

Understanding the Physical Quantity of Number

The most important goal for any teacher of mathematics, in my view, should be the development of curiosity and wonder in students. When students are curious, they become motivated and inspired to learn anything they set out to learn. There is probably no more important time to spark this journey of curiosity than when students are beginning kindergarten. This becomes particularly important if students are in homes where parents think they should get their children ahead in school by teaching mathematical methods to memorize. If students think their role is to remember inflexible sets of rules, they will be hampered in their mathematical journeys. There is a very strong danger that this approach leads them to believe that this is what math is—a set of rules—that they do not need to make sense of, only memorize. In an international survey given to over 13 million students as part of the international PISA testing, from the OECD, it was found that students who took a memorization approach to mathematics were the lowest-achieving students in every country (Boaler & Zoido, 2016). Unfortunately, the elementary school years are often when students develop the idea, from damaging practices such as timed tests (Boaler, 2014), that mathematics is all about memorization. The opposite of a dry, unappealing, memorization approach to mathematics is one that encourages curiosity and wonder.

We invite students to wonder in all of the activities in this book, starting with Big Idea 1. In the Visualize activity, we ask students to discuss times when they have wondered “How many?” and then give students different groups of objects and invite them to work out how many there are.

In our Play activity, we have chosen photographs for students that include several different things to count, and different ways to count them. We have chosen photographs that offer different ways to count in order to help students know that in mathematics there are usually many different ways to see things and varied ways to approach them, and that different approaches can all be correct if students justify their thinking and give reasons for their approach.

In our Investigate activity, we invite students to make their own books. We recommend that students work in pairs. Each pair is given a number for which to make a page for the class counting book and, if possible, a digital camera. Students can then be taken to spaces where they look for numbers in the world. Together the class makes a whole book that they can refer back to during the rest of the year.

Jo Boaler

References

- Boaler, J., & Zoido, P. (2016, November 1). Why math education in the US doesn't add up. *Scientific American*. Retrieved from <https://www.scientificamerican.com/article/why-math-education-in-the-u-s-doesn-t-add-up/>
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Count a Collection

Snapshot

Students stretch their counting capacity by counting collections of classroom objects and coming to agreement with a partner about how many objects there are.



Connection to CCSS
K.CC.4, K.CC.5, K.CC.1

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5–10 min	Ask students what kinds of “How many?” questions they have asked themselves. Tell students that they are going to work with a partner to come to agreement about how many objects are in a given group. Model with a partner how to compare counts and resolve any disagreement.	Collection of objects (fewer than 10) in a container
Explore	15–30 min	Partners work together to count a collection of objects and come to agreement about how many there are.	• Collections of various class-room or every-day objects in containers • Optional: sticky notes or index cards
Discuss	10 min	Gather the class in a circle and invite partners to share how they counted their collections. Partners can demonstrate the ways they counted. Highlight features of counting that you’d like to see others try, such as moving objects to count.	Optional: chart and markers
Extend	Ongoing	Create a counting station with a collection of objects for the class to count in small groups or independently over the course of several days. Provide tools for recording the various counts. Discuss how many are in the collection and come to agreement.	• Collection to count • Tools for recording, such as sticky notes or index cards

To the Teacher

Counting concepts are at the heart of kindergarten mathematics, and educators who both research mathematics learning and write for teachers have made counting central to considering early mathematics. The notion of repeated, ongoing counting of collections of objects is the subject of two books that we recommend: Franke, Kazemi, and Turrou's (2018) *Choral Counting & Counting Collections* and Liu, Dolk, and Fosnot's (2007) unit of study *Organizing and Collecting*, which is part of the Contexts for Learning curriculum series. Both of these resources provide clear ideas for how to incorporate counting collections of classroom and everyday objects into the long-term trajectory of the early childhood classroom. In this activity, we outline a skeleton of what this work can look like, but the research that Franke et al. have conducted in this area makes clear that counting is less an activity than a routine.

One of the key ideas in counting collections is that the size of the collections can and should be tailored to students' development of counting concepts, such as the oral counting sequence and one-to-one correspondence. When students are getting started, collections will probably need to contain fewer than 10 objects, and for some students perhaps no more than 6. But this number can grow toward and beyond 20 objects as students' counting capacity grows. Indeed, we return to this idea in the final big idea in this book with the premise that students can count toward and beyond 100 objects later in the year. But this growth is only possible with ongoing opportunities to count, recount, and learn to organize objects for counting.

