IN THIS CHAPTER

» Fussing with fractions

» Brushing up on basic algebra

» Getting square with geometry

Chapter 1

Getting Down to Basics: Algebra and Geometry

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I know, I know. This is a *calculus* workbook, so what's with the algebra and geometry? Don't worry; I'm not going to waste too many precious pages with algebra and geometry, but these topics are essential for calculus. You can no more do calculus without algebra than you can write French poetry without French. And basic geometry (but not geometry proofs) is critically important because much of calculus involves real-world problems that include angles, slopes, shapes, and so on. So in this chapter — and in Chapter 2 on functions and trigonometry — I give you some quick problems to help you brush up on your skills. If you've already got these topics down pat, you can skip to Chapter 3.

In addition to working through the problems in Chapters 1 and 2 in this book, you may want to check out the great pre-calc review in *Calculus For Dummies,* 2nd Edition.

Fraction Frustration

Many, many math students hate fractions. I'm not sure why, because there's nothing especially difficult about them. Perhaps for some students, fraction concepts didn't completely click when they first studied them, and then fractions became a nagging frustration whenever they came up in subsequent math courses. Whatever the cause, if you don't like fractions, try to get over it. Fractions really are a piece o' cake; you'll have to deal with them in every math course you take.

You can't do calculus without a good grasp of fractions. For example, the very definition of the derivative is based on a fraction called the *difference quotient.* And, on top of that, the symbol for the derivative, $\frac{dy}{dx}$, is a fraction. So, if you're a bit rusty with fractions, get up to speed with the following problems — or else!

EXAMPLE

Q. Solve: $\frac{a}{b} \cdot \frac{c}{d} = ?$

A. $\boldsymbol{\frac{ac}{bd}}$. To multiply fractions, you multiply straight across. You do *not* cross-multiply!

Q. Solve: $\frac{a}{b} \div \frac{c}{d} = ?$

A. $\boldsymbol{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}}$. To divide fractions, you flip the second one, and then multiply.

1 Solve: $\frac{5}{0} = ?$

2 Solve: $\frac{0}{10} = ?$

3 Does $\frac{3a+b}{3a+c}$ equal $\frac{a+b}{a+c}$? Why or why not?

4 Does $\frac{3a+b}{3a+c}$ equal $\frac{b}{c}$? Why or why not?

5 Does $\frac{4ab}{4ac}$ equal $\frac{ab}{ac}$? Why or why not?

6 Does $\frac{4ab}{4ac}$ equal $\frac{b}{c}$? Why or why not?

Misc. Algebra: You Know, Like Miss South Carolina

This section gives you a quick review of algebra basics like factors, powers, roots, logarithms, and quadratics. You absolutely *must* know these basics.

EXAMPLE

Q. Factor $9x^4 - y^6$.

A. $\boldsymbol{9x^4 - y^6 = (3x^2 - y^3)(3x^2 + y^3)}$. This is an example of the single most important factor pattern: $a^2 - b^2 = (a - b)(a + b)$. Make sure you know it!

Q. Rewrite $x^{2/5}$ without a fraction power.

A. $\boldsymbol{\sqrt[5]{x^2}}$ **or** $\boldsymbol{\left(\sqrt[5]{x}\right)^2}$. Don't forget how fraction powers work!

7 Rewrite x^{-3} without a negative power.

8 Does $(abc)^4$ equal $a^4b^4c^4$? Why or why not?

9 Does $(a + b + c)^4$ equal $a^4 + b^4 + c^4$? Why or why not?

10 Rewrite $\sqrt[3]{\sqrt[4]{x}}$ with a single radical sign.

11 Does $\sqrt{a^2+b^2}$ equal $a+b$? Why or why not?

12 Rewrite $\log_a b = c$ as an exponential equation.

13 Rewrite $\log_c a - \log_c b$ with a single log.

14 Rewrite $\log 5 + \log 200$ with a single log and then solve.

15 If $5x^2 = 3x + 8$, solve for x with the quadratic formula.

16 Solve: $|3x+2| > 14$.

17 Solve: $-3^2 - x^0 + \sqrt{0} - |-1| - 1^0 - 0^1 = ?$

18 Simplify $\sqrt[3]{p^6 q^{15}}$.

19 Simplify $\left(\frac{8}{27}\right)^{-4/3}$.

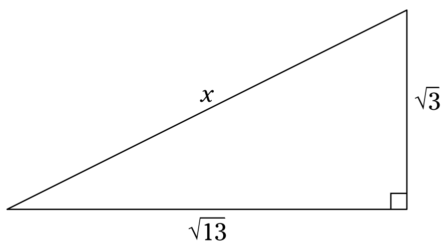
20 Factor $-x^{10}+16$ over the set of integers.

