1

Introduction

We start with the iconic figure (Figure 1.1), which depicts a bulb connected to a battery. Whenever the loop is closed and a full connection is established, the bulb comes on and starts to consume energy provided by the battery. The process is often described as the conversion of the chemical energy stored in the battery into electrical energy that is further released as heat and light by the bulb. The connection between the bulb and battery consists of two wires between the positive and negative terminals of the bulb and battery. These wires are shown as simple straight lines, whereas in real life they are usually coaxial or paired cables that are isolated from the environment.

The purpose of this first chapter is to introduce basic concepts of electrical circuits. In order to understand circuits, such as the one above, we first need to understand electric charge, potential, and current. These concepts provide a basis for recognizing the interactions between electrical components. We further discuss electric energy and power as fundamental variables in circuit analysis. The time and frequency in circuits, as well as related limitations, are briefly considered. Finally, we study conductivity and resistance, as well as resistors, independent sources, and dependent sources as common components of basic circuits.

1.1 Circuits and Important Quantities

An electrical circuit is a collection of components connected via metal wires. Electrical components include but are not limited to resistors, inductors, capacitors, generators (sources), transformers, diodes, and transistors. In circuit analysis, wire shapes and geometric arrangements are not important and they can be changed, provided that the connections between the components remain the same with fixed geometric topology. Wires often meet at intersection points; a connection of two or more wires at a point is called a node. Before discussing how circuits can be represented and analyzed, we first need to focus on important quantities, namely, electric charge, electric potential, and current, as well as energy and power.

1.1.1 Electric Charge

Electric charge is a fundamental property of matter to describe force interactions among particles. According to Coulomb's law, there is an attractive (negative) force between a

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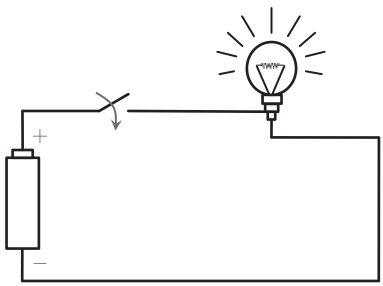


Figure 1.1 A simple circuit involving a bulb connected to a battery. The connection between the bulb and battery is shown via simple lines.

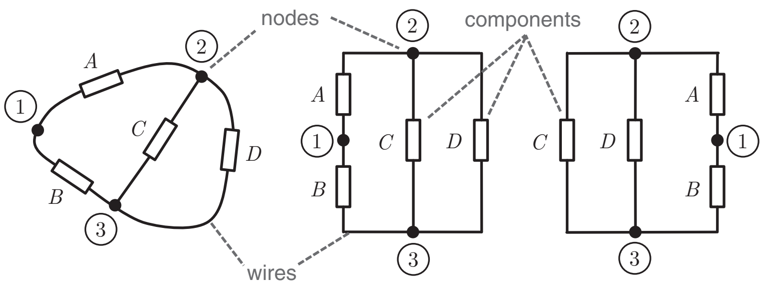


Figure 1.2 A circuit involving connections of four components labeled from *A* to *D*. From the circuit-analysis perspective, connection shapes are not important, and these three representations are equivalent.

proton and an electron given by

$$F_{pe} \approx -\frac{2.3071 \times 10^{-28}}{d^2} \text{ (newton (N))},$$

which is significantly larger than (around 1.2×10^{36} times) the gravity between these particles. In the above, d is the distance between the proton and electron, given in meters (m). This law can be rewritten by using Coulomb's constant

$$k \approx 8.9876 \times 10^9 \text{ (N m}^2/\text{C}^2)$$

as

$$F_{pe} \approx k\frac{q_e q_p}{d^2} \text{ (N)},$$

where

$$q_p \approx +1.6022 \times 10^{-19} \text{ (C)}$$
$$q_e = -q_p \approx -1.6022 \times 10^{-19} \text{ (C)}$$

are the electrical charges of the proton and electron, respectively, in units of coulombs (C). Coulomb's constant enables the generalization of the electric force between any arbitrary charges q_1 and q_2 as

$$F_{12} \approx k\frac{q_1 q_2}{d^2} \text{ (N)},$$

where q_1 and q_2 are assumed to be point charges (theoretically squeezed into zero volumes), which are naturally formed of collections of protons and electrons.

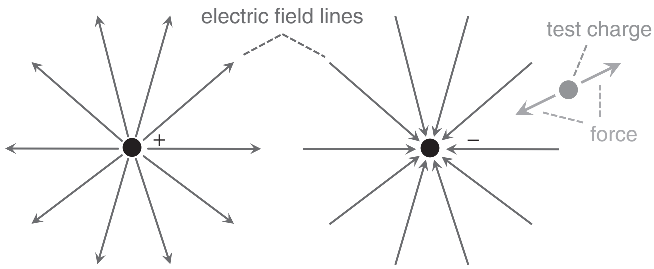


Figure 1.3 Electric field lines created individually by a positive charge and a negative charge. An electric field is assumed to be created whether there is a second test charge or not. If a test charge is located in the field, repulsive or attractive force is applied on it.

The definition of the electric force above requires at least two charges. On the other hand, it is common to extend the physical interpretation to a single charge. Specifically, a stationary charge q_1 is assumed to create an electric field (intensity) that can be represented as

$$E_1 \approx k\frac{q_1}{d^2} \text{ (N/C)},$$

where d is now the distance measured from the location of the charge. This electric field is in the radial direction, either outward (positive) or inward (negative), depending on the type (sign) of the charge. Therefore, we assume that an electric field is always formed whether there is a second test charge or not. If there is q_2 at a distance d, the electric force is now measured as

$$F_{12} = E_1 q_2 \text{ (N)},$$

either as repulsive (if q_1 and q_2 have the same sign) or attractive (if q_1 and q_2 have different signs).

The definition of the electric field is so useful that, in many cases, even the sources of the field are discarded. Consider a test charge q exposed to some electric field E. The force on q can be calculated as

$$F = qE \text{ (N)},$$

without even knowing the sources creating the field. This flexibility further allows us to define the electric potential concept, as discussed below.

1.1.2 Electric Potential (Voltage)

Consider a charge q in some electric field created by external sources. Moving the charge from a position b to another position a may require energy if the movement is opposite to the force due to the electric field. This energy can be considered to be absorbed by the charge. If the movement and force are aligned, however, energy is extracted from the charge. In general, the path from b to a may involve absorption and release of energy, depending on the alignment of the movement and electric force from position to position. In any case, the net energy absorbed/released depends on the start and end points, since the electric field is conservative and its line integral is path-independent.