The collections you provide for students to count should be ordinary classroom objects, such as books, cubes, bears, crayons, markers, counters, tiles, or clothespins. Collections should be offered in some kind of container, such as a bag, bowl, bucket, or tray. You'll want to have plenty of collections ready to offer students when they are ready to try a new group of objects. Be sure these collections have a varied number of objects, depending on what your students might be ready for, so that you can direct students toward collections that will stretch their counting capacities.

Activity

Launch

Launch the activity by asking students, Have you ever wondered, "How many?"? Ask students, What kinds of "How many?" questions have you wondered? Give students a chance to turn and talk to a partner about the kinds of situations where they have wondered, "How many?" Listen in as students talk, and revoice some of



Examples of small collections
for students to count

the questions you hear. You might hear such questions as “How many cookies can I have for dessert?” “How many people are in our class?” “How many stairs are in the staircase?” “How many pets do you have?”

Tell students that asking “How many?” is something that we all do every day. Tell students that today they are going to start figuring out ways to answer those questions by counting groups or collections of objects.

Show students a small group of objects from your classroom. The group should contain more than 5 and fewer than 10 objects. Ask, If my partner and I want to figure out how many there are, what could we do? Model how you and a partner count the group and check with each other by asking, Do we agree? Tell students that they

will be working with a partner to count a collection of objects together and that they should come to agreement about how many there are. This might mean counting the group several times until they can agree.

Explore

Provide each partnership with a group of objects to count. Partners work together to count the group of objects and figure out how many there are. Students work to come to agreement about the count, resolving any differences through recounting together.

As you observe students counting, pay attention to *how* they are counting and look for strategies you might want to have them share with the class. You'll also want to attend to opportunities to sit down with students and help them with the number sequence or to resolve disagreements.

Offer students new collections to count as they finish with and agree on the number objects in the group they've been given. Over time, students can learn to label their count with a sticky note or index card. Students can then get a different collection to count and check whether the label attached is correct.

Discuss

Bring students together in a circle to discuss how many objects they found in their collections and how they know. Invite students to share some of their ways of counting. You'll want to select students to share who have useful or new strategies for counting or who resolved a disagreement about the count in a way that others can learn from. When groups share, ask them to put their collection in the center of the circle and show how they counted.

Notice aloud the structures students use to count and organize for counting, such as moving objects as they count them, lining them up, touching each one, or saying the number name as they touch. By highlighting these counting concepts, others will be able to see new ways they might count their collections the next day. You may want to create a class Ways We Count chart to record some of the strategies students find useful.

Extend

As we said in the To the Teacher section, we see this as an ongoing counting routine that can extend for weeks. However, you can also extend this activity by setting up a counting station on a table in your classroom. Have a collection that students can count and recount at the station that might be slightly larger than what they counted in this activity. In small groups, they can visit the station over the course of several

days and try to figure out how many objects make up the collection. Over time, students can learn to label their count, and at the end of the week, as a class you can discuss, using the various labels students made, how many there might be. You can create a shared visual proof of how many by asking students to model their counting and to recount until the class can agree. In some ways, this activity works like a number talk stretched across a week.

Look-Fors

- **Are students counting with one-to-one correspondence? Where does one-to-one correspondence break down?** One-to-one correspondence refers to assigning one number name to each object, typically accompanied by touching that object. This requires coordinating the rhythm of the counting sequence with the rhythm of touching the objects, and it reflects an understanding that the numbers refer to the objects being counted. However, one-to-one correspondence isn't something that students have or lack wholesale; students can have one-to-one correspondence up to a point and then have it break down. For instance, you might see students count with one-to-one correspondence through six and then the oral counting sequence begins to go faster or slower than their hands, leading to an inaccurate total count. The more students work to stretch their count a bit beyond their threshold for one-to-one correspondence, the further that threshold will extend. Expect that students will need to count repeatedly and that this is active learning work. Use your observations of students' threshold for one-to-one correspondence to choose collections for them to count that are just beyond what they can comfortably count.
- **Where do students struggle with the counting sequence?** You may see students have one-to-one correspondence, but the number names they assign to each object are not in the conventional order. For instance, students may count a set: "1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 15, 18, 19, 15, 18, 19, ..." You may observe gaps in the sequence with skipped numbers, often in the teens; reversals of order; or repetitions. This indicates that students need support with the counting sequence, not the concept of counting. In English, the counting sequence through 12, and to some degree through 20, does not have an internal logic; students simply have to remember these words. Anything that has to do with memory is subject to forgetting. If you see students experiencing difficulties with the counting sequence that create disagreements among partners, you can support them by sitting down to do the counting

