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» Using the number line

» Getting absolute value absolutely right

» Operating on signed numbers: Adding, subtracting, multiplying, and dividing

Chapter 1
Deciphering Signs in Numbers

In this chapter, you practice the operations on signed numbers and figure out how to make these numbers behave the way you want them to. The behaving part involves using some well-established rules that are *good for you*. Heard that one before? But these rules (or *properties*, as they're called in math-speak) are very helpful in making math expressions easier to read and to handle when you're solving equations in algebra.

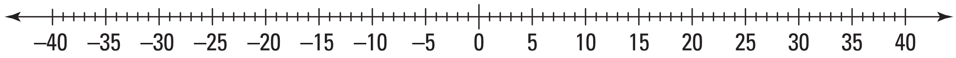
Assigning Numbers Their Place

You may think that identifying that 16 is bigger than 10 is an easy concept. But what about –1.6 and –1.04? Which of these numbers is bigger?

REMEMBER

The easiest way to compare numbers and to tell which is bigger or has a greater value is to find each number's position on the number line. The number line goes from negatives on the left to positives on the right (see Figure 1-1). Whichever number is farther to the right has the greater value, meaning it's bigger.

FIGURE 1-1: A number line.



EXAMPLE

Q. Using the number line in Figure 1-1, determine which is larger, –16 or –10.

A. **–10**. The number –10 is to the right of –16, so it's the bigger of the two numbers. You write that as $-10 > -16$ (read this as "negative 10 is greater than negative 16"). Or you can write it as $-16 < -10$ (negative 16 is less than negative 10).

Q. Which is larger, –1.6 or –1.04?

A. **–1.04**. The number –1.04 is to the right of –1.6, so it's larger.

1 Which is larger, –2 or –8?

2 Which has the greater value, –13 or 2?

3 Which is bigger, –0.003 or –0.03?

4 Which is larger, $-\frac{1}{6}$ or $-\frac{2}{3}$?

Reading and Writing Absolute Value

The *absolute value* of a number, written as $|a|$, is an operation that evaluates whatever is between the vertical bars and then outputs a positive number. Another way of looking at this operation is that it can tell you how far a number is from 0 on the number line — with no reference to which side.

REMEMBER

The absolute value of *a*:

$|a| = a$, if *a* is a positive number ($a > 0$) or if $a = 0$.

$|a| = -a$, if *a* is a negative number ($a < 0$). Read this as "The absolute value of *a* is equal to the *opposite* of *a*."

EXAMPLE

Q. $|4| =$

A. **4**

Q. $|-3| =$

A. **3**

5 $|8| =$

6 $|-6| =$

7 $-|-6| =$

8 $-|8| =$

Adding Signed Numbers

Adding signed numbers involves two different rules, both depending on whether the two numbers being added have the same sign or different signs. After you determine whether the signs are the same or different, you use the absolute values of the numbers in the computation.

REMEMBER

To add signed numbers (assuming that a and b are positive numbers):

- **If the signs are the same:** Add the absolute values of the two numbers together and let their common sign be the sign of the answer.

  $(+a)+(+b)=+(a+b)$ **and** $(-a)+(-b)=-(a+b)$

- **If the signs are different:** Find the difference between the absolute values of the two numbers (subtract the smaller absolute value from the larger) and let the answer have the sign of the number with the larger absolute value. Assume that $|a|>|b|$.

  $(+a)+(-b)=+(a-b)$ **and** $(-a)+(+b)=-(a-b)$

EXAMPLE

Q. $(-6)+(-4)=-(6+4)=$

The signs are the same, so you find the sum and apply the common sign.

A. **−10**

Q. $(+8)+(-15)=-(15-8)=$

The signs are different, so you find the difference and use the sign of the number with the larger absolute value.

A. **−7**

9 $4+(-3)=$

10 $5+(-11)=$

11 $(-18)+(-5)=$

12 $47+(-33)=$

13 $(-3)+5+(-2)=$

14 $(-4)+(-6)+(-10)=$

15 $5+(-18)+(10)=$

16 $(-4)+4+(-5)+5+(-6)=$

Making a Difference with Signed Numbers

You really don't need a new set of rules when subtracting signed numbers. You just change the subtraction problem to an addition problem and use the rules for addition of signed numbers. To ensure that the answer to this new addition problem is the answer to the original subtraction problem, you change the operation from subtraction to addition, and you change the sign of the second number — the one that's being subtracted.

