CHAPTER 1

Early Models

"Nature, and Nature's Laws lay hid in Night.
God said, Let Newton be! And All was Light."

—Alexander Pope

"Beelzebub begat Law
Law begat the Mississippi
The Mississippi begat the System (etc.)"

—Het Groote Tafereel der Dwaasheid
(The Great Mirror of Folly)

The mathematical models used by quants are based on ideas and concepts developed by generations of economists. They in turn were heavily influenced by physics. But is it really possible to model the markets as a kind of physical system, or is quantitative finance more like a set of mathematical tricks for betting on markets? This chapter traces the development of economics; looks at the basic assumptions such as equilibrium and rationality that have shaped both economics and finance; and considers the dual nature of quantitative finance, as exemplified by two men – John Law and Isaac Newton.

In 1705, Scotland was contemplating union with its neighbor England. The English economy was riding high, and Scotland's leaders thought this might be an opportune moment for a merger. However, not everyone thought hooking up was a good idea. One

person who argued against it was the banker, gambler, and social climber John Law. He went so far as to propose an entirely new monetary system for Scotland, which he claimed would go beyond the English system and in a stroke solve his country's monetary problems while boosting trade.

Part of England's success was due to its newly created central bank, the Bank of England, and efficiencies created by the introduction of bank notes. However, Law thought he could do better. According to him, the problem with this new English paper money was not that it was too radical, but that it was not radical enough, since it was still exchangeable for gold. Its supply was therefore determined not by the needs of the economy, but by the quantity of precious metal that happened to be in circulation at the time. In his text *Money and Trade Consider'd with a Proposal for Supplying the Nation with Money*, he argued that Scotland needed a central bank of its own, that issued its own paper currency, but one that was backed only by the state rather than by precious metal. After all, according to this son of an Edinburgh goldsmith, money was just a "Sign of Transmission," like a casino chip, and not a store of real wealth.

The stakes for Law were greater even than the questions of Scottish independence or the meaning of monetary value. Ten years earlier, he had been charged with murder following a duel in London. After being imprisoned, he soon escaped and fled to Amsterdam. For several years he had toured around Europe, supporting himself and his young family by gambling (a trained mathematician, he claimed to have a system), before returning to Scotland. But if that country joined with England, he would have to leave or find himself back in jail.

This time, the dice did not fall in Law's favor. His radical monetary proposal was rejected by parliament, the union with England went ahead, and Law was again on the run from the law.

He set himself up in Paris, playing cards at all the fashionable salons. His system was extremely successful – so much so that he drew the attention of the Chief of Police, M. d'Argenson, who expelled him from the city. Again he hit the road, touring through Germany and Italy in a coach, amassing considerable wealth from his winnings; his prowess at gambling becoming something of a legend. When the "Sun King" Louis XIV died, leaving his country with a massive debt (incurred from wars and the construction of his palace at Versailles) and a bankrupt treasury, Law saw an opportunity and returned to

France. There was a shortage of money, and he had the answer. He quickly won over the regent, Philippe d'Orléans, who took a chance on the Scotsman and appointed him as Controller General of Finances – perhaps with the hope that his "system" would work as well for the economy as it did at cards.

Monetary Alchemy

Law's plan for the country – and he did not lack ambition – consisted of two parts. The first was to set up a state bank financed initially by himself, the Banque Générale, that would issue paper money redeemable in gold or silver. The bank was hugely successful, and its notes soon attracted a premium just for their convenience over coins. The second, which followed two years later, was to establish a company called the Mississippi Company, that would be granted a royal monopoly on trade with Louisiana – a vast region that encompassed the entire Mississippi River Valley.

Neither idea was new. The Bank of England and the Bank of Amsterdam already issued paper receipts for gold that could be traded as money. The Mississippi Company was modeled on the East India Companies of Britain and Holland. Law's brilliant idea was to connect the two, and unleash the alchemical power of paper money. Paper shares in the company could be bought using the paper money produced by the bank, in what seemed like a kind of perpetual-motion machine. In 1718 the bank was nationalized, becoming the Banque Royale; with this royal approval obtained, it was then announced that its notes would no longer be redeemable for precious metal.