Electric potential (voltage) is nothing but the energy considered for a unit charge (1 C) such that it is defined independent of the testing scheme. Specifically, the work done in

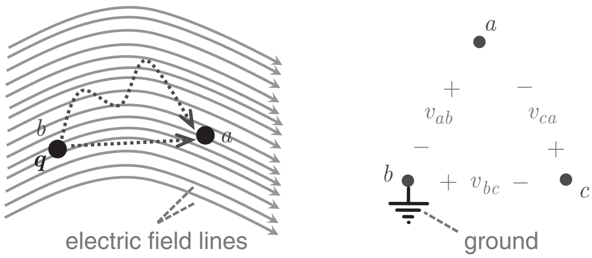


Figure 1.4 Movement of a charge in an electric field created by external sources. The energy absorbed or released by the charge does not depend on the path but depends on the potentials at the start and end points. The electric potential (voltage) is always defined between two points, while selecting a reference point as a ground enables unique voltage definitions at all points.

moving a unit charge from a point b to another point a is called the voltage between a and b. Conventionally, we have

$$v_{ab} = v_a - v_b$$

as the voltage between a and b, corresponding to the work done in moving the charge from b to a. If $v_a > v_b$, then work must be done to move the charge (the energy of the charge increases). On the other hand, if $v_b > v_a$, then the work done is negative, indicating that energy is actually released due to the movement of the charge. The unit of voltage is the volt (V), and 1 volt is 1 joule per coulomb (J/C).

A proper voltage definition always needs two locations and a polarity definition. Considering three separate points a, b, and c, we have

$$v_{ab} = v_a - v_b,$$
$$v_{bc} = v_b - v_c,$$
$$v_{ca} = v_c - v_a,$$

and

$$v_{ab} + v_{bc} + v_{ca} = 0.$$

The equality above is a result of the conservation of the electric energy (conservative electric field). On the other hand, v_a, v_b, and v_c are not yet uniquely defined. In order to simplify the analysis in many cases, a location can be selected as a reference with zero potential. In circuit analysis, such a location that corresponds to a node is called ground, and it allows us to define voltages at all other points uniquely. For example, if $v_b = 0$ in the above, we have $v_a = v_{ab} + v_b = v_{ab}$.

1.1.3 Electric Current

A continuous movement of electric charges is called electric current. Conventionally, the direction of a current flow is selected as the direction of movement of positive charges. The unit of current is the ampere (A), and 1 ampere is 1 coulomb per second (C/s). Formally, we have

$$i(t) = \frac{dq}{dt} \text{ (A)},$$

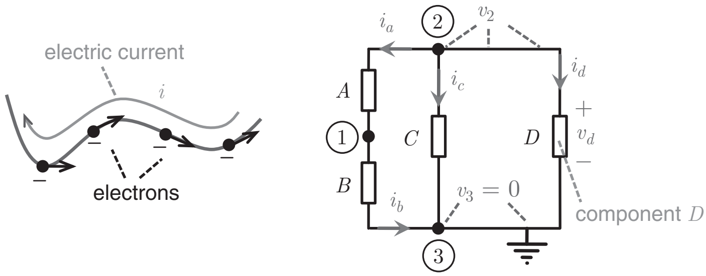


Figure 1.5 On a metal wire, the conventional current direction, which is defined as positive charge flow, is the opposite of the actual electron movements. In a circuit, voltages are defined at the nodes, as well as across components, using the sign convention.

where q and t represent charge and time, respectively. The current itself may depend on time, as indicated in this equation. But, in some cases, we only have steady currents, $i(t) = i$, where i does not depend on time.

Different types of current exist, as discussed in Section 1.2.1. In circuit analysis, however, we are restricted to conduction currents, where free electrons of metals (e.g., wires) are responsible for current flows. Since electrons have negative charges and an electric current is conventionally defined as the flow of positive charges, electron movements and the current direction on a wire are opposite to each other. Indeed, when dealing with electrical circuits, using positive current directions is so common that the actual movement of charges (electrons) is often omitted.

When charges move, they interact with each other differently such that they cannot be modeled only with an electric field. For example, two parallel wires carrying currents in opposite directions attract each other, even though they do not possess any net charges considering both electrons and protons. Similar to the interpretation that electric field leads to electric force, this attraction can be modeled as a magnetic field created by a current, which acts as a magnetic force on a test wire. Electric and magnetic fields, as well as their coupling as electromagnetic waves, are described completely by Maxwell's equations and are studied extensively in electromagnetics.

1.1.4 Electric Voltage and Current in Electrical Circuits

In electrical circuit analysis, charges, fields, and forces are often neglected, while electric voltage and electric current are used to describe all phenomena. This is completely safe in the majority of circuits, where individual behaviors of electrons are insignificant (because the circuit dimensions are large enough with respect to particles), while the force interactions among wires and components are negligible (because the circuit is small enough with respect to signal wavelength). The behavior of components is also reduced to simple voltage–current relationships in order to facilitate the analysis of complex circuits. The limitations of circuit analysis using solely voltages and currents are discussed in Section 1.6.

In an electrical circuit, voltages are commonly defined at nodes, while currents flow through wires and components. A wire is assumed to be perfectly conducting (see Section 1.2.4) such that no voltage difference occurs along it, that is, the voltage is the same on the entire wire. This is the reason why their shapes are not critical.

On the other hand, a voltage difference may occur across a component, depending the type of the component and the overall circuit. For unique representation of a node voltage, a reference node should be selected as a ground. However, the voltage across a component can always be defined uniquely since it is based on two or more (if the component has multiple terminals) points.

In circuit analysis, voltages and currents are usually unknowns to be found. Since they are not known, in most cases, their direction can be arbitrarily selected. When the solution gives a negative value for a current or a voltage, it is understood that the initial assumption is incorrect. This is never a problem at all. For consistency, however, it is useful to follow a sign convention by fixing the voltage polarity and current direction for any given component. In the rest of this book, the current through a component is always selected to flow from the positive to the negative terminal of the voltage.

