This chapter offers a basic introduction to the related definitions, models and computation methods for reliability assessments.

1.1 Definitions of Reliability

According to the standard ISO 8402, reliability is the ability of an item to perform a required function, under given environmental and operational conditions and for a stated period of time without failure. The term "item" refers to either a component or a system. Under different circumstances, the definition of reliability can be interpreted in two different ways:

1.1.1 Probability of Survival

Reliability of an item can be defined as the complement to its probability of failure, which can be estimated statistically on the basis of the number of failed items in a sample. Suppose that the sample size of the item being tested or monitored is n_0. All items in the sample are identical, and subjected to the same environmental and operational conditions. The number of failed items is n_f and the number of the survived ones is n_s, which satisfies

$$n_f + n_s = n_0 \tag{1.1}$$

Reliability Analysis, Safety Assessment and Optimization: Methods and Applications in Energy Systems and Other Applications, First Edition. Enrico Zio and Yan-Fu Li.

The percentage of the failed items in the tested sample is taken as an estimate of the unreliability, $\hat{Q}$,

$$\hat{Q} = \frac{n_f}{n_0} \tag{1.2}$$

Complementarily, the estimate of the reliability, $\hat{R}$, of the item is given by the percentage of survived components in the sample:

$$\hat{R} = \frac{n_s}{n_0} = 1 - \hat{Q} \tag{1.3}$$

Example 1.1

A valve fabrication plant has an average output of 2,000 parts per day. Five hundred valves are tested during a reliability test. The reliability test is held monthly. During the past three years, 3,000 valves have failed during the reliability test. What is the reliability of the valve produced in this plant according to the test conducted?

Solution

The total number of valves tested in the past three years is

$$n_0 = 500 \times 12 \times 3 = 18000$$

The number of failed components is

$$n_f = 3000$$

According to Equation 1.3, an estimate of the valve reliability is

$$\hat{R} = \frac{n_s}{n_0} = \frac{n_0 - n_f}{n_0} = \frac{18000 - 3000}{18000} \approx 0.833$$

1.1.2 Probability of Time to Failure

Let random variable T denote the time to failure. Then, the reliability function at time t can be expressed as the probability that the component does not fail at time t, that is,

$$R(t) = P(T > t) \tag{1.4}$$

Denote the cumulative distribution function (cdf) of T as $F(t)$. The relationship between the cdf and the reliability is

$$R(t) = 1 - F(t) \tag{1.5}$$

Further, denote the probability density function (pdf) of failure time T as $f(t)$. Then, equation (1.5) can be rewritten as

$$R(t) = 1 - \int_0^t f(\xi) d\xi \tag{1.6}$$

Example 1.2

The failure time of a valve follows the exponential distribution with parameter $\lambda = 0.025$ (in arbitrary units of time^{-1}). The value is new and functioning at time $t = 0$. Calculate the reliability of the valve at time $t = 30$ (in arbitrary units of time).

Solution

The pdf of the failure time of the valve is

$$f(t) = \lambda e^{-\lambda t} = 0.025 e^{-0.025t}, t \geq 0$$

The reliability function of the valve is given by

$$R(t) = 1 - \int_0^t 0.025 e^{-0.025\xi} d\xi$$

At time $t = 30$, the value of the reliability is

$$R(30) = 1 - \int_0^{30} 0.025 e^{-0.025\xi} d\xi \approx 0.472$$

In all generality, the expected value or mean of the time to failure T is called the *mean time to failure* (MTTF), which is defined as

$$MTTF = E[T] = \int_0^\infty t f(t) dt \tag{1.7}$$

It is equivalent to

$$MTTF = \int_0^\infty R(t) dt \tag{1.8}$$

Another related concept is the *mean time between failures* (MTBF). MTBF is the average working time between two consecutive failures. The difference between MTBF and MTTF is that the former is used only in reference to a repairable item, while the latter is used for non-repairable items. However, MTBF is commonly used for both repairable and non-repairable items in practice.

The failure rate function or hazard rate function, denoted by $h(t)$, is defined as the conditional probability of failure in the time interval $[t, t+\Delta t]$ given that it has been working properly up to time t, which is given by

$$h(t) = \lim_{\Delta t \to 0} P(T \le t + \Delta t \mid T > t) = \frac{f(t)}{R(t)} \tag{1.9}$$

Furthermore, the cumulative failure rate function, or cumulative hazard function, denoted by $H(t)$, is given by

$$H(t) = \int_0^t h(t)dt \tag{1.10}$$

1.2 Component Reliability Modeling

As mentioned in the previous section, in reliability engineering, the time to failure of an item is a random variable. In this section, we briefly introduce several commonly used discrete and continuous distributions for component reliability modeling.

