BIG IDEA 1

Seeing Patterns inside Numbers

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Numbers make up our world, and they are used throughout our lives, whatever our age, job, or level of interest. But many people develop a narrow relationship with numbers, seeing them as something to use in calculations, rather than as a fascinating set of ideas that can enrich their world. Our first big idea invites students to become captivated by numbers and to get to know numbers deeply. What is enchanting about numbers is that they are all made up of different arrangements, have different factors, can be seen differently, and have their own intricate system to be explored.

When we first came across Brent Vorgey's number visual (Figure 1.1), we were enthralled, as we immediately saw the creativity, beauty, and insights that the visual representations revealed.

In our Visualize activity, we invite students to explore this depiction of numbers and to see what patterns are uncovered by the visual representations. We invite them to see what primes look like and to see the different factors inside numbers. We invite them to investigate patterns among the numbers, seeing what their positioning on the diagram reveals. Also, we invite students to see numbers visually and to develop a realization that numbers contain all sorts of information that make them different from each other, special, and interesting.

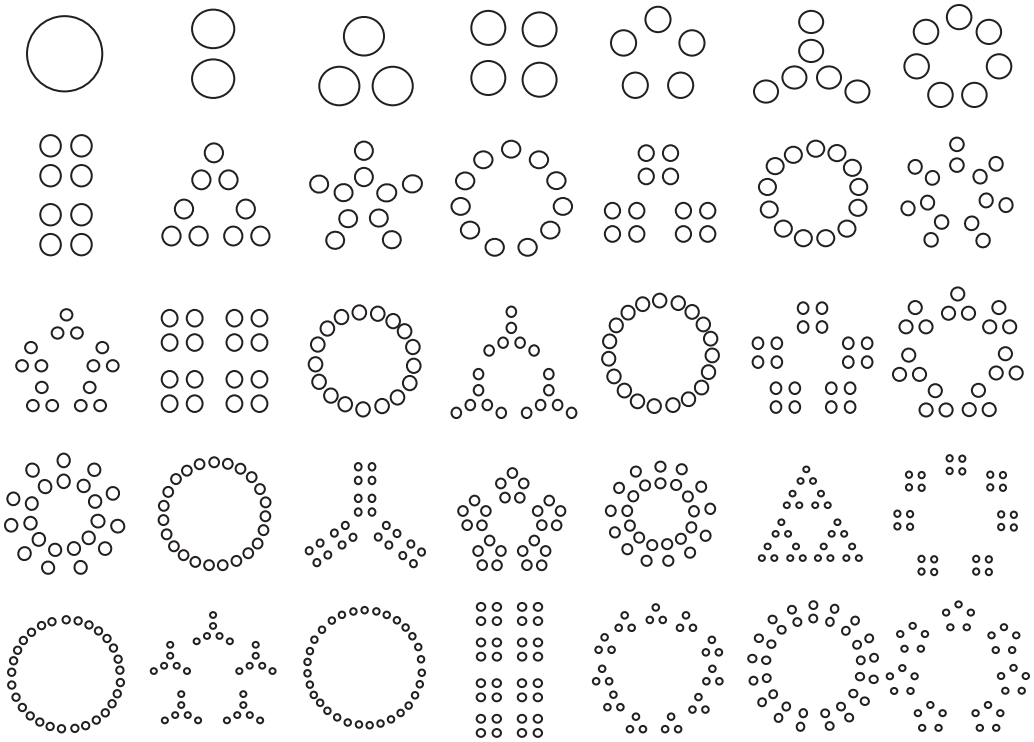


Figure 1.1

In the Play activity, we extend students' time with the number visuals in a more playful setting. Students play a game with the number visual page as a game board, and move between visual and numerical representations. This, as with the other two tasks in this big idea, encourages important connections between different areas of the brain.

In the Investigate activity, we invite students to think carefully about number flexibility. One of the ways that numbers are different from one another is the number of factors they have and the degree of flexibility they give us when using them. For example, 24 is a very flexible number, as it can be broken up in all sorts of different ways. This makes it a useful number for packaging, for designing, and for measuring time. In this activity, we invite students to give value to different numbers according to their flexibility, helping them develop an appreciation for these numbers. The activity also invites students to make equal groups and gives teachers an opportunity to discuss whether students are thinking additively or multiplicatively and what those differences mean.

All three activities give students an opportunity to develop new insights into the numbers that they will use for the rest of their lives.

Brain science tells us that when students are engaging with numbers as symbols, such as the numeral 4, and with numbers as visuals, as shown in Figure 1.2, they are connecting between different areas of the brain, and such connections are critical for mathematics learning and achievement. The activities in this big idea will invite a lot of brain connecting, with students developing pathways that will help them as they go forward in their mathematical careers.

Jo Boaler

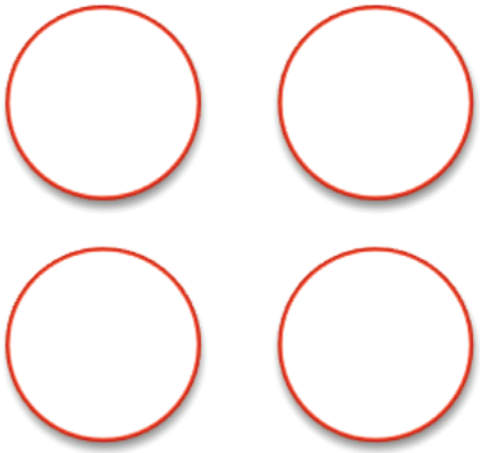


Figure 1.2

Visualizing Numbers

Snapshot

In this activity, students work with the number visual page to explore the patterns that they can see inside of numbers. In this activity, we open the door to understanding factors, multiples, and primes, as well as other number patterns.

Connection to CCSS
4.OA.4

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5 min	Generate multiple ways that numbers can be represented and introduce the number visual page.	Number visual page reproduced for students and one to display
Explore	20+min	Students look for patterns inside the number visual page and color-code them.	Colors for students (colored pencils, markers, or pens)
Discuss	10 min	Discuss the different patterns students found and how their color coding makes the patterns visible.	
Explore	20+min	Student look for patterns shared across the different numbers. Student cut their papers so that they can group or arrange them to show shared patterns.	• Number visual page, one per group • Colors • Scissors • Optional: posters or large paper
Discuss	15 min	Students share the different ways they have grouped the numbers and discuss the shared patterns they have found.	