Money was finally untethered from metal, its value determined instead by the authority of the French crown. A positive consequence was that the state could print as much money as it needed to satisfy the ravenous public appetite for shares, as people flocked from all over the country and abroad to take part in the economic miracle of Law's system. With all this money circulating around at a ferocious rate, the economy boomed. The word "millionaire" came into use for the first time. In 1719 alone, the Company share price vaulted from 500 livres to over 10,000 livres. The dropping of the dead-weight connection to metal also released any restraints on Law's bounding ambitions. In no time he was arranging for the Company to buy the national debt, and have the right to collect taxes. This required

issuing many more shares, and many more paper notes to buy them with. Which is when Law's system started to reveal its flaws.

While Law was certainly correct that money serves as a "Sign of Transmission," its value also depends on the confidence and trust of the community, and he had made the same mistake that he had made as a gambler in Paris, which was to fail to arrange buy-in from all the relevant players. Then it was the Chief of Police, d'Argenson, now it was the business and banking community (which included d'Argenson, who had become a prominent businessman). Rumors began to circulate that Louisiana was not quite the wealth generator it was cracked up to be, and Mississippi Company shareholders began to suspect they were being sold down the river.

The trip down was just as brief and thrilling as the way up. Suffice to say that, as the Company's share price drained away, and the value of the bank's paper notes approached zero, Law was again drummed out of Paris, and the country, and ended up near destitute in Venice. The story ought to serve as a cautionary tale for present-day central bankers. Oh, except that these days no bankers, central or otherwise, ever end up destitute.

Gold Standard

While Law was introducing the French to the benefits, perils, and general excitement of fiat currencies and financial innovation, Isaac Newton was serving as Warden of the Mint in England. Newton is of course best known for his famous contributions to physics, but he worked at the Mint from 1696 until his death in 1727. It is safe to say that his approach to finance was the opposite of Law's. At exactly the same time that Law was arranging to delink the livre from gold or silver, Newton was putting the pound on the gold standard, where it would remain for the next couple of hundred years.[1] While Law was issuing what some considered to be fake money, Newton was sending counterfeiters to their death. One wonders what he would have said about the situation in France, from his position at the Tower of London. Perhaps he felt some sympathy with Law's fall from grace; he did manage to lose £20,000 himself (over £2 million in today's money) on his investment in the South Sea Company, the British version of the Mississippi Company.

[1] By accident. He set the exchange rate for silver too high, so silver coins left the country.

The two certainly had completely different personalities. Here is a portrait of the young John Law by journalist John Flynn: "He got access to the smartest circles. He was a young man of education and culture, handsome, quick-witted, a good athlete excelling at tennis, a graceful dancer, and a redoubtable talker. He spent his mornings in the city, where he got a reputation for skill in speculating in government paper. He passed his afternoons in the parks, his evenings at the opera or theater, and the later hours at the routs, balls, masquerades, and gaming houses. He played for high stakes and won large sums. He was a man with a system. Had he lived in our time he would have been in Wall Street with an infallible formula for beating the market."[2] Perhaps he would have launched a hedge fund, or penned a bestseller about his "system."

Isaac Newton, in contrast, was a decidedly more solitary type. As a child, he showed great talent at making models, such as a working windmill. This skill later came in useful while constructing his own experimental apparatus, including a new design of telescope. He attended Cambridge University, but his most creative period came when the university was closed for two years because of the advancing plague, and Newton returned to his home in Lincolnshire to work alone. It was there that he claimed to have been prompted to discover the law of gravity after seeing an apple fall from a tree. Throughout his life he had a passion for alchemy and mysticism; in fact, most of his output consisted of religious writings, including a 300,000-word tract on the Book of Revelation.[3] He was famously anti-social and incommunicative; if no one showed up for his lectures, he just gave them to the empty room. There is no record of him being an expert dancer, or really fun at parties. As economist John Maynard Keynes wrote, he became instead the "Sage and Monarch of the Age of Reason."[4]

Researchers at Oxford and Cambridge have suggested that Isaac Newton may have had Asperger's Syndrome.[5] There is quite a business in such historical psych evaluations nowadays (see Box 1.1), but this one has a ring of truth about it. Often those with Asperger's Syndrome have a very narrow field of interest, with little curiosity in or appreciation of the bigger picture. They can exhibit intense concentration and understanding, and in many cases there is increased

[2] Flynn (1941).
[3] Manuel (1974).
[4] Keynes (1946).
[5] Muir (2003).

intelligence in areas such as mathematics. Which perhaps would explain why Newton was better with celestial mechanics than the financial sort.