1.1.5 Electric Energy and Power of a Component

Consider a component d with a current i_d and voltage v_d, defined in accordance with the sign convention. If $i_d > 0$, one can assume that positive charges flow from the positive to the negative terminal of the component. In addition, if $v_d > 0$, these positive charges encounter a drop in their potential values, that is, they release energy. This energy must be somehow used (consumed or stored) by the component. Formally, we define the energy of the component as

$$w_d(t) = \int_0^t v_d(t')i_d(t')dt' \text{ (J)},$$

where the time integral is used to account for all charges passing during $0 \leq t' \leq t$, assuming that the component is used from time $t' = 0$. If $w_d(t) > 0$, it is understood that the component consumes net energy during the time interval $[0, t]$. On the other hand, if $w_d(t) < 0$, the component produces net energy in the same time interval. We note that the unit of energy is the joule, as usual.

Energy as defined above provides information in selected time intervals. In many cases, however, it is required to know the behavior (change of the energy) of the component at a particular time. For a device d with a current i_d and voltage v_d, this corresponds to the time derivative of the energy, namely the power of the device, defined as

$$p_d(t) = \frac{dw}{dt} = v_d(t)i_d(t) \text{ (W)}.$$

Specifically, for a given component, its power is defined as the product of its voltage and current. The unit of power is the watt (W), where 1 watt is 1 volt ampere (V A) or 1 joule per second (J/s). If $p(t) > 0$, the component absorbs energy at that specific time. Otherwise (i.e., if $p(t) < 0$), the component produces energy.

Example 1: Electric power and energy are often underestimated. Consider an 80 W bulb, which is on for 24 hours. Using the energy spent by the bulb, how many meters can a 1000 kg object be lifted?

Solution: The energy spent by the bulb is

$$w_b = 24 \times 60 \times 60 \times 80 = 6.912 \times 10^6 \text{ J}.$$

Then, assuming $g = 10\,\text{m/s}^2$, and using $w_p = mgh$ for the potential energy, we have

$$1000 \times 10 \times h = 6.912 \times 10^6 \longrightarrow h = 691.2 \text{ m.}$$

Example 2: There are approximately 12×10^9 bulbs on earth. Assuming an average on period of 6 hours and 50 W average power, find the amount of coal required to produce the same amount of energy for 1 day. Assume that the thermal energy of coal is 3×10^4 J/kg and the efficiency of the conversion of the energy is 100%.

Solution: The required energy for the bulbs per day is

$$w_b = 12 \times 10^9 \times 50 \times 6 \times 60 \times 60 = 1.296 \times 10^{16} \text{ J.}$$

The corresponding amount of coal can be found as

$$13 \times 10^4 \times m_c = 1.296 \times 10^{16} \longrightarrow m_c = 432 \times 10^9 \text{ kg.}$$

Example 3: The voltage and current of a device are given by $v(t) = 100\exp(-3t)$ V and $i(t) = 2[1 - \exp(-3t)]$ A, respectively, as functions of time. Find the maximum power of the device.

Solution: We have

$$\begin{aligned} p(t) = v(t)i(t) &= 200\exp(-3t)[1 - \exp(-3t)] \\ &= 200[\exp(-3t) - \exp(-6t)] \text{ W} \end{aligned}$$

as the power of the device. We note that

$$\begin{aligned} p(0) &= 0, \\ p(\infty) &= 0. \end{aligned}$$

In order to find the maximum point for the power, we use

$$\frac{dp(t)}{dt} = 200[-3\exp(-3t) + 6\exp(-6t)] = 0,$$

leading to

$$16\exp(-6t) = 3\exp(-3t) \longrightarrow \exp(3t) = 2 \longrightarrow t = \ln(2)/3 \text{ s.}$$

Then the maximum power is

$$\begin{aligned} p(\ln(2)/3) &= 200[\exp(-\ln(2)) - \exp(-2\ln(2))] \\ &= 200(1/2 - 1/4) = 50 \text{ W.} \end{aligned}$$

Exercise 1: A device has a power of 60 W when it is active and 10 W when it is on standby. An engineer measures that it spends a total of 2664 kJ energy in 24 hours. How many hours was the device actively used?

1.1.6 DC and AC Signals

Until now, we have considered the time concept in circuits for studying energy (which needs to be defined in time intervals) and power (which may depend on time). In fact, the time dependence of the power of a component corresponds to the time dependence

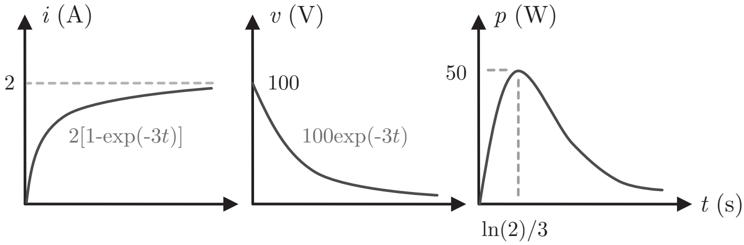


Figure 1.6 Power of a device for given current and voltage across it.

of its voltage and/or current. This brings us to the definition of direct current (DC) and alternating current (AC), which are important terms in describing and categorizing circuits and their components.

DC means a unidirectional flow of electric charges, leading to a current only in a single direction. However, the term ‘DC signal’ is commonly used to describe voltages and other quantities that do not change polarity. DC signals are produced by DC sources, whose voltages or currents are assumed to be fixed in terms of direction and amplitude. Examples of DC sources are batteries and dynamos. Voltage and current values of these sources may have very slight variations with respect to time, which are often neglected in circuit analysis.

AC describes electric currents and voltages that periodically change direction and polarity. This periodicity is generally imposed by AC sources, which may provide voltage and currents in sinusoidal, triangular, square, or other periodic forms. AC is commonly used in all electricity networks, including homes. The reason for its common usage is its well-known advantage when transmitting AC signals over long distances. Specifically, the electric power can be transmitted with less ohmic losses in the AC form in comparison to the DC form. In addition, AC signals can be amplified or reduced easily via transformers, making it possible to use different voltage and current values in different lines of electricity networks and electrical devices. In general, AC circuits have a fixed periodicity and frequency, which is set to 50–60 hertz (Hz = 1/s) in domestic usage. AC signals are also associated with electromagnetic waves (e.g., radiation from electrical components).

AC and DC signals can be converted into each other. The conversion from AC to DC is achieved by rectifiers, while inverters are used to convert DC signals into AC signals. DC to DC and AC to AC converters are also common when the properties but not the types of the signals need to modified.

