

Disappointment Aversion, Asset Pricing and Measuring Asymmetric Dependence

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Abstract

We develop a measure of asymmetric dependence (AD) that is consistent with investors who are averse to disappointment in the utility framework proposed by Skiadas (1997). Using a Skiadas-consistent utility function, we show that disappointment aversion implies that asymmetric joint return distributions impact investor utility. From an asset pricing perspective, we demonstrate that the consequence of these preferences for the realization of a given state results in a pricing kernel adjustment reflecting the degree to which these preferences represent a departure from expected utility behaviour. Consequently, we argue that capturing economically meaningful AD requires a metric that captures the relative differences in the shape of the dependence in the upper and lower tail. Such a metric is better able to capture AD than commonly used competing methods.

1.1 INTRODUCTION

The economic significance of measuring asymmetric dependence (AD), and its associated risk premium, can be motivated by considering a utility-based framework for AD. An incremental AD risk premium is consistent with a marginal investor who derives (dis-)utility from non-diversifiable, asymmetric characteristics of the joint return distribution. The effect of these characteristics on investor utility is captured by the framework developed by Skiadas (1997). In this model, agents rank the preferences of an act in a given state depending on the state itself (state-dependence) as well as the payoffs in other states (non-separability). The agent perceives potentially subjective consequences, such as disappointment and elation, when choosing an act, $b \in \mathcal{B} = \{ \dots, b, c, \dots \}$, in the event that $E \in \Omega = \{ \dots, E, F, \dots \}$ is observed,¹ where \mathcal{B} represents the set of acts that may be chosen on the set of states, $\mathcal{S} = \{ \dots, s, \dots \}$, and Ω represents all possible resolutions of uncertainty and corresponds to the set of events that defines a σ -field on the universal event \mathcal{S} .

¹For example, the event E might represent a major market drawdown.

Within this context, (weak) disappointment is defined as:

$$(b = c \text{ on } E \text{ and } c \geq^{\Omega} = b) \implies b \geq^E c,$$

where the statement ' $b \geq^E c$ ' has the interpretation that, ex ante, the agent regards the consequences of act b on event E as no less desirable than the consequences of act c on the same event (Skiadas, 1997, p. 350). That is, if acts b and c have the same payoff on E , and the consequences of act b are generally no more desirable than the consequences of act c , then the consequence of having chosen b conditional on E occurring is considered to be no less desirable than having chosen c when the agent associates a feeling of elation with b and disappointment with c conditional upon the occurrence of E .

For example, consider two stocks, X and Y , that have identical β s, equal average returns and the same level of dependence in the lower tail. Further, suppose Y displays dependence in the upper tail that is equal in absolute magnitude to the level of dependence in the lower tail, but X has no dependence in the upper tail. In this example, Y is symmetric (but not necessarily elliptical), whereas X is asymmetric, displaying lower-tail asymmetric dependence (LTAD). Within the context of the Capital Asset Pricing Model (CAPM), the expected return associated with an exposure to systematic risk should be the same for X and Y because they have the same β . However, in addition to this, a rational, non-satiable investor who accounts for relative differences in upside and downside risk should prefer Y over X because, conditional on a market downturn event, Y is less likely to suffer losses compared with X . Similarly, a downside-risk-averse investor will also prefer Y over X . These preferences should imply higher returns for assets that display LTAD and lower returns for assets that display upper-tail asymmetric dependence (UTAD), independent of the returns demanded for β .

Now, let the event E represent a major market drawdown and assume that AD is not priced by the market. In the general framework of Skiadas, an investor may prefer Y over X because Y is more likely to recover the initial loss associated with the market drawdown in the event that the market subsequently recovers. Disappointment aversion manifests itself in an additional source of ex-ante risk premium over and above the premium associated with ordinary beta risk because an investor will display greater disappointment having not invested in a stock with compensating characteristics given the drawdown event (that is, holding X instead of Y).²

With regard to preferences in the event that E occurs, a disappointment-averse investor will prefer Y over X because the relative level of lower-tail dependence to upper-tail dependence is greater in X than in Y .³ More generally, this investor prefers an asset displaying joint normality with the market

²An additional risk premium may be required in order to hold either X or Y relative to what the CAPM might dictate. The consequence of holding either X or Y in the event that E occurs is that the investor experiences greater disappointment; losses are larger than what the market is prepared to compensate for because of the greater-than-expected dependence in both the upper and lower tail. This would amount to a risk premium for excess kurtosis. We do not consider this explicitly here.

³We note that a preference for stocks with favourable characteristics during adverse market conditions is consistent with investment decisions made following the marginal conditional stochastic dominance (MCSD) framework developed by Shalit and Yitzhaki (1994). In this framework, expected-utility-maximizing investors have the ability to increase the risk exposure to one asset at the expense of another if the marginal utility change is positive. Shalit and Yitzhaki (1994) show that for a given portfolio, asset X stochastically dominates asset Y if the expected payoff from X conditional on returns less than some level, r , is greater than the equivalent payoff from Y , for all levels of r . Further conditions on the utility function and conditions for general Nth-order MCSD are provided by Denuit *et al.* (2014).

compared with either X or Y as the risk-adjusted loss given event E is lower. A risk premium is required to entice a disappointment-averse investor to invest in either X or Y , and this premium will be greater for X than for Y .

Ang *et al.* (2006) employ a similar rationale based upon Gul's (1991) disappointment-averse utility framework to decompose the standard CRRA utility function into upside and downside utility, which is then proxied by upside and downside β s. In contrast to a Skiadas agent that is endowed with a family of conditional preference relations (one for each event), Gul agents are assumed to be characterized by a single unconditional (Savage) preference relation (Grant *et al.*, 2001). A Skiadis-consistent AD metric conditions on multiple market states, rather than a single condition such as that implied by downside or upside β .