These two contemporaries, Law and Newton, represent two aspects of the relationship between mathematics and finance. Mathematical finance is about using objective, rational, Newtonian models to simulate markets and make predictions about their future evolution. Quants are often described as modern-day wizards, hidden away in secret laboratories, who use mind-bending techniques inspired by areas such as quantum physics and string theory, coupled with the power of massive computers, to find hidden patterns in the markets. As Scott Patterson puts it in his book *The Quants*: "Think of white-coated scientists building ever more powerful devices to replicate conditions at the moment of the Big Bang to understand the forces at the root of creation."[6]

However, these scientists are trying to make money, not discover the next Higgs boson. (Juan Maldacena, Professor of Theoretical Physics at the Institute for Advanced Studies at Princeton and winner of many prizes for his work on such things as black holes, has said that finance is harder than physics. However, he has also given a public lecture in which he uses exchange rates as an analogy to explain the very same boson.) Mathematicians, like Law, are attracted to practical finance because they think they can use a system to beat the market, or even create an entirely new one. As seen later, their financial innovations often amount to creating new forms of credit, which like Law's scheme boost the money supply, at least for a while. In place of paper money, they invent credit default swaps or collateralized debt obligations. ("Make your very own 'credit default swap' and find out how to create money out of thin air!" as guides in a bus tour around the City of London now shout.[7]) They see the markets, with their rhythms and patterns, as a kind of music, which they can shape and control – and would agree with former CitiGroup CEO Chuck Prince who famously said, in the midst of the credit crunch, that "As long as the music is playing, you've got to get up and dance."

As we will see, it is the tension between these two aspects that drives mathematical finance, in both its inventiveness and creativity, and its tendency toward self-destruction.

[6] Patterson (2009, p. 8).
[7] Gitlin (2014).

The Systems of Nature

After his losses in the South Sea debacle, Newton famously said: "I can calculate the movement of the stars, but not the madness of men." While Newton may not have tried to calculate the markets, and preferred chemical alchemy to the financial kind, he probably did more to shape the world of mathematical finance than any other scientist. His law of gravity, coupled with his three laws of motion, provided an archetype for a successful mathematical model that would influence not just areas such as physics and chemistry, but also social sciences including economics, and serve as an inspiration for quants to the present day.

One person who appreciated the power of Newton's approach was Adam Smith. He is of course best known for his book *The Wealth of Nations*,[8] which was the first to present economics as an objective, rational science, separate from areas such as ethics and political science. Some insight into his motivations is provided, however, by an earlier work on astronomy, written around 1758 but not published until after his death, in which his examination of "all the different systems of nature" culminates in a celebration of "The superior genius and sagacity of Sir Isaac Newton." He was less impressed by John Law. As he wrote in *The Wealth of Nations*, "The idea of the possibility of multiplying paper to almost any extent was the real foundation of what is called the Mississippi scheme, the most extravagant project both of banking and stock-jobbing that, perhaps, the world ever saw." (Smith would no doubt have been surprised to learn that we now organize our economies around Law's idea of a fiat currency, which was ahead of its time, rather than Newton's gold standard.)

Smith saw philosophy as a kind of calming device for making sense of the world, with its random events and its John Laws, its "chaos of jarring and discordant appearances." The beauty of Newton's method was the way in which it took a simple idea, such as gravity, and showed how "all the appearances, which he joins together by it, necessarily follow."