1.1.7 Transient State and Steady State

Whether DC or AC, any circuit in real life has a time dependency, at least when switching the circuit on and off. The short-time state, in which variations in voltage and current values are encountered due to outer effects (e.g., switching), is called the transient state. Whether it is a DC or AC circuit, any circuit can be in a transient state before it reaches an equilibrium. A transient state is usually an unwanted state, where voltage, current, and power values involve fluctuations that are not designed on purpose.

In the long term, circuits that are not disturbed by outer effects enter into equilibrium, namely, the steady state. Theoretically, an infinite time is required to pass from transient

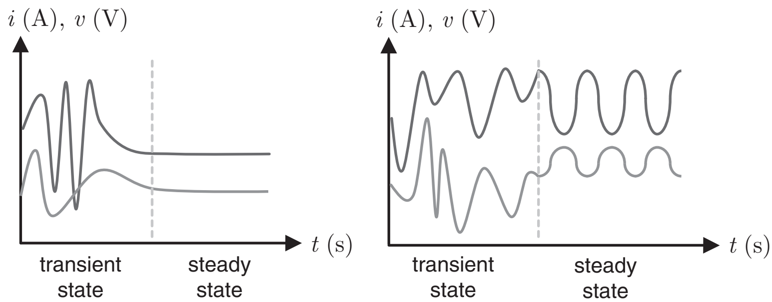


Figure 1.7 Transient state and steady state in DC and AC signals.

state to equilibrium, while most circuits are assumed to reach steady state after a sufficient period (i.e., when fluctuations become negligible). For DC circuits in steady state, voltage and current values are assumed to be constant. In the first few chapters, a steady state is automatically assumed when only resistors and DC sources are considered. In fact, the time needed to pass from transient state to steady state depends on a time constant, which is a contribution of both resistors and energy-storage elements (capacitors and inductors). Hence, circuits with only resistors and DC sources have zero transient time, that is, they can be assumed to be in steady state without any transient analysis. For AC circuits, voltages and currents in steady state oscillate with the time period dictated by the sources. Therefore, we emphasize that the steady state does not indicate constant properties for all circuits.

1.1.8 Frequency in Circuits

When AC sources are involved in a circuit, voltage and current values oscillate with respect to time. In most cases, the periodicity and frequency are fixed, that is, all voltages and currents change at the same rate, while there can be phase differences (delays) between them. The behavior of some components does not rely on the frequency, unless they are exposed to extreme conditions. As an example, resistors behave almost the same in a wide range of frequencies. On the other hand, many components, such as capacitors and inductors, strongly depend on the frequency. With DC sources, corresponding to zero frequency, capacitors/inductors act like open/short circuits, while they become almost the opposite at very high frequencies. Therefore, the behavior of an AC circuit directly depends on the frequency, as discussed extensively in time-harmonic analysis.

1.2 Resistance and Resistors

Resistors (Figure 11.1) are fundamental components in electrical circuits. They are basically energy-consuming elements that are used to control voltage and current values in circuits. In addition, the energy conversion ability of resistors can be useful in various applications, where these elements are directly used for heating and lighting (conventional bulbs). Specifically, the energy consumed by a resistor is usually released as heat, and sometimes as useful light. Resistance is a common property of all metals, and even very conductive wires have resistances, which may need to be included in circuit analysis.

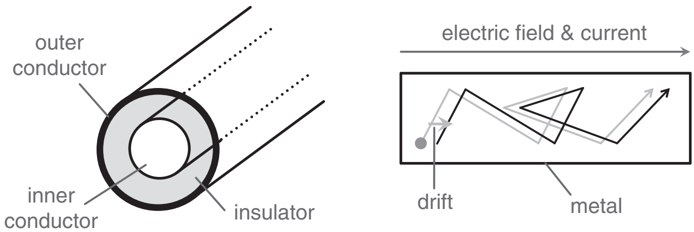


Figure 1.8 Structure of a general coaxial cable and a representation of the drift velocity of an electron under an electric field.

1.2.1 Current Types, Conductance, and Ohm's Law

In order to understand resistance and resistors, first we need to define the conduction current. As described in Section 1.1.3, current is a continuous flow of charges. In electrolytes, gases, and plasmas, currents may be formed by ions, and even by moving protons. In some applications, electrons can be injected from special devices, leading to a current flow in a vacuum. In circuits, however, currents are mostly formed by the conduction of metals.

In good conductors, one or more electrons from each atom is weakly bound to the atom. These electrons can move freely in the metal (especially on the surfaces), while these movements are random if the metal is not exposed to an electric field and potential. Therefore, without any excitation, there is no net flow of charges. When an electric field is applied, however, electrons collectively drift in the opposite direction, leading to a net measurable current. We note that the conventional current direction is also opposite to the movement of electrons, aligning it with the electric field. A simple relation between the current density and electric field intensity can be written by using Ohm's law as

$$J = \sigma E \ \ (\mathrm{A/m^2}),$$

where σ is defined as the conductivity, given in siemens per meter (S/m). In the above equation, J represents the current density, whose surface integral (on the cross-section) gives the overall current flowing through the metal. All materials can be categorized in terms of their conductivity values, as discussed below.

1.2.2 Good Conductors and Insulators

Most metals are good conductors, with conductivity values in the 10^6–10^8 S/m range for a wide band of frequencies. For example, copper has a conductivity of approximately 6×10^7 S/m at room temperature. For all materials, conductivity values depend on temperature and other environmental conditions, as well as the frequency. Sea water is known to be conductive (with around 4–5 S/m conductivity), while its conduction mechanism is based on ions, not free electrons as in metals. Carbon has interesting properties, demonstrating extremely varying conductivity characteristics depending on the arrangement of its atoms. For example, diamond has a very low conductivity (around 10^{-13} S/m), while graphite is as conductive as some metals (greater than 10^5 S/m). A recently popular form of carbon called graphene may have conductivity values as large as 10^8 S/m.