sequence together, but be sure not to take over the job of touching or moving the objects. We encourage you to incorporate some choral counting routines to support students in learning the oral counting sequence (see Franke et al., 2018).

- **Are students moving objects to track their count?** To keep track of the objects that they have already counted and those that they have not yet counted, it is useful for students to move objects as they count. This is one of the first organizing-to-count ideas that students learn, which in the future can also include lining objects up and grouping objects in 2s, 5s, or 10s. If you notice that students are not moving objects and also struggling to agree on a count, you might ask, Is there a way to keep track of which object you've counted? If you do notice students moving objects to count, point this out and ask, Why did you move the objects like that? Invite these students to share their counting strategy with the class during the discussion to highlight how moving helps us keep track of our counting and come to agreement.
- **How are partners resolving disagreements about how many objects are in their collection?** Conceptually, it is important that students recognize that if two people each count the same collection, they should find the same number of objects. Initially, students are likely to simply accept that one person counted eight and the other counted nine. Pose questions to challenge this acceptance and introduce the need to come to agreement, such as, "So, are there eight bears or nine bears?" You will likely need to prompt students to count again to check in order to support them in learning that recounting is a strategy for resolving disagreements or becoming confident in the count. You may also be able to ask, How could we figure out whether it is eight bears or nine bears? Encourage students who disagree to make their counting clear to each other. You might say things like, "Let's show him how you counted. Why don't we put the bears in the middle here. If you count slowly, we can see how you did it." These kinds of moves encourage precision, recounting, and organizing, while at the same time the observing partner learns to follow someone else's count visually. You can also ask them to count together, both touching and counting aloud so that they can each hear where their counts differ and discuss it. If students cannot come to agreement through recounting, bring the collection to the class during the discussion and invite others to help with resolving the disagreement. You might say to the class, "We cannot figure out whether this group has eight bears or nine bears. Can you help us? What can we do to figure it out together?"

Reflect

How can we count a group of objects so that we know how many there are?

References

- Franke, M. L., Kazemi, E., & Turrou, A. C. (2018). *Choral counting & counting collections*. Portsmouth, NH: Stenhouse.
- Liu, N., Dolk, M., & Fosnot, C. T. (2007). *Organizing and collecting: The number system*. Portsmouth, NH: Heinemann.

How Many Do You See?

Snapshot

Students use images to answer the question, How many do you see? in multiple ways.



Connection to CCSS
K.CC.5, K.CC.4, K.CC.1, K.CC.3

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5–10 min	Show students the School Supplies image and ask, How many do you see? Students share how many they saw and what objects they focused on. Highlight that there are many things to count in the picture and many ways to count them.	School Supplies sheet, to display
Play	15–20 min	Partners use a new image to explore the question, “How many do you see?” Partners label their counts on the image and try to find many different ways to count what they see. Students can move on to new images when ready.	- How Many Do You See? images, multiple copies per partnership - Tools for labeling
Discuss	10 min	Beginning with the shared image, students show what they counted and how they counted what they saw. Invite students to share the ways they labeled their counts on the image. Highlight language to make clear what is being counted.	How Many Do You See? images, to display

Activity	Time	Description/Prompt	Materials
Extend	Ongoing	Make a station or center with images for students to count with a partner how many they see. Students label their counts on the sheet.	• Images, either from the set provided here or your own • Tools for labeling • Optional: sheet protectors

To the Teacher

In this activity, we build on the work students are doing counting collections, by counting objects in an image. This is more challenging because students cannot move the objects as they count, and touching an image is not the same experience as touching a three-dimensional object. You will likely find that students can readily count fewer objects in an image than they can in a collection. We draw on Christopher Danielson's (2018) *How Many?* which contains rich images, mostly from the kitchen, for students to count.

