1

Basic concepts

1.1 History

Although this book will not follow a strictly historical development, to 'set the scene' this first chapter will start with a brief review of the most important discoveries that led to the separation of nuclear physics from atomic physics as a subject in its own right, and later work that in its turn led to the emergence of particle physics from nuclear physics.[1]

1.1.1 The origins of nuclear physics

In 1896 Becquerel observed that photographic plates were being fogged by an unknown radiation emanating from uranium ores. He had accidentally discovered *radioactivity*, the fact that some chemical elements spontaneously emit radiation. The name was coined by Marie Curie two years later to distinguish this phenomenon from induced forms of radiation. In the years that followed, radioactivity was extensively investigated, notably by the husband and wife team of Pierre and Marie Curie, and by Rutherford and his collaborators.[2] Other radioactive sources were quickly found, including the hitherto unknown chemical elements polonium and radium,

[1]For a readable and lavishly illustrated account, see Close, Marten, and Sutton (1987). An interesting account of the early period, with descriptions of the personalities involved, is given in Segrè (1980), while a very detailed and scholarly account may be found in Pais (1986).

[2]The 1903 Nobel Prize in Physics was awarded jointly to Henri Becquerel for his discovery and to Pierre and Marie Curie for their subsequent research into radioactivity. Ernest Rutherford had to wait until 1908, when he was awarded the Nobel Prize in Chemistry for his 'investigations into the disintegration of the elements and the chemistry of radioactive substances'.

Nuclear and Particle Physics: An Introduction, Third Edition. B.R. Martin and G. Shaw.

Companion website: www.wiley.com/go/martin/nuclear3

discovered by the Curies in 1897.[3] It was soon established that there were two distinct types of radiation involved, named by Rutherford α and β rays. We know now that β rays are electrons (the name 'electron' had been coined in 1894 by Stoney) and α rays are doubly ionised helium atoms. In 1900 a third type of decay was discovered by Villard that involved the emission of photons, the quanta of electromagnetic radiation, referred to in this context as γ rays. These historical names are still commonly used.

The revolutionary implications of these experimental discoveries did not become fully apparent until 1902. Prior to this, atoms were still believed to be immutable – indestructible and unchanging – an idea with its origin in Greek philosophy and, for example, embodied in Dalton's atomic theory of chemistry in 1804. This causes a big problem: if the atoms in a radioactive source remain unchanged, where does the energy carried away by the radiation come from? Typically, early attempts to explain the phenomena of radioactivity assumed that the energy was absorbed from the atmosphere or, when that failed, that energy conservation was violated in radioactive processes. However, Rutherford had shown in 1900 that the intensity of the radiation emitted from a radioactive source was not constant, but reduced by a factor of two in a fixed time that was characteristic of the source, but independent of its amount. This is called its *half-life*. In 1902, together with Soddy, he put forward the correct explanation, called the *transformation theory*, according to which the atoms of any radioactive element decay with a characteristic half-life, emitting radiation, and in so doing are transformed into the atoms of a different chemical element. The centuries old belief in the immutability of atoms was shattered forever.

An important question not answered by the transformation theory is: which elements are radioactive and which are stable? An early attempt to solve this problem was made by J.J. Thomson, who was extending the work of Perrin and others on the radiation that had been observed to occur when an electric field was established between electrodes in an evacuated glass tube. In 1897 he was the first to definitively establish the nature of these 'cathode rays'. We now know they consist of free electrons, denoted e^- (the superscript denotes the electric charge) and Thomson measured their mass and charge.[4] This gave rise to the speculation that atoms contained electrons in some way, and in 1903 Thomson suggested a model where the electrons were embedded and free to move in a region of positive charge filling the entire volume of the atom – the so-called *plum pudding model*. This model could account for the stability of atoms, but gave no explanation for the discrete wavelengths observed in the spectra of light emitted from excited atoms.

[3]For these discoveries, Marie Curie won a second Nobel Prize in 1911, this time in Chemistry. The honour would presumably have been shared with her husband had he not been killed in a road accident in 1906.

[4]J.J. Thomson received the 1906 Nobel Prize in Physics for his discovery. A year earlier, Philipp von Lenard had received the 1905 Physics Prize for his work on cathode rays.

The plum pudding model was finally ruled out by a classic series of experiments suggested by Rutherford and carried out by his collaborators Geiger and Marsden starting in 1909. This consisted of scattering α particles from very thin gold foils. In the Thomson model, most of the α particles would pass through the foil, with only a few suffering deflections through small angles. However, Geiger and Marsden found that some particles were scattered through very large angles, even greater than 90°. As Rutherford later recalled, 'It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you'.[5] He then showed that this behaviour was not due to multiple small-angle deflections, but could only be the result of the α particles encountering a very small, very heavy, positively charged central *nucleus*. (The reason for these two different behaviours is discussed in Appendix C.)

To explain these results, Rutherford in 1911 proposed the *nuclear model* of the atom. In this model, the atom was likened to a planetary system, with the light electrons (the 'planets') orbiting about a tiny but heavy central positively charged nucleus (the 'sun'). The size of the atom is thus determined by the radii of the electrons' orbits, with the mass of the atom arising almost entirely from the mass of the nucleus. In the simplest case of hydrogen, a single electron orbits a nucleus, now called the *proton* (p), with electric charge $+e$, where e is the magnitude of the charge on the electron, to ensure that hydrogen atoms are electrically neutral. Alpha particles are just the nuclei of helium, while heavier atoms were considered to have more electrons orbiting heavier nuclei. At about the same time, Soddy showed that a given chemical element often contained atoms with different atomic masses but identical chemical properties. He called this *isotopism* and the members of such families *isotopes*. Their discovery led to a revival of interest in *Prout's Law* of 1815, which claimed that all the elements had integer atomic mass in units of the mass of the hydrogen atom, called *atomic weights*. This holds to a good approximation for many elements, like carbon and nitrogen, with atomic weights of approximately 12.0 and 14.0 in these units, but does not hold for other elements, like chlorine, which has an atomic weight of approximately 35.5. However, such fractional values could be explained if the naturally occurring elements consisted of mixtures of isotopes. Chlorine, for example, is now known to consist of a mixture of isotopes with atomic weights of approximately 35.0 and 37.0, giving an average value of 35.5 overall.[6]

Although the planetary model explained the α particle scattering experiments, there remained the problem of reconciling it with the observation of stable atoms. In classical physics, the electrons in the planetary model would be continuously accelerating and would therefore lose energy by radiation, leading to the collapse of the atom. This problem was solved by Bohr in 1913, who revolutionised the study of atomic physics by

[5]Quoted on p. 111 of da C. Andrade (1964).

[6]Frederick Soddy was awarded the 1921 Nobel Prize in Chemistry for his work on isotopes.

JWST916-c01 JWST916-Martin December 20, 2018 13:19 Printer Name: Trim: 254mm × 203mm

applying the newly emerging quantum theory. The result was the Bohr–Rutherford model of the atom, in which the motion of the electrons is confined to a set of discrete orbits. Because photons of a definite energy would be emitted when electrons moved from one orbit to another, this model could explain the discrete nature of the observed electromagnetic spectra when excited atoms decayed. In the same year, Moseley extended these ideas to a study of X-ray spectra and conclusively demonstrated that the charge on the nucleus is $+Ze$, where the integer Z was the atomic number of the element concerned, and implying Z orbiting electrons for electrical neutrality. In this way he laid the foundation of a physical explanation of Mendeleev's periodic table and in the process predicted the existence of no less than seven unknown chemical elements, which were all later discovered.[7]

The phenomena of atomic physics are controlled by the behaviour of the orbiting electrons and are explained in detail by refined modern versions of the Bohr–Rutherford model, including relativistic effects described by the Dirac equation, the relativistic analogue of the Schrödinger equation that applies to electrons, which is discussed in Section 1.2. However, following the work of Bohr and Moseley it was quickly realised that radioactivity was a nuclear phenomenon. In the Bohr–Rutherford and later models, different isotopes of a given element have different nuclei with different nuclear masses, but their orbiting electrons have virtually identical chemical properties because these nuclei all carry the same charge $+Ze$. The fact that such isotopes often have dramatically different radioactive decay properties is therefore a clear indication that these decays are nuclear in origin. In addition, since electrons were emitted in β decays, it seemed natural to assume that nuclei contained electrons as well as protons, and the first model of nuclear structure, which emerged in 1914, assumed that the nucleus of an isotope of an element with atomic number Z and mass number A was itself a tightly bound compound of A protons and $A-Z$ electrons. This provided an explanation of the existence of isotopes and of the approximate validity of Prout's law when applied to isotopes, because the electron mass is negligible compared to that of the proton. However, although this model persisted for some time, it was subsequently ruled out by detailed measurements of the spins of nuclei (cf. Problem 1.1).

The correct explanation of isotopes and nuclear structure had to wait almost twenty years, until a classic discovery by Chadwick, in 1932. His work followed earlier experiments by Irène Curie (the daughter of Pierre and Marie Curie) and her husband Frédéric Joliot. They had observed that neutral radiation was emitted when α particles bombarded beryllium, and later work had studied the energy of protons emitted when paraffin

[7] Niels Bohr received the 1922 Nobel Prize in Physics for his theoretical work on the structure of atoms. Moseley was nominated for the 1915 Nobel Prizes in both Physics and Chemistry for his pioneering use of X-rays, but was tragically killed in World War I in August 1915 at the age of 27, before a decision was made.

was exposed to this neutral radiation. Chadwick refined and extended these experiments and demonstrated that they implied the existence of an electrically neutral particle of approximately the same mass as the proton, called the *neutron* (n).[8] The discovery of the neutron led immediately to the correct formulation of nuclear structure, in which an isotope of atomic number Z and mass number A is a bound state of Z protons and $A - Z$ neutrons. There are no electrons bound inside nuclei.

Finally, to complete this historical account, we must go back to another major result: the discovery of the continuous β - decay spectrum by Chadwick in 1914. At that time, nuclear decays were all viewed as a parent nucleus decaying via α, β, or γ decay to give a daughter nucleus plus either an alpha particle, an electron or a photon, respectively. As each possibility would be a two-body decay, energy and momentum conservation implies that the emitted particle would have a unique energy, depending on the masses of the parent and daughter nucleons, which would be the same for all observed decays of a given type. This behaviour is precisely what is observed for α decays and γ decays and the earliest experiments erroneously suggested the same held for β decays. However, when Chadwick measured the energies of the electrons from samples of nuclei he found that the electrons emitted in a given β-decay process had a continuous energy distribution, as shown in Figure 1.1.

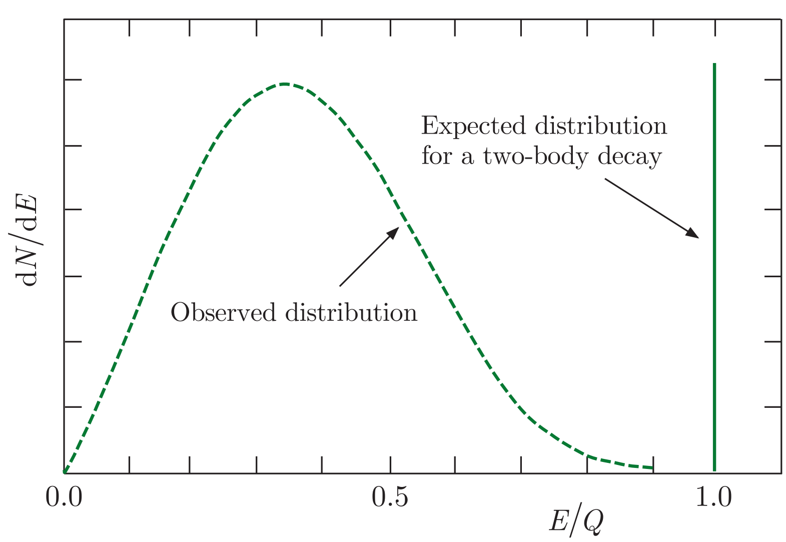

Figure 1.1 The observed electron energy distribution $\mathrm{d}N/\mathrm{d}E$ in β decay (dashed line) as a function of E/Q, where E is the kinetic energy of the electron and Q is the total energy released. Also shown is the expected energy distribution if β decay were a two-body process (solid line).

After a hiatus due to the first world war, various ideas were suggested to explain this unexpected result, including a remarkable proposal by Bohr in 1929 that energy conservation was violated in β decays, but later abandoned by him in favour of the correct hypothesis proposed by Pauli in

[8] James Chadwick received the 1935 Nobel Prize in Physics for his discovery of the neutron. The discovery was not unexpected, because Rutherford had already deduced that the nucleus must include uncharged constituents with masses similar to that of the proton, and had even coined the name 'neutron'. Irène Curie and Frédéric Joliot received the 1935 Nobel Prize in Chemistry for 'synthesising new radioactive elements'.