1.2.1 Discrete Probability Distributions

If random variable X can take only a finite number k of different values $x_1, x_2, \ldots, x_k$ or an infinite sequence of different values $x_1, x_2, \ldots$, the random variable X has a discrete probability distribution. The probability mass function (pmf) of X is defined as the function f such that for every real number x,

$$f(x) = P(X = x) \tag{1.11}$$

If x is not one of the possible values of X, then $f(x) = 0$. If the sequence $x_1, x_2, \ldots$ includes all the possible values of X, then $\sum_i f(x_i) = 1$. The cdf is given by

$$F(x_i) = P(X \le x_i) \tag{1.12}$$

1.2.1.1 Binomial Distribution

Consider a machine that produces a defective item with probability p $(0 < p < 1)$ and produces a non-defective item with probability $1 - p$. Assume the events of defects in different items are mutually independent. Suppose the experiment consists of examining a sample of n of these items. Let X denote the number of defective items in the sample. Then, the random variable X follows a binomial distribution with parameters n and p and has the discrete distribution represented by the pmf in (1.14), shown in Figure 1.1. The random variable with this distribution is said to be a binomial random variable, with parameters n and p,

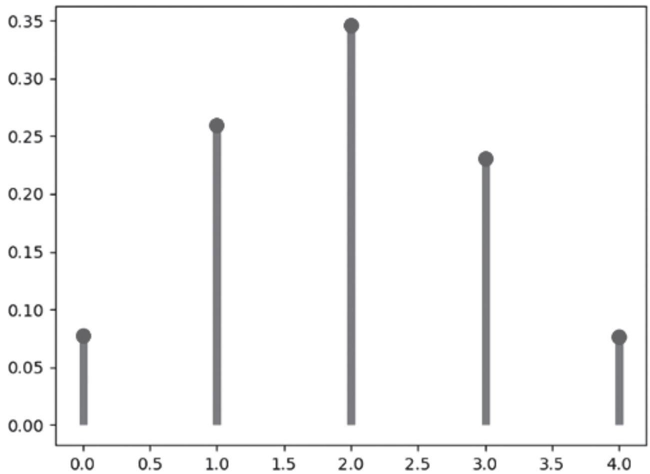

Figure 1.1 The pmf of the binomial distribution with $n=5,\ p=0.4$.

$$f(x)=\begin{cases}\binom{n}{x}p^{x}(1-p)^{n-x}, & for\ x=0,1,\ldots,n,\\ 0, & otherwise.\end{cases} \tag{1.13}$$

The pmf of the binomial distribution is

$$F(x)=\sum_{i=0}^{x}\binom{n}{i}p^{i}(1-p)^{n-i} \tag{1.14}$$

For a binomial distribution, the mean, μ, is given by

$$\mu=np \tag{1.15}$$

and the variance, σ^2, is given by

$$\sigma^2=np(1-p) \tag{1.16}$$

1.2.1.2 Poisson Distribution

Poisson distribution is widely used in quality and reliability engineering. A random variable X has the Poisson distribution with parameter $\lambda, \lambda>0$, the pmf (shown in Figure 1.2) of X is as follows:

$$f(x)=\begin{cases}\dfrac{e^{-\lambda}\lambda^{x}}{x!}, & for\ x=0,1,\ldots,\\ 0, & otherwise.\end{cases} \tag{1.17}$$

The mean and variance of the Poisson distribution are

$$\mu=\sigma^2=\lambda \tag{1.18}$$

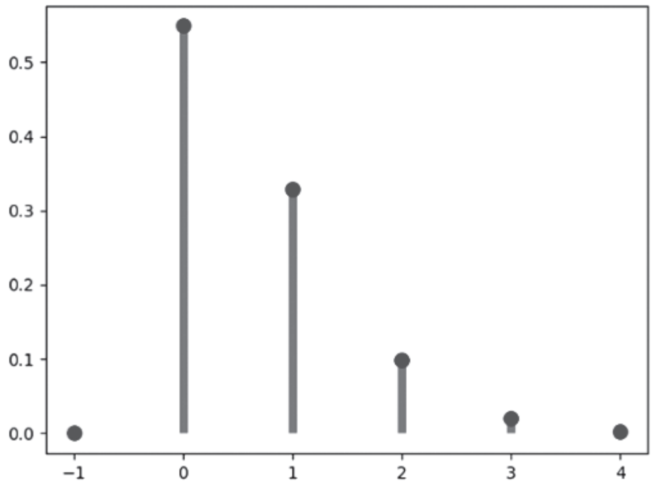

Figure 1.2 The pmf of the Poisson distribution with $\lambda = 0.6$.

1.2.2 Continuous Probability Distributions

We say that a random variable X has a continuous distribution or that X is a continuous random variable if there exists a nonnegative function f, defined on the real line, such that for every interval of real numbers (bounded or unbounded), the probability that X takes a value in an interval $[a,b]$ is the integral of f over that interval, that is,

$$P(a \le X \le b) = \int_a^b f(x)dx. \tag{1.19}$$

If X has a continuous distribution, the function f will be the probability density function (pdf) of X. The pdf must satisfy the following requirements:

$$f(x) \ge 0, \text{ for all } x. \tag{1.20}$$

The cdf of a continuous distribution is given by

$$\int_{-\infty}^{\infty} f(x)dx = 1. \tag{1.21}$$

The mean, μ, and variance, σ^2, of the continuous random variable are calculated by

$$\mu = \int_{-\infty}^{\infty} x f(x) dx \tag{1.22}$$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx.$$