One key feature of these images and those we have presented here is that there are several different things to count, and ways to count them, embedded in a single photo. This accomplishes a number of things simultaneously. First, the images are flexible, with multiple answers and ways of seeing. Second, students have choices, making it possible for them to count smaller or larger groups. Third, these images encourage students to attend to the unit of their count. That is, “three” isn’t enough of an answer for others to understand; we have to know “Three what?” Finally, the best images encourage students to look at the same objects in multiple ways. For some images, students might consider both the parts and the whole, as in a watercolor set with 12 colors in one case. In others, students might see both what is present and what is missing, as with a pencil case that is partially filled.

We have provided some images, and we encourage you to look at those in Danielson's book as well. However, you and your students can make your own images using materials in your classroom or found in your environment. Alternatively, you might find illustrations in favorite picture books that can be used for this activity. When choosing or composing images for this activity, consider the features named in the previous paragraph to make your images flexible and reasonably complex for young children to count in multiple ways.

Activity

Launch

Launch the activity by showing students the School Supplies image on the document camera. Ask, How many do you see? Give students a chance to look and think. Ask students to give a private thumbs-up on their chests when they have something to share, much as they would in a number talk. We encourage plenty of wait time. Ask students to share how many they saw. Students may simply say a number; press them to be clear about the unit by asking questions such as, “Three what?” and having them restate what they saw, naming both the number and the unit. Draw attention to the idea that they can count many different things in this picture, such as paper clips or pencils.

Tell students that today they will be looking at images with a partner and answering the question, How many do you see?

Play

Provide partners a picture from the How Many Do You See? image set. Students use the image to answer the question, How many do you see? Students try to come up with many different things to count and agree on how many there are. Students might label their counts on the image, by circling groups they counted and adding number labels or by writing at the bottom of the sheet the number and word for the objects they counted. To support the later discussion, provide all partnerships with the same first image, but you can give students different images after this if you think they are ready to explore others.

During this work, promote wondering about the images. One way that students might start to look at these images is by first noticing a group of three objects, which is to say that students begin with an answer. But another is to begin with a wonder, such as, How many colors are there? Instead of providing students with your wonders, ask them, What do you wonder about this image? What could you count?

Discuss

Gather students together and show on the document camera the image that everyone counted. Discuss the following questions:

- How many did you see?
- What did you count?

- How did you know how many there were?
- What did you do when you and your partner disagreed?
- How did you record your counting?

As students share, highlight language they use that makes clear what they counted. They may not always have language for what they counted, and this is a prime opportunity to provide words to describe these objects or parts. Invite students to share other images they counted. You might ask, Did anyone count something interesting in another picture? What did you count, and how many were there?

Extend

This activity can be turned into a station or center where you can place a small set of images for students to count together. Partners can discuss an image, pointing out how many they see and what they are counting. You can either provide enough copies for students to work on recording their counts, or place images in sheet protectors so that students can record with dry-erase markers.

Look-Fors

- **Are students specifying what they are counting?** Units are pivotal in this activity, because we cannot take it for granted that we are counting the same things, as we may be when counting collections or in a dot talk. If you notice students only using numbers with one another, you can ask them directly, “Four what?” or you can encourage the partner to ask this question by saying something like, “Ask your partner, ‘Four what?’” Listen for students who are describing what they are counting specifically, such as “12 colors in the paint tray,” and invite them to share this during the discussion.
- **How are students keeping track of their counts?** In the Visualize activity, we discussed the role of moving and organizing objects to count. But in this activity, movement isn’t possible, and students will need other strategies to support them in keeping track. For small quantities, they may need no support; but with larger groups, they may find that marking counted objects with a pencil is useful. Ask questions to help students think about this issue if they seem to be struggling, such as, How can you keep track of which objects you’ve counted? Be sure to highlight innovative strategies during the discussion.
- **How are students labeling their counts?** Producing numerals, and remembering which numerals match to the number words, is a difficult task on its own.