The impact of AD on the utility of an investor who is disappointment-averse in the Skiadas sense is identified using the disappointment-averse utility function proposed by Grant, Kajii and Polak (GKP). Define an outcome $x \in \mathcal{X} = \{ \dots, x, y, z, \dots \}$ such that $b(s) = x$, that is, an act b on state s results in outcome x . A disappointment-averse utility function that is consistent with Skiadas preferences is given by

$$V_{\alpha, \beta_u}^E(b) = \int_{s \in E} v_{\alpha, \beta_u}(b(s), V_{\beta_u}(b)) \mu ds, \tag{1.1}$$

with

$$v_{\alpha, \beta_u}(x, w) = \alpha \varphi(x, w) + (1 - \alpha)w$$

and

$$\varphi_{\beta_u}(x, w) = (x - w) \left(1 + \mathbb{I}_{x < w} \beta_u \right), \tag{1.2}$$

where $\beta_u > -1$ is a disappointment-aversion parameter and \mathbb{I} is an indicator function taking value 1 if the condition in the subscript is true, zero otherwise. The GKP utility function is consistent with Skiadas disappointment⁴ if $\beta_u > \frac{1}{\alpha} - 2 > 0$. The variable $V_{\beta_u}(b)$ solves

$$\int_S \varphi_{\beta_u}(b(s), V_{\beta_u}) \mu ds = 0, \tag{1.3}$$

and can be interpreted as a certainty-equivalent outcome for act b , representing the unconditional preference relation \succeq_{β_u} over the universal event \mathcal{S} . Therefore, for all states s in event E , an agent assigns utility for outcomes $b(s) = x \geq V_{\beta_u}$ and conversely assigns dis-utility to disappointing outcomes $b(s) = x < V_{\beta_u}$, where the dis-utility is scaled by $1 + \beta_u$. The preference, $V_{\alpha, \beta_u}^E(b)$, is then given by a weighted sum of the utility associated with event E , given by the disappointment-averse utility function, $\varphi_{\beta_u}(x, w)$, and the utility associated with the universal event \mathcal{S} , given by the certainty equivalent, w .

The influence of AD on the utility of disappointment-averse investors can be explored using a simulation study. We repeatedly estimate Equation (1.1) using simulated LTAD data and multivariate normal data, where both data sets are mean-variance equivalent by construction. We simulate LTAD using a Clayton copula with a copula parameter of 1, where the asset marginals are assumed to be standard normal. A corresponding symmetric, multivariate normal distribution (MVN) is generated using the same underlying random numbers used to generate the AD data, in conjunction with the sample covariance matrix produced by the Clayton copula data. In this way, we ensure the mean-variance equivalence of the two simulated samples. The mean and variance-covariance matrices of the simulated samples have

⁴Equation (1.1) is also consistent with Gul's representation of disappointment aversion if $\beta_u > 0$. If, in addition, $\alpha > 1/(2 + \beta_u)$, then the conditional preference relation is consistent with Skiadas disappointment (Grant *et al.*, 2001).

the following L^1 - and L^2 -norms: $\|\mu_{AD} - \mu_{MVN}\|_1 < 0.0001$ and $\|\Sigma_{AD} - \Sigma_{MVN}\|_2 < 0.01$. The certainty equivalent is generated using 50,000 realizations of the Clayton sample and the corresponding MVN sample for a given set of utility parameters, (α, β_u) . Given the certainty-equivalent values, we estimate Equation (1.1) 20,000 times, where the realizations of the outcome, x , are re-sampled with each iteration using a sample size of 5,000. The certainty equivalent is computed using market realizations in conjunction with Equation (1.3).