1.2.2.1 Exponential Distribution

A random variable T follows the exponential distribution if and only if the pdf (shown in Figure 1.3) of T is

$$f(t) = \lambda e^{-\lambda t}, t \geq 0, \tag{1.23}$$

where $\lambda > 0$ is the parameter of the distribution. The cdf of the exponential distribution is

$$F(t) = 1 - e^{-\lambda t}, t \geq 0. \tag{1.24}$$

If T denotes the failure time of an item with exponential distribution, the reliability function will be

$$R(t) = e^{-\lambda t}, t \geq 0. \tag{1.25}$$

The hazard rate function is

$$h(t) = \lambda. \tag{1.26}$$

The mean, μ, and variance, σ^2 are

$$\mu = \frac{1}{\lambda} \tag{1.27}$$

$$\sigma^2 = \frac{1}{\lambda^2}.$$

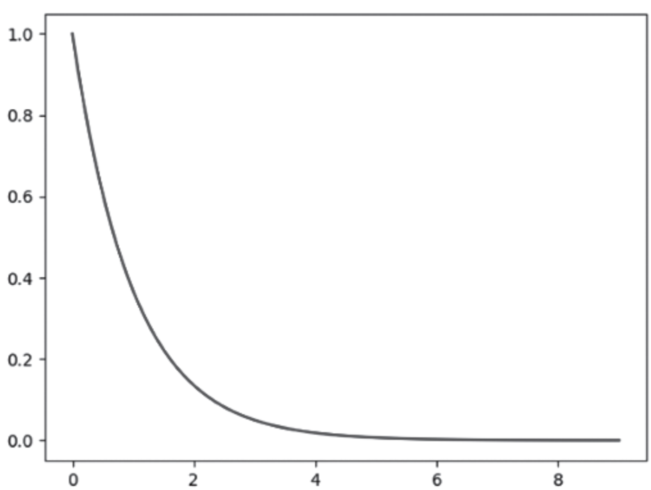

Figure 1.3 The pdf of the exponential distribution with $\lambda = 1$.

1.2.2.2 Weibull Distribution

A random variable T follows the Weibull distribution if and only if the pdf (shown in Figure 1.4) of T is

$$f(t)=\frac{\beta t^{\beta-1}}{\eta^{\beta}}e^{-\left(\frac{t}{\eta}\right)^{\beta}},t\geq 0, \tag{1.28}$$

where $\beta>0$ is the shape parameter and $\eta>0$ is the scale parameter of the distribution. The cdf of the Weibull distribution is

$$F(t)=1-e^{-\left(\frac{t}{\eta}\right)^{\beta}},t\geq 0. \tag{1.29}$$

If T denotes the time to failure of an item with Weibull distribution, the reliability function will be

$$R(t)=e^{-\left(\frac{t}{\eta}\right)^{\beta}},t\geq 0. \tag{1.30}$$

The hazard rate function is

$$h(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1},t\geq 0. \tag{1.31}$$

The mean, μ, and variance, σ^2, are

$$\mu=\eta\Gamma\left(\frac{1+\beta}{\beta}\right), \tag{1.32}$$

$$\sigma^2=\eta^2\left[\Gamma\left(\frac{2+\beta}{\beta}\right)-\left(\Gamma\left(\frac{1+\beta}{\beta}\right)\right)^2\right].$$

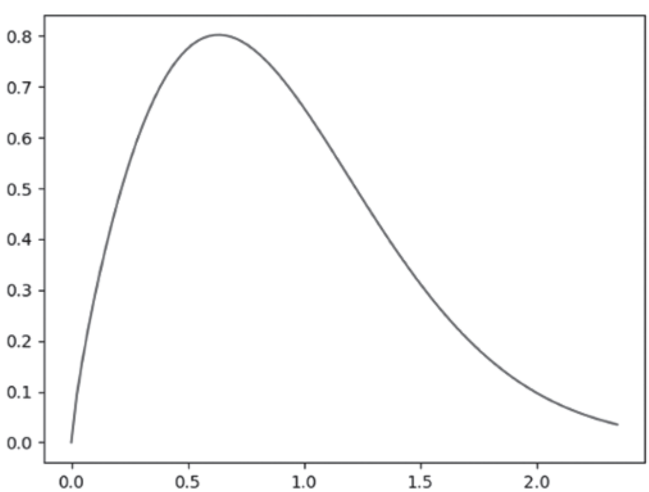

Figure 1.4 The pdf of the Weibull distribution with $\beta=1.79$, $\eta=1$.

1.2.2.3 Gamma Distribution

A random variable T follows the gamma distribution if and only if the pdf (shown in Figure 1.5) of T is

$$f(t)=\frac{\lambda^{\beta}}{\Gamma(\beta)}t^{\beta-1}e^{-\lambda t},t\geq 0, \tag{1.33}$$

where $\beta>0$ is the shape parameter and $\eta>0$ is the scale parameter of the distribution. The cdf of the gamma distribution is

$$F(t)=\frac{\lambda^{\beta}}{\Gamma(\beta)}\int_{0}^{t}x^{\beta-1}e^{-\lambda x}dx,t\geq 0. \tag{1.34}$$

If T denotes the failure time of an item with gamma distribution, the reliability function will be

$$R(t)=\frac{\lambda^{\beta}}{\Gamma(\beta)}\int_{t}^{\infty}x^{\beta-1}e^{-\lambda x}dx,t\geq 0. \tag{1.35}$$

The hazard rate function is

$$h(t)=\frac{t^{\beta-1}e^{-\lambda t}}{\int_{t}^{\infty}x^{\beta-1}e^{-\lambda x}dx},t\geq 0. \tag{1.36}$$

