IN THIS CHAPTER

- **You're only in Chapter 1 and you're already going to get your first calc test**
- **Calculus — it's just souped-up regular math**
- **Zooming in is the key**
- **The world before and after calculus**

Chapter 1
What Is Calculus?

"My best day in Calc 101 at Southern Cal was the day I had to cut class to get a root canal."

— MARY JOHNSON

"I keep having this recurring dream where my calculus professor is coming after me with an axe."

— TOM FRANKLIN, COLORADO COLLEGE SOPHOMORE

"Calculus is fun, and it's so easy. I don't get what all the fuss is about."

— SAM EINSTEIN, ALBERT'S GREAT-GRANDSON

In this chapter, I answer the question "What is calculus?" in plain English, and I give you real-world examples of how calculus is used. After reading this and the following two short chapters, you *will* understand what calculus is all about. But here's a twist: Why don't you start out on the *wrong* foot by briefly checking out what calculus is *not*?

What Calculus Is Not

No sense delaying the inevitable. Ready for your first calculus test? Circle True or False.

> True or False: Unless you actually enjoy wearing a pocket protector, you've got no business taking calculus.
>
> True or False: Studying calculus is hazardous to your health.
>
> True or False: Calculus is totally irrelevant.

False, false, false! There's this mystique about calculus that it's this ridiculously difficult, incredibly arcane subject that no one in their right mind would sign up for unless it was a required course.

Don't buy into this misconception. Sure, calculus is difficult — I'm not going to lie to you — but it's manageable, doable. You made it through algebra, geometry, and trigonometry. Well, calculus just picks up where they leave off — it's simply the next step in a logical progression.

And calculus is not a dead language like Latin, spoken only by academics. It's the language of engineers, scientists, and economists. Okay, so it's a couple steps removed from your everyday life and unlikely to come up at a cocktail party. But the work of those engineers, scientists, and economists has a huge impact on your day-to-day life — from your microwave oven, cell phone, TV, and car to the medicines you take, the workings of the economy, and our national defense. At this very moment, something within your reach or within your view has been impacted by calculus.

So What Is Calculus, Already?

Calculus is basically just very advanced algebra and geometry. In one sense, it's not even a new subject — it takes the ordinary rules of algebra and geometry and tweaks them so that they can be used on more complicated problems. (The rub, of course, is that darn *other* sense in which it *is* a new and more difficult subject.)

Look at Figure 1-1. On the left is a man pushing a crate up a straight incline. On the right, the man is pushing the same crate up a curving incline. The problem, in both cases, is to determine the amount of energy required to push the crate to the top. You can do the problem on the left with regular math. For the one on the right, you need calculus (assuming you don't know the physics shortcuts).

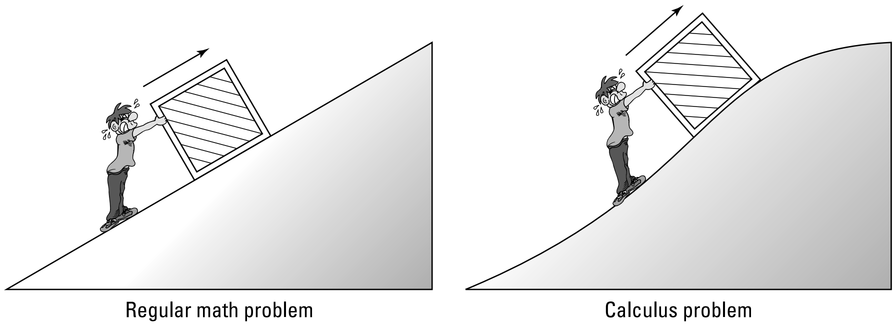

FIGURE 1-1: The difference between regular math and calculus: In a word, it's the *curve*.

For the straight incline, the man pushes with an *unchanging* force, and the crate goes up the incline at an *unchanging* speed. With some simple physics formulas and regular math (including algebra and trig), you can compute how many calories of energy are required to push the crate up the incline. Note that the amount of energy expended each second remains the same.