To the Teacher

The length of this lesson depends largely on how long students would like to explore patterns. We've found that some students want to explore patterns in depth, and they should be given time to do that. Follow your students' lead and interest. This activity can easily be spread across multiple days.

Activity

Launch

Numbers can be represented in lots of different ways. For example, 6 can be written as a numeral, but 6 can also be shown in other ways, as in Figure 1.3.

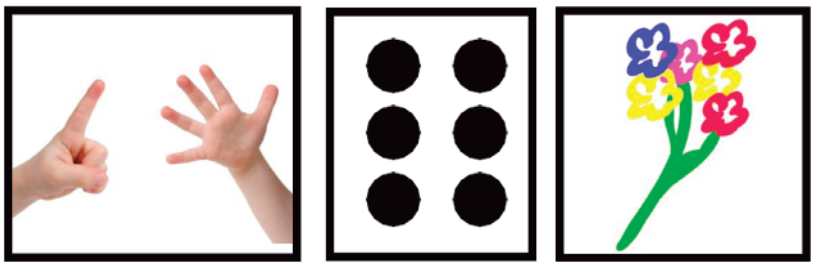


Figure 1.3

When you launch this activity, you may want to share some of these ways with students or have them generate ways that numbers are represented in their world. Give each student a copy of the number visual page and ask them to notice what numbers are shown. Have them record the actual number value by each visual. There are patterns all over this page. Ask students, what patterns do you notice?

Explore

Ask students to investigate the patterns in the number visual page.

- What patterns do you see?

Provide students with colors (colored pencils, pens, or markers).

- How can you use color to show the patterns within these numbers?

Students might notice equal-size groups within some numbers. For instance, 4, 8, 12, 16, 20, 24, 28, and 32 all have square clusters of 4 inside them. Students might notice that some numbers have no groups inside them; numbers like 11, 13, 17, and 19 are circles. Students might notice how some numbers grow outward from a central pattern. For instance, 6 has a group of 3 in the center, and each corner has been added onto with one dot. Students might also notice multiple numbers inside one

number. For instance, 18 has three groups of 6, but also has 9 pairs. Some of these patterns are shown in Figure 1.4 as an example of how students might use color to highlight different patterns they see.

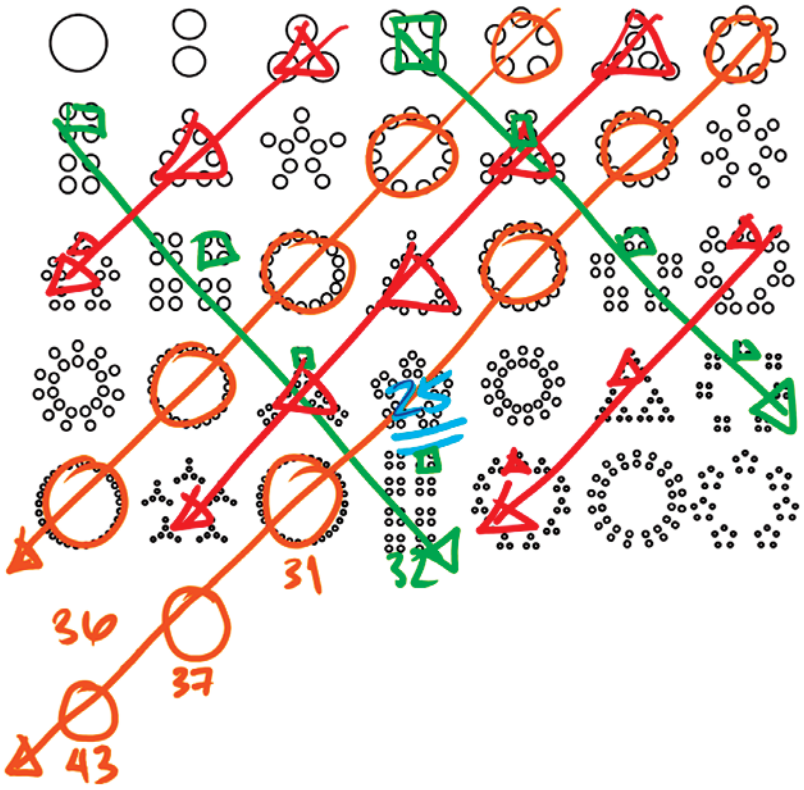


Figure 1.4

Discuss

Ask students to share the different ways they have color-coded their numbers to reveal patterns. What do different ways of coloring show? You may want to focus discussion on a single number to compare the different patterns inside. For instance, you could look at the different patterns inside of 12 that different ways of coloring make clear.

What do different numbers have in common? If you focused on a particular number, you might ask, What other numbers are like this number? How are they alike? If students notice the clusters within each number, give them the term *factor* to describe these clusters. For instance, if students see the three clusters of 4 inside 12, you can say that 4 is a factor of 12, or that 12 has 4 as a factor.

Explore

Now ask students to return to their color-coded number visuals and look for patterns that different numbers share. Provide students with a new number visual page and scissors to cut this new page apart so that they can group, sort, or web numbers by common features and color-code those features. Students should work with a partner or in a small group to find patterns.

- What patterns do different numbers share?
- How can you group or arrange the numbers to show what numbers have in common?

You might have students glue or tape their arrangements onto a poster to make sharing easier. This way, they could label the groups or the relationships between the numbers.

Discuss

Ask students to share the patterns they notice between numbers. You may want to have students hang their posters and do a gallery walk, or ask each group to share what they have found. In either case, discuss as a class the following questions:

- What patterns do different numbers share?
- What are you wondering now about these numbers?
- What are you wondering about the numbers we haven't looked at yet?

If students notice the clusters that different number share, be sure to tell them that we say that they share a factor. If students notice the circles and the lack of clusters within, be sure to probe what this means. You can name these numbers, where no equal groups are possible, as *prime*.