In the same book, Smith makes his first mention of the invisible hand. However, the passage was about the tendency for polytheistic religions to interpret events as being caused by gods: "the invisible hand of Jupiter." It was only later that he attributed this miraculous

[8] Smith (1776).

power to the markets. In *The Theory of Moral Sentiments*, he used the term in the context of wealth distribution: the rich "divide with the poor the produce of all their improvements. They are led by an invisible hand to make nearly the same distribution of the necessaries of life, which would have been made, had the earth been divided into equal portions among all its inhabitants, and thus without intending it, without knowing it, advance the interest of the society." (We wonder if he asked the poor.) Finally, and most famously, the phrase pops up again in *The Wealth of Nations*, in which – in a section on trade policy – an individual is again "led by an invisible hand to promote an end which was no part of his intention."

No one paid any attention to the metaphor until 1948, when Chicago School economist Paul Samuelson published his textbook *Economics*, which would go on to become the best-selling economics textbook of all time, translated into over 40 languages.[9] As he paraphrased: "Every individual, in pursuing only his own selfish good, was led, as if by an invisible hand, to achieve the best good for all, so that any interference with free competition by government was almost certain to be injurious." Which is when widespread use of the term, both in academic papers and general use, suddenly took off.[10]

Box 1.1 On the Couch

As mentioned above, it's unreliable to psychoanalyze people who aren't around to lie down on the couch, and sometimes it's annoying – as in the 2014 film *The Imitation Game*, in which Benedict Cumberbatch, the actor playing mathematician Alan Turing, might as well have worn a button saying "Hi, I have Asperger's!" Also, we're not psychologists and have no idea what we're talking about. But Adam Smith does seem worth a look.

From our case notes, it seems that tales abound of Smith's bizarre character. Friendly and good-tempered, he was also, according to one friend, "the most absent man in Company that I ever saw, Moving his Lips and talking to himself, and Smiling."[11] He did things like absentmindedly walk into a tanning pit, from which he needed to be rescued, or go for a stroll in his nightgown and end up 15 miles outside town. He was frequently ill and his doctors diagnosed him as a

[9] Samuelson (1973).
[10] Kennedy (2005).
[11] Alexander Carlyle, quoted in Özler (2012).

hypochondriac. He had no known serious romantic relationships, and lived with his mother (his father died two months after he was born) until she died at the age of 90, just six years before his own death in 1790. As his biographer Dugald Stewart noted, Smith was "certainly not fitted for the general commerce of the world, or for the business of active life."[12]

Usually these quirks are presented as the harmless foibles of a genius – but there does seem to be a connection with this invisible hand business.

As UCLA's Şule Özler wrote in the journal *Psychoanalytic Review*, Smith was financially dependent first on family income, and then on "rich businessmen, gentry, intellectuals, and aristocrats for teaching positions and his pension."[13] And there is a striking contrast between his life and his economic theories. "Denying his reality of lifelong dependence on his mother and benefactors, Smith appears to have idealized independence," according to Özler. The invisible hand, after all, only works if everyone acts independently to further their own interests, without collusion. There is no room for things like money, power, or the fact that we can be financially dependent on one another.

Smith found solace and refuge in Newtonian laws, which treated people as independent atoms, and he turned the market into a kind of parental figure that always knows what is right. Rather like a lot of modern economics then (whose practitioners often have about as much experience as Smith of "the general commerce of the world").

Rational Mechanics

Smith's work was influential on the USA at the time of its formation – the Founders were early readers of his work – and remains so today. Economist George Akerlof describes the "central ideology" of the United States as conforming to "the fundamental view of Adam Smith," which even today "drives huge amounts of policy" (he should know, being married to Federal Reserve Chair Janet Yellen).[14] According to this picture, the market is made up of firms and individuals acting to further their self-interest by buying and selling. If a good or service is too expensive, then more suppliers enter the market, supply increases, and competition drives the price down to its

[12]Hamilton (1858, p. 77).
[13]Özler (2012).
[14]Fleischacker (2002), Kiladze (2015).

"natural" level, which serves as a "center of repose." If instead the price is too low, then suppliers go broke or leave the market, and the price goes up: "The natural price, therefore, is, as it were, the central price, to which the prices of all commodities are continually gravitating." The invisible hand is the market version of gravity.