Geometry: When Am I Ever Going to Need It?

You can use calculus to solve many real-world problems that involve two- or three-dimensional shapes and various curves, surfaces, and volumes — such as calculating the rate at which the water level is falling in a cone-shaped tank or determining the dimensions that maximize the volume of a cylindrical soup can. So the geometry formulas for perimeter, area, volume, surface area, and so on will come in handy. You should also know things like the Pythagorean Theorem, proportional shapes, and basic coordinate geometry, like the midpoint and distance formulas.

EXAMPLE

Q. What's the area of the triangle in the following figure?



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A. $\mathbf{\frac{\sqrt{39}}{2}}$.

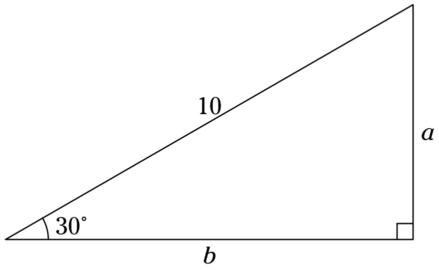
$$\begin{aligned} Area_{triangle} &= \frac{1}{2} base \cdot height \\ &= \frac{1}{2} \cdot \sqrt{13}\sqrt{3} \\ &= \frac{\sqrt{39}}{2} \end{aligned}$$

Q. How long is the hypotenuse of the triangle in the previous example?

A. $\boldsymbol{x = 4}$.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 &= a^2 + b^2 \\ x^2 &= \sqrt{13}^2 + \sqrt{3}^2 \\ x^2 &= 13 + 3 \\ x^2 &= 16 \\ x &= 4 \end{aligned}$$

21 Fill in the two missing lengths for the sides of the triangle in the following figure.



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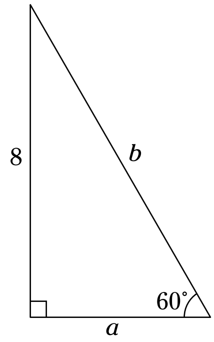
22 What are the lengths of the two missing sides of the triangle in the following figure?



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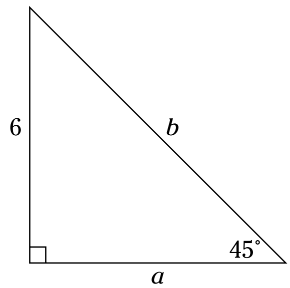
23 Fill in the missing lengths for the sides of the triangle in the following figure.



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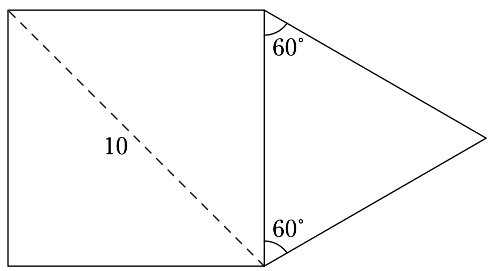
24 **a.** What's the total area of the pentagon in the following figure (the shape on the left is a square)?

b. What's the perimeter?



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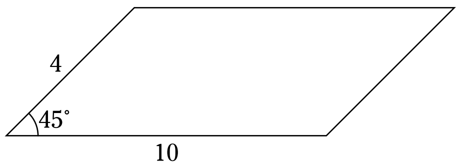
25 Compute the area of the parallelogram in the following figure.



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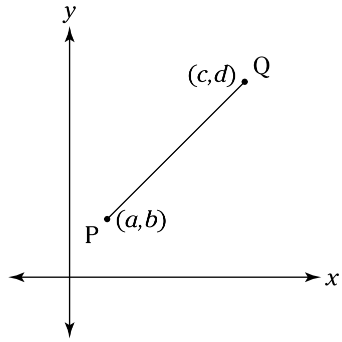
26 What's the slope of $\overline{PQ}$?