The mean, μ, and variance, σ^2, are

$$\mu=\frac{\beta}{\lambda} \tag{1.37}$$
$$\sigma^2=\frac{\beta}{\lambda^2}.$$

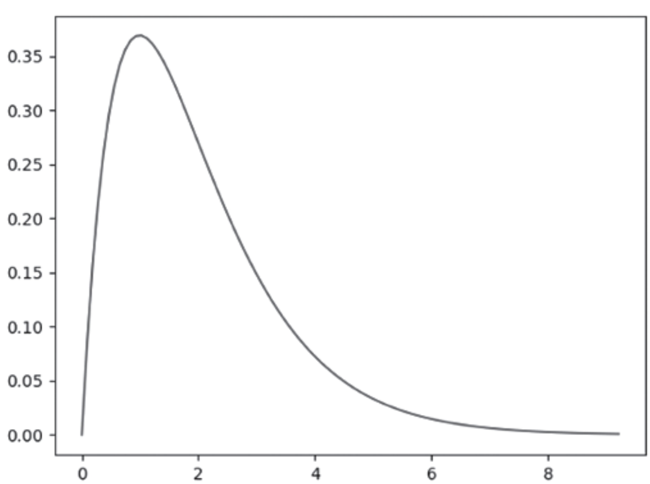

Figure 1.5 The pdf of the gamma distribution with $\beta=1.99$, $\lambda=1$.

1.2.2.4 Lognormal Distribution

A random variable T follows the lognormal distribution if and only if the pdf (shown in Figure 1.6) of T is

$$f(t) = \frac{1}{\sigma t\sqrt{2\pi}} \exp\left[-\frac{1}{2\sigma^2}(\ln t - \mu)^2\right], t > 0, \tag{1.38}$$

where $\sigma > 0$ is the shape parameter and $\mu > 0$ is the scale parameter of the distribution. Note that the lognormal variable is developed from the normal distribution. The random variable $X = \ln T$ is a normal random variable with parameters μ and σ. The cdf of the lognormal distribution is

$$F(t) = \Phi\left(\frac{\ln t - \mu}{\sigma}\right), t > 0, \tag{1.39}$$

where $\Phi(x)$ is the cdf of a standard normal random variable. If T denotes the failure time of an item with lognormal distribution, the reliability function of T will be

$$R(t) = 1 - \Phi\left(\frac{\ln t - \mu}{\sigma}\right), t > 0. \tag{1.40}$$

The hazard rate function is

$$h(t) = \frac{f(t)}{1 - \Phi\left(\frac{\ln t - \mu}{\sigma}\right)}, t > 0. \tag{1.41}$$

The mean, μ, and variance, σ^2, are

$$\begin{aligned} \mu &= e^{\mu + \sigma^2/2}, \\ \sigma^2 &= e^{2\mu + \sigma^2}\left(e^{\sigma^2} - 1\right). \end{aligned} \tag{1.42}$$

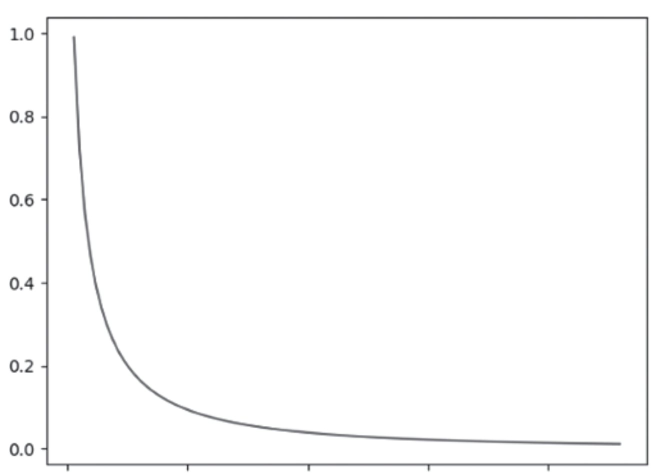

Figure 1.6 The pdf of the lognormal distribution with $\mu = 0$, $\sigma = 0.954$.

Example 1.3

The random variable of the time to failure of an item, T, follows the following pdf:

$$f(t)=\begin{cases}\frac{1}{6000}, & 0\le t\le 6000,\\ 0, & otherwise.\end{cases}$$

where t is in days and $t\ge 0$.

a) What is the probability of failure of the item in the first 100 days?
b) Find the MTTF of the item.

Solution

a) The cdf of the random variable is

$$F(t)=\begin{cases}\frac{t}{6000}, & 0\le t\le 6000,\\ 0, & otherwise.\end{cases}$$

The probability of failure in the first 100 days is

$$P(T\le 100)=F(100)=\frac{100}{6000}\approx 0.017.$$

b) The MTTF of the item is

$$\text{MTTF}=\text{E}[T]=\int_0^{6000}\frac{6000-t}{6000}dt=3000\text{ days}.$$

1.2.3 Physics-of-Failure Equations

Different from the traditional reliability assessment approach, the Physics-of-Failure (P-o-F) represents an approach to reliability assessment based on modeling and simulation of the physical processes leading to the occurrence of failures in an item [2]. The P-o-F approach begins within the first stages of the design of the item. A model is constructed based on the customer's requirements, service environment, and stress analysis [1]. Once the models are established, a reliability assessment can be conducted on the item.