JWST916-c01 JWST916-Martin December 20, 2018 13:19 Printer Name: Trim: 254mm × 203mm

1930. Pauli proposed that an additional, and hitherto unknown, neutral particle was emitted in β decays and shared the energy released with the electron. This particle had to be very light, since the most energetic electrons in the observed continuous distribution carried off almost all the energy released in the decay, as can be seen in Figure 1.1; it had also to interact so weakly with matter that it invariably escaped detection. Despite this, its existence was rapidly accepted, largely because of its crucial role in the highly successful theory of β decay proposed in 1932 by Fermi, who used the name *neutrino* (meaning 'little neutral one') for the new particle after his close friend and colleague Amaldi jokingly suggested it to distinguish Pauli's particle from Chadwick's 'big neutral one', the neutron.[9]

In conclusion, by 1932 physicists had arrived at a model of the nucleus in which an isotope of atomic number Z and mass number A is a bound state of Z protons and $A - Z$ neutrons. Later workers, including Heisenberg, another of the founders of quantum theory, applied quantum mechanics to the nucleus, now viewed as a collection of neutrons and protons, collectively called *nucleons*. In this case, however, the force binding the nucleus is not the electromagnetic force that holds electrons in their orbits, but a much stronger force that does not depend on the charge of the nucleon (i.e. is charge-independent) and with a very short effective range. This binding interaction is called the *strong nuclear force*. In addition, there is a third force, much weaker than the electromagnetic force, called the *weak interaction,* responsible for β decays, where neutrinos as well as electrons are emitted. These ideas form the essential framework of our understanding of the nucleus today. Nevertheless, there is still no single theory that is capable of explaining all the data of nuclear physics and we shall see that different models are used to interpret different classes of phenomena.

1.1.2 The emergence of particle physics: hadrons and quarks

By the early 1930s, the nineteenth century view of atoms as indivisible elementary particles had been replaced and a smaller group of subatomic particles now enjoyed this status: electrons, protons and neutrons. To these we must add two electrically neutral particles: the photon (γ) and the neutrino (ν). However, this simple picture was not to last, because of the discovery of many new subatomic particles, initially in cosmic rays and later in experiments using particle accelerators.

We start with cosmic rays, which may be conveniently divided into two types: *primaries*, which are high-energy particles, mostly protons,

[9] The neutrino was eventually detected, but not until very much later, by Reines and Cowan in 1956. A brief description of their experiment is given in Section 2.1.1 of Martin and Shaw (2017) and in more detail in Chapter 12 of Trigg (1975). Frederick Reines shared the 1995 Nobel Prize in Physics for his work in neutrino physics, particularly for the detection of the neutrino.

incident on the Earth's atmosphere from all directions in space; and *secondaries*, which are produced when the primaries collide with nuclei in the Earth's atmosphere, with some penetrating to sea level. It was among these secondaries that the new particles were discovered, mainly using a detector devised by C.T.R. Wilson, called the *cloud chamber*. It consisted of a vessel filled with air almost saturated with water vapour and fitted with an expansion piston. When the vessel was suddenly expanded, the air was cooled and became supersaturated. Droplets were then formed preferentially along the trails of ions left by charged particles passing through the chamber. Immediately after the expansion, the chamber was illuminated by a flash of light and the tracks of droplets so revealed were photographed before they had time to disperse. The use of these chambers in cosmic ray studies led to many important discoveries, including, in 1932, the detection of *antiparticles*, to be discussed in Section 1.2.[10] However, the birth of particle physics as a new subject, distinct from atomic and nuclear physics, dates from 1947 with the discovery of *pions* and of *strange particles* by cosmic ray groups at Bristol and Manchester Universities, respectively. We will consider these in turn.

The discovery of pions was not unexpected, since Yukawa had famously predicted their existence in a theory of the strong nuclear forces proposed in 1934. We will return to this in Section 1.5. Here we will simply note that the range of the nuclear force required the pions to have a mass of around one seventh of the proton mass, while the charge independence of the nuclear force required there to be three charge states, denoted π^+, π^- π^0, with charges $+e, -e$ and zero, respectively. This gave rise to a search for these particles in cosmic ray secondaries, and in 1936 Anderson and Neddermeier discovered new subatomic particles that were initially thought to be pions, but are now known to be particles called *muons*. As we shall see in Chapter 3, muons are rather like heavy electrons and, like both electrons and neutrinos, do not interact via the strong force that holds the nucleus together. Charged pions with suitable properties were finally detected in 1947 using photographic emulsions containing a silver halide. The ionisation energy deposited by a charged particle passing through the emulsion causes the formation of a latent image, and the silver grains resulting from subsequent development form a visual record of the path of the particle. The neutral pion was detected somewhat later in 1950.[11] Pions interact with each other and with nucleons via forces comparable in strength to the strong nuclear interaction between nucleons and in future we will refer to all such forces as *strong interactions*, reserving the term

[10]Wilson built the first cloud chamber in 1911 and shared the 1927 Nobel Prize in Physics. Victor Hess discovered cosmic rays in 1912, by making a series of balloon flights and showing that the intensity of radiation increased at high altitudes, indicating an extraterrestrial origin. He shared the 1936 Nobel Prize in Physics.

[11]The 1949 Nobel Prize in Physics was awarded to Hideki Yukawa for his prediction of the pion and in 1950 the Nobel Prize in Physics was awarded to Cecil Powell for his leading role in its discovery.

strong nuclear interaction to the special case of nucleon–nucleon interactions. Particles that interact by the strong force are now called *hadrons*. Thus pions and nucleons are examples of hadrons, while electrons, muons and neutrinos are not.

Further work using cloud chambers to detect cosmic ray secondaries led to the discovery in 1947 by Rochester and Butler of new particles, named *kaons*, which, in contrast to the discovery of pions, was totally unexpected. Kaons were almost immediately recognised as a completely new form of matter, because they had supposedly 'strange' properties, which will be discussed further in Section 3.3. Other *strange particles* with similar properties were discovered, and in 1953 it was realised that these properties were precisely what would be expected if they were hadrons with nonzero values of an hitherto unknown quantum number, given the name *strangeness* by Gell-Mann, which was conserved in strong and electromagnetic interactions, but not necessarily conserved in the so-called *weak interactions* responsible for β decay. Non-strange particles like the pions and nucleons have zero values of strangeness. This led Gell-Mann, and independently Zweig, to suggest that hadrons were composed of more fundamental particles called *quarks* (q), together with their antiparticles. Three quarks were required at the time, denoted u, d, and s, with fractional electric charges $+2e/3$, $-e/3$, and $-e/3$, respectively. In particular, ordinary matter, i.e. protons and nucleons are composed of u and d quarks only, while the strange particles also contain s quarks. The latter is called the *strange quark* and the strangeness quantum number merely reflects the number of strange quarks and/or antiquarks present.

The 1950s also saw technological developments that enabled high-energy beams of particles to be produced in laboratories, and these rapidly replaced cosmic rays as the source of the high-energy particles required to create new particles in collisions. At the same time, cloud chambers were largely superseded by bubble chambers, a more efficient device in which charged particles were detected by the trail of bubbles left along their tracks through a superheated liquid, rather than droplets in a super-cooled gas.[12] By the mid-1960s this had resulted in the discovery of many more unstable particles and the above *quark model* had considerable success in understanding the properties of the observed hadrons, as we shall see in Section 3.3,[13] but because no free quarks were detected experimentally, there was initially considerable scepticism for this interpretation. We now know that there is a fundamental reason why quarks cannot

[12] Many beautiful pictures of events observed in both cloud and bubble chambers may be found in Close Marten, and Sutton (1987). Donald Glaser was awarded the 1960 Nobel Prize in Physics for his invention of the bubble chamber and Luis Alvarez received the 1968 prize for its further development and use in discovering new subatomic particles.

[13] Murray Gell-Mann received the 1969 Nobel Prize in Physics for 'contributions and discoveries concerning the classification of elementary particles and their interactions'. For the origin of the word 'quark', he cited the now famous quotation 'Three quarks for Muster Mark' from James Joyce's book *Finnegans Wake*. George Zweig had suggested the name 'aces'. Subsequently, more than three quarks were discovered, as we shall see.

be observed as free particles (it is discussed in Section 5.1), but at the time many physicists looked upon quarks as a convenient mathematical description, rather than physical particles. However, evidence for the existence of quarks as real particles began to emerge in 1969 from a series of experiments analogous to those of Rutherford and his co-workers, where high-energy beams of electrons and neutrinos were scattered from nucleons. (These experiments are discussed in Section 5.5.) Analysis of the angular distributions of the scattered particles confirmed that the nucleons were themselves bound states of point-like charged entities, with properties consistent with those hypothesised in the quark model, including their fractional electric charges. This is essentially the picture today, where elementary particles are considered to be a small number of fundamental physical entities, including quarks, the electron, neutrinos, the photon and a few others we shall meet, but no longer nucleons.

1.1.3 The standard model of particle physics

Following the discovery of quarks, an 'in principle' complete theory of elementary particles gradually emerged, called, rather prosaically, the *standard model*. This aims to explain all the phenomena of particle physics, except those due to gravity, in terms of the properties and interactions of a small number of *elementary* (or *fundamental*) *particles*, which are now defined as being point-like, without internal structure or excited states. Particle physics thus differs from nuclear physics in having a single theory to interpret its data. Here we restrict ourselves to a brief outline of the standard model, which will be developed in more detail later in Chapters 3, 5, 6, and 7.

An elementary particle is characterised by, amongst other things, its mass, its electric charge and its *spin*. The latter is a permanent angular momentum possessed by all particles in quantum theory, even when they are at rest. Spin has no classical analogue and is not to be confused with the use of the same word in classical physics, where it usually refers to the angular momentum of extended objects. The maximum value of the spin angular momentum about any axis is $S\hbar\,(\hbar \equiv h/2\pi)$, where h is Planck's constant and S is the *spin quantum number*, or *spin* for short. It has a fixed value for particles of any given type (for example $S = 1/2$ for electrons) and general quantum mechanical principles restrict the possible values of S to be 0, 1/2, 1, 3/2, …. Particles with half-integer spin are called *fermions* and those with integer spin are called *bosons*. There are two families of elementary fermions in the standard model: the quarks, which interact via strong forces, and the *leptons*, including electrons, muons, and neutrinos, which do not. In addition, there is a family of spin-1 bosons, which act as force carriers in the theory, and a spin-0 particle, called the *Higgs boson*, which plays a key role in understanding the origin of elementary particle masses within the theory.

The above particles interact via four forces of nature. In decreasing order of strength, these are the strong interaction, which binds the quarks together into hadrons; the electromagnetic interaction between the charged leptons and quarks; the weak interaction responsible for β decay; and gravity. Although an understanding of all four forces will ultimately be essential in a complete theory, gravity is so weak that it can be neglected in nuclear and particle physics at presently accessible energies. Because of this, we will often refer in practice to the three forces of nature. The standard model specifies the origin of these three forces. In classical physics the electromagnetic interaction is propagated by electromagnetic waves, which are continuously emitted and absorbed. While this is an adequate description at long distances, at short distances the quantum nature of the interaction must be taken into account. In quantum theory, the interaction is transmitted discontinuously by the exchange of photons, which are members of the family of fundamental spin-1 bosons of the standard model. Photons are referred to as the *gauge bosons*, or 'force carriers', of the electromagnetic interaction. The use of the word 'gauge' originates from the fact that the electromagnetic interaction possesses a fundamental symmetry called *gauge invariance*. For example, Maxwell's equations of classical electromagnetism are invariant under a specific transformation of the electromagnetic fields, called a gauge transformation. This property is common to all the three interactions of nature we will be discussing and has profound consequences, but we will not need its details in this book.[14] The weak and strong interactions are also mediated by the exchange of spin-1 gauge bosons. For the weak interaction these are the W^+, W^-, and Z^0 bosons (again the superscripts denote the electric charges) with masses about 80–90 times the mass of the proton. For the strong interaction, the force carriers are called *gluons*. There are eight gluons, all of which have zero mass and are electrically neutral.

In addition to the elementary particles of the standard model, there are other important particles we will be studying. These are the *hadrons*, the bound states of quarks. Nucleons are examples of hadrons, but there are several hundred more, not including nuclei, most of which are unstable and decay by one of the three interactions. For example, the charged pions $\pi^\pm$ decay via the weak interaction with a lifetime of about 10^{-8} s, while the neutral pion π^0 decays via the electromagnetic interaction with a lifetime of about 10^{-17} s. The existence of quarks was first inferred from the properties of hadrons, as we have seen, and they remain particularly important because free quarks are unobservable in nature. Hence to deduce properties of quarks we are forced to study hadrons. An analogy would be if we had to deduce the properties of nucleons by exclusively studying the properties of nuclei.

Since nucleons are bound states of quarks and nuclei are bound states of nucleons, the properties of nuclei should in principle be deducible from

---

[14] A brief description of gauge invariance and some of its consequences is given, for the interested reader, in Appendix D.

the properties of quarks and their interactions, that is, from the standard model. Although there has been some progress in this direction, in practice this is still beyond present calculational techniques and often nuclear and particle physics are treated as two almost separate subjects. However, there remain some connections between them and in introductory treatments it is still useful to present both subjects together.

The remaining sections of this chapter are devoted to introducing some of the basic theoretical tools needed to describe the phenomena of both nuclear and particle physics, starting with a key concept in the latter: antiparticles.