For the curving incline, on the other hand, things are constantly changing. The steepness of the incline is *changing* — and not just in increments like it's one steepness for the first 3 feet then a different steepness for the next 3 feet. It's *constantly changing.* And the man pushes with a *constantly changing* force — the steeper the incline, the harder the push. As a result, the amount of energy expended is also changing, not every second or every thousandth of a second, but *constantly changing* from one moment to the next. That's what makes it a calculus problem. By this time, it should come as no surprise to you that calculus is described as "the mathematics of change." Calculus takes the regular rules of math and applies them to fluid, evolving problems.

For the curving incline problem, the physics formulas remain the same, and the algebra and trig you use stay the same. The difference is that — in contrast to the straight incline problem, which you can sort of do in a single shot — you've got to break up the curving incline problem into small chunks and do each chunk separately. Figure 1-2 shows a small portion of the curving incline blown up to several times its size.

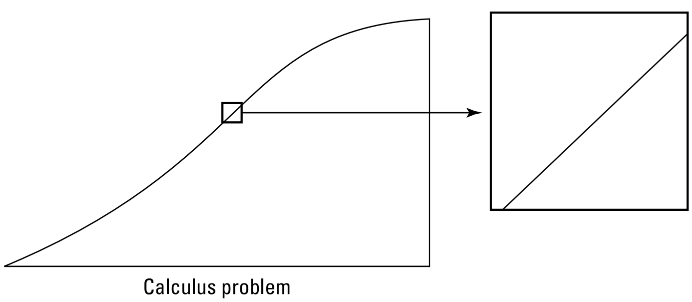

FIGURE 1-2: Zooming in on the curve — voilà, it's straight (almost).

When you zoom in far enough, the small length of the curving incline becomes practically straight. Then, because it's straight, you can solve that small chunk just like the straight incline problem. Each small chunk can be solved the same way, and then you just add up all the chunks.

That's calculus in a nutshell. It takes a problem that can't be done with regular math because things are constantly changing — the changing quantities show up on a graph as curves — it zooms in on the curve till it becomes straight, and then it finishes off the problem with regular math.

What makes the invention of calculus such a fantastic achievement is that it does what seems impossible: it zooms in *infinitely.* As a matter of fact, everything in calculus involves infinity in one way or another, because if something is constantly changing, it's changing infinitely often from each infinitesimal moment to the next.

Real-World Examples of Calculus

So, with regular math you can do the straight incline problem; with calculus you can do the curving incline problem. Here are some more examples.

With regular math you can determine the length of a buried cable that runs diagonally from one corner of a park to the other (remember the Pythagorean theorem?). With calculus you can determine the length of a cable hung between two towers that has the shape of a *catenary* (which is different, by the way, from a simple circular arc or a parabola). Knowing the exact length is of obvious importance to a power company planning hundreds of miles of new electric cable. See Figure 1-3.

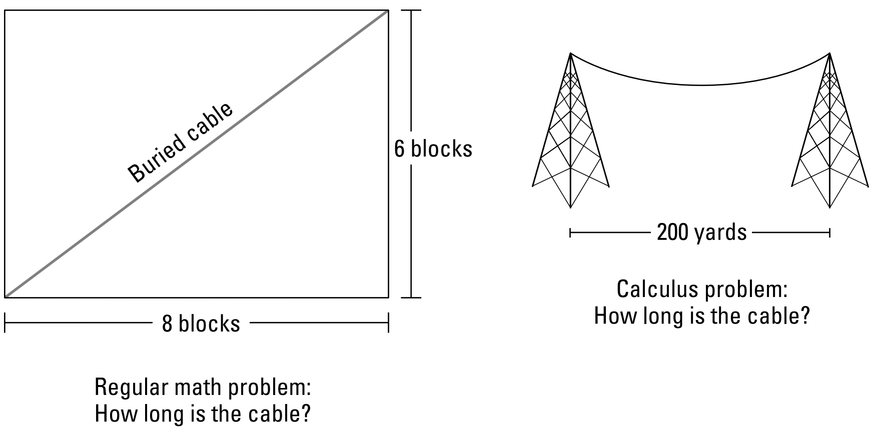

FIGURE 1-3: Without and with calculus.