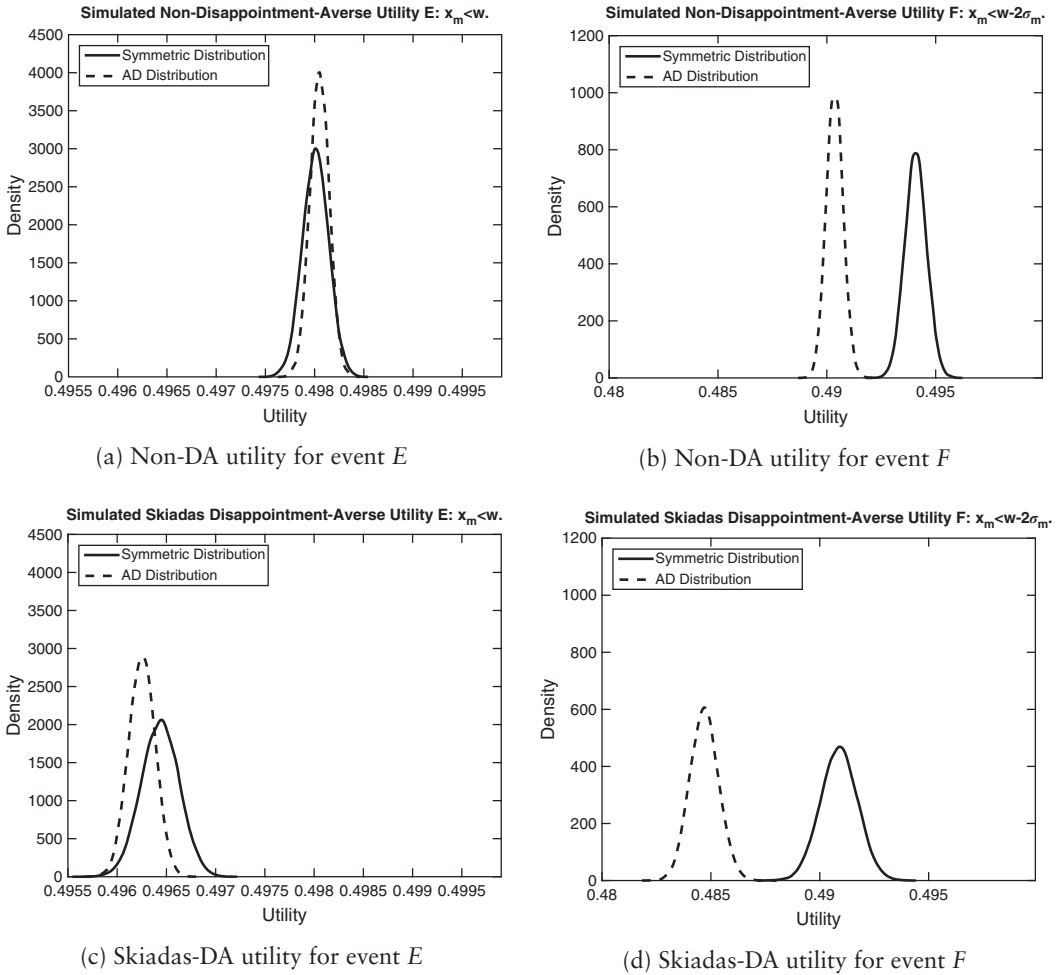


FIGURE 1.1 Simulated densities of GKP utility functions calculated when returns are symmetrically distributed (MVN) and asymmetrically distributed. Non-disappointment-averse utility is described by the GKP utility function (1.1) with $\alpha = 0.5$ and $\beta = 0$. Skiadas disappointment-averse utility is described with $\alpha = 0.5$ and $\beta = 1$. Each of these two utility functions is calculated for both AD and symmetric distributions for two different conditioning events, E and F . The event E is the event that the market return is less than the certainty-equivalent market return, w_m , and event F is the event that the market return is lower than the certainty-equivalent market return, w_m , less two market return standard deviations.

We consider two sets of utility parameters: disappointment aversion, given by $\alpha = 0.5$ and $\beta_u = 0.5$, and no disappointment aversion, given by $\alpha = 0.5$ and $\beta_u = 0$.⁵ We define two events: E , the event that the market return is less than the certainty-equivalent market return, w_m , and F , the event that the market return is lower than the certainty-equivalent market return, w_m , less two market return standard deviations. The density of Equation (1.1) for event E is given in Figure 1.1(a) and (c). If an investor is not disappointment-averse, then their utility is similar regardless of the return distribution for event E . The utility of a disappointment-averse investor drops for both AD and symmetric distributions, with lower utility for the AD distribution than the symmetric distribution.

Further into the lower tail, the realizations of the AD distribution are much further away from the certainty equivalent than those of the symmetric distribution. Therefore, the utility of event F is less than that for event E . In addition, the utility of the disappointment-averse investor is lower for the AD distribution than for the symmetric distribution (Figure 1.1(b) and (d)). That is, as the level of tail dependence that defines our event, F , becomes even more pronounced, an investor displaying aversion to disappointing outcomes will experience lower net utility compared with an investor whose preferences are defined over an event spanning a much wider range of market realizations (event E , for example). Furthermore, the characteristics of the joint return distribution will ultimately dictate the value of the certainty equivalent, which in turn impacts the overall level of utility via the weighting $(1 - \alpha)w$. Therefore, to capture economically meaningful AD requires a metric that captures the relative differences in the shape of the dependence in the upper and lower tail.

1.2 FROM SKIADAS PREFERENCES TO ASSET PRICES

The implication of Skiadas-style preferences is that the ranking of the preferences of an act in a given state depends on the state itself (state-dependence) as well as on the payoffs at other states (non-separability). Following Skiadas (1997), disappointment aversion therefore uniquely satisfies

$$u(b) = A[f(b, u(b))], \quad b \in B, \quad (1.4)$$

where u is an unconditional utility, f is non-increasing in its last argument representing the conditional utility given some fixed partition, \mathcal{F} , and $A : L \rightarrow \mathbb{R}$ is an increasing mapping where L is the set of all random variables. Hence, the subjective consequences that define the conditional utility function associated with the outcome of a random lottery are captured by the aggregator function, A .

Skiadas (1997) shows that for arbitrary probability, \mathbb{P} , the pair (U, \mathbb{P}) admits an additive representation if, for every event D ,

$$b \succeq^D c \Leftrightarrow \int_D U(b) d\mathbb{P} \geq \int_D U(c) d\mathbb{P}, \quad b, c \in B,$$

if U is of the form $U : \Omega \times B \rightarrow \mathbb{R}$.

Under certain conditions, the aggregate consequence of these preferences for the realization of a given state results in a pricing kernel adjustment, reflecting the degree to which these preferences represent a departure from expected utility behaviour. To consider the Skiadas preferences in an asset-pricing

⁵We retain $\alpha = 0.5$, meaning that although the agent does not display either Skiadas (1997) or Gul (1991) disappointment aversion conditional on E , the net utility continues to be a weighted average of the local utility and the certainty equivalent. This implies that if all returns are equal to the asset's certainty equivalent, then $x - w$ in the expression for φ is zero. Therefore, $\alpha\varphi = 0$, but $(1 - \alpha)w$ is non-zero, so the agent continues to generate some utility in this instance.

framework, we draw upon the insights of Kraus and Sagi (2006) and the derivations therein. Let $\mathcal{F} = (\mathcal{F}_1, \dots, \mathcal{F}_T)$ be a sequence of sigma algebras over T periods, such that $\mathcal{F}_1 = \{\Omega, \emptyset\}$, $\mathcal{F}_t \subseteq \mathcal{F}_{t+1}$ and \mathcal{F}_T contain all subsets of Ω .

Unique partitions of Ω , denoted \mathcal{A}_t , are assumed to generate each of the \mathcal{F}_t filtrations. Elements of \mathcal{F}_t are referred to as date- t events, while arbitrary atoms of the date- t partition, $a_t \in \mathcal{A}_t$, are referred to as date- t macro states, where $a_{t+1} \in \mathcal{A}_{t+1} \implies a_{t+1} \subseteq a_t$ for one and only one $a_t \in \mathcal{A}_t$. State prices are computed by maximizing the expected utility over all future $t + 1$ macro states, a_{t+1} for a given pair of date- t consumption, c_t and date $t + 1$ realization of wealth, w_{t+1}^i . The expected utility is given by

$$\sum_{a_{t+1} \subseteq a_t} \pi(a_{t+1}|a_t) U_t^{g_t^i}(c_t, w_{t+1}^i, a_{t+1}) = u_{g_t^i}(c_t) + \beta \sum_{a_{t+1} \subseteq a_t} \pi(a_{t+1}|a_t) \varphi_t^{g_t^i}(V_{t+1}^{g_t^i \star}, \dots, V_{t+1}^{g_n^i \star}), \quad (1.5)$$