1.2 Relativity and antiparticles

Elementary particle physics is also called high-energy physics. One reason for this is that if we wish to produce new particles in a collision between two other particles, then because of the relativistic mass–energy relation $E = mc^2$, energies are needed at least as great as the rest masses of the particles produced. The second reason is that to explore the structure of a particle requires a probe whose wavelength λ is smaller than the structure to be explored. By the de Broglie relation $\lambda = h/p$, this implies that the momentum p of the probing particle, and hence its energy, must be large. For example, to explore the internal structure of the proton using electrons requires wavelengths that are much smaller than the radius of the proton, which is roughly 10^{-15} m. This in turn requires electron energies that are greater than 10^3 times the rest energy of the electron, implying electron velocities very close to the speed of light. Hence any explanation of the phenomena of elementary particle physics must take account of the requirements of the theory of special relativity, in addition to those of quantum theory. There are very few places in particle physics where a nonrelativistic treatment is adequate, whereas the need for a relativistic treatment is much less in nuclear physics.

Constructing a quantum theory that is consistent with special relativity leads to the conclusion that for every charged particle of nature, there must exist an associated particle, called an *antiparticle*, with the same mass as the corresponding particle. This important prediction was first made by Dirac and follows from the solutions of the equation he postulated to describe relativistic electrons.[15] The *Dirac equation* for a particle of mass m and momentum $\mathbf{p}$ moving in free space is of the form

$$i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t} = H(\mathbf{r},\hat{\mathbf{p}})\Psi(\mathbf{r},t), \tag{1.1}$$

[15] Paul Dirac shared the 1933 Nobel Prize in Physics with Erwin Schrödinger. The somewhat cryptic citation stated 'for the discovery of new productive forms of atomic theory'.

where we use the notation $\mathbf{r} = (x_1, x_2, x_3) = (x, y, z)$, $\hat{\mathbf{p}} = -i\hbar\boldsymbol{\nabla}$ is the usual quantum mechanical momentum operator and the Hamiltonian was postulated by Dirac to be

$$H = c\,\boldsymbol{\alpha}\cdot\hat{\mathbf{p}} + \beta mc^2. \tag{1.2}$$

The coefficients $\boldsymbol{\alpha}$ and β are determined by the requirement that the solutions of (1.1) are also solutions of the free-particle *Klein-Gordon equation*

$$-\hbar^2\frac{\partial^2\Psi(\mathbf{r},t)}{\partial t^2} = -\hbar^2c^2\boldsymbol{\nabla}^2\Psi(\mathbf{r},t) + m^2c^4\Psi(\mathbf{r},t), \tag{1.3}$$

which follows from making the usual quantum mechanical substitutions $\mathbf{p}\to -i\hbar\boldsymbol{\nabla}$ and $E\to i\hbar\partial/\partial t$ in the relativistic mass–energy relation $E^2 = p^2c^2 + m^2c^4$. This leads to the conclusion that $\boldsymbol{\alpha}$ and β cannot be ordinary numbers; their simplest forms are 4×4 matrices. Thus the solutions of the Dirac equation are four-component wavefunctions (called *spinors*) with the form[16]

$$\Psi(\mathbf{r},t) = \begin{pmatrix}\psi_1(\mathbf{r},t)\\ \psi_2(\mathbf{r},t)\\ \psi_3(\mathbf{r},t)\\ \psi_4(\mathbf{r},t)\end{pmatrix}. \tag{1.4}$$

The interpretation of (1.4) is that the four components describe the two spin states of a negatively charged electron with positive energy and the two spin states of a corresponding particle having the same mass, but with negative energy. Two spin states arise because in quantum mechanics the projection in any direction of the spin vector of a spin-1/2 particle can only result in one of the two values $\pm 1/2$, referred to as 'spin up' and 'spin down', respectively. The two energy solutions arise from the two solutions of the relativistic mass–energy relation $E = \pm(p^2c^2 + m^2c^4)^{1/2}$. The negative-energy states can be shown to behave in all respects as positively charged electrons called *positrons*, but with *positive* energy.[17] The positron is referred to as the *antiparticle* of the electron. The discovery of the positron by Anderson in 1933, with all the predicted properties, was a spectacular verification of Dirac's prediction, as was the much later discovery of the antiproton in 1955.[18]

Although Dirac originally made his prediction for electrons, the result holds for all charged particles and is true whether the particle is an elementary particle or a hadron. If we denote a particle by P, then the antiparticle is in general written with a bar over it, i.e. $\bar{P}$. For example,

---

[16] The details may be found in many quantum mechanics books, e.g. pp. 475–477 of Schiff (1968).

[17] See, for example, Chapter 1 of Martin and Shaw (2017).

[18] Carl Anderson shared the 1936 Nobel Prize in Physics for the discovery of the positron and Emilio Segrè and Owen Chamberlain were awarded the 1959 prize for their discovery of the antiproton.

the antiparticle of the proton p is the antiproton $\bar{p}$, with negative electric charge, and associated with every quark, q, is an antiquark, $\bar{q}$. However, for some very common particles the bar is usually omitted. Thus, for example, in the case of the positron e^+, the superscript denoting the charge makes explicit the fact that the antiparticle has the opposite electric charge to that of its associated particle. The argument does not extend to neutral particles in general and while some have distinct antiparticles, others do not. For example, the neutron has a non-zero magnetic moment, as we shall see below, and there is a distinct antiparticle, the *antineutron* $\bar{n}$, which has a magnetic moment equal in magnitude to that of the neutron, but opposite in sign. On the other hand, neither the photon γ nor the neutral pion π^0 has a distinct antiparticle.

Electric charge is just one example of a quantum number that has equal and opposite values for particles and antiparticles. We will meet others later. When brought together, particle–antiparticle pairs, each of mass m, can annihilate, releasing their combined rest energy $2mc^2$ as photons or other particles. There is a symmetry between particles and antiparticles, and it is a convention to call the electron the particle and the positron its antiparticle. This reflects the fact that normal matter contains electrons rather than positrons.

Finally, we note that among the many successful predictions of the Dirac equation is that for magnetic moments. A charged particle with spin necessarily has an intrinsic magnetic moment $\boldsymbol{\mu}$, and it can be shown from the Dirac equation that a point-like spin-1/2 particle of charge q and mass m has a magnetic moment $\boldsymbol{\mu} = (q/m)\,\mathbf{S}$, where $\mathbf{S}$ is its spin vector.[19] Magnetic moment is a vector, and the value μ tabulated is the z component of $\boldsymbol{\mu}$ when the z component of spin has is maximum value, i.e. $\mu = q\hbar/2m$. This is a test of the elementarity of a spin-1/2 particle and the measured magnetic moment of the electron is compatible with this assumption. However, the experimental values for the proton and neutron are

$$\boldsymbol{\mu}_p = 2.79e\,\mathbf{S}/m_p \quad \text{and} \quad \boldsymbol{\mu}_n - 1.91e\,\mathbf{S}/m_n,$$

which do not obey the Dirac prediction, reflecting the fact that and the proton and neutron are not point-like, elementary particles.[20]

1.3 Space-time symmetries and conservation laws

Symmetries and the invariance properties of the underlying interactions play an important role in physics. Some lead to conservation laws that are universal. Familiar examples are translational invariance, leading to

[19]There is a small correction to this predicted value, of order one part in a thousand, which we ignore in this simple account. See, for example, Section 9.6 of Mandl and Shaw (2010).

[20]The proton magnetic moment was first measured by Otto Stern in 1933 using a molecular beam method that he developed and for this he received the 1943 Nobel Prize in Physics.

JWST916-c01 JWST916-Martin December 20, 2018 13:19 Printer Name: Trim: 254mm × 203mm

the conservation of linear momentum; and rotational invariance, leading to conservation of angular momentum. The latter plays an important role in nuclear and particle physics as it leads to a scheme for the classification of states based, among other quantum numbers, on their spins. This is similar to the scheme used to classify states in atomic physics.[21] Another very important invariance that we have briefly mentioned is gauge invariance. This fundamental property of all three interactions restricts their forms in a profound way. In its simplest form, it predicts zero masses for all elementary particles. However, there are theoretical solutions to this problem whose experimental verification is described in Section 6.5.

In nuclear and particle physics we need to consider additional symmetries of the interactions and the conservation laws that follow. In the remainder of this section we discuss three space–time symmetries that we will need in later chapters – *parity*, *charge conjugation*, and *time-reversal*.

1.3.1 Parity

Parity was first introduced in the context of atomic physics by Wigner in 1927.[22] It refers to the behaviour of a state under a spatial reflection, i.e. $\mathbf{r} \to -\mathbf{r}$. If we consider a single-particle state, represented for simplicity by a nonrelativistic wavefunction $\Psi(\mathbf{r}, t)$, then under the parity operator $\hat{P}$,

$$\hat{P}\Psi(\mathbf{r}, t) \equiv P\Psi(-\mathbf{r}, t). \tag{1.5}$$

Applying the operator again gives

$$\hat{P}^2\Psi(\mathbf{r}, t) = P\hat{P}\Psi(-\mathbf{r}, t) = P^2\Psi(\mathbf{r}, t), \tag{1.6}$$

implying $P = \pm 1$. If the particle is an eigenfunction of linear momentum $\mathbf{p}$, i.e.

$$\Psi(\mathbf{r}, t) \equiv \Psi_{\mathbf{p}}(\mathbf{r}, t) = \exp[i(\mathbf{p} \cdot \mathbf{r} - Et)/\hbar], \tag{1.7}$$

then

$$\hat{P}\Psi_{\mathbf{p}}(\mathbf{r}, t) = P\Psi_{\mathbf{p}}(-\mathbf{r}, t) = P\Psi_{-\mathbf{p}}(\mathbf{r}, t) \tag{1.8}$$

and so a particle at rest, with $\mathbf{p} = \mathbf{0}$, is an eigenstate of parity. The eigenvalue $P = \pm 1$ for a particle at rest is called the *intrinsic parity*, or just the *parity*, of the particle. Parity is a multiplicative quantum number, and thus for many-particle systems the appropriate generalisation of (1.5) is

$$\hat{P}\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots, t) \equiv P_1 P_2 \cdots \Psi(-\mathbf{r}_1, -\mathbf{r}_2, \ldots, t),$$

with one intrinsic parity factor P_1, P_2, $\cdots$ occurring for each particle present.

[21] These points are explored in more detail in, for example, Chapter 5 of Martin and Shaw (2017).

[22] Eugene Wigner shared the 1963 Nobel Prize in Physics, principally for his work on symmetries.

The strong and electromagnetic interactions, but not the weak interactions, are invariant under parity, that is, the interaction Hamiltonian of the system, and hence the equation of motion, remains unchanged under a parity transformation on the position vectors of all particles in the system. Parity is therefore conserved, by which we mean that the total parity quantum number remains unchanged in the interaction. Compelling evidence for parity conservation in the strong and electromagnetic interactions comes from the suppression of transitions between nuclear states that would violate parity conservation. Such decays are not absolutely forbidden, because the Hamiltonian responsible for the transition will always have a small admixture due to the weak interactions between nucleons. However, the observed rates are extremely small compared to analogous decays that do not violate parity and are entirely consistent with the transitions being due to this very small weak interaction component. The evidence for nonconservation of parity in the weak interaction will be discussed in detail in Section 7.1.

In addition to intrinsic parity, there is a contribution to the total parity if the particle has an orbital angular momentum l. In this case its wavefunction is a product of a radial part R_{nl} and an angular part $Y_l^m(\theta, \phi)$:

$$\Psi_{lmn}(\mathbf{x}) = R_{nl} Y_l^m(\theta, \phi), \tag{1.9}$$

where n and m are the principal and magnetic quantum numbers and $Y_l^m(\theta, \phi)$ is a spherical harmonic. It is straightforward to show from the relations between Cartesian (x, y, z) and spherical polar coordinates (r, θ, ϕ), that is,

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta, \tag{1.10}$$

where the parity transformation $\mathbf{r} \to -\mathbf{r}$ implies

$$r \to r, \quad \theta \to \pi - \theta, \quad \phi \to \pi + \phi, \tag{1.11}$$

and from this it can be shown that

$$Y_l^m(\theta, \phi) \to Y_l^m(\pi - \theta, \pi + \phi) = (-)^l Y_l^m(\theta, \phi). \tag{1.12}$$

Equation (1.12) may easily be verified directly for specific cases; for example, for the first three spherical harmonics,

$$Y_0^0 = \left(\frac{1}{4\pi}\right)^{1/2}, \quad Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2}\cos\theta, \quad Y_1^{\pm1} = \left(\frac{3}{8\pi}\right)^{1/2}\sin\theta\, e^{\pm i\phi}. \tag{1.13}$$

Hence

$$\hat{P}\,\Psi_{lmn}(\mathbf{r}) = P\Psi_{lmn}(-\mathbf{r}) = P(-)^l\Psi_{lmn}(\mathbf{r}), \tag{1.14}$$

that is, $\Psi_{lmn}(\mathbf{r})$ is an eigenstate of parity with eigenvalue $P(-1)^l$.

An analysis of the Dirac equation (1.1) for relativistic electrons shows that it is invariant under a parity transformation only if $P(e^+)P(e^-) = -1$. This is a general result for all fermion–antifermion pairs, so it is a convention to assign $P = +1$ to all leptons and $P = -1$ to their antiparticles. We will see in Chapter 3, that in strong and electromagnetic interactions quarks can only be created as part of a quark–antiquark pair, so the intrinsic parity of a single quark cannot be measured. For this reason, it is also a convention to assign $P = +1$ to quarks. Since quarks are fermions, it follows from the Dirac result that $P = -1$ for antiquarks. The intrinsic parities of hadrons then follow from their structure in terms of quarks and the orbital angular momentum between the constituent quarks, using (1.14). This will be explored in Chapter 3 as part of the discussion of the quark model.