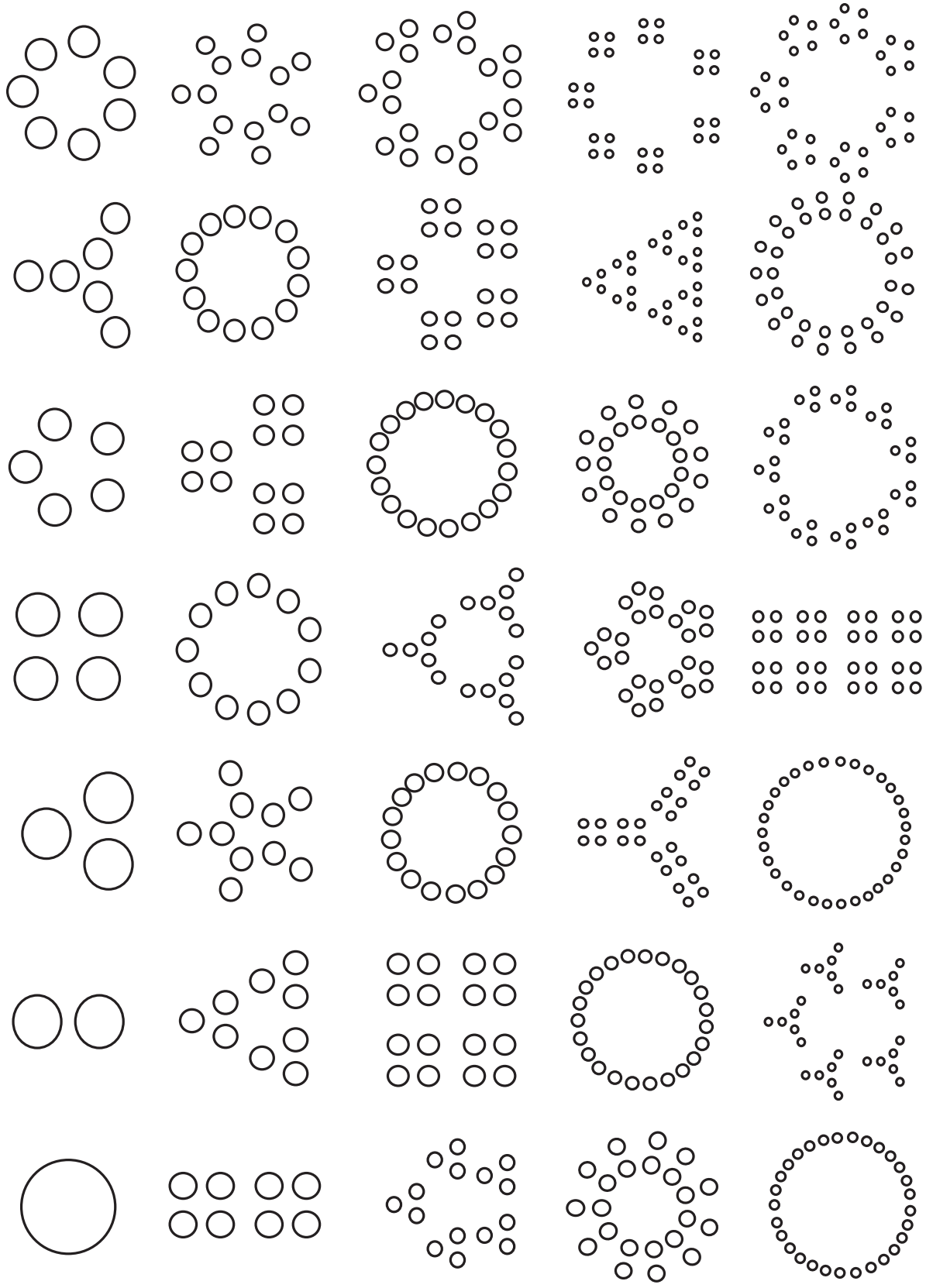
Look-Fors

- **Do students notice that numbers are inside of other numbers?** For instance, does anyone see three clusters of 4 inside of 12? One goal of this activity is for students to see the building blocks of numbers.
- **Do students notice that some numbers are made only of individual dots?** This is the beginning of noticing primes.

- **Are students thinking multiplicatively or additively?** Although numbers can be broken apart through addition, we want to push students to notice patterns of equal groups. This is an interesting point of discussion.
- **Are students noticing that some numbers have similar building blocks?** This is the beginning of noticing common factors.

Reflect

What do you think is the most interesting number on this page? Why?



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What Could It Be?

Snapshot

Students play a game using the number visuals to further explore the idea of factors. Pairs of students each try to claim four squares in a row on a number game board, while thinking both visually and numerically.

Connection to CCSS
4.OA.4

Agenda

Activity	Time	Description/Prompt	Materials
Launch	10 min	Introduce the idea of a number part and teach students the rules of today's game.	• Visual Number Part Cards (one deck per partnership) • Game boards (at least one per partnership) • Marking tools: pencils, colors, or chips
Play	20+ min	Students play What Could It Be? in partnerships.	
Discuss	10 min	Discuss the strategies students developed playing the game.	

To the Teacher

Today's game uses the number visuals students explored in the Visualize activity. Different game boards are provided that pose different degrees of challenge; the board with larger numbers and the board where the values are not in order are more challenging. Students will play in pairs, and may play with one board initially and then want to try a more challenging board.

Activity

Launch

Display one of the Visual Number Part Cards or zoom in on one of the visual numbers so that only a portion is visible to students. Tell students that this is just a part of a larger number. Ask the class, What number could this be? You may want to ask student to partner to share ideas and reasoning. Collect from the class some possible answers and the reasoning that supports them. You might also want to ask, What number could it not be? Why not?

Introduce today's game by showing students a game board. We suggest starting with the 1–36 game board. Without fully playing the game with students, explain the rules, showing students how they could have marked the number visual they just discussed as a class.