where $0 < \beta < 1$ is a constant, $\pi(a_{t+1}|a_t)$ is the conditional probability of realizing macro state a_{t+1} given current macro state a_t , $U_t(c_t, w_{t+1}^i, a_{t+1})$ is the contribution of (c_t, w_{t+1}^i) to the agent's utility in state a_{t+1} , $u(c_t)$ is the time-independent utility of date- t consumption, g_t^i is the agent's current preference state and V_{t+1}^* is the indirect utility function for date $t + 1$ realization of wealth, given by

$$V_{t+1}^*(w_{t+1}^i, a_{t+1}) \equiv \max_{(c_t, \tilde{w}_{t+1}^i) \in B(w_{t+1}^i)} \sum_{a_{t+1} \subseteq a_t} \pi(a_{t+1}|a_t) U_t(c_t, w_{t+1}^i, a_{t+1}),$$

where $B(w_{t+1}^i)$ is the agent's budget set. The aggregator, φ_t , accounts for the date $t + 1$ preference states, g_1^i, \dots, g_n^i , conditional on attaining macro state a_{t+1} . When the aggregator, φ_t , is chosen to be consistent with agents displaying hyperbolic absolute risk aversion,⁶ the system of time $(t + 1)$ state-prices can be derived from the solution to the agent's maximum utility optimization problem:

$$\begin{aligned} \phi(a_{t+1}|a_t) &= \pi(a_{t+1}|a_t) \tilde{M}_{t+1} \\ &= \pi(a_{t+1}|a_t) \beta \left(\frac{\tilde{C}_{t+1}}{C_t} \right)^{-\gamma} \left(\frac{\tilde{R}_{t+1}^R}{R_t^R} \right)^\gamma \left[1 - \frac{\tilde{\delta}_{t+1}}{\tilde{Q}_{t+1} + 1} \right]^{-\gamma}. \end{aligned} \quad (1.6)$$

Here, \tilde{M}_{t+1} is the state-price deflator, C_t is aggregate market consumption, R_t^R is a measure of aggregate relative risk aversion, \tilde{C}_{t+1} and \tilde{R}_{t+1}^R are random variables reflecting aggregate consumption and risk aversion at time $t + 1$ conditional upon information at date t , γ and β are constants, and \tilde{Q}_{t+1} is a function of the aggregate variables as well as a wealth-consumption ratio. The variable $\tilde{\delta}_{t+1} \equiv \delta(a_{t+1}|a_t)$ is a state-dependent function representing the aggregate departure from expected utility behaviour. With $\tilde{\delta}_{t+1} = 0$, \tilde{M}_{t+1} reduces to the Lucas (1978) model under certain simplifying assumptions on the relation between aggregate risk aversion and aggregate consumption.

If, in Equation (1.6), we set $\delta(a_{t+1}|a_t) = f(g_1^i, \dots, g_n^i)$, where f is defined in Equation (1.4), we see that deviations from expected utility depend on the collective incremental experiences associated with state a_{t+1} being realized. This observation has several implications for measuring AD, in that any measure of AD will need to suppose that two incremental characteristics matter for asset pricing. First, it must measure AD over and above the level of dependence that is consistent with ordinary beta. This supposes that an incremental risk premium may be required to hold an asset that displays LTAD with the market beyond what would typically be expected if the assets were jointly normal. The consequence of holding a tail-dependent asset is that the investor experiences a sense of disappointment that losses are larger than what the market is prepared to compensate for. Second, any measure of

⁶Chosen by Kraus and Sagi (2006) for tractability.

AD must incorporate differences in tail dependence across the upper and lower tail. This is consistent with an investor preferring UTAD to LTAD, as a stock with UTAD is more likely to recover the initial loss associated with market drawdowns in the event that the market subsequently bounces. The consequence of the investor holding a LTAD asset can therefore be expected to elicit a sense of disappointment that they did not invest in a stock with compensating characteristics (i.e., UTAD) given the drawdown event.

1.3 CONSISTENTLY MEASURING ASYMMETRIC DEPENDENCE

To measure the relevant characteristics embodied within Skiadas's framework of preferences, we propose a metric that captures the asymmetry of dependence in the upper and lower tail, across a range of market events, over and above the level of dependence that is consistent with ordinary beta. We measure AD using an adjusted version of the J statistic, originally proposed by Hong *et al.* (2007). J^{Adj} is a non-parametric and β -invariant statistic that measures AD using conditional correlations across opposing sample exceedances. Several alternative metrics have been used to assess non-linearities in the dependence between asset returns, including extreme value theory (Poon *et al.*, 2004), higher-order moments (Harvey and Siddique, 2000), downside beta (Ang *et al.*, 2006), copula function parameters (Genest *et al.*, 2009; Low *et al.*, 2013) and the J statistic itself. However, many of these metrics have difficulty capturing the level and price of AD in asset return distributions independently of other price-sensitive factors such as the CAPM beta.

To illustrate, we concoct an approximate AD distribution by simulating $N = 25,000$ pairs of random variables (x, y) where $x_i \sim N(\mu_S, \sigma_S)$ and $y_i = \beta x_i + \epsilon_i$, where $\epsilon_i \sim N(0, (x_i + \mu_S)^\alpha)$, with $\mu_S = 0.25$ and $\sigma_S = 0.15$. When $\alpha = 0$, no AD is present and (x, y) are bivariate normal with linear dependence equal to β . Higher LTAD is proxied by increasing $\alpha > 0$, and higher UTAD is proxied by decreasing $\alpha < 0$. A sample of $N = 500$ simulated data points is given in Figure 1.2.