REMEMBER

To subtract two signed numbers:

$a-(+b)=a+(-b)$ **and** $a-(-b)=a+(+b)$

EXAMPLE

Q. $(-8)-(-5)=$

Change the problem to $(-8)+(+5)=$

A. **–3**

Q. $6-(+11)=$

Change the problem to $6+(-11)=$

A. **–5**

17 $5-(-2)=$

18 $-6-(-8)=$

19 $4-87=$

20 $0-(-15)=$

21 $2.4-(-6.8)=$

22 $-15-(-11)=$

Multiplying Signed Numbers

When you multiply two or more numbers, you just multiply them without worrying about the sign of the answer until the end. Then to assign the sign, just count the number of negative signs in the problem. If the number of negative signs is an even number, the answer is positive. If the number of negative signs is odd, the answer is negative.

REMEMBER

The product of two signed numbers:

$(+)(+)=+$ **and** $(-)(-)=+$

$(+)(-)=-$ **and** $(-)(+)=-$

The product of more than two signed numbers:

$(+)(+)(+)(-)(-)(-)(-)$ has a *positive* answer because there are an *even* number of negative factors.

$(+)(+)(+)(-)(-)(-)$ has a *negative* answer because there are an *odd* number of negative factors.

EXAMPLE

Q. $(-2)(-3)=$

There are two negative signs in the problem.

A. **+6**

Q. $(-2)(+3)(-1)(+1)(-4)=$

There are three negative signs in the problem.

A. **−24**

23 $(-6)(3)=$

24 $(14)(-1)=$

25 $(-6)(-3)=$

26 $(6)(-3)(4)(-2)=$

27 $(-1)(-1)(-1)(-1)(-1)(2)=$

28 $(-10)(2)(3)(1)(-1)=$

Dividing Signed Numbers

The rules for dividing signed numbers are exactly the same as those for multiplying signed numbers — as far as the sign goes. (See "Multiplying Signed Numbers" earlier in this chapter.) The rules do differ, though, because you have to divide, of course.

REMEMBER

When you divide signed numbers, just count the number of negative signs in the problem — in the numerator, in the denominator, and perhaps in front of the problem. If you have an even number of negative signs, the answer is positive. If you have an odd number of negative signs, the answer is negative.

EXAMPLE

Q. $\frac{-36}{-9} =$

A. +**4**. There are two negative signs in the problem, which is even, so the answer is positive.

Q. $\frac{-(-3)(-12)}{4} =$

A. **–9**. There are three negative signs in the problem, which is odd, so the answer is negative.

29 $\frac{-22}{-11} =$

30 $\frac{24}{-3} =$

31 $\frac{-3(-4)}{-2} =$

32 $\frac{(-5)(2)(3)}{-1} =$

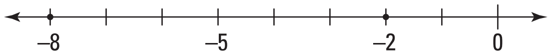
33 $\frac{(-2)(-3)(-4)}{(-1)(-6)} =$

34 $\frac{-1{,}000{,}000}{1{,}000{,}000} =$

Answers to Problems on Signed Numbers

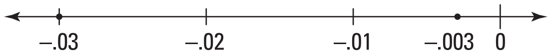
This section provides the answers (in bold) to the practice problems in this chapter.

1. Which is larger, –2 or –8? The answer is **–2 is larger.** The following number line shows that the number –2 is to the right of –8. So –2 is bigger than –8 (or $-2 > -8$).

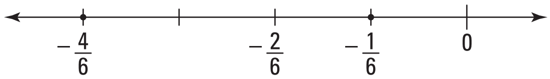


2. Which has the greater value, –13 or 2? **2 is greater**. The number 2 is to the right of –13. So 2 has a greater value than –13 (or $2 > -13$).

3. Which is bigger, –0.003 or –0.03? **–0.003 is bigger**. The following number line shows that the number –0.003 is to the right of –0.03, which means –0.003 is bigger than –0.03 (or $-0.003 > -0.03$).