There is often confusion between the velocity of electricity, velocity of electrons, and the drift velocity of electrons. In a typical metal without any excitation, electrons move randomly with a high (Fermi) velocity. These movements are of high speed (e.g., 1.57×10^6 m/s for copper). However, due to their random nature, no net current flows along the metal. When the metal is exposed to a voltage difference, leading to an electric field, electrons continue their random movements, while they tend to drift in the opposite direction to the electric field. The corresponding drift velocity is usually very low (e.g., only 10^{-5} m/s for a typical copper wire). On the other hand, the current measured along a wire is due to this drift velocity. Obviously, when AC sources are involved, electrons do not drift only in a single direction, but oscillate back and forth (in addition to high-velocity random movements) with the frequency of the signal. Since circuits are usually small with respect to wavelength, drift movements of electrons are almost synchronized through the entire circuit. Finally, the velocity of the electricity along a wire is not related to any actual movement of electrons. It is related to the speed of the electromagnetic wave through the wire (similar to sea waves that are not movements of water molecules). This speed is comparable to the speed of light in a vacuum, but it is reduced by a velocity factor depending on the properties of the material.

In general, materials with low conductivity values are called insulators. Wood, glass, rubber, air, and Teflon are well-known insulators in real-life applications. Insulators are also natural parts of all circuits, for example for isolating components and wires from each other, as well as the parts of electrical components. Since they are not electrically active, however, they are not considered directly in circuit analysis. For example, when considering wires in circuits, we assume perfectly conducting metals without any insulator, while in real life, electrical wires have shielded or coaxial structures with layers of conducting metals and insulating materials separating them.

1.2.3 Semiconductors

As their name suggests, semiconductors conduct electricity better than insulators and worse than good conductors. In addition, the conductivity of semiconductors can be altered by externally modifying their material properties permanently (via chemical processes) and temporarily (via electrical bias), making them suitable for controlling electricity. Silicon is the best-known semiconductor, and has been used in producing diverse components of integrated circuits. The key chemical operation is called doping, that is, modifying the conductivity of semiconductors by introducing impurities into their crystal lattice structures. This way, different (e.g., n-type, p-type) kinds of semiconductors can be produced and used to form junctions that enable control over electric current and voltage. Engineers use many different types of semiconducting devices, such as diodes and transistors, to construct modern circuits. These special components are discussed in Chapter 8.

1.2.4 Superconductivity and Perfect Conductivity

Perfect conductivity is a theoretical limit when the conductivity of a metal becomes infinite, that is, $\sigma \to \infty$. In this case, if a current J exists along the metal, $E \to 0$ and there is no potential difference over it. Therefore, a perfect conductor does not dissipate power while conducting electricity. Perfect conductivity is an idealized property as all metals actually have finite conductivity, while some metals can be assumed to be perfect conductors to simplify their modeling. In circuit analysis, all wires are assumed to be perfect

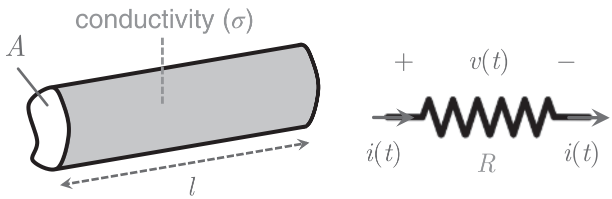


Figure 1.9 The resistance of a rod with conductivity σ is often approximated as $R = l/(\sigma A)$, where l and A are the length and cross-section area, respectively, of the rod. In circuit analysis, a resistor is a two-terminal device that usually consumes energy.

conductors (with no voltage drop across them), while any resistance due to imperfect conductivity can be modeled as a resistor component.

Under the perfect conductivity assumption, the electric field is zero anywhere on a metal. This also means that all charges are distributed on the surface of the metal. For electromagnetic fields, where electric and magnetic currents are coupled, a zero electric field leads to a zero magnetic field. On the other hand, perfect conductivity does not enforce any assumption on a static magnetic field. Specifically, a static magnetic field inside a perfect conductor does not violate Maxwell's equations.

Recently, superconductors have become popular due to their potential applications. Similar to perfect conductivity, superconductivity can be described as a limit case when the electrical conductivity goes to infinity. On the other hand, this infinite conductivity cannot be explained simply by electron movements, and quantum effects need to be considered to understand how a metal can become a superconductor. In a superconductor, magnetic fields are expelled toward its surface, making it different from theoretical perfect conductivity. Superconductivity is achieved in real life by cooling down special materials below critical temperatures.

1.2.5 Resistors as Circuit Components

As mentioned above, resistors are fundamental components of circuits. Given a resistor, the voltage–current relationship (obeying the sign convention) can be written as

$$v(t) = Ri(t),$$

where $R \geq 0$ is called the resistance. In general, the resistance of a structure depends on its dimensions and is inversely proportional to the conductivity of the material. The simple relationship above for the definition of the resistance is also commonly called Ohm's law. The unit of resistance is the ohm (Ω), and 1 ohm is 1 volt per ampere (V/A). The power of a resistor can be found from

$$p(t) = v(t)i(t) = Ri(t)i(t) = R[i(t)]^2 \geq 0,$$

which is always nonnegative. Therefore, resistors cannot produce energy themselves. In some cases, it is useful to use conductance, defined as

$$G = \frac{1}{R},$$
$$i(t) = Gv(t).$$

The unit of the conductance is the siemens (S), and 1 siemens is 1 ampere per volt (A/V).

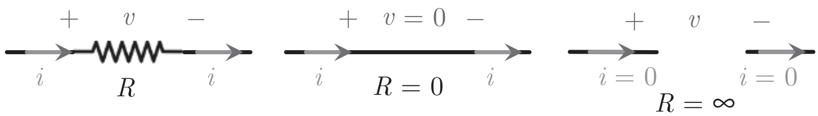


Figure 1.10 Short circuit and open circuit can be interpreted as special cases of resistors, with zero and infinite resistance values, respectively.

Resistors in real life are made of different materials, including carbon. In addition to standard resistors with fixed resistance values, there are also adjustable resistors, such as rheostats, which can be useful in different applications. The resistance of a fixed resistor also demonstrates nonlinear behaviors, that is, it may change with temperature, which may rise during the use of the resistor, leading to a complicated relationship between the voltage and current. In circuit analysis, however, these nonlinear behaviors are often discarded, and a fixed resistor has always the same resistance value.

Two limit cases of resistors are of particular interest in circuit analysis. When $R \to 0$, indicating a lack of resistance, we have a short circuit. Specifically, in a short circuit, we have

$$R = 0,$$
$$v(t) = 0,$$

while $i(t)$ can be anything (may not be zero). While all wires with zero resistances can be categorized as short circuits, this definition is often used to indicate a direct connection between two points that are not supposed to be connected. For many components and devices, having a short circuit means a failure or breakdown. At the other extreme case, two points without a direct connection between them can be interpreted as a resistor with infinite resistance. Such a case is called an open circuit, which can be defined as

$$R = \infty,$$
$$i(t) = 0,$$

while $v(t)$ can be anything (may not be zero). Any two points without a direct connection in a circuit can be interpreted as an open circuit, while this definition is again used to indicate a special case, particularly a breakdown of a connection.