This view of society as a collection of atomistic individuals, each pursuing their economic self-interest, was modeled directly after Newton's view of nature as a mechanistic, law-bound system.[15] Just as Newton had showed that a wide range of phenomena were all explained by the law of gravity, Smith had shown that market behavior could be explained by what was later known as the law of supply and demand. However, there was an important difference, for the theory lacked what Smith had so admired in Newton's work, namely the ability to make accurate predictions. It was qualitative rather than quantitative; descriptive rather than predictive. This problem would be addressed by a new generation of "neoclassical" economists in the late 19th century, including William Stanley Jevons in England and Léon Walras in France, who aimed to put the field on a solid mathematical footing, and turn it into a kind of "rational mechanics" for society. Their work would pave the way for the development of quantitative finance.

Any model is a simplification of reality, and the neoclassical economists had to make some rather sweeping assumptions in order to make progress. The most basic of these was that people act to optimize their own utility – defined rather hazily as whatever makes them happy – but not that of other people. As Francis Edgeworth put it in 1881, "the first principle of Economics is that every agent is actuated only by self-interest."[16] People also had a fixed set of preferences. So if they liked cereal for breakfast, they didn't suddenly swap over to eating toast. And people always acted in a completely rational fashion.

Thus was born the notion of *homo economicus*, or rational economic man. While these assumptions had obvious flaws – surely we do change our minds? – they did allow economists to construct elegant mathematical models of the economy.

[15]Greene (1961, p. 88).
[16]Edgeworth (1881, p. 16).

Finding Equilibrium

An obvious difference between economics and physics was that physical quantities could be measured in well-defined units, while "utility" was rather vague and no one knew what its units were ("utils" was suggested). However, Jevons argued that in reality we can never directly measure a force like gravity, only its effects. Even if utility could not be directly measured, or even defined, we could infer it from market prices. Today, it is fair to say that quants do not suffer from lack of data – we have more information about markets than we have about other things that we wish to predict, such as the weather.

Another problem was that, while the atoms of physics are believed to have the same properties everywhere in the universe, people – who are the atoms of the economy – show a high degree of variability. According to Jevons, though, what counted was the behavior of "the single average individual, the unit of which population is made up."[17] This meant that the agents in the economy – i.e., individuals and firms – could be treated as if they were all the same. The idea was inspired by the "social physics" of the 19th-century Belgian scientist Adolphe Quetelet, who wrote of *l'homme moyen*, or the average man.[18] He claimed that "the greater the number of people observed, the more do peculiarities, whether physical or moral, become effaced, and allow the general facts to predominate, by which society exists and is preserved."[19]

Here we have a social science version of the probabilist's Law of Large Numbers. This mathematical law states that the average of a large number of trials will converge to the expected value of a single trial. A die has 21 spots and six sides, giving an expected throw of $21/6 = 3.5$. As you roll the dice more frequently, the average will converge to this expected value. The idea that the expected behavior of humans, as with dice, is all that matters in the long run could be an explanation behind Isaac Asimov's fictional character Hari Seldon in the *Foundation* series of novels. Professor Seldon is one of the creators of "psychohistory," a science that makes predictions about the future based on the statistics of large groups of people. When asked

[17]Jevons (1957).
[18]Quetelet (1842).
[19]Quoted in Bernstein (1998, p. 160).

"Can you prove that this mathematics is valid?" he replies, "Only to another mathematician." Nobel Laureate Paul Krugman says that he became interested in economics thanks to Hari Seldon and his ability to predict mankind's actions.[20] Of course, it's all poppycock, but entertaining reading nonetheless.

Whatever the equations governing man's economic behavior, the neoclassical economists faced a rather daunting computational problem. One way to make it tractable was to assume that prices were at equilibrium. Jevons compared the price mechanism to the motion of a pendulum, which came to rest at the ideal balance between supply and demand. Even if one could not compute the daily permutations of the markets, it should be possible to compute the average equilibrium position to which the invisible hand was pushing them. Furthermore, it made sense that markets should be at or near equilibrium; since if prices were too low or too high, then this would imply that market participants were not making rational decisions. The assumption of equilibrium was therefore also tied up with the idea of rationality.