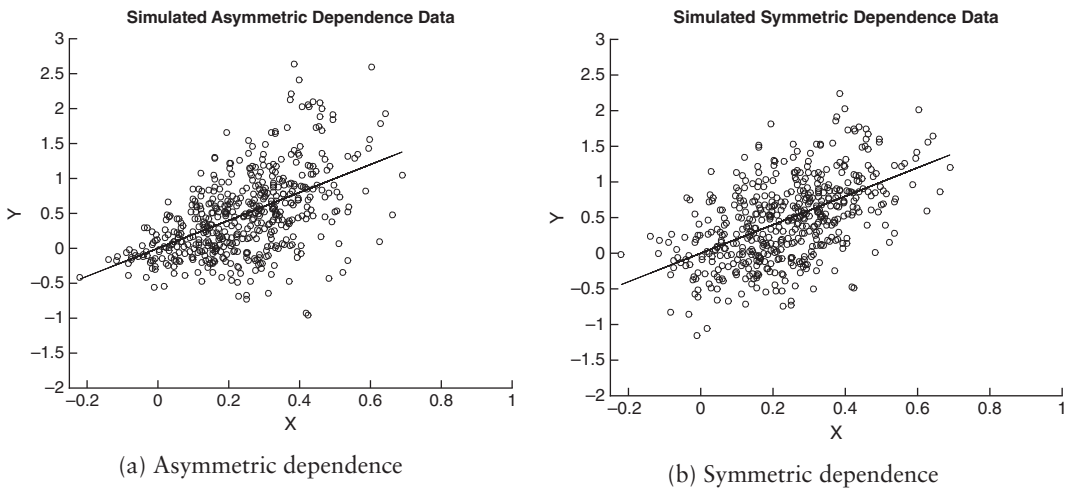


FIGURE 1.2 Scatter plot of simulated bivariate data with asymmetric dependence (a) and symmetric dependence (b) that is used to test different downside-risk metrics. The $N = 500$ sample is a random draw of bivariate data (x, y) where $x_i \sim N(\mu_S, \sigma_S)$ and $y_i = \beta x_i + \epsilon_i$, where $\epsilon_i \sim N(0, (x_i + \mu_S)^\alpha)$, with $\mu_S = 0.25$, $\sigma_S = 0.15$ and $\beta = 2.0$. In (a), $\alpha = 2$ so the sample displays LTAD. In (b), $\alpha = 0$ so no AD is present and (x, y) are bivariate normal with linear dependence equal to β . Higher LTAD is proxied by increasing $\alpha > 0$, and higher UTAD is proxied by decreasing $\alpha < 0$.

Ordinary least-squares estimates of the CAPM beta and the downside beta, and IFM estimates⁷ of the Clayton copula parameter of LTAD, are provided in Figure 1.3 for various combinations of α and β .

The CAPM beta and the downside beta are largely insensitive to AD and their estimates of linear dependence are not confounded by the presence of AD.⁸ The Clayton copula parameter is unable to uniquely identify either the presence or level of AD or of linear dependence. This seems to be due to the fact that the Clayton copula parameter attempts to fit both dimensions of dependence with a single parameter. As a result, the copula measure of AD is sensitive to the value of linear dependence and to the value of α . Almost all Archimedean copulae, including multi-parameter copulae, will similarly be unable to determine AD separately from linear dependence, unless one parameter is especially dedicated to estimating linear dependence. To the best of our knowledge, a copula with these characteristics is yet to be described in the literature.

Further, downside and upside β s are also likely to be confounded with the CAPM β , so that any risk premium empirically associated with downside β , upside β , or even the difference in upside and downside β , is likely to reflect both the compensation for systematic risk and asymmetries in upside and downside risk. Ang *et al.* (2006) are careful to avoid this confounding by ensuring that the CAPM β and the upside/downside β s are not included in the same cross-sectional regression.

1.3.1 The Adjusted J Statistic

The J statistic of Hong *et al.* (2007) is able to identify AD and allows the use of critical values to establish a hypothesis test on the presence of AD. We introduce the β -invariant adjusted J statistic, in order to establish the AD premium separately from the CAPM β premium while retaining the integrity of the dependence structure. We obtain β -invariance by unitizing β for each data set before a modified version of the J statistic is computed. In particular, given $\{R_{it}, R_{mt}\}_{t=1}^T$ (Figure 1.4(a)), we first let $\hat{R}_{it} = R_{it} - \beta R_{mt}$ (Figure 1.4(b)), where R_{it} and R_{mt} are the continuously compounded return on the i th asset and the market, respectively, and $\beta_{\hat{R}_{it}, R_{mt}} = \text{cov}(R_{it}, R_{mt}) / \sigma_{R_{mt}}^2$. This initial transformation sets $\beta_{\hat{R}_{it}, R_{mt}} = 0$, making it possible to standardize the data without contaminating the linear relation between the variables (Figure 1.4(c)).⁹ Standardization yields R_{mt}^S and \hat{R}_{it}^S and ensures that the standard deviation of the market model residuals, a measure of idiosyncratic risk, is identical for all data sets.¹⁰ We then re-transform the data to have $\hat{\beta}_{\hat{R}_{it}, R_{mt}} = 1$ by letting $\tilde{R}_{mt} = R_{mt}^S$ and $\tilde{R}_{it} = \hat{R}_{it}^S + R_{mt}^S$ (Figure 1.4(d)). Therefore, all data display the same β after these transformations,¹¹ forcing the output of J^{Adj} to be invariant to the overall level of linear dependence, as well as being independent of idiosyncratic risk.

⁷For full details of the inference function for margins (IFM) method of estimating copula parameters, see Joe (1997).

⁸The unadjusted J statistic of Hong *et al.* (2007) is similar to the difference between upside and downside beta, $\beta^+ - \beta^-$, if only one exceedance ($\delta = 0$) is used. The notable difference is that the J statistic determines the squared differences in correlations, whereas the upside/downside β s scale the unsquared differences by market semi-variance. The adjustment of the J statistic, described in Section 1.3.1, removes the influence of β altogether.

⁹We are careful to avoid look-ahead bias by ensuring that at time t , only historical data up to time t is employed to estimate the $\beta_{\hat{R}_{it}, R_{mt}}$ used to standardize the data.

¹⁰From the market model, the total variance of a stock's returns can be written as $\sigma_T^2 = \beta^2 \sigma_M^2 + \sigma_\epsilon^2$, where σ_M^2 is the market's variance and σ_ϵ^2 is the variance of the idiosyncratic component of returns. Since we set $\beta = 0$, $\sigma_T^2 = \sigma_\epsilon^2$. Hence, standardizing at this point is equivalent to dividing out the idiosyncratic component of transformed returns.

¹¹At this point, $\tilde{R}_{mt} \sim N(0, 1)$ whereas $\tilde{R}_{it} \sim N(0, \sqrt{2})$ assuming marginal distributions are normal. This holds for all stocks.