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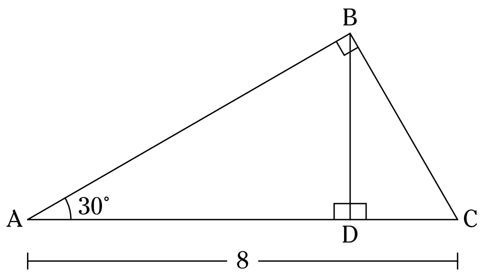
27 How far is it from P to Q in the figure from Problem 26?

28 What are the coordinates of the midpoint of $\overline{PQ}$ in the figure from Problem 26?

29 What's the length of altitude of triangle *ABC* in the following figure?



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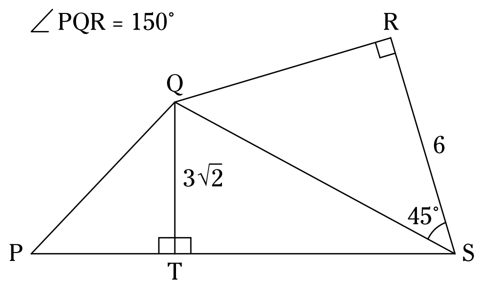
30 What's the perimeter of triangle *ABD* in the figure for Problem 29?

31 What's the area of quadrilateral *PQRS* in the following figure?



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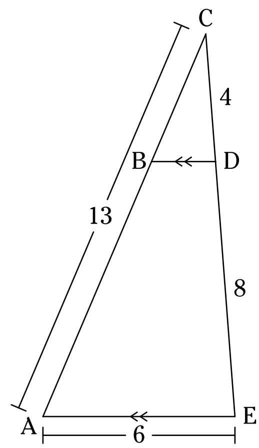
32 What's the perimeter of triangle *BCD* in the following figure?



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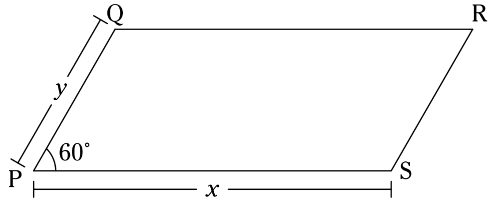
33 What's the ratio of the area of triangle *BCD* to the area of triangle *ACE* in the figure for Problem 32?

34 In the following figure, what's the area of parallelogram *PQRS* in terms of *x* and *y*?



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Solutions for This Easy, Elementary Stuff

1. Solve: $\frac{5}{0}=?$ $\mathbf{\frac{5}{0}}$ **is undefined!** Don't mix this up with something like $\frac{0}{8}$, which equals zero.

   Here's a great way to think about this problem and fractions in general. Consider the following simple division or fraction problem: $\frac{8}{2}=4$. Note the *multiplication* problem implicit here: 2 times 4 is 8. This multiplication idea is a great way to think about how fractions work. So in the current problem, you can consider $\frac{5}{0}=$ ____, and use the multiplication idea: 0 times ______ equals 5. What works in the blank? Nothing, obviously, because 0 times anything is 0. The answer, therefore, is undefined.

   Note that if you think about these two fractions as examples of slope $\left(\frac{rise}{run}\right)$, $\frac{5}{0}$ has a rise of 5 and a run of 0, which gives you a *vertical* line that has sort of an infinite steepness or slope (that's why it's undefined). Or just remember that it's impossible to drive up a vertical road, so it's impossible to come up with a slope for a vertical line. The fraction $\frac{0}{8}$, on the other hand, has a rise of 0 and a run of 8, which gives you a *horizontal* line that has no steepness at all and thus has the perfectly ordinary slope of zero. Of course, it's also perfectly ordinary to drive on a horizontal road.

2. Solve: $\frac{0}{10}=?$ $\mathbf{\frac{0}{10}=0}$**.** (See the solution to Problem 1 for more information.)

3. Does $\frac{3a+b}{3a+c}$ equal $\frac{a+b}{a+c}$? **No.** You can't cancel the 3s.

   WARNING

   You can't cancel in a fraction unless there's an unbroken chain of multiplication running across the entire numerator and the entire denominator — like with $\frac{4ab^2c(x+y)}{5apqr(x^2-y)}$ where you can cancel the *a*s (but only the *a*s). (Note that the addition and subtraction inside the parentheses don't break the multiplication chain.) But, you may object, can't you cancel $4x^2$ from the five terms in $\frac{8x^3-12x^2y+16x^5}{8x^2p-4x^2q^2}$, giving you $\frac{2x-3y+4x^3}{2p-q^2}$? Yes you can, but that's because that fraction can be factored into $\frac{4x^2(2x-3y+4x^3)}{4x^2(2p-q^2)}$, resulting in a fraction where there is an unbroken chain of multiplication across the entire numerator and the entire denominator. Then, the $4x^2$s cancel.