This activity provides a real context for practicing this work. Although we don't want students to get bogged down in too much handwriting labor, asking students to label even one of their counts will support them in making connections between images of objects, number words, and numerals. Students may also invent interesting ways of labeling with circles, lines, and arrows, which are useful to share with others during the discussion.

Reflect

What makes counting “how many” in a picture different from counting a collection?

Reference

Danielson, C. (2018). *How many?* Portland, ME: Stenhouse.



School Supplies



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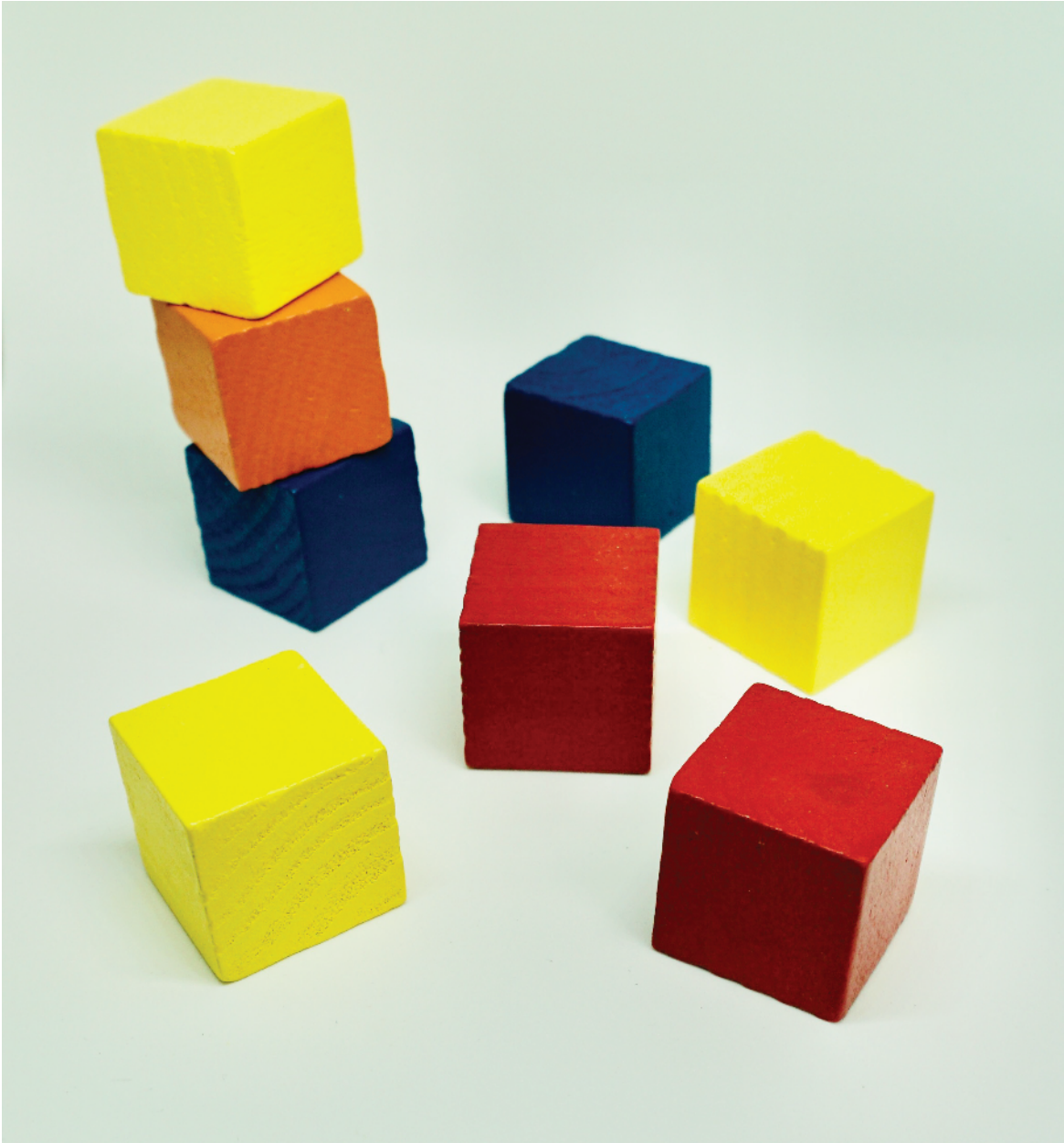
How Many Do You See?