Intrinsic Value

As economics developed in the 20th century, concepts such as rationality and equilibrium remained at the heart of the theory. In the 1960s, economists Kenneth Arrow and Gérard Debreu created a model of an idealized market economy, and famously showed that it would reach a kind of optimal equilibrium (a result that did not displease their sponsors at the US Department of Defense, at a time when the country was embroiled in an ideological conflict with its communist foes[21]). But to prove its results, its authors had to assume that market participants act rationally to maximize their utility, not just now but also in the future. Since the future is unknown, this means they have to know what is the best course of action for every possible future state of the world – something which implied infinite computational capacity. The Arrow–Debreu model of the economy

[20]We find this a bit disturbing. But not as disturbing as Alan Greenspan's extreme fondness for Ayn Rand. As he wrote in *The Age of Turbulence*, "Ayn Rand became a stabilizing force in my life… I was intellectually limited until I met her" (Greenspan, 2007).

[21]Bockman (2013, p. 47).

served as the theoretical foundation for general equilibrium models, versions of which are used today to determine the effects of policy changes on the economy.

Unfortunately, these models – despite being "aesthetically beautiful" to theoreticians[22] – turned out to be little better at predicting the economy than random guessing (which is why they are not used by quants). Psychohistory they weren't. However, the University of Chicago's Eugene Fama came forward with a convenient excuse for why economists were doing such a poor job of predicting the future, at least for markets. His efficient market hypothesis portrayed the market as a swarm of "rational profit maximizers" who drive the price of any security to its "intrinsic value." It was therefore impossible to beat or out-predict the market, because any information would already be priced in. The invisible hand of the market was the epitome of rationality. This leads to the weird situation where individuals are assumed to be able to make perfect predictions (Arrow–Debreu), but this in turn means that no one can predict the markets (Fama).

This would normally be the point at which most investors turned their backs on too much theorizing – as ex-Fidelity fund manager Peter Lynch told *Fortune* magazine, "Efficient markets? That's a bunch of junk, crazy stuff" – but it is precisely the elegance of this "result" that excites the academic economists.[23] As discussed further below, the efficient market idea formed the backbone of academic models used in risk analysis, and much of quantitative finance in general. As with Adam Smith and the neoclassical economists, the central idea was of the market at equilibrium, with the invisible hand constantly restoring it to what Smith called a "tone of tranquillity and composure."

Quants in general have a somewhat conflicted attitude toward the efficient market hypothesis. If it were really true, then they would be out of a job. On the contrary, many quants came out of the Chicago School of Economics, or were otherwise influenced by Fama and his academic accolades, so at least pay lip service to the idea.[24] From a quant survey we performed at wilmott.com, some 43% of

[22]Haldane (2014).
[23]Para (1995).
[24]E.g., Cliff Asness (co-founder of AQR Capital Management) (Patterson, 2009, p. 265).

respondents described it as true. One way to square the circle is for quants to see themselves as enforcers of efficiency, whose job it is to drive prices to their correct level – even if that means driving them off a cliff. (We'll give our own verdict on the theory in the next chapter, but basically, Lynch is right.)

The assumptions of neoclassical economists therefore had a dual nature. On the one hand, they were designed to make the economy mathematically tractable. It is obviously easier to model people who are selfish, have fixed preferences, and are completely rational than it is to model people who are influenced by the opinions of others, change their minds for no reason, and make puzzling and bizarre life choices. On the other hand, they shaped the way that we see and model the economy – as a beautifully rational, stable, and efficient system – which as we'll see, shaped the economy itself.

Of course, no one – even business school lecturers – thinks that people are perfectly rational, or that markets are perfectly stable or uniform, or that models are perfect. Much work has been done exploring deviations from these assumptions. As we will see, though, the models used in finance continue to treat the world as a very rational, stable, and symmetric place – and this has as much to do with aesthetics, mathematical ego, and the desire to impress and intimidate as it does with making money. In the next chapter, we look at how these elegant but unrealistic assumptions and formulas were made to seem compatible with markets that often appear to be driven more by chaos than by reason – more Law than Newton.