Play

Game Directions

- Set up the game space by placing a game board between you and your partner. Put the card deck face down between you. Each player will need a pencil, color, or a set of chips to mark the board.
- Partners take turns drawing a Visual Number Part Card. The player drawing the card must figure out, "What number could it be?" and share their reasoning with their partner. Then the player can mark (with an X, color, or chip) the number they chose.
- Players take turns drawing, reasoning, and marking numbers until one player marks four squares in a row, horizontally, vertically, or diagonally.

As students play, you'll want to walk around and see what kind of reasoning students are using, what possibilities they come up with, and whether they may want a more challenging board. Students can play repeatedly on the same board. You may decide you want to change partnerships midway through the play time so that students can try their strategies out on someone new.

Discuss

Bring students together to discuss the strategies they developed during the game. Discuss the following questions:

- How did you decide what numbers a picture might represent?
- How did you then choose which of those numbers you would mark?
- What made the game hard? Did you make any mistakes? What did you learn from those mistakes?
- Which numbers were easiest to capture? Which were hardest? Why do you think that is?

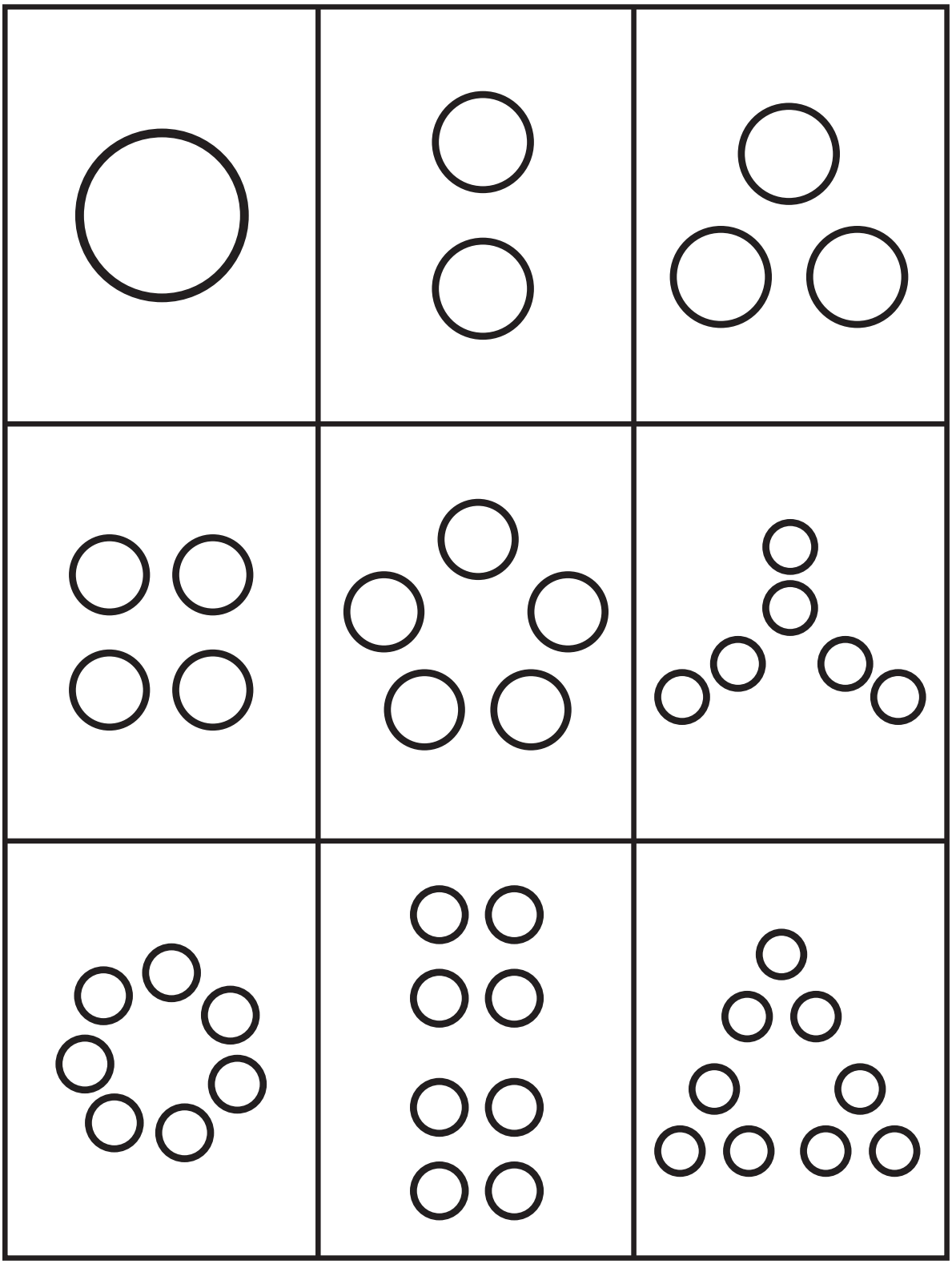
Look-Fors

- **How are students reasoning about the numbers each picture could represent? Are they applying thinking about factors?** The portions on the card should support students in beginning to generate multiples and think about how the factors can be used to build a larger number.
- **Do students recognize which kinds of pictures could be prime numbers?** The primes on the game board are particularly challenging, and students will need to notice that only some cards could be used to capture those numbers.

Reflect

How would you make an easy game board? A very challenging game board?