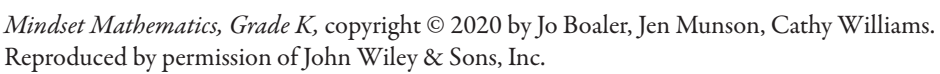
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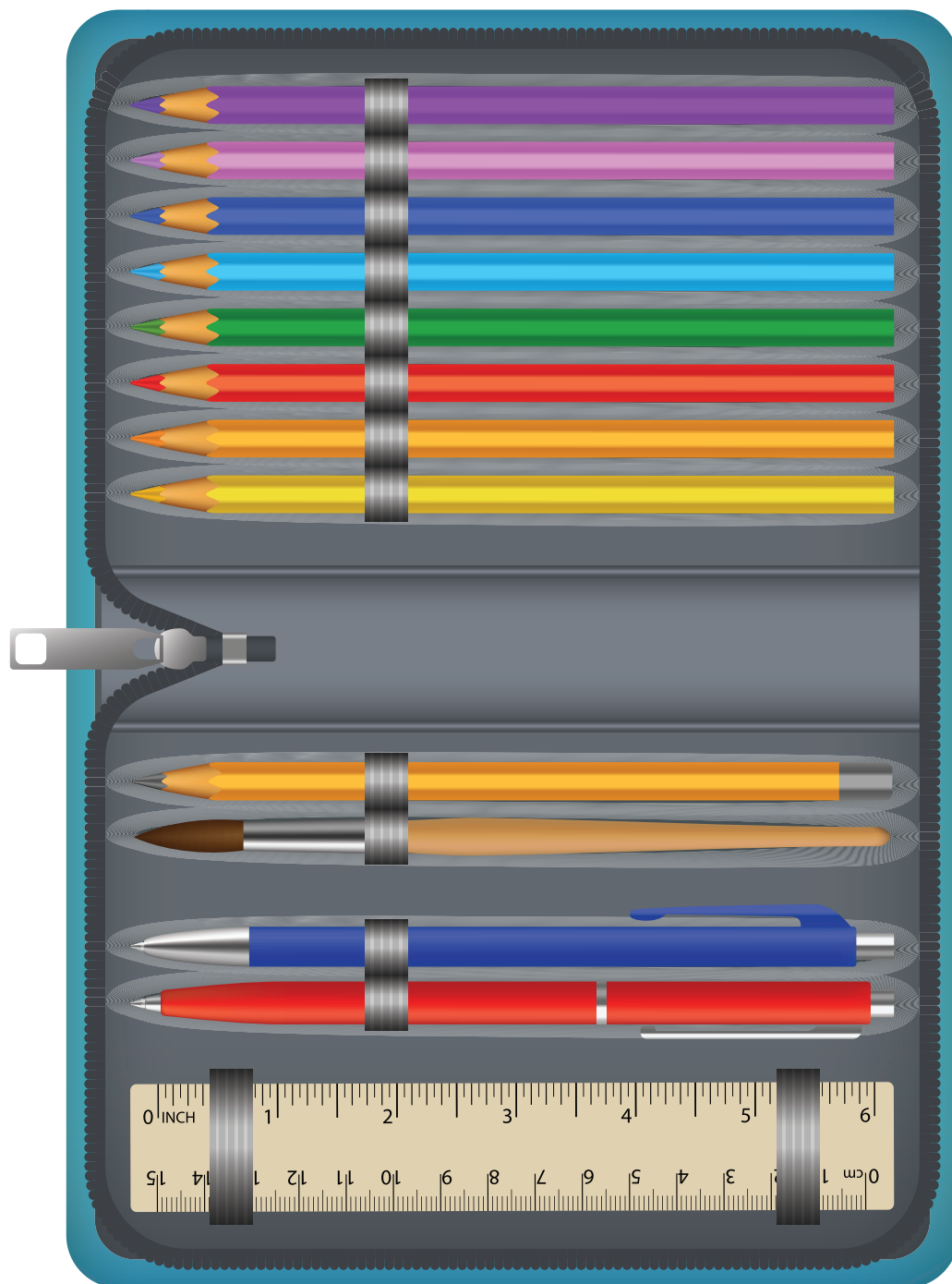


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How Many Do You See?