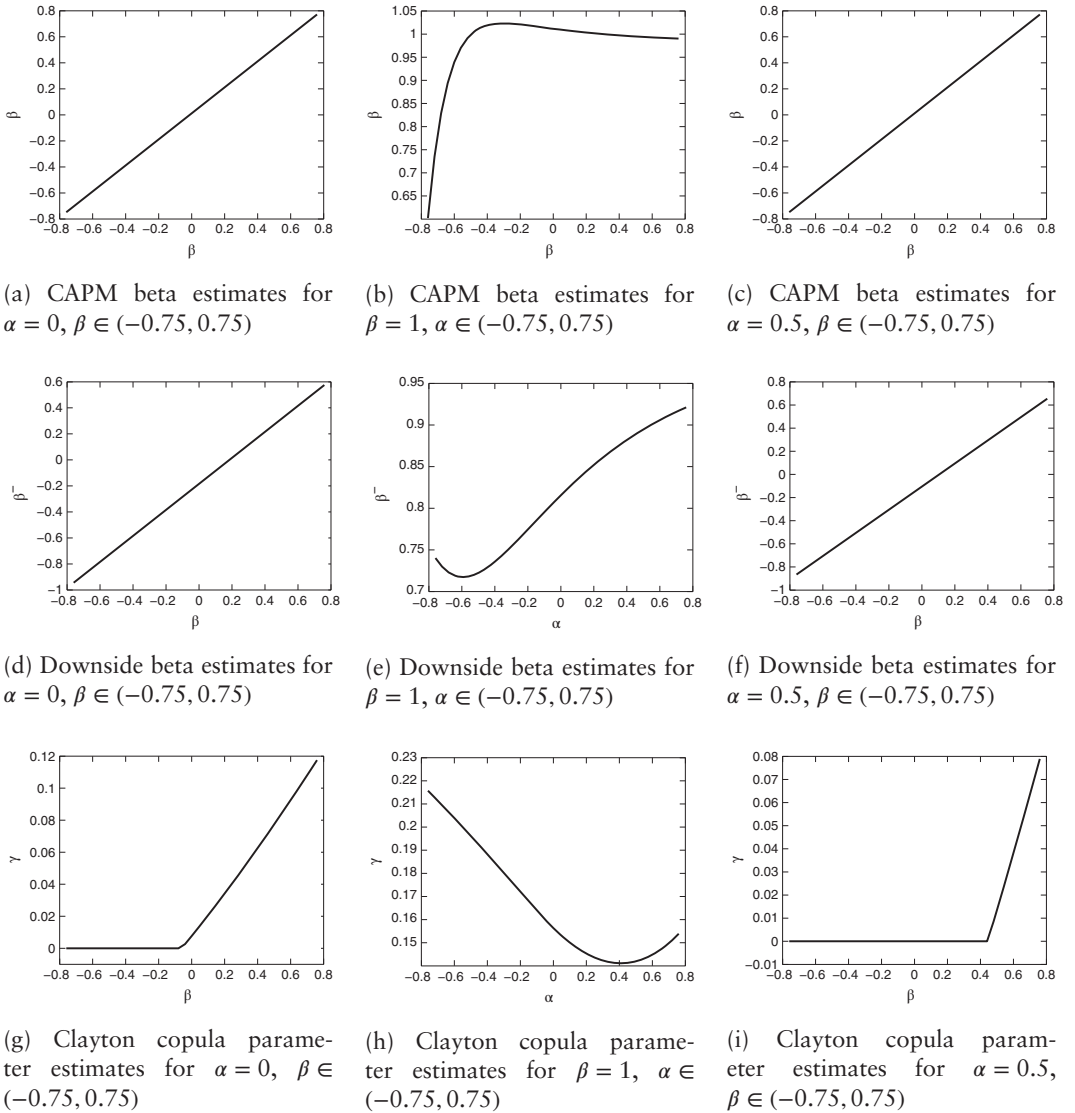


FIGURE 1.3 Estimates of linear dependence and AD. We estimate the CAPM beta, downside beta and the Clayton copula parameter using $N = 10,000$ simulated pairs of data (x, y) , where $y_i = \beta x_i + \epsilon_i$, with $x_i \sim N(0.25, 0.15)$ and $\epsilon_i \sim N(0, (x_i + 0.25)^\alpha)$. Higher levels of linear dependence are incorporated with higher values of β and higher levels of LTAD are incorporated with higher levels of α . Figure parts (a), (d) and (g) provide estimates for varying levels of linear dependence but with no AD ($\alpha = 0$). Figure parts (b), (e) and (h) provide estimates for varying degrees of AD with constant linear dependence ($\beta = 1$). Figure parts (c), (f) and (i) provide estimates for varying degrees of linear dependence with constant AD ($\alpha = 0.5$).