4. Does $\frac{3a+b}{3a+c}$ equal $\frac{b}{c}$? **No.** You can't cancel the 3as. (See the warning in Problem 3.) You can also just test this problem with numbers: Does $\frac{3\cdot4+5}{3\cdot4+6}=\frac{5}{6}$? No, they're not equal, and thus the canceling doesn't work.

5. Does $\frac{4ab}{4ac}$ equal $\frac{ab}{ac}$? **Yes.** You can cancel the 4s because the entire numerator and the entire denominator are connected with multiplication.

6. Does $\frac{4ab}{4ac}$ equal $\frac{b}{c}$? **Yes.** You can cancel the 4*a*s.

7 Rewrite x^{-3} without a negative power. $\boldsymbol{\frac{1}{x^3}}$.

8 Does $(abc)^4$ equal $a^4b^4c^4$? **Yes.** Exponents do distribute over multiplication.

9 Does $(a+b+c)^4$ equal $a^4+b^4+c^4$? **No!** Exponents do *not* distribute over addition (or subtraction).

TIP

When you're working a problem and can't remember the algebra rule, try the problem with numbers instead of variables. Just replace the variables with simple, round numbers and work out the numerical problem. (Don't use 0, 1, or 2 because they have special properties that can mess up your test.) Whatever works for the numbers will work with variables, and whatever doesn't work with numbers won't work with variables. Watch what happens if you try this problem with numbers:

$$(3+4+6)^4 \stackrel{?}{=} 3^4+4^4+6^4$$
$$13^4 \stackrel{?}{=} 81+256+1{,}296$$
$$28{,}561 \neq 1{,}633$$

10 Rewrite $\sqrt[3]{\sqrt[4]{x}}$ with a single radical sign. $\boldsymbol{\sqrt[12]{x}}$.

11 Does $\sqrt{a^2+b^2}$ equal $a+b$? **No!** The explanation is basically the same as for Problem 9. Consider this: If you turn the root into a power, you get $\sqrt{a^2+b^2}=\left(a^2+b^2\right)^{1/2}$. But because you *can't* distribute the power over addition, $\left(a^2+b^2\right)^{1/2} \neq \left(a^2\right)^{1/2}+\left(b^2\right)^{1/2}$, or $a+b$, and thus $\sqrt{a^2+b^2} \neq a+b$.

12 Rewrite $\log_a b = c$ as an exponential equation. $\boldsymbol{a^c = b}$.

13 Rewrite $\log_c a - \log_c b$ with a single log. $\boldsymbol{\log_c \frac{a}{b}}$.

14 Rewrite $\log 5+\log 200$ with a single log and then solve.
$\boldsymbol{\log 5+\log 200=\log(5\times 200)=\log 1{,}000=3}$.

REMEMBER

When you see "log" without a base number, the base is 10.

15 If $5x^2=3x+8$, solve for x with the quadratic formula. $\boldsymbol{x=\frac{8}{5}}$ **or** $\boldsymbol{-1}$.

Start by rearranging $5x^2=3x+8$ into $5x^2-3x-8=0$ because when solving a quadratic equation, you want just a zero on one side of the equation.

The quadratic formula tells you that $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Plugging 5 into a, −3 into b, and −8 into c gives you $x=\frac{-(-3)\pm\sqrt{(-3)^2-4(5)(-8)}}{2\cdot 5}=\frac{3\pm\sqrt{9+160}}{10}=\frac{3\pm 13}{10}=\frac{16}{10}$ or $\frac{-10}{10}$, so $x=\frac{8}{5}$ or −1.

(16) Solve: $|3x+2|>14$. $\boldsymbol{x<-\frac{16}{3}\ \cup\ x>4}$.

1. **Turn the inequality into an equation:**

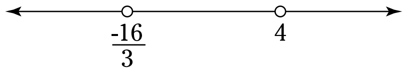
$$|3x+2|=14$$

2. **Solve the absolute value equation.**

$$\begin{aligned} 3x+2&=14 \\ 3x&=12 \\ x&=4 \end{aligned} \qquad \text{or} \qquad \begin{aligned} 3x+2&=-14 \\ 3x&=-16 \\ x&=-\frac{16}{3} \end{aligned}$$

3. **Place both solutions on a number line (see the following figure).**

(You use hollow dots for > and <; if the problem had involved ≥ or ≤, you would use solid dots.)