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Making a Counting Book

Snapshot

The class works together to construct a counting book using photographs they take of numbers in their world.



Connection to CCSS
K.CC.4, K.CC.5, K.CC.1, K.CC.3, K.CC.6

Agenda

Activity	Time	Description/Prompt	Materials
Launch	10 min	Show students one or more examples of counting books and discuss how the authors show each number. Tell students that they are going to make a class counting book. Decide as a class what features each page should have.	Examples of counting books
Explore	Multiple days	Assign each partnership one or more numbers for which to make a page for the class counting book. Provide partners with a digital camera, if possible, and take students to spaces where they can look for these numbers in the world. Partners then take photos and construct pages for each number they are asked to represent. Construct a class counting book.	- Digital camera, per partnership, or tools for sketching, such as clipboard and colors - Tools for constructing book pages, such as access to computers or construction paper, glue sticks, and colors, depending on the type of book you construct

Activity	Time	Description/Prompt	Materials
Discuss	15 min	Read aloud your class counting book. For each page, ask the class how they see the number represented. Ask the partners who constructed the page how they chose the image to represent the number.	Class counting book
Extend	Ongoing	Make the class counting book available for students to read and take home to share with families. Consider adding pages to the counting book as students extend their counting, particularly for decade numbers.	• Class counting book • Tools for adding to your class book (varies depending on the type of book you constructed)

To the Teacher

In this investigation, we invite students to coconstruct a counting book as a class, an activity inspired by a dear colleague and Pre-K/K teacher, Thora Balk. Counting books are a staple of early childhood, and many excellent examples exist, such as Donald Crews's (1995) *Ten Black Dots*, Lois Ehlert's (1990) *Fish Eyes*, and Zoran Milich's (2007) *City 123*. The most useful counting books, from a math learning perspective, include clear, easy-to-count images in bold colors and with simple shapes. These images are coupled with numerals, and sometimes the number word. It is useful when the number highlights the unit being counted, such as Ehlert's "1 green fish." Milich's book goes one step further, showing the counting sequence all at once with a growing string of dots. We encourage you to read some of these books aloud in advance of launching this investigation so that students are familiar with the genre and can think about the features they might want to include in their own book.

In advance you'll need to decide the range of your class counting book, taking into account how many students you have and the numbers they are comfortable working with. Your book could be 1–10, 1–12, 1–20, or go even higher. Students will work in pairs to construct one or more pages for the book, and we encourage you to assign numbers to students mindful of the demands of each number. For instance, it would make sense that the partnership producing the page for number 1 also produce another page, but a partnership making a page for 13 might only make that page.

It is our hope that to construct a page for the book, students can take photographs in the world to represent their number, whether they stage the photos by making a collection of objects in the classroom or go on a number hunt on the playground looking for preexisting groups of objects to represent their number. For instance, in one classroom, we saw students find twigs on the ground with five branches to represent 5 or a set of monkey bars to show 8.



Examples of student photos

The form that your class book takes will depend on your resources and time. You can ask students to physically produce pages, pasting photos on paper and hand-writing labels that can then go in sheet protectors in a binder. You could also scan these physical sheets to print multiple copies of such a book. Or you can use technology to produce a book through a photobook printing service online, or even make a virtual book with slides. However you decide to produce it, consider how you can make the class book available for students to read after it is constructed.

Activity

Launch

Launch the activity by reminding students of all they have been counting with objects and in pictures in the Visualize and Play activities. Show students one or more examples of a counting book, and talk about how these books are organized counting up, with (usually) a page for each number. Show students one page for a particular number, such as 5, and ask, What do the authors put on this page to show 5? Give students a chance to turn and talk, and then take some student ideas. The author may show an image, the numeral, and maybe the number word. Be sure students see that each page follows the same format.

Tell students that they are going to make a counting book as a class, with partners in charge of different numbers. Ask the class, What should we show on each page to help our readers understand the number? Should we have an image? The numeral? The word? Anything else? Come to agreement as a class about what each page should look like. We encourage you to include at least an image and the numeral.