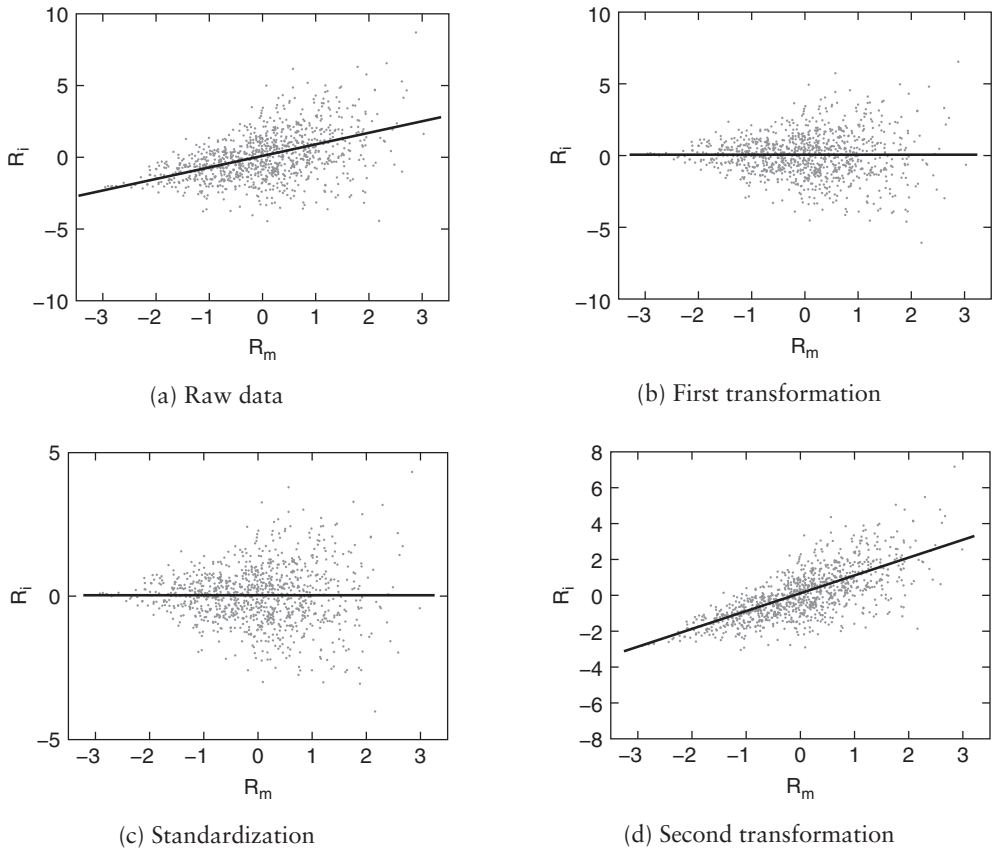


FIGURE 1.4 J^{Adj} data transformations. To calculate the J^{Adj} statistic with a random sample, $\{R_{it}, R_{mt}\}_{t=1}^T$, as in (a), we let $\hat{R}_{it} = R_{it} - \beta R_{mt}$ where R_{it} is the continuously compounded return on the i th asset, R_{mt} is the continuously compounded return on the market and $\beta = \text{cov}(R_{it}, R_{mt}) / \sigma_{R_{mt}}^2$. This transformation forces $\beta_{\hat{R}_{it}, R_{mt}} = 0$, as in (b). We standardize the transformed data, yielding R_{mt}^S and \hat{R}_{it}^S in (c). Finally, we re-transform the data to have $\hat{\beta} = 1$ by letting $\tilde{R}_{mt} = R_{mt}^S$ and $\tilde{R}_{it} = \hat{R}_{it}^S + R_{mt}^S$ in (d). The solid line through the middle of each plot is given to illustrate how the linear trend changes with each transformation.

J^{Adj} is given by

$$J^{Adj} = [\text{sign}([\tilde{\rho}^+ - \tilde{\rho}^-]\mathbf{1})]T(\tilde{\rho}^+ - \tilde{\rho}^-)' \tilde{\Omega}^{-1}(\tilde{\rho}^+ - \tilde{\rho}^-), \quad (1.7)$$

for $\tilde{\rho}^+ = \{\tilde{\rho}^+(\delta_1), \tilde{\rho}^+(\delta_2), \dots, \tilde{\rho}^+(\delta_N)\}$ and $\tilde{\rho}^- = \{\tilde{\rho}^-(\delta_1), \tilde{\rho}^-(\delta_2), \dots, \tilde{\rho}^-(\delta_N)\}$, where $\mathbf{1}$ is an $N \times 1$ vector of ones, $\tilde{\Omega}$ is an estimate of the variance-covariance matrix (Hong *et al.*, 2007) for the difference vector $(\tilde{\rho}^+ - \tilde{\rho}^-)$ and

$$\tilde{\rho}^+(\delta) = \text{corr}(\tilde{R}_{mt}, \tilde{R}_{it} | \tilde{R}_{mt} > \delta, \tilde{R}_{it} > \delta), \quad (1.8)$$

$$\tilde{\rho}^-(\delta) = \text{corr}(\tilde{R}_{mt}, \tilde{R}_{it} | \tilde{R}_{mt} < -\delta, \tilde{R}_{it} < -\delta). \quad (1.9)$$

The null hypothesis for the significance of the adjusted J statistic is that dependence is symmetric across the joint distribution, that is: $\rho^+(\delta_i) = \rho^-(\delta_i)$, $i = 1, \dots, N$. Under the null, $|J^{Adj}| \sim \chi^2_N$ following Hong *et al.* (2007).¹² Where dependence is symmetric across upper and lower tails, J^{Adj} will be near zero. Conversely, any strong asymmetries in dependence between upper and lower tails will result in a significant, non-zero J^{Adj} . A positive (negative) J^{Adj} is indicative of UTAD (LTAD), over and above the tail dependence implied by ordinary β .

We demonstrate the suitability of the adjusted J statistic in capturing LTAD and UTAD, as well as the β -invariance of J^{Adj} in Figure 1.5, estimated using the same simulations as above. In its own right,