1.3 Independent Sources

All circuits are excited with AC and DC sources. Among these, independent sources are defined as energy-delivering devices whose voltage or current values are fixed at a given value, independent of the rest of the circuit. Two types of independent sources are used in circuit analysis: voltage and current sources.

An ideal voltage source is defined as

$$v(t) = v_o(t),$$

where $v_o(t)$ is given and independent of other parts of the circuit. If the voltage source is DC, we further have $v(t) = v_o$ as a constant. We note that the current through a voltage

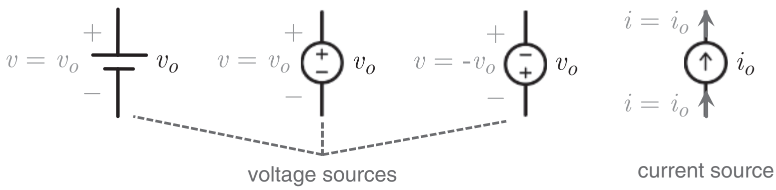


Figure 1.11 There are alternative symbols to show voltage and current sources; circular representations are used in this book. For any source, the polarity should be clearly indicated. In addition to sources with constant values (DC sources), the source values v_o and i_o may depend on time (AC sources).

source, $i(t)$, can be anything (not necessarily zero). In fact, if a voltage source is delivering energy, $i(t)$ must be nonzero.

An ideal current source is defined as

$$i(t) = i_o(t),$$

where $i_o(t)$ is given and independent of other parts of the circuit. Once again, if the current source is DC, we further have $i(t) = i_o$ as a constant. We note that the voltage across a current source, $v(t)$, can be anything (not necessarily zero).

In real life, batteries and dynamos can be considered as independent voltage sources. On the other hand, an independent current source, which has a predetermined current value no matter what the rest of the circuit does, is usually designed using a voltage source and some other components (e.g., diodes, transistors, and/or OP-AMPs). In this book, we always show an independent source as a single and ideal device, without detailed structures and any internal resistances. If a source has a nonideal resistance (e.g., nonzero resistance for a voltage source or finite resistance for a current source) it can be shown as a separate component in addition to the ideal part of the source.

Under normal circumstances, voltage and current sources provide energy to their circuits. However, depending on the rest of the circuit, a voltage or current source may consume energy, which is a perfectly valid scenario. A source that consumes energy indicates that there is at least one other source that delivers energy. For a given isolated circuit, the sum of powers of all components must be zero due to the conservation of energy.

1.4 Dependent Sources

Dependent sources are also energy-delivering devices, where, unlike independent sources, the voltage or current provided depends on another voltage or current in the circuit. While dependent sources are not frequently used practice, they are very common in circuit analysis for modeling a component, (e.g., transistors and OP-AMPs). Therefore, we assume that dependent sources exist as individual components of circuits, while the actual circuit structure may not be exactly the same. There are four types of dependent sources, which can be listed as follows.

- Voltage-controlled voltage source (VCVS): A voltage source whose voltage depends on another voltage in the circuit, i.e., $v_{ds} = A_{ds} v_o$, where A_{ds} is a unitless quantity.

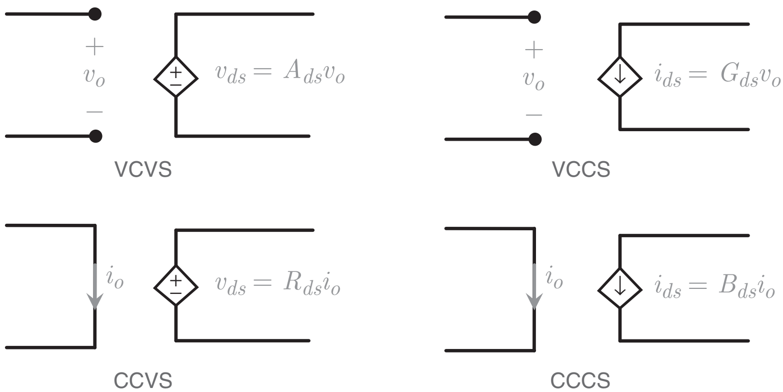


Figure 1.12 Dependent sources have fixed voltage/current values, depending on some other voltage/current values in the circuit.

- Voltage-controlled current source (VCCS): A current source whose current depends on a voltage in the circuit, i.e., $i_{ds} = G_{ds}v_o$, where G_{ds} is measured in siemens.
- Current-controlled voltage source (CCVS): A voltage source whose voltage depends on a current in the circuit, i.e., $v_{ds} = R_{ds}i_o$, where R_{ds} is measured in ohms.
- Current-controlled current source (CCCS): A current source whose current depends on another current in the circuit, i.e., $i_{ds} = B_{ds}i_o$, where B_{ds} is unitless.

The polarization of the voltage/current of a dependent source, as well as the reference voltage/current and the linkage constant (A_{ds}, B_{ds}, G_{ds}, R_{ds}), are given with the definition of the source. Similarly to independent sources, dependent sources can be DC or AC, depending on the reference voltage/current, v_o and i_o.

1.5 Basic Connections of Components

In any given circuit, components are connected via wires that intersect at nodes. Considering multiple components, two basic types of connection may occur: series and parallel.

If a common current flows through the components, they are connected in series. Hence, such components share the same current. If a common voltage is applied on the components, they are connected in parallel. Hence, such components share the same voltage. In general, series and parallel connections occur together, also with other types of connections, leading to a complex network.

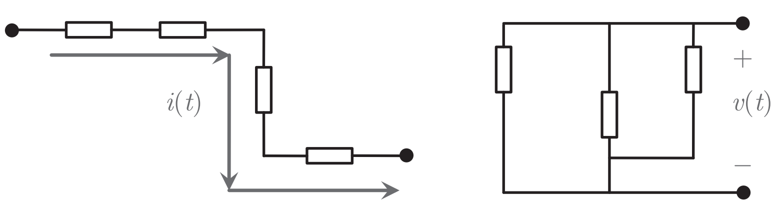


Figure 1.13 Series and parallel connections, where the current and voltage are common values, respectively, for the components.