1.3.2 Charge conjugation

Charge conjugation is the operation of changing a particle into its antiparticle. Like parity, it gives rise to a multiplicative quantum number that is conserved in strong and electromagnetic interactions, but violated in the weak interaction. In strong interactions this can be tested experimentally, by, for example, measuring the rates of production of positive and negative mesons in $p\bar{p}$ annihilations, and is found to hold.

In discussing charge conjugation, we will need to distinguish between states such as the photon γ and the neutral pion π^0 that do not have distinct antiparticles and those such as the π^+ and the neutron that do. Particles in the former class we will collectively denote by a, and those of the latter type will be denoted by b. It is also convenient at this point to extend our notation for states. Thus we will represent a state of type a having a wavefunction Ψ_a by $|a, \psi_a\rangle$ and similarly for a state of type b.[23] Then under the charge conjugation operator $\hat{C}$,

$$\hat{C}|a, \Psi_a\rangle = C_a|a, \Psi_a\rangle \quad \text{and} \quad \hat{C}|b, \Psi_b\rangle = |\bar{b}, \Psi_{\bar{b}}\rangle, \tag{1.15}$$

where C_a is a phase factor analogous to the phase factor in (1.5).[24] Applying the operator twice, in the same way as for parity, leads to $C_a = \pm 1$. From the first equation in (1.15), we see that states of type a are eigenstates of $\hat{C}$ with eigenvalues ± 1, called their *C parities*. As an example, consider the π^0. This decays via the electromagnetic interaction to two photons: $\pi^0 \to \gamma\gamma$. The C parity of the photon follows directly from the invariance of Maxwell's equations under charge conjugation and is

[23] This is part of the so-called 'Dirac notation' in quantum mechanics. However, we will only need the notation and not the associated mathematics.

[24] A phase factor C_b could also have been inserted in the second equation of (1.15), but it is straightforward to show that the relative phase of the two states b and $\bar{b}$ cannot be measured, and so a phase introduced in this way would have no physical consequences. (See Problem 1.4.)

$C_\gamma = -1$[25] and hence $C_{\pi^0} = C_\gamma^2 = 1$. It follows that the decay $\pi^0 \to \gamma\gamma\gamma$ is forbidden by C invariance. The experimental limit for the ratio of rates $\pi^0 \to 3\gamma/\pi^0 \to 2\gamma$ is less than 3×10^{-8}, which is strong evidence for C invariance in electromagnetic interactions. The evidence for the violation of C invariance in the weak interaction is discussed in detail in Chapter 7.

If a state contains particles that have distinct antiparticles, it can only be an eigenstate of $\hat{C}$ if they are present as particle–antiparticle pairs. As an example of this, consider a $\pi^+\pi^-$ pair with orbital angular momentum L between them. We then have

$$\hat{C}|\pi^+\pi^-; L\rangle = (-1)^L|\pi^+\pi^-; L\rangle, \tag{1.16}$$

because interchanging the pions reverses their relative positions in the spatial wavefunction. The same factor occurs for spin-1/2 fermion pairs $f\bar{f}$, but in addition there are two other factors. The first is $(-1)^{S+1}$, where S is the total spin of the pair. This follows directly from the structure of the spin wavefunctions:

$$\left.\begin{array}{ll} \uparrow_1\uparrow_2 & S_z = 1 \\ \frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 + \downarrow_1\uparrow_2) & S_z = 0 \\ \downarrow_1\downarrow_2 & S_z = -1 \end{array}\right\} \quad S = 1 \tag{1.17a}$$

and

$$\frac{1}{\sqrt{2}}(\uparrow_1\downarrow_2 - \downarrow_1\uparrow_2) \quad S_z = 0 \qquad S = 0, \tag{1.17b}$$

where $\uparrow_i(\downarrow_i)$ represents particle i having spin 'up' ('down') in the z direction. A second factor (-1) arises whenever fermions and antifermions are interchanged. This has its origins in quantum field theory.[26] Combining these factors, finally we have

$$\hat{C}|f\bar{f}; J, L, S\rangle = (-1)^{L+S}|f\bar{f}; J, L, S\rangle, \tag{1.18}$$

for fermion–antifermion pairs having total, orbital, and spin angular momentum quantum numbers J, L, and S, respectively.

1.3.3 Time reversal

Time-reversal invariance is defined as invariance under the transformation

$$t \to t' = -t, \tag{1.19}$$

leaving all position vectors unchanged. Like parity and charge conjugation invariance, it is a symmetry of the strong and electromagnetic

[25] A proof of this is given in Section 5.4.1 of Martin and Shaw (2017). An alternative argument is that electromagnetic fields are produced by moving electric charges, which change sign under charge conjugation, and hence $C_\gamma = -1$.

[26] See, for example, pp. 249–250 of Gottfried and Weisskopf (1986).

interactions, but is violated by the weak interactions. However, unlike parity and charge conjugation, there is no associated quantum number that is conserved when weak interactions are neglected. To understand this we consider the transformation of a single-particle wavefunction, which must satisfy

$$|\Psi(\mathbf{r},t)|^2 \xrightarrow{T} |\Psi'(\mathbf{r},t)|^2 = |\Psi(\mathbf{r},-t)|^2 \tag{1.20}$$

if the system is T invariant, so that the probability of finding the particle at position $\mathbf{r}$ at time $-t$ becomes the probability of finding it at position $\mathbf{r}$ at time t in the transformed system. In addition, since in classical mechanics linear and angular momentum change sign under (1.19), we would expect the same result

$$\mathbf{p} \xrightarrow{T} \mathbf{p}' = -\mathbf{p}; \qquad \mathbf{J} \xrightarrow{T} \mathbf{J}' = -\mathbf{J} \tag{1.21}$$

to hold in quantum mechanics by the correspondence principle. Hence a free-particle wavefunction

$$\Psi_{\mathbf{p}}(\mathbf{r},t) = \exp[i(\mathbf{p}\cdot\mathbf{r} - Et)/\hbar],$$

corresponding to momentum $\mathbf{p}$ and energy $E = p^2/2m$, must transform into a wavefunction corresponding to momentum $-\mathbf{p}$ and energy E, i.e.

$$\Psi_{\mathbf{p}}(\mathbf{r},t) \xrightarrow{T} \Psi'_{\mathbf{p}}(\mathbf{r},t) = \Psi_{-\mathbf{p}}(\mathbf{r},t) = \exp[-i(\mathbf{p}\cdot\mathbf{r} + Et)/\hbar]. \tag{1.22}$$

A suitable transformation that satisfies both (1.20) and (1.22) is

$$\Psi(\mathbf{r},t) \xrightarrow{T} \Psi'(\mathbf{r},t) = \Psi^*(\mathbf{r},-t) \equiv \hat{T}\,\Psi(\mathbf{r},t), \tag{1.23}$$

where we have introduced the time reversal operator $\hat{T}$ by analogy with the parity operator $\hat{P}$ introduced in (1.5). However, quantum mechanical operators $\hat{O}$ that correspond to physical observables must be both linear

$$\hat{O}(\alpha_1\Psi_1 + \alpha_2\Psi_2) = \alpha_1(\hat{O}\,\Psi_1) + \alpha_2(\hat{O}\,\Psi_2) \tag{1.24a}$$

(to ensure that the superposition principle holds) and Hermitian

$$\int (\hat{O}\Psi_1)^*\Psi_2\,\mathrm{d}x = \int \Psi_1^*(\hat{O}\,\Psi_2)\,\mathrm{d}x \tag{1.24b}$$

(to ensure that the eigenvalues of $\hat{O}$, that is, observable quantities, are real), where $\Psi_{1,2}$ are arbitrary wavefunctions and $\alpha_{1,2}$ are arbitrary complex numbers. In contrast, the definition (1.23) implies

$$\hat{T}(\alpha_1\Psi_1 + \alpha_2\Psi_2) = \alpha_1^*(\hat{T}\,\Psi_1) + \alpha_2^*(\hat{T}\,\Psi_2) \neq \alpha_1(\hat{T}\,\Psi_1) + \alpha_2(\hat{T}\,\Psi_2)$$

for complex α_1 and α_2, and one easily verifies that (1.24b) is also not satisfied by $\hat{T}$. Thus the time reversal operator does not correspond to a physical observable and there is no observable analogous to parity that is conserved as a consequence of T invariance.

Although T invariance does not give rise to a conservation law, it does lead to a relation between any reaction and the 'time-reversed' process related to it by (1.19). Thus reactions like

$$a(\mathbf{p}_a, m_a) + b(\mathbf{p}_b, m_b) \to c(\mathbf{p}_c, m_c) + d(\mathbf{p}_d, m_d) \tag{1.25a}$$

and their time-reversed counterparts

$$c(-\mathbf{p}_c, -m_c) + d(-\mathbf{p}_d, -m_d) \to a(-\mathbf{p}_a, -m_a) + b(-\mathbf{p}_b, -m_b), \tag{1.25b}$$

in which the initial and final states are interchanged and the particle momenta ($\mathbf{p}_a$ etc.) and z components of their spins (m_a etc.) are reversed in accordance with (1.21), are related. In particular, if weak interactions are neglected, the rates for reactions (1.25a) and (1.25b) must be equal.

A more useful relation between reaction rates can be obtained if we combine time reversal with parity invariance. Under the parity transformation (1.5), momenta $\mathbf{p}$ change sign while orbital angular momenta $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ do not. If we assume the same behaviour holds for spin angular momenta, then

$$\mathbf{p} \xrightarrow{P} \mathbf{p}' = -\mathbf{p}; \qquad \mathbf{J} \xrightarrow{P} \mathbf{J}' = \mathbf{J} \tag{1.26}$$

under parity. The parity-transformed reaction corresponding to (1.25b) is

$$c(\mathbf{p}_c, -m_c) + d(\mathbf{p}_d, -m_d) \to a(\mathbf{p}_a, -m_a) + b(\mathbf{p}_b, -m_b) \tag{1.25c}$$

so that if both P and T invariance holds, all three reactions (1.25a, 1.25b, and 1.25c) must have the same rate. If we average over all spin projections

$$m_i = -S_i, -S_i + 1, \quad \ldots \quad S_i \qquad (i = a, b, c, d),$$

where S_i is the spin of particle i, then reactions (1.25a) and (1.25c) differ only by the interchange of initial and final states. Consequently, the rates for the reactions

$$i \equiv a(\mathbf{p}_a) + b(\mathbf{p}_b) \leftrightarrow c(\mathbf{p}_c) + d(\mathbf{p}_d) \equiv f \tag{1.27}$$

should be equal, provided that we average over all possible spin states. This relation is called the *principle of detailed balance* and has been accurately confirmed experimentally in a variety of strong and electromagnetic reactions.

Finally, although the weak interaction is not invariant under the above transformations, there is a general result, called the *CPT theorem*, which states that under very general conditions *any* relativistic field theory is invariant under the combined operation of CPT, taken in any order. Among other things, CPT invariance predicts that the masses and lifetimes of a particle and its antiparticle must be exactly equal. These predictions are consistent with all known data. The measured mass of the positron, for example, is equal to the mass of the electron within an experimental uncertainty of better than one part in 10^8.

1.4 Interactions and Feynman diagrams

We now turn to a discussion of particle interactions and how they can be described by the very useful pictorial methods of Feynman diagrams.

1.4.1 Interactions

Reactions involving elementary particles and/or hadrons are conveniently summarised by 'equations' in analogy to chemical reactions, in which the different particles are represented by symbols, which usually, but not always, have a superscript to denote their electric charge. For example, in the interaction

$$\nu_e + n \to e^- + p, \tag{1.28}$$

a neutrino ν_e (the subscript is explained in Section (3.1.3)) collides with a neutron n to give an electron e^- and a proton p, while the equation

$$e^- + p \to e^- + p \tag{1.29}$$

represents an electron and proton interacting to give the same particles in the final state, but in general travelling in different directions. In such equations, conserved quantum numbers must have the same total values in initial and final states.

Particles may be transferred from initial to final states and vice versa, when they become antiparticles. Thus starting from the process

$$\pi^- + p \to \pi^- + p, \tag{1.30a}$$

and taking the proton from the initial state to an antiproton in the final state and the negatively charged pion in the final state to a positively charged pion in the initial state, we obtain

$$\pi^+ + \pi^- \to p + \bar{p}. \tag{1.31}$$

It follows that if (1.30a) does not violate any relevant quantum numbers, then neither does reaction (1.31) and so is also in principle an allowed reaction. The qualification is needed because although (1.31) does not violate any quantum numbers, energy conservation leads to a minimum total energy below which it cannot proceed, because the two pions must have enough energy to create the heavier (i.e. having greater mass) $p\bar{p}$ pair.

The interactions (1.29) and (1.30a), in which the particles remain unchanged, are examples of *elastic scattering*, in contrast to reactions (1.28) and (1.31), where the final-state particles differ from those in the initial state. Collisions between a given pair of initial particles do not always lead to the same final state, but can lead to different final states with different probabilities. For example, the collision of a negatively charged pion and a proton can give rise to elastic scattering (1.30a) and a variety of other reactions, such as

$$\pi^- + p \to n + \pi^0 \quad \text{and} \quad \pi^- + p \to p + \pi^- + \pi^- + \pi^+, \tag{1.30b}$$

depending on the initial energy. In particle physics it is common to also refer (rather imprecisely) to such interactions as 'inelastic' scattering.