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4. **Test a number from each of the three regions on the line (left of the left dot, between the dots, and right of the right dot) in the original inequality.**

For this problem you can use −10, 0, and 10.

$$\begin{aligned} |3\cdot(-10)+2| &\overset{?}{>} 14 \\ |-28| &\overset{?}{>} 14 \\ 28 &\overset{?}{>} 14 \end{aligned}$$

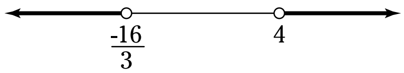
True, so you shade the left-most region.

$$\begin{aligned} |3\cdot(0)+2| &\overset{?}{>} 14 \\ 2 &\overset{?}{>} 14 \end{aligned}$$

False, so you don't shade the middle region.

$$\begin{aligned} |3\cdot(10)+2| &\overset{?}{>} 14 \\ |32| &\overset{?}{>} 14 \\ 32 &\overset{?}{>} 14 \end{aligned}$$

True, so you shade the region on the right. The following figure shows the result. *x* can be any number where the line is shaded. That's your final answer.



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5. **You may also want to express the answer symbolically.**

Because *x* can equal a number in the left region *or* a number in the right region, this is an *or* solution which means *union* ($\cup$). When you want to include everything from both regions on the number line, you want the union of the two regions. So, the symbolic answer is

$$x < -\frac{16}{3} \quad \cup \quad x > 4$$

(You can write the above using the word "or" instead of the union symbol.) If only the middle region were shaded, you'd have an *and* or *intersection* problem ($\cap$). Using the above number line points, for example, you would write the middle-region solution like this:

$$x > -\frac{16}{3} \quad \cap \quad x < 4$$

(You can use the word "and" instead of the intersection symbol.) Note that in this solution (whether you use "and" or the intersection symbol) the two inequalities overlap or intersect in the middle region. You can avoid the intersection issue by simply writing the solution as

$$-\frac{16}{3} < x < 4$$

REMEMBER

You say "to-*may*-to," I say "to-*mah*-to."

While we're on the subject of absolute value, don't forget that $\sqrt{x^2} = |x|$. $\sqrt{x^2}$ does *not* equal $\pm x$.

17 Solve: $-3^2 - x^0 + \sqrt{0} - |-1| - 1^0 - 0^1 = ?$ **The answer is −12.**

Funny looking problem, eh? It's just meant to help you review a few basics. Take a look at the six terms:

Don't forget, $-3^2 = -9$. If you want to square a negative number, you have to put it in parentheses: $(-3)^2 = 9$. Next, anything to the zero power (including a variable) equals 1. That takes care of the second and fifth chunks of the problem. The square root of zero is just zero, of course, because zero squared equals zero. And you know that the absolute value of −1 is 1; you just have to be careful not to goof up with all those negative signs and subtraction signs. Finally, zero to any *positive* power equals zero. That does it:

$$\begin{aligned} & -3^2 - x^0 + \sqrt{0} - |-1| - 1^0 - 0^1 \\ &= -9 - 1 + 0 - 1 - 1 - 0 \\ &= -12 \end{aligned}$$

18 Simplify $\sqrt[3]{p^6q^{15}}$. **The answer is p^2q^5.**

Most people prefer working with power rules to working with root rules, so that's the way I solve the problem here. First, rewrite the root as a power: $\sqrt[3]{p^6q^{15}} = \left(p^6q^{15}\right)^{1/3}$. Now, just distribute the power to the p^6 and the q^6, and then use the power-to-a-power rule:

$$\begin{aligned} &\left(p^6q^{15}\right)^{1/3} \\ &= \left(p^6\right)^{1/3}\left(q^{15}\right)^{1/3} \\ &= p^{6(1/3)}q^{15(1/3)} \\ &= p^2q^5 \end{aligned}$$

19 Simplify $\left(\frac{8}{27}\right)^{-4/3}$. **The answer is $\frac{81}{16}$.**

I'll give you the longer version of the solution and then show you a shortcut. First, use the definition of a negative exponent to rewrite the problem as $\frac{1}{\left(\frac{8}{27}\right)^{4/3}}$. Next, change the power to a root: $\frac{1}{\sqrt[3]{\frac{8}{27}}^{\,4}}$ (instead, you could first distribute the fraction power to the numerator and denominator). The rest shouldn't be too bad: $\frac{1}{\sqrt[3]{\frac{8}{27}}^{\,4}} = \frac{1}{\left(\frac{\sqrt[3]{8}}{\sqrt[3]{27}}\right)^4} = \frac{1}{\left(\frac{2}{3}\right)^4} = \frac{1}{\left(\frac{16}{81}\right)} = \frac{81}{16}$.