Visual Number Part Cards



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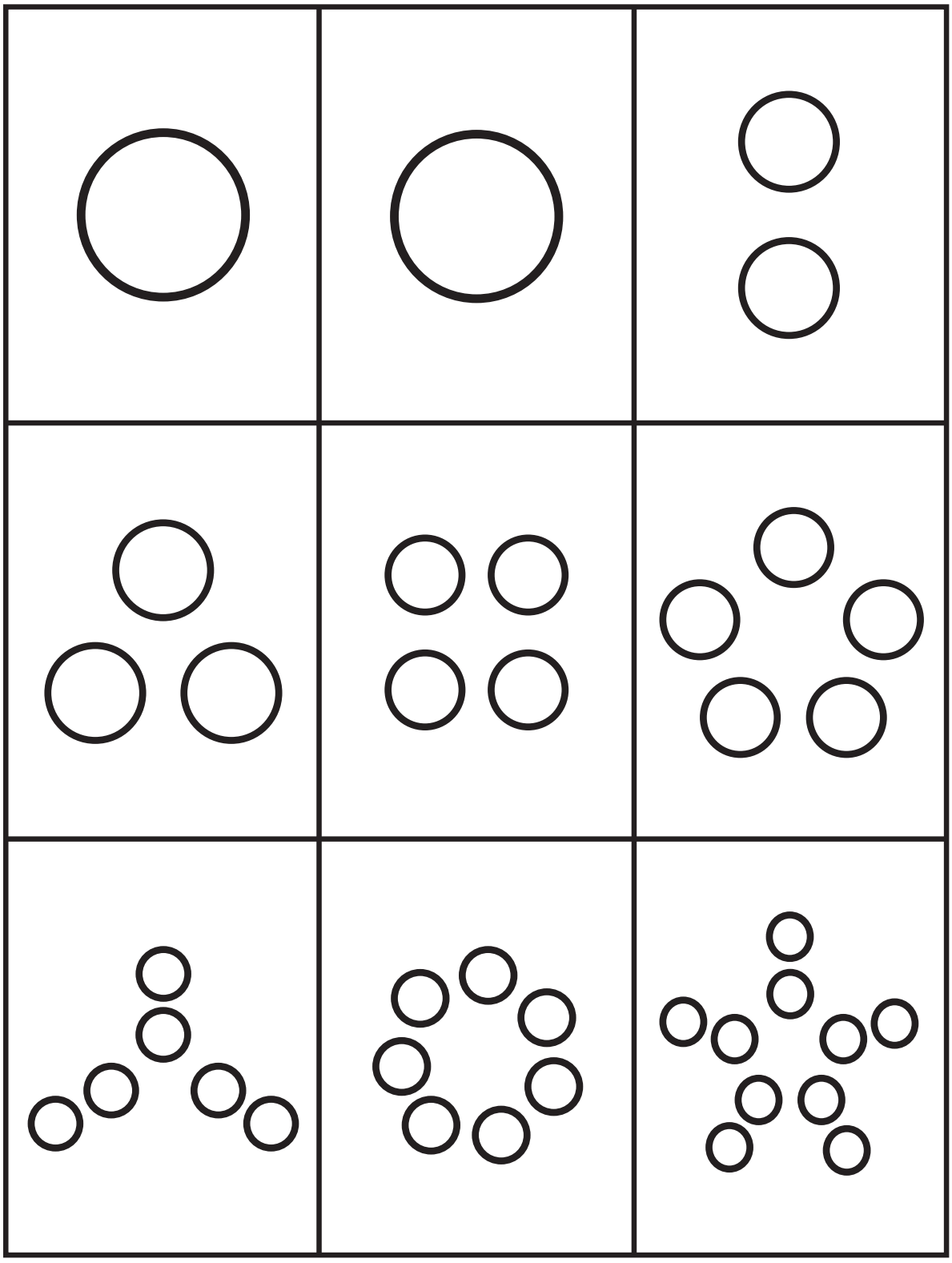
Visual Number Part Cards



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Game Board 1

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36

1. _____ x _____ = _____
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4. _____ x _____ = _____
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11. _____ x _____ = _____
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15. _____ x _____ = _____

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Game Board 2

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	45	46	48	49	50	51
52	54	55	56	60	63	64	65
69	70	72	73	75	77	80	81

1. _____ x _____ = _____
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3. _____ x _____ = _____
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23. _____ x _____ = _____
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Game Board 3

37	3	9	13	27	19	22	48
10	31	39	45	15	81	60	28
17	34	20	21	18	30	23	14
38	72	27	69	29	40	31	32
33	12	1	36	11	26	50	2
41	64	80	16	4	49	55	42
52	75	5	56	7	63	6	65
35	70	18	8	13	77	54	51

1. _____ x _____ = _____
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How Flexible Is a Number?

Snapshot

In this activity, we extend the work students have been doing with the number visuals and focus on the importance of factors for making numbers useful and flexible. Making equal groups is a central idea for number flexibility and is a different way of decomposing numbers than students may often use.

Connection to CCSS
4.OA.4

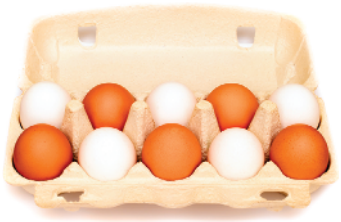
Agenda

Activity	Time	Description/Prompt	Materials
Launch	5 min	Ask why some numbers are used frequently in our world and some are not. Set up the investigation of number flexibility.	Optional: items in packages or photos to show
Explore	20+ min	Students work in small groups to determine the relative flexibility of a set of numbers and arrange them in a line.	• Visual Number Part Cards, in small sets for each group • Optional: cubes, tiles, chips, or other manipulatives
Discuss	25+ min	The class works together to construct a shared continuum of flexibility. The discussion closes with developing a definition of flexibility.	• Line on the wall, with ends labeled "Inflexible" and "Most Flexible" • Tape or pushpins for posting numbers
Extend	25+ min	Students investigate a number (36–100) of their choosing and determine where to place it on the continuum.	Additional paper

To the Teacher

In the work students do to solve problems, they often identify meaningful and useful ways to decompose numbers. These ways sometimes use equal groups, such as when students break 18 into two groups of 9. But often students use other ways to decompose, like place value (18 = 10 + 8) or compensation (18 = 20 – 2). In today's activity, the goal is for students to think about equal groups as a useful way to decompose, and to see those equal groups as factors. You might notice some students thinking simply about the additive ways that a number can be decomposed—for instance, that 18 is 17 + 1, 16 + 2, 15 + 3, and so on. This kind of thinking will make it difficult for students to see the flexibility of numbers based on factors. Encourage students to think about the context of the task: Why are some numbers often used in making packages of items while others aren't? You might encourage students to draw pictures of these packages as part of their investigation to help them visualize what makes these numbers useful and flexible. Flexibility can often be seen in the arrangement of the objects in arrays.