1.2.3.1 Paris' Law for Crack Propagation

Paris' law is a crack growth equation that gives the rate of growth of a fatigue crack [3]. The stress intensity factor K characterizes the load around a crack tip and the rate of crack growth is experimentally shown to be a function of the range of the stress intensity ΔK experienced in a loading cycle (shown in Figure 1.7). The Paris' equation describing this is

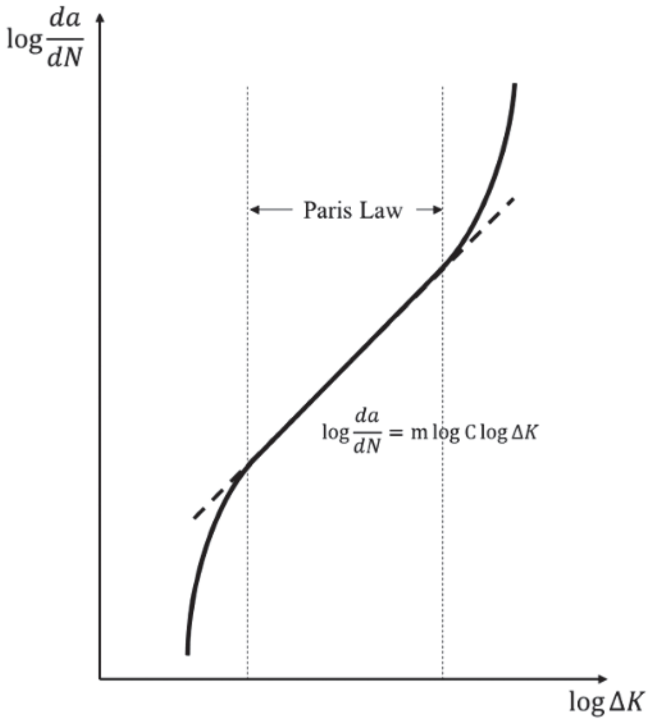

Figure 1.7 Illustration of Paris Law.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K)^m, \tag{1.43}$$

where a is the crack length and $\frac{\mathrm{d}a}{\mathrm{d}N}$ is the fatigue crack growth for a load cycle N. The material coefficients C and m are obtained experimentally and their values depend on environment, temperature, and stress ratio. The stress intensity factor range has been found to correlate with the rate of crack growth in a variety of different conditions, which is the difference between the maximum and minimum stress intensity factors in a load cycle, defined as

$$\Delta K = K_{max} - K_{min}. \tag{1.44}$$

1.2.3.2 Archard's Law for Wear

The Archard's wear equation is a simple model used to describe sliding wear, which is based on the theory of asperity contact [4]. The volume of the removed debris due to wear is proportional to the work done by friction forces. The Archard's wear equation is given by

$$Q = \frac{KWL}{H}, \tag{1.45}$$

where Q is the total volume of the wear debris produced, K is a dimensionless constant, W is the total normal load, L is the sliding distance, and H is the hardness of the softest contacting surfaces. It is noted that WL is proportional to the friction forces. K is obtained from experimental results and it depends on several parameters, among which are surface quality, chemical affinity between the material of two surfaces, surface hardness process, etc.

1.3 System Reliability Modeling

The methods to model and estimate the reliability of a single component were introduced in Section 1.2. Compared with the single component case, the system reliability modeling and assessment is more complicated. The term 'system' is used to indicate a collection of components working together to perform a specific function. The reliability of a system depends not only on the reliability of each component but also on the structure of the system, the interdependence of its components, and the role of each component within the system, etc. To compute the reliability of the system, it is essential to construct the model of the system, representing the above characteristics.

The conventional approaches typically assume that the components and the system have two states: perfect working and complete failure [5]. Below, we introduce the reliability models of a binary state system with specific structures. Details about the multi-state system can be found in Chapter 3.

1.3.1 Series System

In a series system, all components must operate successfully for the system to function or operate successfully. It implies that the failure of any component will cause the entire system to fail. The reliability block diagram of a series system is shown in Figure 1.8.

Let R_i be the reliability of the ith component, $i = 1,2,\ldots,n,$, and R_s be the reliability of the system. Let x_i be the event that the ith component is operational and let x be the event that indicates system is operational. The reliability of the series system can be calculated by

$$R_s = P(x) = P(x_1, x_2, \ldots, x_n). \tag{1.46}$$

Assume all the components in the series system are independent; if so, the reliability of the system can be expressed as

$$R_s = \prod_{i=1}^{n} R_i. \tag{1.47}$$

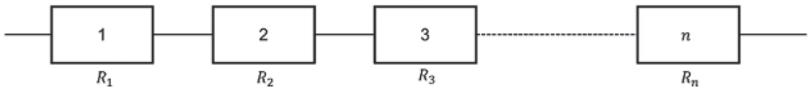

Figure 1.8 Reliability block diagram of a series system.

Considering that the component reliability is a number between 0 and 1, we have the following relationship

$$R_s <= min\{R_1, R_2, \ldots, R_n\}. \tag{1.48}$$

1.3.2 Parallel System

In a parallel system, the system functions or operates successfully when at least one component function is working. It implies that the failure of all components will cause the entire system to fail. The reliability block diagram of a parallel system is shown in Figure 1.9.