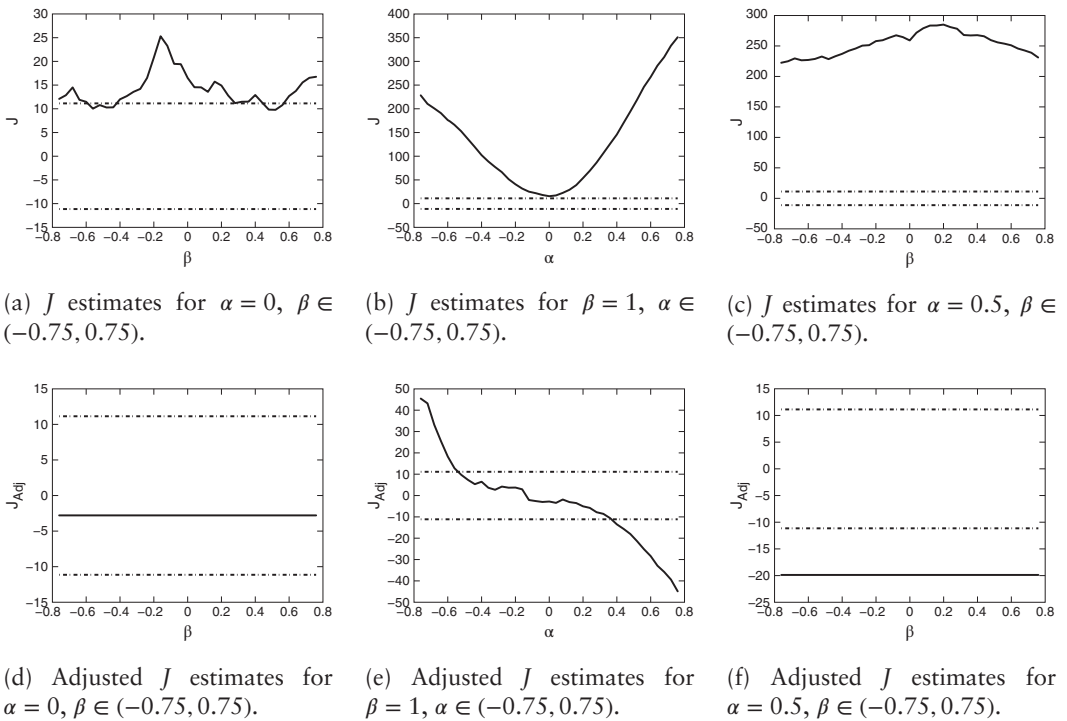


FIGURE 1.5 Estimates of linear dependence and AD. We estimate the J statistic (Hong *et al.*, 2007) and the adjusted J statistic using $N = 10,000$ simulated pairs of data (x, y) , where $y_i = \beta x_i + \epsilon_i$, with $x_i \sim N(0.25, 0.15)$ and $\epsilon_i \sim N(0, (x_i + 0.25)^\alpha)$. Higher levels of linear dependence are incorporated with higher values of β and higher levels of LTAD are incorporated with higher levels of α . Figure parts (a) and (d) provide estimates for varying levels of linear dependence but with no AD ($\alpha = 0$). Figure parts (b) and (e) provide estimates for varying degrees of AD with constant linear dependence ($\beta = 1$). Figure parts (c) and (f) provide estimates for varying degrees of linear dependence with constant AD ($\alpha = 0.5$).

¹²The transformations described represent (non-singular) affine transformations that may ultimately be expressed as linear transformations (Webster, 1995). Birkhoff and Lane (1997) show that a non-singular linear transformation of the space, V , is an isomorphism of the vector space, V , to itself. The assumptions used by Hong *et al.* (2007) to derive an asymptotic distribution for the J statistic therefore hold for the transformed returns $\{\tilde{R}_{1t}, \tilde{R}_{2t}\}$. $|J^{Adj}| \sim \chi^2_N$ then follows the proof described in Hong *et al.* (2007).

J^{Adj} captures both LTAD and UTAD between a stock and the market. To isolate upside and downside risk for the purposes of our regression analysis, we compute

$$J^{Adj+} = J^{Adj} \mathbb{I}_{J^{Adj} > 0}, \quad (1.10)$$

$$J^{Adj-} = J^{Adj} \mathbb{I}_{J^{Adj} < 0}. \quad (1.11)$$

We capture a family of conditional preferences, consistent with those of the Skiadas agent, by employing a range of exceedances in the calculation of J^{Adj} . Adjusting J to be β -invariant enables identification of the price paid by disappointment-averse agents in addition to the ordinary β risk premium. J^{Adj-} and J^{Adj+} capture disappointment and elation premia distinctly.

Further, as a non-parametric measure of AD, the J^{Adj} statistic facilitates the separation of the actual price of tail dependence from the effect of non-normal marginal return characteristics. J^{Adj} is also consistent with the work of Stapleton and Subrahmanyam (1983) and Kwon (1985), who suggest a means of deriving a linear relation between β and expected return without the need for multivariate normal assumptions. J^{Adj} is also consistent with the evidence that correlations tend to be larger in the lower tail of the joint return distribution compared with the upper tail (Longin and Solnik, 2001; Ang and Chen, 2002). LTAD exists provided that dependence in the lower tail exceeds dependence in the upper tail. Normality in the opposite tail is not required by this definition, which precludes parametric alternatives such as the H statistic (Ang and Chen, 2002) for the purposes of our investigation.

Another advantage of transforming the data in the way described above is that the standard deviation of market model residuals is forced to be the same across data sets. Controlling for the effects of idiosyncratic risk is important given (and despite) the debate over whether idiosyncratic risk is relevant in an asset-pricing context (Goyal and Santa-Clara, 2003; Bali *et al.*, 2005). It is sometimes argued that idiosyncratic risk should be priced whenever investors fail to hold sufficiently diversified portfolios (Merton, 1987; Campbell *et al.*, 2001; Fu, 2009). However, when tail risk is characterized by dependence that increases during down markets, the ability to diversify will be affected and the ability to protect the portfolio from risk will be reduced. Hence, downside risk may be mistakenly identified as idiosyncratic risk. Where this occurs, we expect idiosyncratic risk to increase as downside risk increases. Standardizing market model residuals allows us to distinguish between downside risk and other firm-specific risks.

Note that because tail risk is estimated by analysing the difference in correlation beyond N exceedances, the occurrence of net AD may be contingent upon a relatively small number of positive or negative joint returns. As a result, any measure of AD will suffer from a high likelihood of Type II errors, making it difficult to detect AD unless large data sets are utilized. Consequently, we present conservative estimates of AD between equity returns and the market.

1.4 SUMMARY

Skiadas (1997) offers an alternative framework to the standard von Neumann–Morgenstern expected utility theory, in which subjective consequences (disappointment, elation, regret, etc.) are incorporated indirectly through the properties of the decision maker's preferences rather than through explicit inclusion among the formal primitives.

Individuals with Skiadas preferences are endowed with a family of conditional preference relations, one for each event (Grant *et al.*, 2001). Preferences are state-dependent, as in the Gul (1991) framework, and because consequences are treated implicitly through the agent's preference relations, preferences can be regarded as 'non-separable' in that the ranking of an act given an event may depend on subjective consequences of these acts outside the event.

We demonstrate that AD influences the utility of disappointment-averse investors and establish the conditions under which this implies a market price for LTAD and UTAD. Using a comprehensive

set of simulations, we demonstrate that many of the commonly employed risk metrics are unable to adequately capture the salient distributional characteristics of AD. We further propose a β -invariance metric to capture AD consistent with Skiadas preferences and demonstrate its suitability using simulated AD data sets.

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