The shortcut is to use the fact that when you have a fraction raised to a negative power, you can flip the fraction and make the power positive, like this $\left(\frac{8}{27}\right)^{-4/3} = \left(\frac{27}{8}\right)^{4/3}$. Then proceed as follows: $\left(\frac{27}{8}\right)^{4/3} = \frac{27^{4/3}}{8^{4/3}} = \frac{\sqrt[3]{27}^{\,4}}{\sqrt[3]{8}^{\,4}} = \frac{3^4}{2^4} = \frac{81}{16}$.

20 Factor $-x^{10}+16$ over the set of integers. $\boldsymbol{(4-x^5)(4+x^5)}$.

To factor $-x^{10}+16$, you use the oh-so-important a^2-b^2 rule. a^2-b^2 factors into $(a-b)(a+b)$. Make sure you know this factoring rule (and the corresponding FOILing rule, which is the factoring rule in reverse). Whenever you see a binomial with a subtraction sign (in the current problem, you have to switch the two terms to see the subtraction sign), ask yourself whether you can rewrite the binomial as $(\quad)^2-(\quad)^2$, in other words, as something squared minus something else squared. If you can, then the first blank is your a, and the second blank is your b.

The binomial in this problem can be rewritten as $(4)^2-\left(x^5\right)^2$. Now just plug the 4 into the a and the x^5 into the b in $(a-b)(a+b)$, and you're done.

21 Fill in the two missing lengths for the sides of the triangle. **$a=5$ and $b=5\sqrt{3}$.**

This is a 30°-60°-90° triangle.

22 Fill in the two missing lengths for the sides of the triangle.

$\boldsymbol{a} = \mathbf{\frac{8}{\sqrt{3}}}$ **or** $\mathbf{\frac{8\sqrt{3}}{3}}$

$\boldsymbol{b} = \mathbf{\frac{16}{\sqrt{3}}}$ **or** $\mathbf{\frac{16\sqrt{3}}{3}}$

Another 30°-60°-90° triangle.

23 Fill in the two missing lengths for the sides of the triangle. $\boldsymbol{a} = \mathbf{6}$ **and** $\boldsymbol{b} = \mathbf{6\sqrt{2}}$**.**

Make sure you know your 45°-45°-90° triangle.

24 **a.** What's the total area of the pentagon? $\mathbf{50 + \frac{25\sqrt{3}}{2}}$.

The square is $\frac{10}{\sqrt{2}}$ by $\frac{10}{\sqrt{2}}$ (because half a square is a 45°-45°-90° triangle), so the area is $\frac{10}{\sqrt{2}} \cdot \frac{10}{\sqrt{2}} = \frac{100}{2} = 50$. The equilateral triangle has a base of $\frac{10}{\sqrt{2}}$, or $5\sqrt{2}$, so its height is $\frac{5\sqrt{6}}{2}$ (because half of an equilateral triangle is a 30°-60°-90° triangle). So the area of the triangle is $\frac{1}{2}\left(5\sqrt{2}\right)\left(\frac{5\sqrt{6}}{2}\right) = \frac{25\sqrt{12}}{4} = \frac{50\sqrt{3}}{4} = \frac{25\sqrt{3}}{2}$. The total area is thus $50 + \frac{25\sqrt{3}}{2}$.

b. What's the perimeter? **The answer is $\mathbf{25\sqrt{2}}$.**

The sides of the square are $\frac{10}{\sqrt{2}}$, or $5\sqrt{2}$, as are the sides of the equilateral triangle.

The pentagon has five sides, so the perimeter is $5 \cdot 5\sqrt{2}$, or $25\sqrt{2}$.