Denote F_s as the probability of failure of the system. Denote F_i as the probability of failure of component i. The system reliability can be expressed as

$$R_s = 1 - F_s = 1 - \prod_{i=1}^{n} F_i = 1 - \prod_{i=1}^{n} [1 - R_i]. \tag{1.49}$$

It follows that

$$R_s \geq max\{R_1, R_2, \ldots, R_n\}. \tag{1.50}$$

1.3.3 Series-parallel System

A series-parallel system consists of m subsystems that are connected in series, with n_i units connected in parallel in each subsystem, $i = 1, \ldots, m$. The reliability block diagram of a series-parallel system is shown in Figure 1.10.

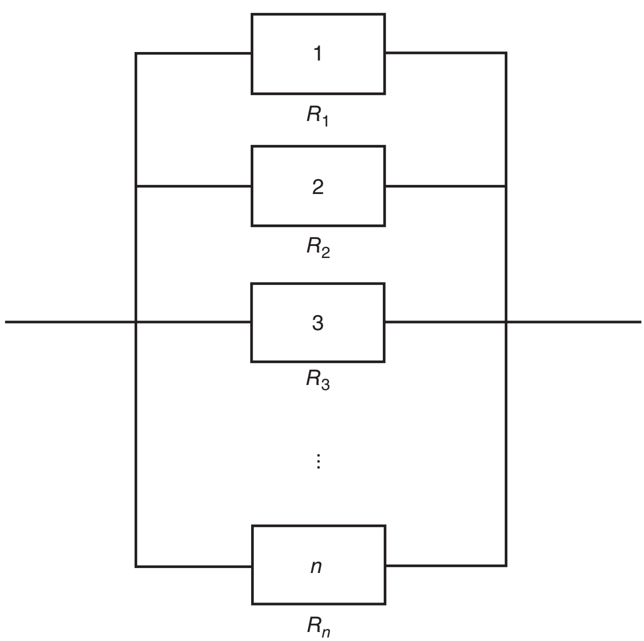

Figure 1.9 Reliability block diagram of a parallel system.

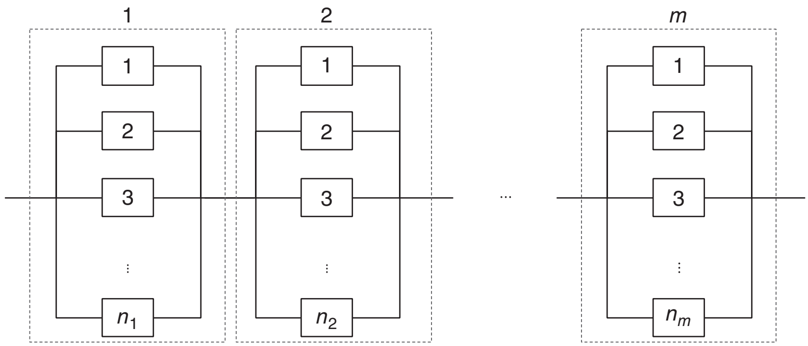

Figure 1.10 Reliability block diagram of a series-parallel system.

Denote R_{ij} as the reliability of component j in subsystem i, $1 \le i \le m, 1 \le j \le n_i$. Let R_i be the reliability of the subsystem i, $1 \le i \le m$. First, the reliability of each subsystem is derived as for the parallel system, that is,

$$R_i = 1 - \prod_{j=1}^{n_i} \left(1 - R_{ij}\right), i = 1, 2, \ldots, m. \tag{1.51}$$

The reliability of the series-parallel system is, then,

$$R_s = \prod_{i=1}^{m} R_i = \prod_{i=1}^{m} \left(1 - \prod_{j=1}^{n_i} \left(1 - R_{ij}\right)\right). \tag{1.52}$$

1.3.4 K-out-of-n System

For a system composed of n components, the system is operational if and only if at least k of the n components are operational. We call this type of system as k-out-of-n: G system, where G is short for Good. For a system composed of n components, the system fails if and only if at least k of the n components are failed. We call this type of system a k-out-of-n: F system. According to the definition, the series system is a 1-out-of-n: F system, where F is short for Failed. The parallel system is a 1-out-of-n: G system. We will mainly present the reliability of the k-out-of-n: G system here.

Assume that the n components are identical and independent. Denote R as the reliability of each component, F as the unreliability of each component, $F = 1 - R$. Let P_i be the probability so that exactly i components are functional. In a k-out-of-n: G system, the number of functional components follows the binomial distribution with parameter n and R. The probability that exactly i components are functional, P_i, is

$$P_i = \binom{n}{i} R^i F^{n-i}, i = 0, 1, 2, \ldots, n \tag{1.53}$$

The reliability of the system is the probability that the number of functional components is greater than or equal to k. Thus, the system reliability, R_s, is calculated by

$$R_s = \sum_{i=k}^{n} P_i = \sum_{i=k}^{n} \binom{n}{i} R^i F^{n-i}. \tag{1.54}$$

If the components are not identical, the system reliability should be calculated by enumerating all combinations of working components.

1.3.5 Network System

There are systems that can be represented by network diagrams, for example, gas networks, telecommunications networks, and power networks. A network system consists of a set of nodes and links. All the nodes and links have a probability of failure.

1.4 System Reliability Assessment Methods

There are many reliability assessment approaches developed to compute the reliability of complex systems, e.g. networks. Path-set and cut-set methods, decomposition and factorization methods, and binary decision diagram (BDD) are four commonly used methods, and we will introduce them in this section.