4. Which is larger, $-\frac{1}{6}$ or $-\frac{2}{3}$? $\mathbf{-\frac{1}{6}}$ **is larger**. The number= $-\frac{2}{3} = -\frac{4}{6}$, and $-\frac{4}{6}$ is to the left of $-\frac{1}{6}$ on the following number line. So $-\frac{1}{6}$ is larger than $-\frac{2}{3}$ (or $-\frac{1}{6} > -\frac{2}{3}$).



5. $|8| = \mathbf{8}$ because $8 > 0$.

6. $|-6| = \mathbf{6}$ because $-6 < 0$ and 6 is the opposite of –6.

7. $-|-6| = \mathbf{-6}$ because $|-6| = 6$ as in the previous problem.

8. $-|8| = \mathbf{-8}$ because $|8| = 8$.

9. $4 + (-3) = \mathbf{1}$ because 4 is the greater absolute value.
$4 + (-3) = +(4 - 3) = 1$

10. $5 + (-11) = \mathbf{-6}$ because –11 has the greater absolute value.
$5 + (-11) = -(11 - 5) = -6$

11. $(-18) + (-5) = \mathbf{-23}$ because both of the numbers have negative signs; when the signs are the same, find the sum of their absolute values. $(-18) + (-5) = -(18 + 5) = -23$

12. $47 + (-33) = \mathbf{14}$ because 47 has the greater absolute value.
$47 + (-33) = +(47 - 33) = 14$

13. $(-3) + 5 + (-2) = \mathbf{0}$
$(-3) + 5 + (-2) = [(-3) + 5] + (-2) = (2) + (-2) = 0$

14. $(-4) + (-6) + (-10) = \mathbf{-20}$
$(-4) + (-6) + (-10) = -(4 + 6) + (-10) = (-10) + (-10) = -(10 + 10) = -20$

15 $5+(-18)+(10)=\mathbf{-3}$

$5+(-18)+(10)=-(18-5)+10=-(13)+10=-(13-10)=-3$

Or you may prefer to add the two numbers with the same sign first, like this:

$5+(-18)+(10)=(5+10)+(-18)=15+(-18)=-(18-15)=-3$

You can do this because order and grouping (association) don't matter in addition.

16 $(-4)+4+(-5)+5+(-6)=\mathbf{-6}$

$(-4)+4+(-5)+5+(-6)=[(-4)+4]+[(-5)+5]+(-6)=0+0+(-6)=-6$

17 $5-(-2)=\mathbf{7}$

$5-(-2)=5+(+2)=7$

18 $-6-(-8)=\mathbf{2}$

$-6-(-8)=-6+(+8)=8-6=2$

19 $4-87=\mathbf{-83}$

$4-87=4+(-87)=-(87-4)=-83$

20 $0-(-15)=\mathbf{15}$

$0-(-15)=0+15=15$

21 $2.4-(-6.8)=\mathbf{9.2}$

$2.4-(-6.8)=2.4+6.8=9.2$

22 $-15-(-11)=\mathbf{-4}$

$-15-(-11)=-15+11=-(15-11)=-4$

23 $(-6)(3)=\mathbf{-18}$ because the multiplication problem has one negative, and 1 is an odd number.

24 $(14)(-1)=\mathbf{-14}$ because the multiplication problem has one negative, and 1 is an odd number.

25 $(-6)(-3)=\mathbf{18}$ because the multiplication problem has two negatives, and 2 is an even number.

26 $(6)(-3)(4)(-2)=\mathbf{144}$ because the multiplication problem has two negatives.

27 $(-1)(-1)(-1)(-1)(-1)(2)=\mathbf{-2}$ because the multiplication problem has five negatives.

28 $(-10)(2)(3)(1)(-1)=\mathbf{60}$ because the multiplication problem has two negatives.

29 $-22/_{-11}=\mathbf{2}$ because the division problem has two negatives.

30 $24/_{-3}=\mathbf{-8}$ because the division problem has one negative.

31 $\frac{-3(-4)}{-2}=\mathbf{-6}$ because three negatives result in a negative.

32 $\frac{(-5)(2)(3)}{-1}=\mathbf{30}$ because the division problem has two negatives.

33 $\frac{(-2)(-3)(-4)}{(-1)(-6)}=\mathbf{-4}$ because the division problem has five negatives.

34 $\frac{-1{,}000{,}000}{1{,}000{,}000}=\mathbf{-1}$ because the division problem has one negative.