Explore

Assign each partnership one or more numbers for which to make a book page. Provide students with digital cameras. If you don't have access to digital cameras, you can ask students to sketch what they see instead. If you decide on this route, you'll need to provide students with clipboards and colors. Students can capture multiple pictures for their number(s). As a class, you'll need to decide whether students can include multiple pictures on their pages or whether they should choose the clearest or most interesting one for the page.

Take students to places where they are able to look for numbers in the world. This could be your classroom, the school, a playground, a park, or the community. Give students time to explore. They will do a lot of counting, estimating, and comparing in the process.

Once back in the classroom, provide students with access to the materials that match the way you'd like to construct the book, whether those tools are physical (such as construction paper and glue sticks) or digital. Support students in thinking about how to label their images, and remind them of the agreement the class came to about what features they need to include. You may want to support students in constructing their pages in small groups over several days.

Discuss

Read your class book aloud. For each page, ask students to come up and show how they see each number in the images that students created. Ask the creators of each page to share something about how they chose this image or saw it in the world. After you've read your book, ask, What do you think makes our book a helpful counting book?

Extend

Make your class book available in a station or classroom library to be read by students. You could allow students to check it out to take home and share with their families, or read it aloud at a family event at school.

Consider adding on to your book later in the year as students become more comfortable with larger numbers. You could choose to add numbers consecutively (such as 21, 22, 23, 24 . . .) or to highlight only the decade numbers (30, 40, 50, 60 . . .). You could add a page for 100 as a class on the 100th day of school.

Look-Fors

- **Are students thinking expansively about where they might find numbers in their world?** Where students can look for numbers in the world depends on what spaces you take them to for number hunting. However, in every space, some numbers are more obvious, and others are more subtle. For instance, on the playground it may be that students see four swings more readily than a cluster of four acorns on the ground or four points on a poplar leaf. Encourage students to look expansively for examples of their numbers and take many photographs. One of the central goals of this investigation is to promote seeing the world mathematically and noticing numbers everywhere. Even if students quickly find one example of their number, which may be the case for students representing numbers less than 5, challenge them to look for hidden places where they might find their number. This is a good argument for allowing students to include more than one photo on their number page,

so both they and the reader can see multiple ways that the number might be represented in the world.

- **As students look for their numbers in the world, are they counting with precision to check?** As students look in the world for their number, they will likely develop hunches about where their number might be found. These hunches are the foundation for estimation. Students might see a stack of books or a branch and think, “That looks about right.” It is worth making these hunches explicit and naming them as estimation, as in, “You saw that stack and thought it looked like about six books. You *estimated* it was about six books. That was a useful way of looking for 6.” The next step is for students to check their estimates. Valuing the estimate itself means that even when the stack of six books turns out to have five or seven books in it, you can highlight this mathematically useful practice. Encourage students to check their estimates more than once, particularly with items where touching or keeping track of the count may be harder, as with monkey bars high above their heads or the branches on a twig.
- **Are students using any comparative language as they look for their numbers in the world?** Embedded in this counting work are opportunities for students to develop ideas about number comparisons and to connect these to their estimates. When students check the numbers they see in their world, they will likely find times when what they see is more or less than the number they were looking for. In these moments, do you notice students spontaneously using comparative language? Students may use language like *too much*, *too big*, *too many*, *more*, *less*, *not enough*, or *too small*. Ask students questions about their reasoning, such as, How do you know? If students simply reject something they counted as not right, press them to make comparisons by asking questions like, How do you know? Or, Is it more or less than your number?
- **Are students attending to the features of a counting book that make numbers clear?** When your class defines the structure of your counting book, students will need to attend to that structure to create a page for the book. They will need to focus on the elements that need to be included and whether there is any format to how they are arranged. This is a form of patterning that we often don’t talk about in mathematics, but structured books are patterns. In essence, we are asking students to define and then reproduce the pattern in the form of their pages, and this is far from simple work. Invite students to think about the pattern of the book and

the reason for its elements, rather than just telling them what to produce and how. Ask, What parts does your page need? Where do we need to place those parts so that they fit with the other pages in the book? What did we agree on as a class?

Reflect

Where do we see numbers in our world?

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