25 Compute the area of the parallelogram. **The answer is $\mathbf{20\sqrt{2}}$.**

The height of the parallelogram is $\frac{4}{\sqrt{2}}$, or $2\sqrt{2}$, because its height is one of the legs of a 45°-45°-90° triangle. The parallelogram's base is 10. So, because the area of a parallelogram equals base times height, the area is $10 \cdot 2\sqrt{2}$, or $20\sqrt{2}$.

26 What's the slope of $\overline{PQ}$? $\mathbf{\frac{\boldsymbol{d} - \boldsymbol{b}}{\boldsymbol{c} - \boldsymbol{a}}}$**.** Remember that $slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$.

27 How far is it from *P* to *Q*? $\mathbf{\sqrt{(\boldsymbol{c} - \boldsymbol{a})^2 + (\boldsymbol{d} - \boldsymbol{b})^2}}$**.** Remember that $distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

28 What are the coordinates of the midpoint of $\overline{PQ}$? $\mathbf{\left(\frac{\boldsymbol{a} + \boldsymbol{c}}{2}, \frac{\boldsymbol{b} + \boldsymbol{d}}{2}\right)}$**.** The midpoint of a segment is given by the average of the two *x* coordinates and the average of the two *y* coordinates.

29 What's the length of altitude of triangle *ABC*? $\mathbf{2\sqrt{3}}$**.**

There are a few ways to solve this problem, all of which use your knowledge of 30°-60°-90° triangles. Here's a quick and easy way. Triangle *ABC* is a 30°-60°-90° triangle, and the short leg of a 30°-60°-90° triangle is half as long as its hypotenuse, so $\overline{BC}$ is 4. Triangle *BCD* is another 30°-60°-90° triangle, so its short leg is half as long as its hypotenuse. That gives $\overline{DC}$ a length of 2. Then, because $\overline{BD}$ is the long leg of 30°-60°-90° triangle *BCD*, it's $\sqrt{3}$ times its short leg. That gives you the answer of $2\sqrt{3}$, for altitude $\overline{BD}$.

30 What's the perimeter of triangle *ABD*? $\mathbf{6+6\sqrt{3}}$.

Triangle *ABD* is yet another 30°-60°-90° triangle, so its hypotenuse is twice as long as its short leg, $\overline{BD}$. That gives you a length of $4\sqrt{3}$ for $\overline{AB}$. Next, $\overline{AD}$ is 8 − 2, or 6. The perimeter of triangle *ABD* is therefore $6+2\sqrt{3}+4\sqrt{3}$, or $6+6\sqrt{3}$.

31 What's the area of quadrilateral *PQRS*? $\mathbf{27+9\sqrt{3}}$.

Piece o' cake. Begin with triangle *QRS*, which you can see is a 45°-45°-90° triangle. The legs of a 45°-45°-90° triangle are equal, so $\overline{QR}$ is 6, and the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times either leg, so $\overline{QS}$ is $6\sqrt{2}$.

Now you see that the hypotenuse of triangle *TQS* is twice as long as its short leg, $\overline{QT}$, which tells you that triangle *TQS* is a 30°-60°-90° triangle. That makes $\angle TQS$ 60°, and you also get the length of $\overline{TS}$, which, since it's the long leg of 30°-60°-90° triangle *TQS*, has to be $\sqrt{3}$ times as long as its short leg, $\overline{QT}$. So $\overline{TS}$ is $3\sqrt{6}$.

Next, since $\angle PQR$ is 150°, and angles *TQS* and *SQR* are 60° and 45°, respectively, you subtract to get 45° for $\angle PQT$. That makes triangle *PQT* a 45°-45°-90° triangle, and thus $\overline{PT}$, like $\overline{QT}$, is $3\sqrt{2}$.

Now you have everything you need to figure the area of the quadrilateral. The area of a right triangle equals half the product of its legs, so here's the final math:

$$\begin{aligned}Area_{Quad\ PQRS} &= area_{\triangle PQT} + area_{\triangle TQS} + area_{\triangle QRS}\\ &= \frac{1}{2}(3\sqrt{2})(3\sqrt{2}) + \frac{1}{2}(3\sqrt{6})(3\sqrt{2}) + \frac{1}{2}(6)(6)\\ &= 9 + \frac{1}{2}(9\sqrt{12}) + 18\\ &= 9 + 9\sqrt{3} + 18\\ &= 27 + 9\sqrt{3}\end{aligned}$$

Make sure you know your 30°-60°-90° and 45°-45°-90° triangles!