Activity



Launch

Launch by telling students that some numbers are used a lot in our world, and some numbers rarely show up. In the grocery store, students might notice that eggs come in a carton of 12, and bottled water is also sold in cases of 12. Pose the questions, Why not 11? or 13? Collect some ideas from students about why this might be.

What makes a number so flexible that it can be used in lots of different ways? What makes a number inflexible? Tell students that they will be working together to investigate these questions. The class will make a display of how flexible or inflexible students think these numbers are, using evidence about each number.

Explore

Students work in groups and receive a set of numbers from the Visual Number Part Cards. Be sure that each group gets a diverse set of numbers, a mix of primes and composites. For instance, a group could get 2, 7, 8, 25, and 29. Make sure each group gets at least one of these numbers: 8, 12, 16, 18, 20, 24, 30.

Students work together to investigate how flexible each number is. You may want to provide tiles, chips, cubes, or additional paper. Students record their evidence of flexibility on the number cards or additional paper.

Then students in each group work to come to consensus about how to rank the flexibility of the numbers in their set. Students then arrange their numbers in a line to show the most flexible on one end and inflexible on the other end. They might decide that some numbers are equally flexible. Whatever their argument, the group should be prepared to justify their findings.

Discuss

Bring the groups together to cocreate a display on the board or one wall of your classroom. Have a horizontal line on your wall with labels at either end for "Inflexible" and "Most Flexible." The goal of this discussion is to create a continuum of number flexibility, building on ideas around factors, multiples, and primes. Students need to convince the class where the numbers their group investigated should go on this continuum. You may want to have each group present in turn, convincing the

class where their numbers should go in relationship to the numbers that have already been placed. Students may conclude that some numbers need to be moved. To move a number, students need to convince their classmates where it should be placed. Alternately, you may want to have each group place one number, give their reasoning, and field any questions from the class. Rotate from group to group until all numbers have been presented, explained, and agreed on.

Close the discussion by coming up with a class definition, based on all that you have discussed, of what qualities define "inflexible" and "most flexible. Add these definitions to the labels on your continuum.

Extend

Ask groups to now choose a number between 36 and 100. Have students create a number image for that number, investigate the number's flexibility, and decide as a group where it should be placed on the class continuum. Groups can share these numbers and justify their placement in a second discussion. You could decide to set students a goal of finding a particularly flexible or inflexible number in this extension.

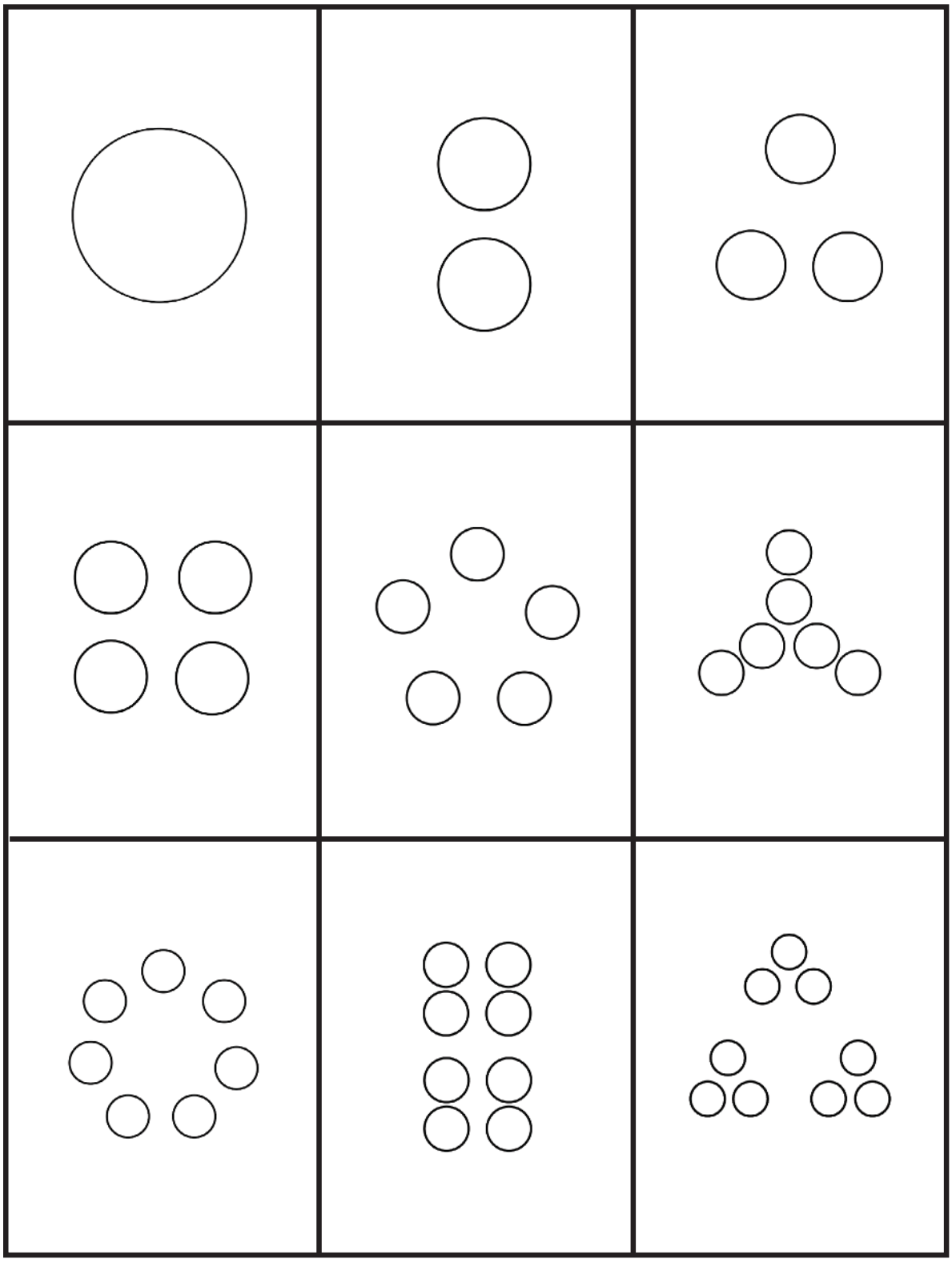
Look-Fors

- **Are students thinking multiplicatively or additively?** The goal of this lesson is to be thinking about making equal groups and the many ways you can break a number into equal groups.
- **Are students focusing on the number of factors a given number has?** This is one criterion for flexibility.
- **Are students noticing that some factors are themselves composites and some are prime?** You may want to probe why this matters in making numbers flexible or not.