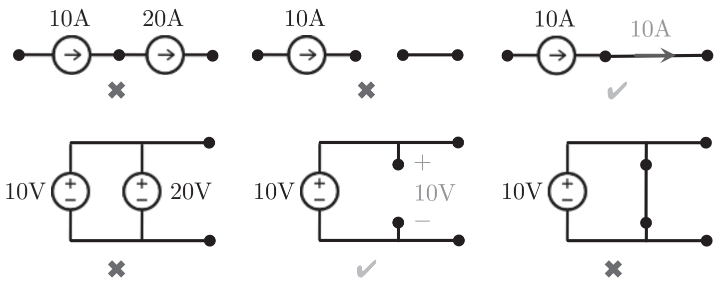


Figure 1.14 Some possible and impossible configurations using ideal components.

Considering ideal components, some of the connections are impossible. Some basic examples of possible and impossible scenarios are as follows.

- A 10 A current source and a 20 A current source cannot be connected in series.
- A 10 A current source and an open circuit cannot be connected in series.
- A 10 A current source and a short circuit can be connected in series. If these are the only components of the circuit (with a full connection on both terminals), no voltage occurs across the current source; hence, it does not produce any power.
- A 10 V voltage source and a 20 V voltage source cannot be connected in parallel.
- A 10 V voltage source and an open circuit can be connected in parallel. If these are the only components of the circuit, no current flows through the voltage source; hence, it does not produce any power.
- A 10 V voltage source and a short circuit cannot be connected in parallel.

In order to understand why a connection may not be possible, one can directly use the definition of the components. For example, consider a series connection of 10 A and 20 A current sources. The 10 A source indicates that 10 A is passing through the line. On the other hand, the 20 A source, by definition, needs 20 A current to flow in the same line. Therefore, there is an inconsistency, since a wire cannot have different current values at the same time. Similar inconsistencies can be found for all impossible cases. Such impossible configurations are not due to a modeling incapability in circuit analysis; they actually correspond to physically impossible practices in real life. Consider another example involving a parallel connection of two voltage sources with different values. In real life, this configuration never exists since voltage sources have internal resistances, while the wires between them are also not perfectly conducting, leading to a voltage drop. Therefore, a more realistic model of the physical scenario would require a resistor between the voltage sources, leading to a perfectly valid circuit that can be analyzed. All impossible configurations described above have similar missing components, which can be added to convert them into possible scenarios.

Impossible scenarios rarely occur, even when we consider ideal components in circuit analysis. In general, many circuits have multiple components and connections, where the voltage and current values become consistent. In order to find relations between voltage and current values, we use basic rules, namely Kirchhoff's laws, as described in the next chapter. These rules, which are based on the conservation of energy and charge, provide the necessary equations to relate different voltage and current values.

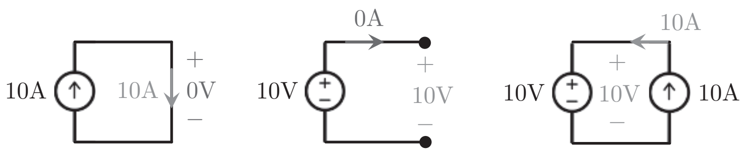


Figure 1.15 Some possible circuits involving only one or two components that are connected consistently. In the first and second circuits, where there is a current and a voltage source, the sources do not produce any power. However, they still provide the current and voltage values, in accordance with their definitions. In the third circuit, the voltage source absorbs power (100 W), while the current source delivers power (100 W).

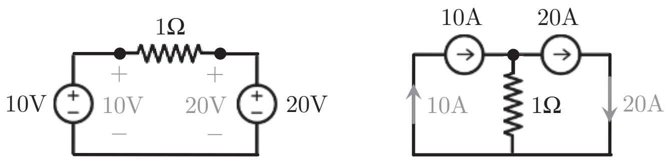
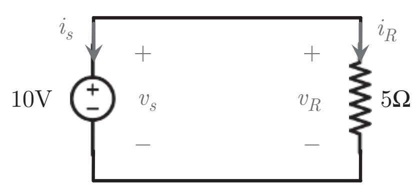


Figure 1.16 Two simple circuits that can be interpreted incorrectly as impossible. In fact, both two circuits are possible and they involve consistent voltage and current values. In the first circuit, a voltage drop (by an amount of 20 − 10 = 10 V) exists across the resistor. In the second circuit, a nonzero current (20 − 10 = 10 A) flows through the resistor. These values can easily be found by applying Kirchhoff's laws, as described in the next chapter.

At this stage, we can start analyzing some simple circuits, just by considering the definitions of the components.

Example 4: Consider a circuit involving a 10 V voltage source connected to a 5 Ω resistor.

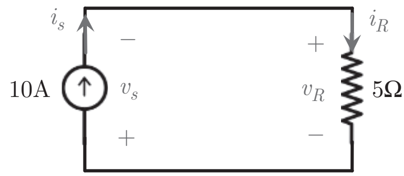


Note that voltage and current directions are defined arbitrarily, but they must obey the sign convention. We can analyze the circuit as follows.

- Using the definition of the voltage source: $v_s = 10$ V.
- Using the definition of the voltage between two points and considering that the voltage is fixed along a wire: $v_R = v_s = 10$ V.
- Using the definition of the resistor (Ohm's law): $i_R = v_R/5 = 2$ V.
- Using the definition of the current: $i_s = -i_R = -2$ A.
- Using the definition of the power: $p_s = v_s i_s = -20$ W.
- Using the definition of the power: $p_R = v_R i_R = 20$ W.

The signs of power values indicate that the voltage source delivers energy, while the resistor consumes the same amount of energy. As expected, we have $p_s + p_R = 0$ due to the conservation of energy.

Example 5: Consider a circuit involving a 10 A current source connected to a 5 Ω resistor.

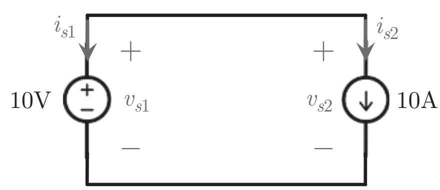


In this case, the current through the circuit is determined via the current source, while the voltage value is found by applying Ohm's law. We can analyze the circuit as follows.