32 What's the perimeter of triangle *BCD*? $\mathbf{10\frac{1}{3}}$.

To do this problem and the next one, you first have to establish that the two triangles are similar (the same shape). Because segments $\overline{BD}$ and $\overline{AE}$ are parallel, angles *BDC* and *AED* are corresponding angles and are therefore congruent. And the two triangles share angle *C*. Thus, by the AA (angle-angle) theorem, triangles *BCD* and *ACE* are similar.

To get the length of $\overline{BC}$, you could use similar triangle proportions, but it's a little bit quicker to use the side-splitter theorem, which tells you that $\frac{BC}{AB}=\frac{4}{8}$. Since the ratio equals $\frac{4}{8}$, you can set $\overline{BC}$ equal to 4*x* and $\overline{AB}$ equal to 8*x*. They add up to 13, so you have $4x+8x=13$, or $x=\frac{13}{12}$. Plugging that into 4*x* gives you $\frac{13}{3}$ for the length of $\overline{BC}$.

Now all you need to finish is the length of $\overline{BD}$. Did you fall for the nasty trap in this problem? When you see the 4 and the 8 along the right side of triangle *ACE*, it's easy to make the mistake of thinking that $\overline{BD}$ and $\overline{AE}$ will be in the same 4-to-8 or 1-to-2 ratio and conclude that $\overline{BD}$ therefore equals 3. But $\overline{BD}$ and $\overline{AE}$ are not in a 1-to-2 ratio. To get $\overline{BD}$, you have to use a similar triangle proportion like the following:

$$\frac{\text{right side of } \Delta BCD}{\text{right side of } \Delta ACE} = \frac{\text{base of } \Delta BCD}{\text{base of } \Delta ACE}$$

$$\frac{CD}{CE} = \frac{BD}{AE}$$

$$\frac{4}{12} = \frac{BD}{6}$$

Cross multiplication gives you a length of 2 for $\overline{BD}$.

Adding up the three sides (4, $\frac{13}{3}$, and 2) gives you the perimeter.

33 What's the ratio of the area of triangle *BCD* to the area of triangle *ACE* in the figure for Problem 32? $\mathbf{\frac{1}{9}}$ **or 1 : 9.**

If you know the appropriate theorem for this problem, the problem's a snap. If you don't know the theorem, the problem's very hard. You could also get tripped up if you thought you needed the areas of the two triangles (you don't), and you could be thrown off by the trap referred to in Problem 32.

All you need is the theorem that tells you that the ratio of the areas of similar figures is equal to the square of the ratio of any of their corresponding sides. For this problem, the theorem tells you that

$$\frac{Area_{\Delta BCD}}{Area_{\Delta ACE}} = \left(\frac{CD}{CE}\right)^2 = \left(\frac{4}{12}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

(Note that you did not need to know the altitudes of the triangles or their areas in order to compute the ratio of their areas.)

In plain English, the idea is simply that if you take any 2-D shape and blow it up to, say, 4 times its height, its area will grow 4^2, or 16 times. By the way, if you blow up a 3-D shape, say, 4 times its height, its volume will grow 4^3, or 64 times.

34 What's the area of parallelogram *PQRS*? $\mathbf{\frac{\sqrt{3}}{2}xy}$**.**

When you see a 60° angle in a problem, one of the first things you should consider is the 30°-60°-90° triangle. Sure enough, that's the key to this problem.

All you need to do is to drop an altitude from *Q* straight down to base $\overline{PS}$, making a right angle with $\overline{PS}$. Call the point where the altitude meets the base point *T*. Triangle *PQT* contains a 60° angle and a 90° angle, so it has to be a 30°-60°-90° triangle. The short leg of a 30°-60°-90° triangle is half as long as its hypotenuse, so $\overline{PT}$ is half of $\overline{PQ}$, or $\frac{1}{2}y$. Then, because the long leg of a 30°-60°-90° triangle is $\sqrt{3}$ times as long as its short leg, altitude $\overline{QT}$ is $\sqrt{3}\cdot\frac{1}{2}y = \frac{\sqrt{3}}{2}y$.

Now that you have the altitude and the base of the parallelogram, you just plug them into the parallelogram area formula to get your answer:

$$\begin{aligned} Area_{\text{parallelogram } PQRS} &= base \cdot height \\ &= x \cdot \frac{\sqrt{3}}{2}y \end{aligned}$$