Similar considerations apply to nuclear physics, but the term *inelastic scattering* is reserved for the case where the final state is an excited state of the parent nucleus A, which subsequently decays, for example via photon emission, i.e.

$$a + A \to a + A^*; \qquad A^* \to A + \gamma, \tag{1.32}$$

where a is a projectile and A^* is an excited state of A. A useful shorthand notation used in nuclear physics for the general reaction $a + A \to b + B$ is $A(a, b)B$. It is usual in nuclear physics to further subdivide types of interactions according to the underlying mechanism that produced them. We will return to this in Section 2.9, as part of a more general discussion of nuclear reactions.

Finally, many particles are unstable and spontaneously decay to other, lighter (i.e. having less mass) particles. An example of this is the free neutron (i.e. one not bound in a nucleus), which decays by the β-decay reaction

$$n \to p + e^- + \bar{\nu}_e, \tag{1.33}$$

with a mean lifetime of about 900 seconds.[27] The same notation can also be used in nuclear physics. For example, many nuclei decay via the β-decay mechanism. Thus, denoting a nucleus with Z protons and N neutrons as (Z, N), we have

$$(Z, N) \to (Z - 1, N + 1) + e^+ + \nu_e. \tag{1.34}$$

This is also a weak interaction. This reaction is effectively the decay of a proton bound in a nucleus. Although a *free* proton cannot decay by the beta decay $p \to n + e^+ + \nu_e$ because it violates energy conservation (the final-state particles have greater total mass than the proton), a proton bound in a nucleus can decay because of its binding energy. The explanation for this is given in Chapter 2.

1.4.2 Feynman diagrams

The forces producing all of the above interactions are due to the exchange of particles and a convenient way of illustrating this is to use *Feynman diagrams*. There are mathematical rules (the *Feynman rules*) and techniques associated with these that enable them to be used to calculate the

[27] The reason why this decay involves an antineutrino rather than a neutrino will become clear in Chapter 3.

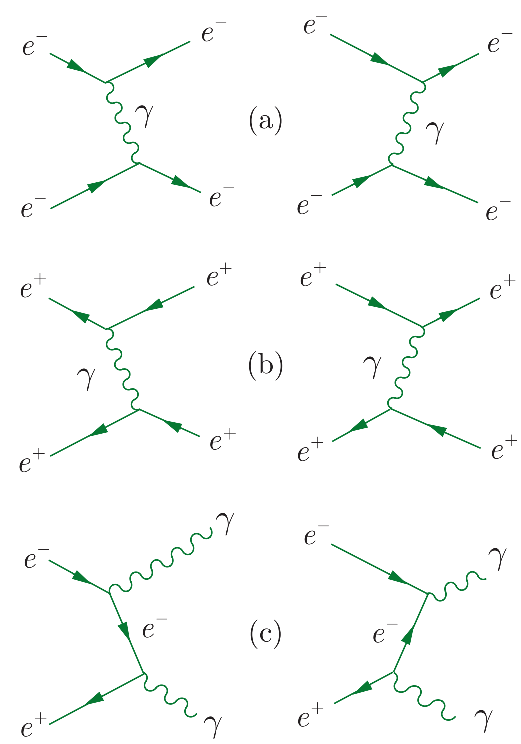

Figure 1.2 Some examples of electromagnetic processes: (a) single-photon exchange in $e^- + e^- \to e^- + e^-$; (b) single-photon exchange in $e^+ + e^+ \to e^+ + e^+$; (c) electron–positron annihilation producing two photons. Time runs from left to right and in each case the two diagrams are related by 'time ordering', as explained in the text.

quantum mechanical probabilities for given reactions to occur, but in this book Feynman diagrams will only be used as a convenient very useful pictorial description of reaction mechanisms.

We first illustrate them at the level of elementary particles for the case of electromagnetic interactions, which arise from the emission and/or absorption of photons. For example, the dominant interaction between two electrons is due to single photon exchange processes in which the photon is emitted by one of the electrons and absorbed by the other. This mechanism, which gives rise to the familiar Coulomb interaction at large distances, is illustrated in the Feynman diagrams of Figure 1.2a.

In such diagrams, we will use the convention that particles in the initial state are shown on the left and particles in the final state are shown on the right, i.e. the time axis runs from left to right. Spin-1/2 fermions (such as the electron) are drawn as solid lines and photons are drawn as wiggly lines. Arrows pointing to the right indicate that the solid lines represent electrons. In the case of photon exchange between two positrons, which is shown in Figure 1.1b, the arrows on the antiparticle (in this case the positron) lines are conventionally shown as pointing to the left. In interpreting these diagrams, it is important to remember that the direction of the arrows on fermion lines does not indicate the particle's direction of motion, but merely whether the fermions are particles or antiparticles, and that particles in the initial state are always to the left of the vertex and particles in the final state are always to the right. Pairs of electrons and positrons can also annihilate to produce photons in the final state and an example of this is shown in Figure 1.2c, which corresponds to the process $e^+ + e^- \to \gamma + \gamma$.

The two diagrams shown in Figure 1.2c are closely related. If the lines of the first diagram were made of rubber, we could imagine deforming them so that the top vertex occurred after, instead of before, the bottom vertex, and it became the second diagram. The pairs of diagrams shown in each of Figure 1.2a and b are related in the same way. For any given process, diagrams related in this way are called different 'time orderings'. In practice, it is usual to draw only one time ordering (e.g. the left-hand diagram in Figure 1.2a) leaving the other(s) implied, and we shall usually follow this practice in what follows. In contrast, for any given process, pairs of diagrams that are not related in this way are referred to as 'topologically distinct' and must both be retained.

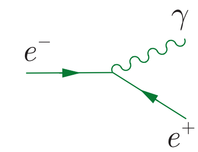

Figure 1.3 The forbidden vertex $e^- \to e^+ + \gamma$.

A feature of the above diagrams is that they are constructed from combinations of simple three-line vertices. This is characteristic of electromagnetic processes. Each vertex has a line corresponding to a single photon being emitted or absorbed, while one fermion line has the arrow pointing towards the vertex and the other away from the vertex, guaranteeing charge conservation at the vertex, which is one of the rules of Feynman diagrams.[28] For example, a vertex like Figure 1.3 would correspond to a process in which an electron emitted a photon and turned into a positron. This would violate charge conservation and is therefore forbidden.

Feynman diagrams can also be used to describe the fundamental weak and strong interactions which, as mentioned in Section 1.1.3, are mediated by the massive W^+, W^- and Z^0 bosons and the massless gluons, respectively. This is illustrated by Figure 1.4a and b, which show contributions to the elastic weak scattering reaction $e^- + \nu_e \to e^- + \nu_e$ due to the exchange of single Z^0 and W bosons, and by Figure 1.4c, which shows the exchange of a gluon g (represented by a coiled line) between two quarks q, which is a strong interaction.

Feynman diagrams that involve hadrons can also be drawn. As illustrations, Figure 1.5a shows the decay of a neutron via an intermediate W boson and Figure 1.5b and c denote the exchange of a neutral and a charged pion, respectively (shown as a dashed line), between a proton and a neutron. Pion exchange diagrams of this type form the basis of Yukawa's theory of nuclear forces mentioned earlier. He was then able to predict the pion mass by using a fundamental relation between the mass of the exchanged particle and the range of the resulting force discussed in the following two subsections.

We turn now to consider in more detail the relation between exchanged particles and forces.

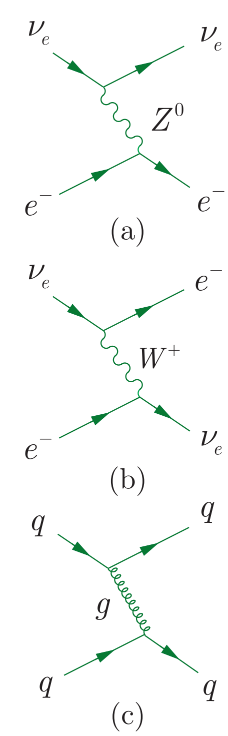

Figure 1.4 Contributions to the elastic weak scattering reaction $e^- + \nu_e \to e^- + \nu_e$ by (a) the exchange of a Z^0 boson and (b) the exchange of a W boson; (c) gluon exchange contribution to the strong interaction $q + q \to q + q$.

[28] Compare Kirchhoff's laws in electromagnetism.

1.5 Particle exchange: forces and potentials

This section starts with a discussion of the important relationship between forces and particle exchanges and then relates this to potentials. Although the idea of a potential has its greatest use in nonrelativistic physics, nevertheless it is useful to illustrate concepts and is used in later sections as an intermediate step in relating theoretical Feynman diagrams to measurable quantities. The results can be extended to more general situations.

1.5.1 Range of forces

At each vertex of a Feynman diagram, charge is conserved by construction. We will see later that depending on the nature of the interaction (strong, weak or electromagnetic), other quantum numbers are also conserved. However, it is easy to show that energy and momentum cannot in general be conserved simultaneously.

Consider the general case of a reaction $A+B\to A+B$ mediated by the exchange of a particle X, as shown in Figure 1.6. In the rest frame of the incident particle A, the lower vertex represents the *virtual* process ('virtual' because X does not appear as a real particle in the final state),

$$A(M_A c^2, \mathbf{0}) \to A(E_A, \mathbf{p}_A c) + X\,(E_X, -\mathbf{p}_A c)\,, \tag{1.35}$$

where E_A is the *total* energy of the final particle A and $\mathbf{p}_A$ is its 3-momentum.[29] Thus, if we denote by P_A the 4-momentum for particle A,

$$P_A = (E_A/c, \mathbf{p}_A) \tag{1.36}$$

and

$$P_A{}^2 = E_A{}^2/c^2 - \mathbf{p}_A{}^2 = M_A{}^2c^2. \tag{1.37}$$

Applying this to the diagram and imposing momentum conservation gives

$$E_A = \left(p^2c^2 + M_A{}^2c^4\right)^{1/2} \text{ and } E_X = \left(p^2c^2 + M_X{}^2c^4\right)^{1/2}, \tag{1.38}$$

where $p = |\mathbf{p}_A|$. The energy difference between the final and initial states is given by

$$\begin{aligned} \Delta E = E_X + E_A - M_A c^2 &\to 2pc, \qquad p\to\infty \\ &\to M_X c^2, \quad p \to 0 \end{aligned} \tag{1.39}$$

and thus $\Delta E \geq M_X c^2$ for all p, i.e. energy is not conserved. However, by the energy–time uncertainty principle, such an energy violation is allowed, but only for a time $\tau \approx \hbar/\Delta E$, so we immediately obtain

$$R \equiv \hbar/M_X c \tag{1.40}$$

as the approximate value of the maximum distance over which X can propagate before being absorbed by particle B. The distance R is called

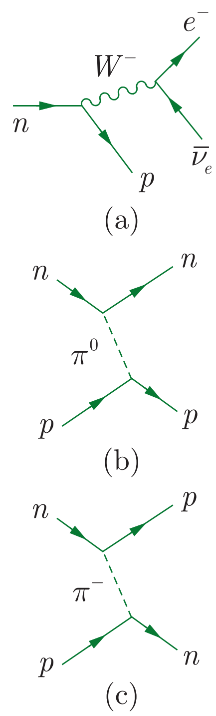

Figure 1.5 (a) The decay of a neutron via an intermediate W boson; (b) single π^0 exchange in the reaction $p+n\to p+n$; (c) single π^- exchange in the reaction $p+n\to n+p$.

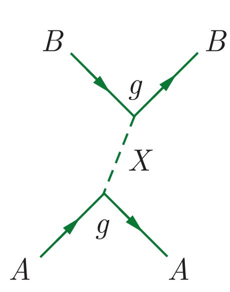

Figure 1.6 Exchange of a particle X in the reaction $A+B\to A+B$.

[29] A résumé of relativistic kinematics is given in Appendix B.

the *range* of the interaction, which was the sense of the word used in Section 1.1.1.

The electromagnetic interaction has an infinite range because the exchanged particle is a massless photon. In contrast, the weak interaction is associated with the exchange of very heavy particles – the W and Z bosons. These lead to ranges that from (1.40) are approximately $R_{W,\,Z} \approx 2 \times 10^{-18}$ m. The fundamental strong interaction between quarks has infinite range because, like the photon, gluons have zero mass. On the other hand, the strong nuclear force has a much shorter range of approximately $(1-2) \times 10^{-15}$ m, corresponding to the exchange of pions with the mass predicted by Yukawa, as illustrated in Figure 1.5a and b. We will return briefly to the relation between these two different manifestations of the strong interaction in Section 8.1.