1.4.1 Path-set and Cut-set Method

A path set P is a set of components, which by functioning ensures that the system is functioning. A path set is said to be minimal if it cannot be reduced without losing its status as a path set. A cut set K is a set of components, which by failing causes the system to fail. A cut set is said to be minimal if it cannot be reduced without losing its status as a cut set. We refer to these minimal sets as minimal path and cut sets or vectors (MPSs, MPVs and MCSs, MCVs).

Consider the minimal path sets of the system, $P_1, P_2, \ldots, P_p$, and the minimal cut sets of the system, $K_1, K_2, \ldots, K_k$. The reliability of the system is given by the union of all minimal path sets. The unreliability is given by the probability that at least one minimal cut set occurs.

Example 1.4

Consider a bridge structure with five edges, $E_1, \ldots, E_5$, as shown in Figure 1.11:

a) Find the minimal path sets and the minimal cut sets of the system.
b) Calculate the reliability of the system if the reliability of each component is R.

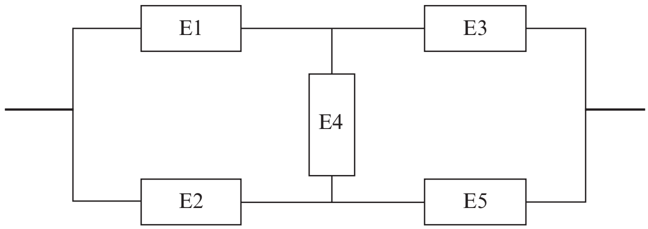

Figure 1.11 Bridge system.

Solution

a) The minimal path sets are

$$P_1 = \{1,3\}, P_2 = \{2,5\}, P_3 = \{1,4,5\}, P_4 = \{2,3,4\}.$$

The minimal cut sets are

$$K_1 = \{1,2\},\ K_2 = \{3,5\},\ K_3 = \{1,4,5\},\ K_4 = \{2,3,4\}.$$

b) The reliability of the system is calculated by the union of the path sets:

$$= 2R^2 + 2R^3 - 5R^4 + 2R^5.$$

1.4.2 Decomposition and Factorization

The decomposition method begins by selecting a critical component, denoted by x, which is an important component of the complex system structure. The reliability of the system can be calculated by the conditional probability:

$$R_s = P(\text{system functional}|x)R(x) + P(\text{system functional} \mid \bar{x})(1 - R(x)). \qquad (1.55)$$

The factorization method is developed based on the decomposition method, which is used in a network system. Denote e as a critical edge in the network G. The reliability of the network is

$$R_s = P(G \text{ functional}|e)R_e + P(G \text{ functional} \mid \bar{e})(1 - R_e). \qquad (1.56)$$

1.4.3 Binary Decision Diagram

Binary decision diagram (BDD) is used to represent a Boolean function. A Boolean function can be represented as a rooted, directed, acyclic graph, which consists of several nodes and two terminal nodes. The two terminal nodes are labeled 0 (FALSE) and 1 (TRUE). Each node u is labeled by a Boolean variable x_i and has two child nodes called low child and high child. The edge from a node to a child represents an assignment of the value FALSE (or TRUE, respectively) to variable x_i. The advantage of BDD in reliability assessment is that its accuracy and efficiency are high [6]. The algorithm to compute the probability of a gate from a BDD is based on the Shannon Decomposition, which is defined by recursive equations.

Example 1.5

Calculate the reliability of the bridge system in Figure 1.11, if the reliability of each component is R.

Solution

The block decision diagram of the bridge system is shown in Figure 1.12.
The reliability of the system is

$$R_s = R^3 + R^3(1-R) + R^4(1-R) + 2R^3(1-R)^2 + R^2(1-R) + R^3(1-R)^2$$
$$+R^3(1-R)^2 + R^2(1-R)^3 = 2R^2 + 2R^3 - 5R^4 + 2R^5.$$

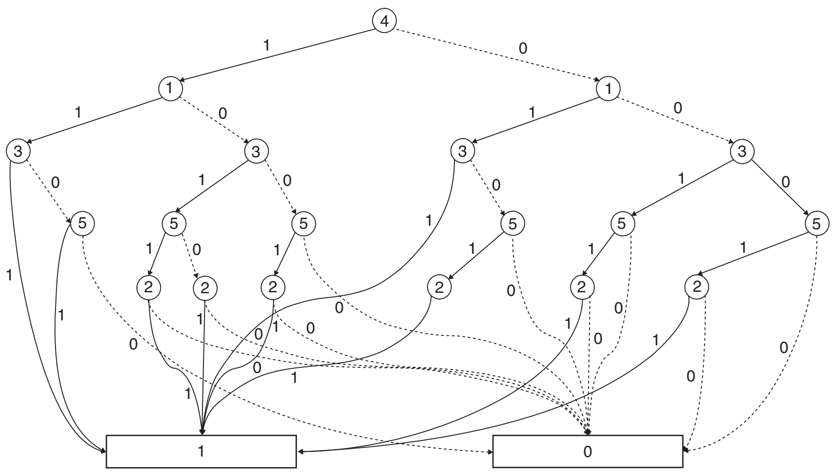

Figure 1.12 Block decision diagram of the bridge system.