Reflect

If you wanted to find an inflexible number, how would you do it? If you wanted to find a very flexible number, how would you do it?

Visual Number Part Cards



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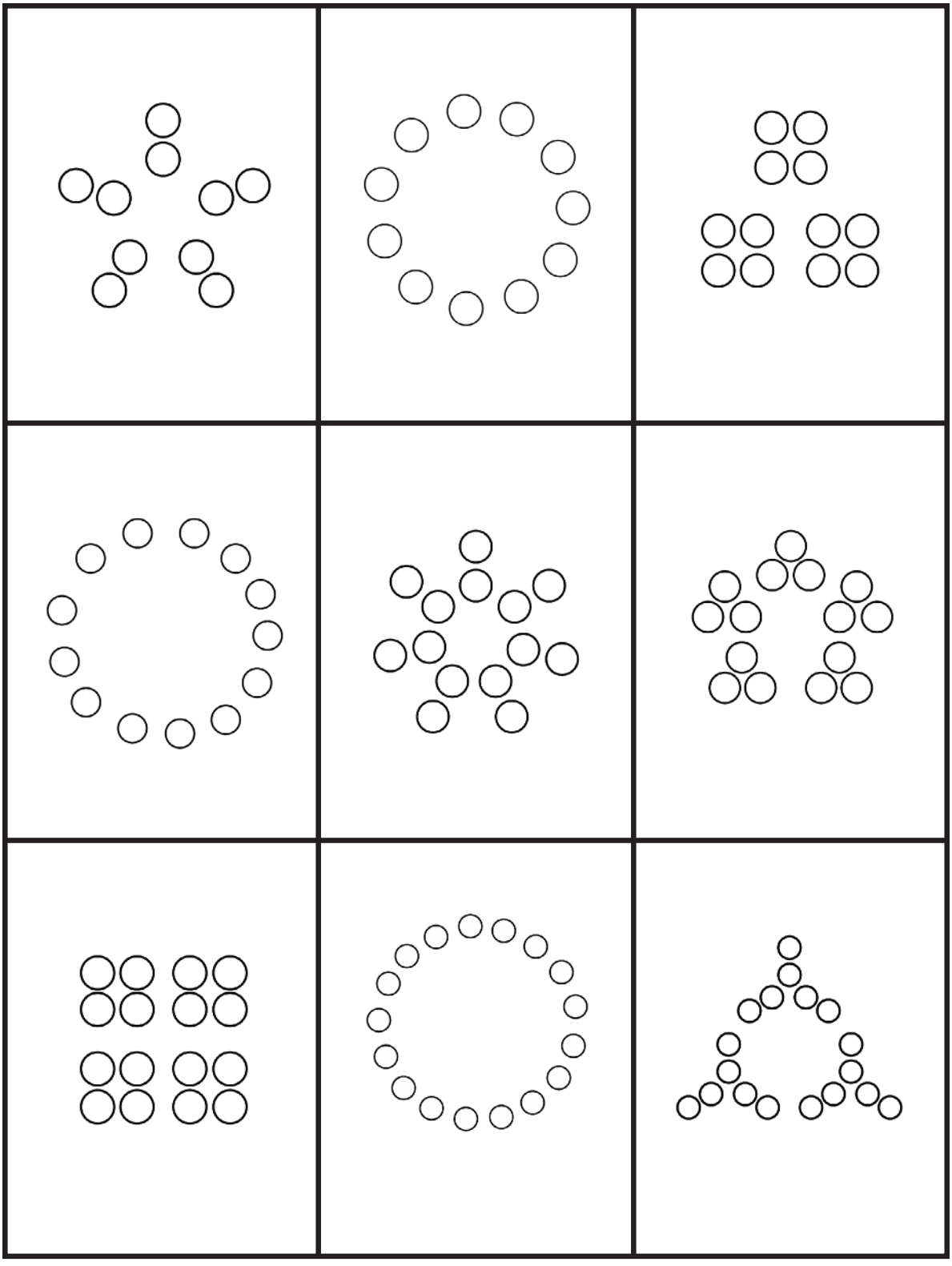