1.5.2 The Yukawa potential

In the limit that M_A becomes large, we can regard B as being scattered by a static potential of which A is the source. This potential will in general be spin dependent, but its main features can be obtained by neglecting spin and considering X to be a spin-0 boson, in which case it will obey the Klein–Gordon equation,

$$-\hbar^2 \frac{\partial^2 \phi(\mathbf{r}, t)}{\partial t^2} = -\hbar^2 c^2 \nabla^2 \phi(\mathbf{r}, t) + {M_X}^2 c^4 \phi(\mathbf{r}, t). \tag{1.41}$$

The static solution of this equation satisfies

$$\nabla^2 \phi(\mathbf{r}) = \frac{{M_X}^2 c^2}{\hbar^2} \phi(\mathbf{r}), \tag{1.42}$$

where $\phi(\mathbf{r})$ is interpreted as a static potential. For $M_X = 0$ this equation is the same as that obeyed by the electrostatic potential, and for a point charge $-e$ interacting with a point charge $+e$ at the origin, the appropriate solution is the Coulomb potential

$$V(r) = -e\,\phi(r) = -\frac{e^2}{4\pi\varepsilon_0} \frac{1}{r}, \tag{1.43}$$

where $r = |\mathbf{r}|$ and ϵ_0 is the dielectric constant. The corresponding solution in the case where ${M_X}^2 \neq 0$ is easily verified by substitution to be

$$V(r) = -\frac{g^2}{4\pi} \frac{e^{-r/R}}{r}, \tag{1.44}$$

where R is the range defined earlier and g, the so-called *coupling constant,* is a parameter associated with each vertex of a Feynman diagram and represents the basic strength of the interaction. For simplicity, we have assumed equal strengths for the coupling of particle X to the particles A and B.

The form of $V(r)$ in (1.44) is called a *Yukawa potential*, after the physicist who in 1935 first introduced the idea of forces due to the exchange of massive particles. As $M_X \to 0$, $R \to \infty$ and the Coulomb potential is recovered from the Yukawa potential, while for very large masses the interaction is approximately point-like (zero-range). It is conventional to introduce a dimensionless parameter α_X by

$$\alpha_X = g^2/4\pi\hbar c, \tag{1.45}$$

which characterises the strength of the interaction. For the electromagnetic interaction this is denoted

$$\alpha \equiv e^2/4\pi\varepsilon_0\hbar c \approx 1/137 \tag{1.46}$$

and is called the *fine structure constant* because it determines the magnitude of the fine structure seen in atomic spectral lines.

In deriving the above potential we implicitly assumed point-like particles, so that (1.41) can be used for all $r \neq 0$. For composite particles, at distances smaller or of the same order as the size of the particles, it breaks down and the interaction must be modified to take account of the particle's size and structure. This is important for the strong forces between hadrons, whose range is not much bigger than the size of the hadrons themselves, so that the strong nuclear force, for example, is only dominated by pion exchange over a limited range and is significantly modified at distances less than 10^{-15}m. The strong nuclear interaction is actually a complicated effect that has its origins in the fundamental strong interactions between the quark distributions within the two hadrons. Similarly, two neutral atoms also experience an electromagnetic interaction (the van der Waals force), which has its origins in the fundamental Coulomb forces, but is of a much shorter range. Although an analogous mechanism is not in fact responsible for the nuclear strong interaction, it is a useful reminder that the force between two *distributions* of particles can be much more complicated than the forces between the individual components. We will return to this point when we discuss the nature of the nuclear potential in more detail in Section 8.1.

1.6 Observable quantities: cross-sections and decay rates

We have mentioned earlier that Feynman diagrams can be translated into probabilities for a process by using a set of mathematical rules (the *Feynman Rules*) that can be derived from the quantum theory of the underlying interaction. In the case of the electromagnetic interaction, the theory is called Quantum Electrodynamics (QED) and is spectacularly successful in explaining experimental results.[30] We will not pursue this

[30]Richard Feynman, Sin-Itiro Tomonoga, and Julian Schwinger shared the 1965 Nobel Prize in Physics for their work on formulating quantum electrodynamics. The Feynman rules are introduced in an accessible way in Griffiths (1987) or more rigorously in Mandl and Shaw (2010).

in detail in this book, but rather will show in principle their relation to *observables*, i.e. quantities that can be measured, concentrating on the cases of two-body scattering reactions and decays of unstable states.

1.6.1 Amplitudes

The intermediate step is the *amplitude* $\mathcal{M}$, the modulus squared of which is directly related to the probability of the process occurring. To get a qualitative idea of the structure of $\mathcal{M}$, we will use nonrelativistic quantum mechanics and assume that the coupling constant g^2 is small compared to $4\pi\hbar c$, so that the interaction is a small perturbation on the free particle solution, which will be taken as plane waves.

In lowest-order perturbation theory, the probability amplitude for a particle with initial momentum $\mathbf{q}_i$ to be scattered to a final state with momentum $\mathbf{q}_f$ by a potential $V(\mathbf{r})$ is proportional to[31]

$$\mathcal{M}(\mathbf{q}) = \int V(\mathbf{r}) \exp\left(i\mathbf{q}\cdot\mathbf{r}/\hbar\right) \mathrm{d}^3\mathbf{r}, \tag{1.47}$$

where $\mathbf{q} \equiv \mathbf{q}_i - \mathbf{q}_f$ is the momentum transfer. The integration may be done using polar coordinates. Taking $\mathbf{q}$ in the z direction gives

$$\mathbf{q}\cdot\mathbf{r} = |\mathbf{q}|\, r\cos\theta \tag{1.48}$$

and

$$\mathrm{d}^3\mathbf{r} = r^2 \sin\theta \,\mathrm{d}\theta \,\mathrm{d}r \,\mathrm{d}\phi, \tag{1.49}$$

where $r \equiv |\mathbf{r}|$. For the Yukawa potential, the integral (1.47) gives

$$\mathcal{M}(\mathbf{q}^2) = \frac{-g^2\hbar^2}{|\mathbf{q}|^2 + M_X{}^2c^2}. \tag{1.50}$$

In deriving (1.50) for the scattering amplitude, we have used potential theory, treating the particle A as a static source. The particle B then scatters through some angle without loss of energy, so that $|\mathbf{q}_i| = |\mathbf{q}_f|$ and the initial and final energies of particle B are equal, $E_i = E_f$. While this is a good approximation at low energies, at higher energies the recoil energy of the target particle cannot be neglected, so that the initial and final energies of B are no longer equal. A full relativistic calculation taking account of this is beyond the scope of this book, but the result is surprisingly simple. Specifically, in lowest-order perturbation theory, one obtains

$$\mathcal{M}(q^2) = \frac{g^2\hbar^2}{q^2 - M_X{}^2c^2}, \tag{1.51}$$

[31] This is called the Born approximation. For a discussion, see, for example, Section 10.2.2 of Mandl (1992) or pp. 397–399 of Gasiorowicz (1974).

where

$$q^2 \equiv (E_f - E_i)^2/c^2 - (\mathbf{q}_f - \mathbf{q}_i)^2 \tag{1.52}$$

is the squared four-momentum transfer. In the low-energy limit, $E_i = E_f$ and (1.51) reduces to (1.50). However, in contrast to (1.50), which was derived in the rest frame of particle A, the form (1.51) is explicitly Lorentz invariant and holds in all inertial frames of reference. It is thus also called the *invariant amplitude*.

In the zero-range approximation, (1.51) reduces to a constant. To see this, we note that this approximation is valid when the range $R = \hbar/M_X c$ is very small compared to the de Broglie wavelengths of all the particles involved. In particular, this implies $q^2 \ll M_X^2 c^2$, and neglecting q^2 in (1.51) gives

$$\mathcal{M}(q^2) = -G, \tag{1.53a}$$

where the constant G is given by

$$\frac{G}{(\hbar c)^3} = \frac{1}{\hbar c}\left(\frac{g}{M_X c^2}\right)^2 = \frac{4\pi\alpha_X}{(M_X c^2)^2} \tag{1.53b}$$

and the right-hand side has the dimensions of inverse energy squared. Thus we see that in the zero-range approximation, the resulting point interaction between A and B is characterised by a single dimensioned coupling constant G and not g and M_X separately.

In the above discussion, we have assumed for simplicity that the exchanged particle is a spin-zero meson. However, by far the most important application of the zero-range approximation is to the weak interaction, in which the exchanged particle is a heavy spin-1 boson. In particular, most observed low-energy weak processes, including β - decay, are dominated by W-exchange; if the spin structure of the interaction is taken into account, the effective low-energy coupling corresponding to (1.53b) is called the *Fermi coupling constant*, and is given by

$$\frac{G_F}{(\hbar c)^3} = \frac{\sqrt{2}}{\hbar c}\left(\frac{g_W}{M_W c^2}\right)^2 = \frac{4\sqrt{2}\pi\alpha_W}{(M_W c^2)^2}, \tag{1.54}$$

where g_W is the weak coupling constant, $\alpha_W = g_W^2/4\pi\hbar c$ and $M_W \approx 80\,\text{GeV}/c^2$ is the mass of the W-boson. As we shall see, this approximation is extremely useful in weak interactions, and the Fermi coupling constant, measured in various processes, is given by

$$G_F/(\hbar c)^3 = 1.166 \times 10^{-5}\ \text{GeV}^{-2}. \tag{1.55}$$

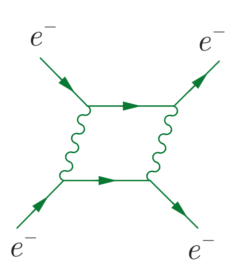

Figure 1.7 Two-photon exchange in the reaction $e^- + e^- \to e^- + e^-$.

The amplitude (1.50) corresponds to the exchange of a single particle, as shown, for example, in Figure 1.6. It is also possible to draw more complicated Feynman diagrams that correspond to the exchange of more than one particle. An example of such a diagram for elastic e^-e^- scattering, where two photons are exchanged, is shown in Figure 1.7. Multiparticle exchange corresponds to higher orders in perturbation theory and higher powers of the appropriate coupling constant g^2.

The number of vertices in any diagram is called the *order n*, and for electromagnetic processes, when the amplitude associated with any given Feynman diagram is calculated, it always contains a factor of $\alpha^{n/2}$. Since the probability is proportional to the square of the modulus of the amplitude, the former will contain a factor α^n. The probability associated with the single-photon exchange diagrams of Figure 1.2 thus contain a factor of α^2 and the contribution from two-photon exchange is of order α^4. As $\alpha \approx 1/137$, the latter is usually very small compared to the contribution from a single-photon exchange. This is a general feature of electromagnetic interactions: because the fine structure constant is very small, in many cases only the lowest-order diagrams that contribute to a given process need be taken into account, and more complicated higher-order diagrams with more vertices can to a good approximation be ignored.

1.6.2 Cross-sections

For scattering reactions, the simplest observable is the *cross-section*. In *fixed-target experiments*, a beam of mono-energetic particles is directed on to a stationary target and the rates of production of various particles are measured. It is clear that these rates will be proportional to: (i) the number N of particles in the target illuminated by the beam and (ii) the rate per unit area at which beam particles cross a small surface placed in the beam at rest with respect to the target and perpendicular to the beam direction. This rate is called the *flux* and is given by

$$J = n_b \upsilon_i, \tag{1.56}$$

where n_b is the number density of particles in the beam and υ_i is the magnitude of their velocity in the rest frame of the target. Hence the rate W_r at which a specific reaction r occurs in a particular experiment can be written in the form

$$W_r = JN\sigma_r, \tag{1.57a}$$

where σ_r, the constant of proportionality, is called the *cross-section* for reaction r. If the beam has a cross-sectional area S, its intensity is $I = JS$ and so an alternative expression for the rate is

$$W_r = N\sigma_r I/S = I\sigma_r n_t t, \tag{1.57b}$$

where n_t is the number of target particles per unit volume and t is the thickness of the target. If the target consists of an isotopic species of atomic mass M_A in atomic mass units (these are defined in Section 1.7 below), then $n_t = \rho N_A/M_A$, where ρ is the density of the target and N_A is Avogadro's constant. Thus, (1.57b) may be written

$$W_r = I\sigma_r\,(\rho t)\,N_A/M_A, \tag{1.57c}$$

where (ρt) is a measure of the amount of material in the target, expressed in units of mass per unit area. The form (1.57c) is particularly useful for the case of thin targets commonly used in experiments (such as those of Rutherford and his collaborators) to reduce the probability of multiple scattering.

The basic equation (1.57a) is frequently written in the form

$$W_r = L\sigma_r, \tag{1.58a}$$

where the product

$$L \equiv J\,N \tag{1.58b}$$

is called the *luminosity*. The luminosity has dimensions $[\text{length}]^{-2}[\text{time}]^{-1}$ and contains all the dependencies on the densities and geometries of the beam and target. The cross-section is independent of these factors, but characteristic of the particular reaction r. It follows from (1.58a) that σ_r has the dimensions of an area and the rate per target particle $J\sigma_r$ at which the reaction occurs is equal to the rate at which beam particles would hit a surface of area σ_r placed in the beam at rest with respect to the target and perpendicular to the beam direction. Since the area of such a surface is unchanged by a Lorentz transformation in the beam direction, the cross-section is the same in all reference frames related by such a transformation. In particular, the cross-sections in the *laboratory frame*, in which the target particles are at rest, and the *centre-of-mass frame*, in which the colliding particles have equal but opposite momenta, are identical.