You can calculate the area of the flat roof of a home with ordinary geometry. With calculus you can compute the area of a complicated, nonspherical shape like the dome of the Minneapolis Metrodome. Architects designing such a building need to know the dome's area to determine the cost of materials and to figure the weight of the dome (with and without snow on it). The weight, of course, is needed for planning the strength of the supporting structure. Check out Figure 1-4.

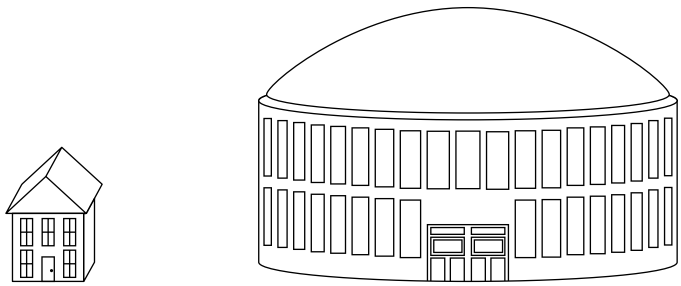

FIGURE 1-4: Sans and avec calculus.

With regular math and some simple physics, you can calculate how much a quarterback must lead his receiver to complete a pass. (I'm assuming here that the receiver runs in a *straight* line and at a *constant* speed.) But when NASA, in 1975, calculated the necessary "lead" for aiming the Viking I at Mars, it needed calculus because both the Earth and Mars travel on *elliptical* orbits (of different shapes) and the speeds of both are *constantly changing* — not to mention the fact that on its way to Mars, the spacecraft is affected by the different and *constantly changing* gravitational pulls of the Earth, the moon, Mars, and the sun. See Figure 1-5.

You see many real-world applications of calculus throughout this book. The differentiation problems in Part 4 all involve the steepness of a curve — like the steepness of the curving incline in Figure 1-1. In Part 5, you do integration problems like the cable-length problem shown back in Figure 1-3. These problems involve breaking up something into little sections, calculating each section, and then adding up the sections to get the total. More about that in Chapter 2.

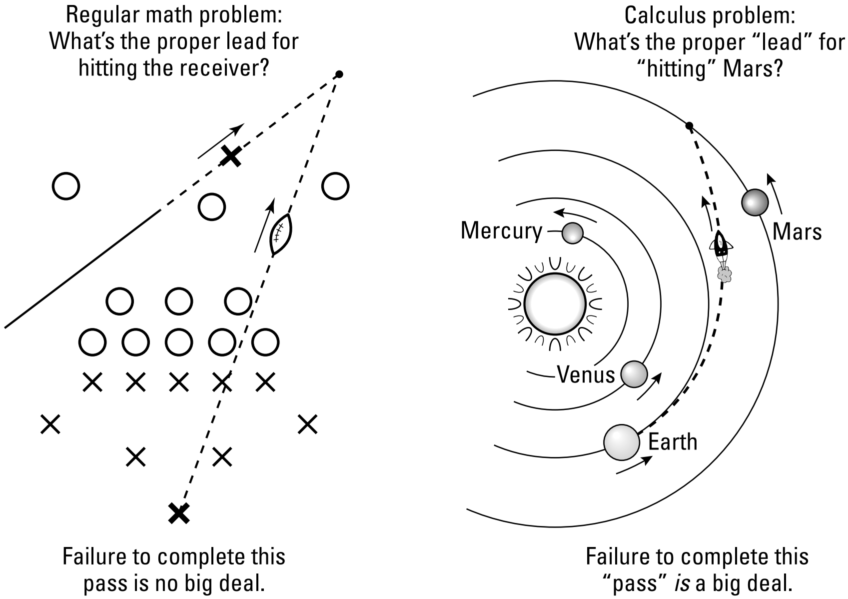

FIGURE 1-5: B.C.E. (Before the Calculus Era) and C.E. (the Calculus Era).