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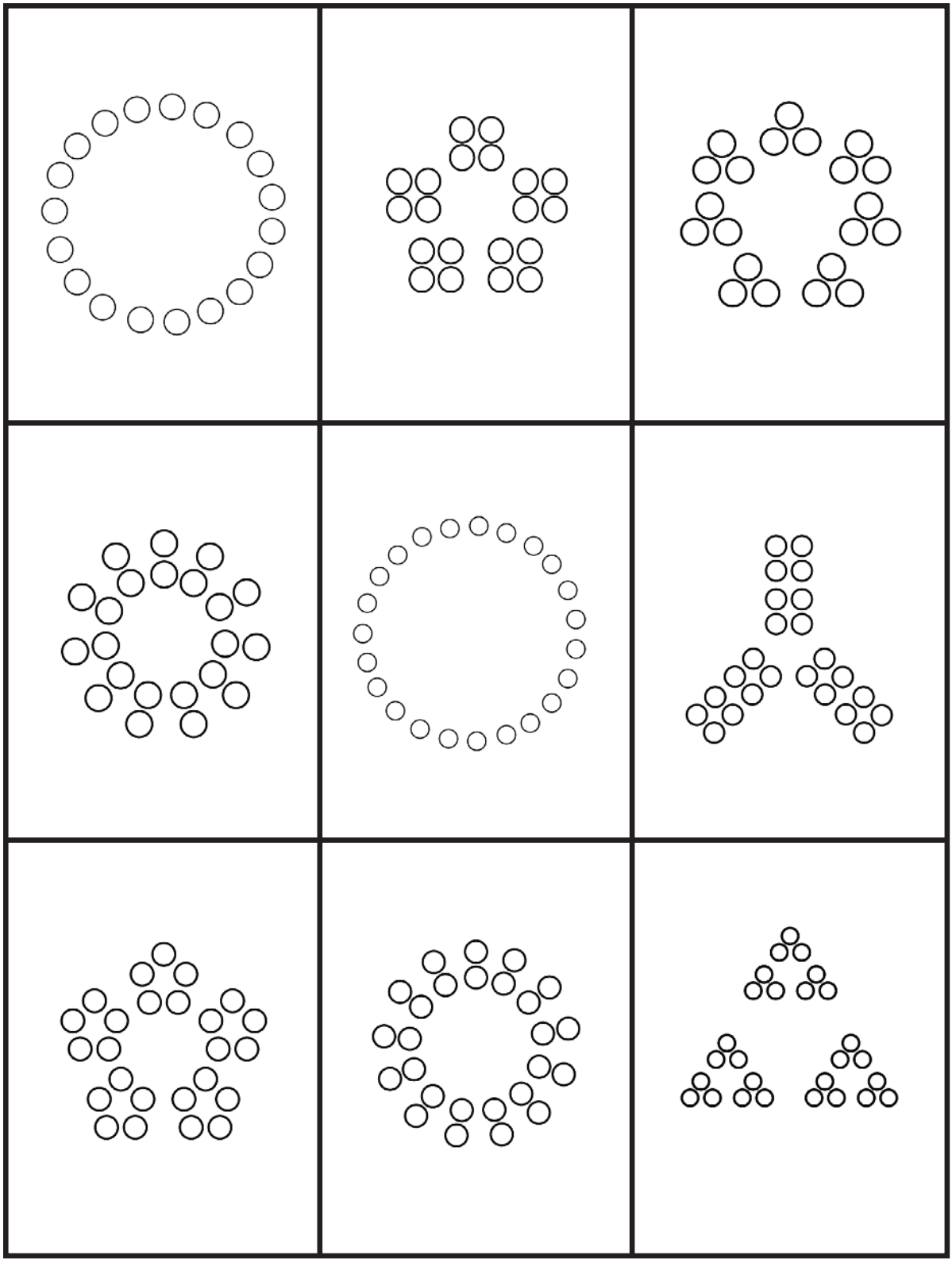

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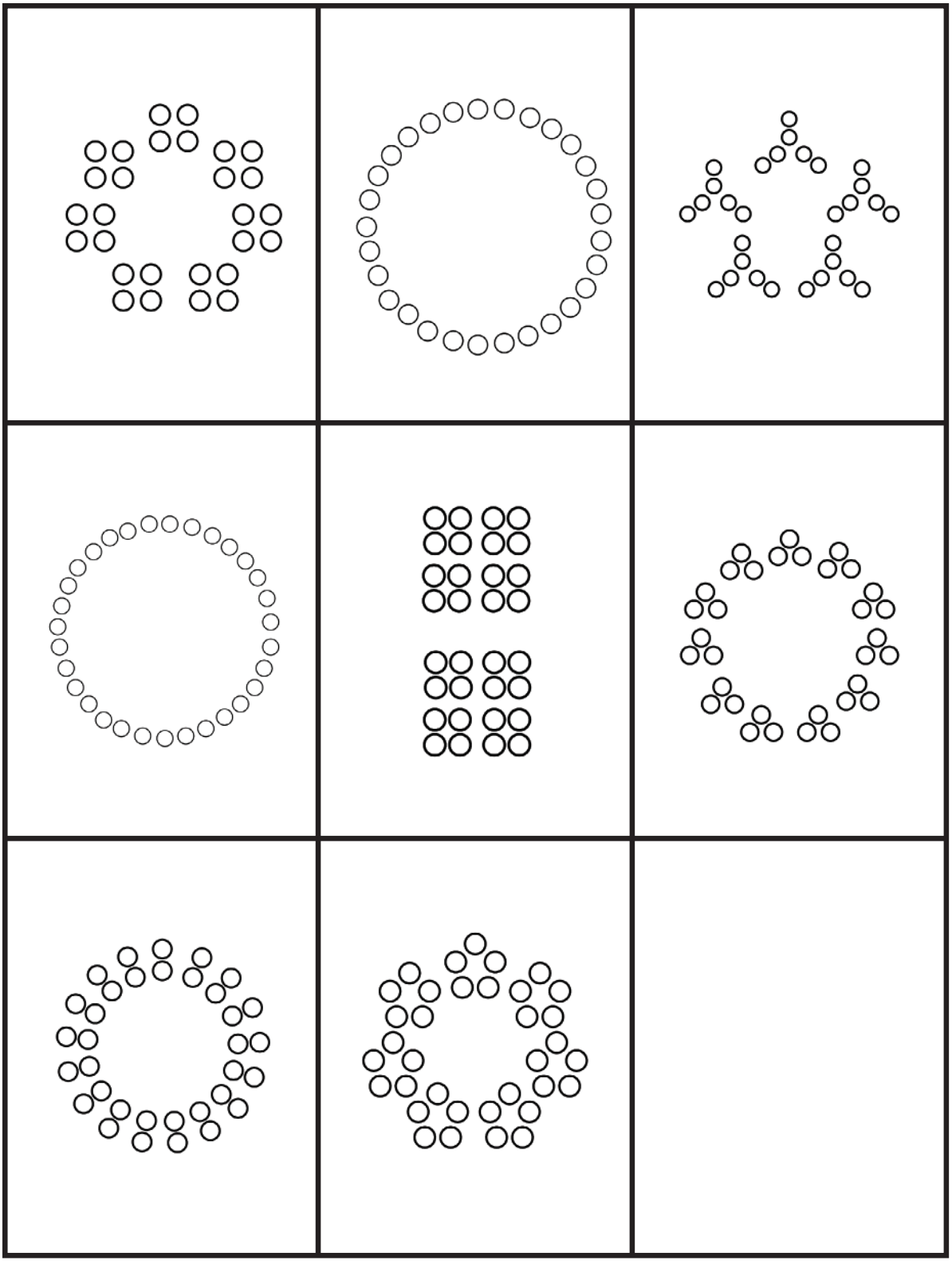

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