In scattering experiments, the target is not always stationary and in the case of a *colliding beam experiment*, the 'target' is itself another beam. Equations (1.58a) and (1.58b) still hold, but it is more convenient to express the luminosity in a different form. For example, consider an experiment with beams of particles of types 1 and 2, which may or may not be identical, travelling in opposite directions and made to collide at a given point. The beams are not usually continuous, but composed of bunches, and we define N_1 and N_2 to be the numbers of particles per bunch in the two beams and f to be the frequency of collisions between bunches. Since the beams do not have sharp cutoffs at their edges, we will assume Gaussian shaped beams with transverse dimensions s_x and s_y, which yields a cross-sectional area $A = 4\pi s_x s_y$. In one collision between bunches, a particle in beam 1 crosses N_2/A particles in beam 2, regarded as the target, and because it is in a bunch of N_1 particles, the luminosity is

$$L = fN_1N_2/A. \tag{1.58c}$$

A related quantity that is also used is the *integrated luminosity*, defined as the integral of L over the time during which the experiment is performed. Hence the product of the integrated luminosity and the cross-section σ_r gives the total number of events corresponding to the reaction

r that would be observed in the experiment, assuming 100% detection efficiency.

The quantity σ_r is better named the *partial cross-section*, because it is the cross-section for a particular reaction r. Other types of cross-section are also of interest. One is the *total cross-section* σ_{tot}, defined by

$$\sigma_{\text{tot}} \equiv \sum_r \sigma_r, \tag{1.59}$$

where the summation is over all allowed reactions. Another is of particular interest for describing the angular distributions observed in reactions with only two particles in the final state, like the elastic scattering processes (1.29) and (1.30a). In such reactions, if one particle, referred to as the scattered particle, is emitted in a particular direction, then the direction of the other particle is determined by energy–momentum conservation (see Problem 1.14), so that it is sufficient to describe the angular distribution of the scattered particle alone. This is done by specifying another useful quantity, the *differential cross-section* $\mathrm{d}\sigma_r(\theta, \phi)/\mathrm{d}\Omega$, defined by

$$\mathrm{d}W_r \equiv JN\frac{\mathrm{d}\sigma_r(\theta,\phi)}{\mathrm{d}\Omega}\mathrm{d}\Omega, \tag{1.60}$$

where $\mathrm{d}W_r$ is the measured rate for the scattered particles to be emitted into an element of solid angle $\mathrm{d}\Omega = \mathrm{d}\cos\theta\,\mathrm{d}\phi$ in the direction (θ, ϕ), as shown in Figure 1.8. The partial cross-section σ_r is obtained on integrating the differential cross-section over all angles, i.e.

$$\sigma_r = \int_0^{2\pi} \mathrm{d}\phi \int_{-1}^{1} \frac{\mathrm{d}\sigma_r(\theta,\phi)}{\mathrm{d}\Omega}\,\mathrm{d}\cos\theta. \tag{1.61}$$

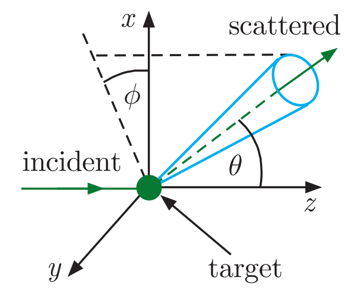

Figure 1.8 Geometry of the differential cross-section. A beam of particles, shown in green, is incident along the z axis and collides with a stationary target at the origin. The differential cross-section is proportional to the rate for particles to be scattered into a small solid angle $\mathrm{d}\Omega$ in the direction (θ, ϕ) defining the cone shown in blue.

1.6.3 The basic scattering formulas

The next step is to write the cross-sections in terms of the scattering amplitude $\mathcal{M}(\mathbf{q}^2)$ appropriate for describing the scattering of a nonrelativistic spinless particle from a potential. To do this it is convenient to consider a single beam particle interacting with a single target particle and to confine the whole system in an arbitrary large volume V (which cancels in the final result). The incident flux is then given by

$$J = n_b \upsilon_i = \upsilon_i / V \tag{1.62}$$

and since the number of target particles is $N = 1$, the differential rate is

$$\mathrm{d}W_r = \frac{\upsilon_i}{V}\frac{\mathrm{d}\sigma_r(\theta,\phi)}{\mathrm{d}\Omega}\mathrm{d}\Omega. \tag{1.63}$$

In quantum mechanics, provided the interaction is not too strong, the transition rate for any process is given in perturbation theory by the Born approximation[32]

$$\mathrm{d}W_r = \frac{2\pi}{\hbar}\left|\int \psi_r^* V(\mathbf{r})\psi_{\mathbf{i}}\mathrm{d}^3\mathbf{r}\right|^2 \rho\left(E_f\right). \tag{1.64}$$

The term $\rho(E_f)$ is the *density-of-states factor* (see below) and we take the initial and final state wavefunctions to be plane waves:

$$\psi_i = \frac{1}{\sqrt{V}}\exp(i\mathbf{q_i}\cdot\mathbf{r}/\hbar), \qquad \psi_f = \frac{1}{\sqrt{V}}\exp(i\mathbf{q_f}\cdot\mathbf{r}/\hbar), \tag{1.65}$$

where the final momentum $\mathbf{q}_f$ lies within a small solid angle $\mathrm{d}\Omega$ located in the direction (θ, ϕ) (see Figure 1.8). Then, by direct integration,

$$\mathrm{d}W_r = \frac{2\pi}{\hbar V^2}|\mathcal{M}(\mathbf{q}^2)|^2\rho(E_f), \tag{1.66}$$

where $\mathcal{M}(\mathbf{q}^2)$ is the scattering amplitude defined in (1.47).

The density of states $\rho(E_f)$ that appears in (1.64) is defined so that the number of possible final states with energy lying between E_f and $E_f + \mathrm{d}E_f$ is $\rho(E_f)\mathrm{d}E_f$. It is given by[33]

$$\rho(E_f) = \frac{V}{(2\pi\hbar)^3}{q_f}^2\frac{\mathrm{d}q_f}{\mathrm{d}E_f}\mathrm{d}\Omega, \tag{1.67}$$

where, nonrelativistically,

$$\mathrm{d}q_f/\mathrm{d}E_f = 1/\upsilon_f. \tag{1.68}$$

If we use (1.66), (1.67) and (1.68) in (1.63), we have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{4\pi^2\hbar^4}\frac{{q_f}^2}{\upsilon_i\upsilon_f}|\mathcal{M}(\mathbf{q}^2)|^2. \tag{1.69}$$

Although this result has been derived in the laboratory system, it is also valid in the centre-of-mass system.

The only place where nonrelativistic kinematics have been explicitly used in obtaining (1.69) is in the derivation of the density-of-states factor, so to have a formula that is also true for the general two-body relativistic scattering process $a + b \to c + d$, we have to re-examine the derivative (1.68) using relativistic kinematics. In this case we can use

$$E_f = E_c + E_d = \left(q_f^2c^2 + m_c^2c^4\right)^{1/2} + \left(q_f^2c^2 + m_d^2c^4\right)^{1/2} \tag{1.70}$$

to give

$$\frac{\mathrm{d}E_f}{\mathrm{d}q_f} = q_fc^2\left(\frac{1}{E_c} + \frac{1}{E_d}\right), \tag{1.71}$$

[32] This equation is a form of the *Second Golden Rule* in quantum mechanics. It is discussed in Section A.3.

[33] The derivation is given in detail in Section A.2.

which, using the relativistic relation $\mathbf{v} = \mathbf{p}c^2/E$ (see Eq. (B.9) of Appendix B) and noting that in the centre-of-mass system $\mathbf{p}_c = -\mathbf{p}_d$, yields

$$\frac{\mathrm{d}q_f}{\mathrm{d}E_f} = \frac{1}{\upsilon_f}, \tag{1.72}$$

where υ_f is the modulus of the relative velocity of particles c and d. Thus the general interpretation of (1.69) is that $q_f = |\mathbf{q}_c| = |\mathbf{q}_d|$ is the centre-of-mass momentum of the final-state particles and $\upsilon_{i,f}$ are the relative velocities in the centre-of-mass of particles a and b, and c and d, respectively.

All the above is for spinless particles, so finally we have to generalise (1.69) to include the effects of spin. Suppose the initial-state particles a and b have spins S_a and S_b and the final-state particles c and d have spins S_c and S_d. The total numbers of spin substates available to the initial and final states are g_i and g_f, respectively, given by

$$g_i = (2S_a + 1)(2S_b + 1) \quad \text{and} \quad g_f = (2S_c + 1)(2S_d + 1). \tag{1.73}$$

If the initial particles are unpolarised (which is the most common case in practice), then we must average over all possible initial spin configurations (because each is equally likely) and sum over the final configurations. Thus, (1.69) becomes

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{g_f}{4\pi^2\hbar^4}\frac{{q_f}^2}{\upsilon_i\upsilon_f}\left|\mathcal{M}_{fi}\right|^2, \tag{1.74}$$

where

$$\left|\mathcal{M}_{fi}\right|^2 \equiv \overline{|\mathcal{M}(\mathbf{q}^2)|^2} \tag{1.75}$$

and the bar over the amplitude denotes a spin-average of the squared matrix element.

1.6.4 Unstable states

In the case of an unstable state, the observable of interest is its *lifetime at rest* τ. However, decays are quantum mechanical processes, governed by statistics, so that individual particles of a given type do not always have the same lifetime. In particle physics, it is usual to specify the *mean lifetime*, averaged over a large number of observed decays, whereas in nuclear physics it is usual to specify the *half-life* $t_{1/2}$, defined as the time for half the nuclei in a sample containing many nuclei to decay. In either case, the fact that these are independent of the number N of nuclei or particles in the sample implies that that the average decay rate is proportional to N, i.e. the *activity*

$$\mathcal{A} \equiv -\mathrm{d}N/\mathrm{d}t = \lambda N, \tag{1.76}$$

where λ is called the *decay constant*. Integrating this equation leads immediately to the *exponential decay law*

$$N(t) = N_0 \exp(-\lambda t), \tag{1.77}$$

where N_0 is the initial number of nuclei, i.e. the number at $t = 0$. The mean lifetime is then given by

$$\tau \equiv \frac{\int tN(t)\mathrm{d}t}{\int N(t)\mathrm{d}t} = \frac{\int_0^\infty t\exp(-\lambda t)\mathrm{d}t}{\int_0^\infty \exp(-\lambda t)\mathrm{d}t} = \frac{1}{\lambda} \qquad (1.78)$$

The half-life also follows directly from (1.77) and is given by

$$t_{1/2} = \ln 2/\lambda = \tau \ln 2. \qquad (1.79)$$

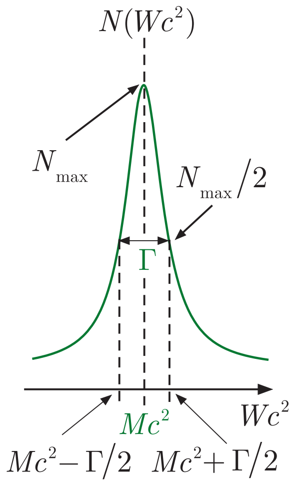

Figure 1.9 The Breit–Wigner formula (1.82).

In this book, the term *lifetime* will stand for the mean lifetime in the rest frame of the decaying particle, both for radioactive nuclei and unstable hadrons, unless explicitly stated otherwise. An equivalent quantity is the *natural decay width*, given by $\Gamma = \hbar/\tau$, which is also a measure of the rate of the decay reaction. In general, an initial unstable state will decay to several final states and in this case we define Γ_f as the *partial width* for a specific final state f and

$$\Gamma = \sum_f \Gamma_f \qquad (1.80)$$

as the *total decay width*, while

$$B_f \equiv \Gamma_f/\Gamma \qquad (1.81)$$

is defined as the *branching ratio* for decay to the state f.

The energy distribution of an isolated unstable state decaying to a final state f has the *Breit–Wigner* form

$$N_f(W) \propto \frac{\Gamma_f}{(W-M)^2c^4 + \Gamma^2/4}, \qquad (1.82)$$

where M is the mass of the decaying state and $N_f(W)\mathrm{d}W$ is the number of events in which the invariant mass of the decay products lies between W and $W + \mathrm{d}W$.[34] The Breit–Wigner formula is shown in Figure 1.9 and is the same formula that describes the widths of atomic and nuclear spectral lines. It is a symmetrical bell-shaped curve with a maximum at $W = M$ and a full width Γ at half the maximum height of the curve, whose height is proportional to the number of events with invariant mass W.

If an unstable state is produced in a scattering reaction by the mechanism $1 + 2 \to R \to f$ illustrated in Figure 1.10, then the scattering cross-section for that reaction will show an enhancement described by the same Breit–Wigner formula. In this case, the unstable state is referred to as a *resonance* and the mechanism of Figure 1.10 is called a *resonance*

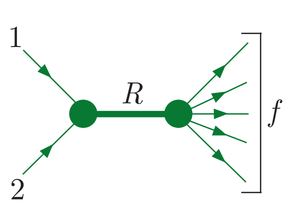

Figure 1.10 Formation and decay of a resonance R in the reaction $1 + 2 \to f$.

[34] Proofs of the various Breit–Wigner formulas quoted in this section are quite lengthy. See, for example, Appendix B of Martin and Shaw (2017).

JWST916-c01 JWST916-Martin December 20, 2018 13:19 Printer Name: Trim: 254mm × 203mm

formation. In the vicinity of a resonance of mass M and width Γ, the cross-section for the reaction $i \to f$ has the form

$$\sigma_{fi} \propto \frac{\Gamma_i \Gamma_f}{\left(E - Mc^2\right)^2 + \Gamma^2/4}, \tag{1.83a}$$

where $E = Wc^2$ is the total energy of the system in the centre-of-mass frame. In this case the overall constant can be calculated and if the resonance particle has spin j and the spins of the initial particles are S_1 and S_2, then

$$\sigma_{fi} = \frac{\pi\hbar^2}{q_i^{\,2}} \frac{2j+1}{(2S_1+1)(2S_2+1)} \frac{\Gamma_i \Gamma_f}{\left(E - Mc^2\right)^2 + \Gamma^2/4}, \tag{1.83b}$$

where q_i is the magnitude of the initial particle momenta in the centre-of-mass frame.