1.5 Exercises

1 Consider an electrical generating system with two engines, E_1, E_2, and three generators, G_1, G_2, G_3, each one with rate equal to 30 kVA. The system fails when the generators fail to supply at least 60 kVA. The structure of the system is shown in Figure 1.13.

a. Find the minimal cut sets of the system.
b. Estimate the unreliability of the system for one-month operation, given that the failure rate for each engine is $5\times10^{-6}h^{-1}$ and that for each generator is $10^{-5}h^{-1}$.

2 Consider the reliability of the following system consisting of five components in Figure 1.14. All the components are identical and independent from each other. The reliability of components i is R_i. Let R_s be the reliability of the system. Give the reliability formulation of the system.

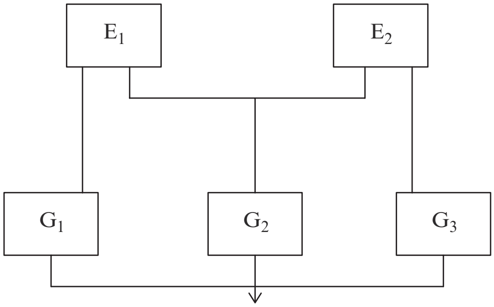

Figure 1.13 Electrical generating system.

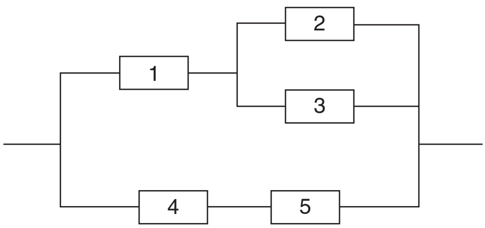

Figure 1.14 Reliability block diagram of the system.

3 The system has $N = 4$ components. Each component has three states: ($M \in \{0,1,2\}$). Let x_i denote the state of component i: then, we have the probability $P(x_i \geq 1) = 0.7$, $P(x_i = 2) = 0.5$, for $i = 1,2,3,4$. Give the following system structure function,

a. $\phi(x) = \min(x_1, (x_2 + x_3), x_4)$.

b. Find all minimal path and cut vectors (MPVs and MCVs) of the system.

c. Calculate system reliability $R = \Pr(\phi(x) \geq 1)$.

4 The power grid structure is shown in Figure 1.15 below. There are three substations: A is the power supplier that generates electric power to be transmitted to the substations B and C, which are the power consumers. Assume that the substations are always working but the power transmission lines may fail. The overall power grid works only if all the following conditions are satisfied:

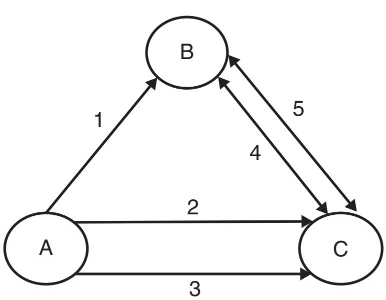

Figure 1.15 Diagram of the power grid structure.

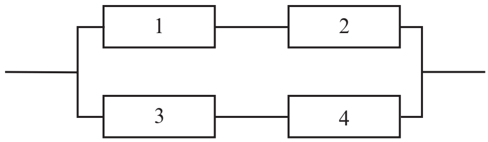

Figure 1.16 Reliability block diagram of the system.

i. Both substations B and C have power input.

ii. At least two outgoing transmission lines of A are working.

Then

a. Build a BDD for the power grid system.
b. Estimate the unreliability of the system for one-month operation by BDD, given that the failure rate for lines 1, 2, 3 is $\lambda_1 = 510^{-6}$ h^{-1} and for lines 4, 5 is $\lambda_2 = 10^{-5}$ h^{-1}.

5 Consider the series-parallel system in Figure 1.16. The components 1, 2, 3, and 4 are independent from each other and have exponential reliabilities with failure rates λ_1, λ_2, λ_3 and λ_4, respectively. Assuming that λ_1 and $\lambda_4 = \lambda_2$, calculate the system mean time to failure (MTTF) expression in terms of λ_2 and λ_3.

6 A manufacturer performs a test on a ceramic capacitor and finds that it experiences failures exponentially distributed in time, with rate $\lambda = 510^{-4}$ failures per hour. To retain operation performance of the ceramic capacitor, an instantaneous and imperfect maintenance activity is performed at an interval of 10^3 hours. The reliability after maintenance is 0.98. Calculate the average availability and the instantaneous availability at time 1.210^3 hours.

References

1 Zio, E. (2007). *An Introduction to the Basics of Reliability and Risk Analysis*, Vol. 13. World scientific.

2 Matic, Z. and Sruk, V. (2008, June). The physics-of-failure approach in reliability engineering. In *ITI 2008-30th International Conference on Information Technology Interfaces* (pp. 745–750). IEEE.

3 Paris, P. and Erdogan, F. (1963). A critical analysis of crack propagation laws.

4 Magnee, A. (1995). Generalized law of erosion: Application to various alloys and intermetallics. *Wear* 181: 500–510.

5 Barlow, R.E. and Proschan, F. (1975). *Statistical Theory of Reliability and Life Testing: Probability Models*. Florida State Univ Tallahassee.

6 Rauzy, A. (2008). Binary decision diagrams for reliability studies. In: *Handbook of Performability Engineering*, (ed. K.B. Misra) 381–396. London: Springer.