- Using the definition of the current source: $i_s = 10$ A.
- Using the definition of the current: $i_R = i_s = 10$ A.
- Using the definition of the resistor (Ohm's law): $v_R = 5i_R = 50$ V.
- Using the definition of the voltage between two points and considering that the voltage is fixed along a wire: $v_s = -v_R = -50$ V.
- Using the definition of the power: $p_s = v_s i_s = -500$ W.
- Using the definition of the power: $p_R = v_R i_R = 500$ W.

Similarly to the previous example, the current source delivers energy, while the resistor consumes the same amount of energy.

Example 6: Consider a circuit involving a 10 V voltage source connected to a 10 A current source.

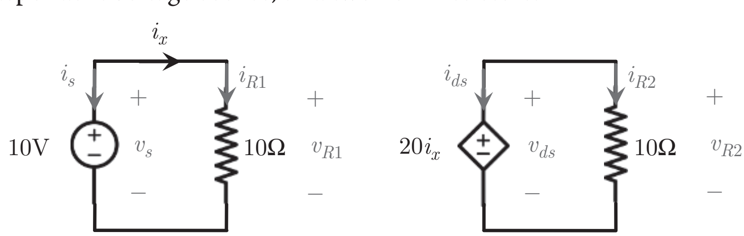


We can analyze the circuit as follows.

- Using the definition of the voltage source and voltage: $v_{s2} = v_{s1} = 10$ V.
- Using the definition of the current source and current: $i_{s1} = -i_{s2} = -10$ A.
- Using the definition of the power: $p_{s1} = v_{s1} i_{s1} = -100$ W and $p_{s2} = v_{s2} i_{s2} = 100$ W.

In this circuit, the voltage source produces energy, while the current source (despite also being a source) consumes energy. Once again the total power is zero due to the conservation of energy.

Example 7: Consider the following circuit involving a 10 V voltage source, a current-dependent voltage source, and two 10 Ω resistors.



We can analyze the circuit as follows. First, using the definition of voltage, we have

$$v_{R1} = v_s = 10 \text{ V}.$$

Then, using Ohm's law, we get

$$i_{R1} = v_{R1}/10 = 10/10 = 1 \text{ A},$$
$$i_s = -i_{R1} = -1 \text{ A},$$

and

$$i_x = i_{R1} = 1 \text{ A}.$$

The definition of the current-dependent voltage source leads to

$$v_{ds} = 20i_x = 20 \text{ V}.$$

Therefore, using the definition of voltage again, we derive

$$v_{R2} = v_{ds} = 20 \text{ V}.$$

Using Ohm's law once again, we have

$$i_{R2} = v_{R2}/10 = 2 \text{ A}$$

and

$$i_{ds} = -i_{R2} = -2 \text{ A}.$$

Finally, the powers of all components can be found:

$$p_{R1} = v_{R1}i_{R1} = 10 \times 1 = 10 \text{ W},$$
$$p_{R2} = v_{R2}i_{R2} = 20 \times 2 = 40 \text{ W},$$
$$p_s = v_s i_s = 10 \times (-1) = -10 \text{ W},$$
$$p_{ds} = v_{ds}i_{ds} = 20 \times (-2) = -40 \text{ W}.$$

We note that both the independent and dependent source provides energy to the circuit, while both resistors consume.

1.6 Limitations in Circuit Analysis

A type of circuit analysis, which is used throughout this book, is based on lumped-element models. Specifically, we assume that the behavior of components and their interactions with each other can be described by voltage–current relations given by the descriptions of the components. In addition, we assume that all elements demonstrate their ideal characteristics, being independent of outer conditions. All these assumptions rely on two constraints on the sizes of the components and circuits.

- The circuit components are large enough to omit individual behaviors of protons and electrons. Hence, without dealing with individual particles, their bulk behaviors (i.e., voltage and current) are used directly to model the components.
- The circuit components and circuits are much smaller than the wavelength of the signals. For example, oscillations in the current and voltage values through a wire are synchronized and no phase accumulation occurs. In addition, voltage/current phase differences are well defined in all components.

Obviously, lumped-element models fail when the size constraints are not satisfied. For example, in circuits larger than the wavelength, connections may need to be modeled as transmission lines, where wave equations are solved. Some circuits may need the full application of Maxwell's equations to precisely describe the electromagnetic interactions of components with each other.

Depending on the complexity of the circuit model, further assumptions are often made to simplify the analysis of circuits. In this book, we accept the following assumptions that are also common in the circuit analysis literature.

- The voltage–current relationship defined for a component does not depend on outer conditions (temperature, pressure, light, etc.). This also means that the circuit behaves always the same (e.g., change in resistance due to rising temperature as the circuit is used is omitted).
- The voltage–current relationship defined for a component does not depend on other components. For example, a resistor of 10 Ω always satisfies Ohm's law as $v_R = 10i_R$, independent of other elements used in the same circuit. We also ignore cross-talk of circuits and their parts, other than the linkage through well-defined dependent sources.
- All components are ideal and we omit secondary effects, such as the resistance of a voltage source, inductance of a capacitor, or capacitance of a resistor. If these effects cannot be neglected, they can be represented as individual components. For example, the leakage of a capacitor can be represented by a resistor connected in parallel to the capacitor.

Despite all these limitations and assumptions, circuit analysis methods presented in this book are widely accepted and used to analyze diverse circuits and electrical devices. In many cases, lumped elements are used as starting models before more complicated analysis techniques are applied.

1.7 What You Need to Know before You Continue

Before proceeding to the next chapter, we summarize a few key points that need to be known to understand the higher-level topics.

- Sign convention: In this book, the current through a component is selected to flow from the positive to the negative side of the voltage.
- Steady state: For DC circuits in steady state, voltage and current values are assumed to be constant. In the first few chapters, steady state is automatically assumed when only resistors and DC sources are considered.
- Short circuit and open circuit: Short circuit and open circuit can be interpreted as special cases of resistors, with zero and infinite resistance values, respectively.
- Sources: There are alternative symbols to show voltage and current sources; circular representations are used in this book. DC/AC types are indicated in the context of source values.
- Energy conservation: For a given isolated circuit, the sum of powers of all components must be zero due to the conservation of energy.

- Series connection: Components that share the same current are connected in series.
- Parallel connection: Components that share the same voltage are connected in parallel.
- Impossible configurations: Some connections of ideal components are not allowed due to inconsistency of voltage and current values enforced by the definitions of components.

In the next chapter, we start with the most basic tools, namely Kirchhoff's laws, to analyze circuits.