Finally, in the common situation where $\Gamma \ll Mc^2$, the *narrow width approximation* is often used to simplify calculations. There are different forms of this, but one is to make the replacement

$$\frac{1}{\left(E - Mc^2\right)^2 + \Gamma^2/4} \to \frac{2\pi}{\Gamma}\delta(E - Mc^2) \tag{1.84a}$$

in (1.83a), where δ is the Dirac delta function,[35] and the constant $2\pi/\Gamma$ multiplying it is chosen to ensure that both sides of (1.84a) give the same result, to a very good approximation, when integrated over all energies. Alternatively, if we instead express the Breit–Wigner formula as a function of E^2 rather than E, (1.84a) becomes

$$\frac{4\left(Mc^2\right)^2}{\left(E^2 - \left(Mc^2\right)^2\right)^2 + \left(Mc^2\right)^2\Gamma^2} \to \frac{4\pi\left(Mc^2\right)}{\Gamma}\delta\left(E^2 - \left(Mc^2\right)^2\right), \tag{1.84b}$$

where we have used the standard result

$$|a|\delta[a(x-a)] = \delta(x-a) \tag{1.85}$$

for any constant a to write $\delta(E - Mc^2)$ in terms of E^2.

The above formulas assume one is dealing with an isolated resonance. In practice, there may be several overlapping resonances and nonresonant processes contributing to the same reaction that must be taken into account. An example of resonance formation in $\pi^- p$ interactions is given in Figure 1.11, which shows the $\pi^- p$ total cross-section in the centre-of-mass energy range (1.4–2.4 GeV). (The units used in the plots will become clear after the next section.) Two enhancements can be seen that

[35] The delta function is defined by the conditions $\int f(x)\delta(x-a)\mathrm{d}x \equiv f(a)$, where a is a real constant, implying $\delta(x-a) = 0$ if $x \neq a$, and $\int_b^c f(x)\delta(x-a)\mathrm{d}x \equiv 1,\ b < a < c$. It is discussed, for example, in Section 13.3.2 of Martin and Shaw (2015), where derivations of its important properties, including (1.85), may be found.

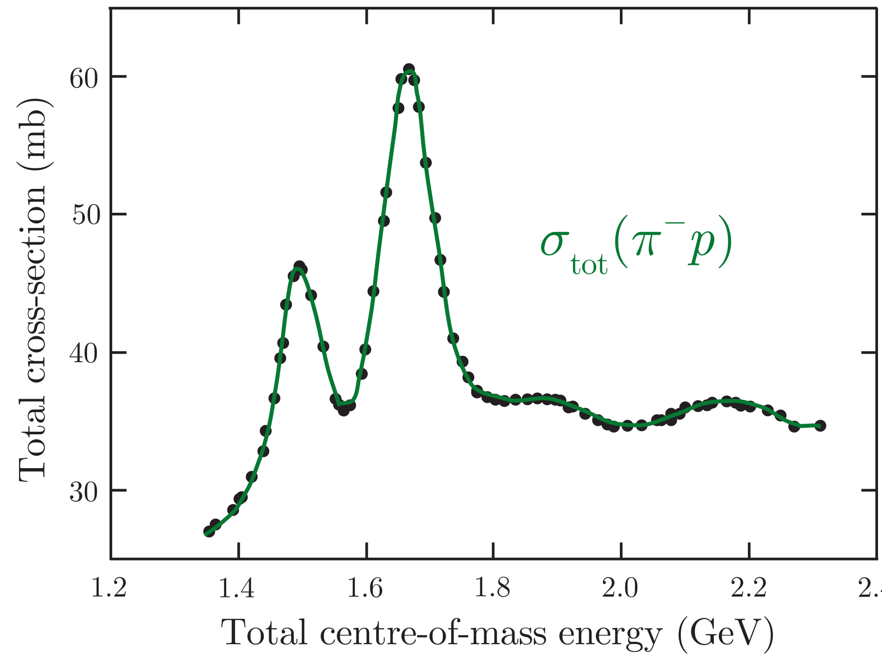

Figure 1.11 Total cross-sections for $\pi^- p$ interactions. Source: Data from Carter *et al.* (1968).

are of the approximate Breit–Wigner resonance form and there are two other maxima at higher energies. In principle, the mass and width of a resonance may be obtained by using a Breit–Wigner formula and varying M and Γ to fit the cross-section in the region of the enhancement. In practice more sophisticated methods are used that simultaneously fit a wide range of data, including differential cross-sections, and also take account of nonresonant contributions to the scattering. The widths obtained from such analyses are of the order of 100 MeV, with corresponding interaction times of order 10^{-23} s, which are typical of hadrons, which decay by strong interactions, as we shall see in Chapter 3. Resonances are also a prominent feature of interactions in nuclear physics and we will return to this in Section 2.9 when we discuss nuclear reaction mechanisms.

1.7 Units

Many branches of science introduce special units that are convenient for their own purposes. Nuclear and particle physics are no exceptions. Distances tend to be measured in femtometres or, equivalently, *fermis*, with 1 fm $\equiv 10^{-15}$ m. In these units, the radius of the proton is about 0.8 fm. The range of the strong nuclear force between protons and neutrons is of order 1–2 fm, while the range of the weak force is of order 10^{-3}fm. For comparison, the radii of atoms are of order 10^5 fm. A common unit for area is the *barn*, defined by $1\,\text{b} = 10^{-28}\text{m}^2$. For example, the total cross-section for pp scattering (a strong interaction) is a few tens of millibarns (mb) (compare also the $\pi^- p$ total cross-section in Figure 1.11), whereas the same quantity for νp scattering (a weak interaction) is a few tens of femtobarns (fb), depending on the energies involved. Nuclear cross-sections are very much larger and increase approximately like $A^{2/3}$, where A is the total number of nucleons in the nucleus.

Energies are invariably specified in terms of the electron volt, eV, defined as the energy required to raise the electric potential of an electron or proton by one volt. In S.I. units, $1\,\text{eV} = 1.6 \times 10^{-19}$ joules. The units $1\text{ keV} = 10^3\text{eV}$, $1\,\text{MeV} = 10^6\text{ eV}$, $1\text{ GeV} = 10^9\text{ eV}$ and $1\text{ TeV} = 10^{12}\text{ eV}$ are also in general use. In terms of these units, atomic ionisation energies are typically a few eV, the energies needed to bind nucleons in heavy nuclei are typically 7–8 MeV per particle and the highest particle energies produced in the laboratory are of order of a few TeV for protons. Momenta are specified in eV/c, MeV/c, etc.

In order to create a new particle of mass M, an energy at least as great as its rest energy Mc^2 must be supplied. The rest energies of the electron and proton are 0.51 MeV and 0.94 GeV respectively, whereas the W and Z^0 bosons have rest energies of 80 GeV and 91 GeV, respectively. Correspondingly their masses are conveniently measured in MeV/c^2 or GeV/c^2, so that, for example,

$$M_e = 0.51\,\text{MeV}/c^2, \quad M_p = 0.94\text{ GeV}/c^2,$$
$$M_W = 80.4\,\text{GeV}/c^2, \quad M_Z = 91.2\text{ GeV}/c^2.$$

In S.I. units, $1\text{ MeV/c}^2 = 1.78 \times 10^{-30}$ kg. In nuclear physics it is also common to express masses in *atomic mass units* (u), defined as 1/12th the mass of the commonest isotope of carbon: $1\,\text{u} = 1.661 \times 10^{-27}\text{kg} = 931.5$ MeV/c^2.

Although experimental results are expressed in the above units, it is usual in particle physics to make theoretical calculations in units chosen such that $\hbar \equiv h/2\pi = 1$ and $c = 1$ (called *natural units*) and many books do this. The corresponding equations in practical units can then be regained, if required, by using the method of dimensions to restore the suppressed factors of $\hbar$ and c. However, as this book is about both nuclear and particle physics, practical units will be used throughout, the sole exception being in Appendix D. A table giving numerical values of fundamental and derived constants, together with some useful conversion factors, is given inside the rear cover of the book.

Problems 1

1.1 (a) For what combinations of atomic number Z and mass number A is the associated neutral atom a boson, and for what combinations is it a fermion?

(b) How would these results be modified if the nucleus were a bound state of protons and electrons, as initially proposed prior to the discovery of the neutron?

1.2 Verify that the spherical harmonic $Y_1{}^1 = \sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$ is an eigenfunction of parity with eigenvalue $P = -1$.

1.3 A proton and antiproton at rest in an S-state annihilate to produce $\pi^0\pi^0$ pairs. Show that this reaction cannot be a strong interaction.

1.4 Suppose that an intrinsic C-parity factor is introduced into the second equation of (1.15), which then becomes

$$\hat{C}|b, \psi_b\rangle = C_b|\bar{b}, \psi_{\bar{b}}\rangle.$$

Show that the eigenvalue corresponding to any eigenstate of $\hat{C}$ is independent of C_b, so that C_b cannot be measured.

1.5 (a) In the absence of free electric charges, the scalar potential ϕ may be set to zero and the electromagnetic field described solely by the vector potential $\mathbf{A}(\mathbf{r}, t)$. The latter is related to the electric field $\mathbf{E}$ by $\mathbf{E} = -\partial\mathbf{A}/\partial t$ and transforms under charge conjugation as $\hat{C}\mathbf{A}(\mathbf{r}, t) = C_\gamma \mathbf{A}(\mathbf{r}, t)$, where C_γ is the C-parity of the photon. If the electromagnetic interaction is invariant under charge conjugation, deduce the value of C_γ.

(b) Use Maxwell's equations to deduce how the magnetic field $\mathbf{B}(\mathbf{r}, t)$ transforms under P and T?

1.6 Show that a particle of spin $\mathbf{J}$ can only have a non-zero electric dipole moment if time-reversal invariance is violated.

1.7 Use the principle of detailed balance applied to the reactions $pp \rightleftarrows \pi^+ d$ to deduce that the spin of the π^+ may be found from the expression

$$S_\pi = \frac{1}{2}\left[\frac{4R}{3}\left(\frac{p_p}{p_\pi}\right)^2 - 1\right],$$

where $p_{p,\pi}$ are the magnitudes of the proton and pion momenta and

$$R = \frac{\mathrm{d}\sigma(pp \to \pi^+ d)/\mathrm{d}\Omega}{\mathrm{d}\sigma(\pi^+ d \to pp)/\mathrm{d}\Omega},$$

where the differential cross-sections are at the same total centre-of-mass energy and both beams and projectiles are unpolarised.

1.8 Consider the strong interaction $\pi^- d \to nn$, where d is a spin–1 S-wave bound state of a proton and a neutron called the deuteron and the initial pion is at rest. Deduce the intrinsic parity of the negative pion.

1.9 Write down equations in symbol form that describe the following interactions:

(a) elastic scattering of an electron antineutrino and a positron;

(b) inelastic production of a pair of neutral pions in proton–proton interactions;

(c) the annihilation of an antiproton with a neutron to produce three pions.

1.10 Draw two topologically distinct Feynman diagrams that can contribute to each of the following processes in lowest order: (a) $\gamma + e^- \to \gamma + e^-$; (b) $e^+ + e^- \to e^+ + e^-$; (c) $\nu_e \bar{\nu}_e$ elastic scattering.

JWST916-c01 JWST916-Martin December 20, 2018 13:19 Printer Name: Trim: 254mm × 203mm

1.11 Draw a fourth-order Feynman diagram for the reaction (a) $\gamma + \gamma \to e^+ + e^-$ and (b) $e^+ + e^- \to e^+ + e^-$.

1.12 Show that the Yukawa potential of Eq. (1.44) is the only spherically symmetric solution of the static Klein–Gordon equation (1.41) that vanishes as r goes to infinity.

1.13 Verify by explicit integration that

$$\mathcal{M}(q^2) = -g^2\hbar^2(|\mathbf{q}|^2 + m^2c^2)^{-1}$$

is the scattering amplitude (1.47) corresponding to the Yukawa potential (1.44).

1.14 A high-energy electron of momentum k is scattered through 90° by a stationary proton and the recoil proton is emitted at an angle θ' to the initial electron direction. Derive an expression for θ' in terms of k and the proton mass m. You may assume the energies are sufficiently large for the electron masses to be neglected throughout.

1.15 A thin ('density' of 1 mg cm^{-2}) target of ^{24}Mg ($M_A = 24.3$ atomic mass units) is bombarded with a 10 nA beam of alpha particles. A detector subtending a solid angle of 2×10^{-3} sr records 20 protons per second. If the scattering is isotropic, determine the cross-section for the $^{24}\text{Mg}(\alpha,p)^{27}\text{Al}$ reaction.

1.16 The cross-section for photon scattering from free electrons when $E_\gamma \ll m_e c^2$ is given *in natural units* by

$$\sigma = \frac{8\pi\alpha^2}{3m_e^2}.$$

What is the value of σ in mb?