BIG IDEA 1

Taking Apart Prisms and Polygons

The mathematical concepts at the heart of this big idea are area and volume. Although these ideas call for students to learn through objects, holding them in their hands and exploring with them, many students are asked only to memorize formulas and so do not develop an understanding of area, volume, or the differences between them. In our Youcubed summer camp, we gave the students an activity with sugar cubes; they were invited to build different sized larger cubes with the sugar cubes. When we interviewed the students a year after they attended the camp, one of the boys told us that he now thinks about the sugar cubes every time he learns about volume, as they gave him a physical representation of a $1 \times 1 \times 1$ cube. His experience holding the cubes and building with them contributed to a deep understanding of volume that was powerful and enduring for him. In our Investigate activity, we invite the students to build with very similar cubes—snap cubes. In all of the activities, students are asked to build with two- and three-dimensional shapes.

In the Visualize activity, we ask students to find different ways to take apart two-dimensional complex shapes as they work to find area. We have used shapes that require students to reason about how to determine the area when its boundary doesn't fit exactly on a square grid. As students reason through determining the area, they also need to break the shape into other shapes they are familiar with, such as triangles and rectangles. This type of thinking is foundational for later work in geometry and calculus.

In the Play activity, we ask students to determine the area of a complex piece of artwork that is made from different polygons. We like to connect mathematics and art in our books, as it is important for students to see that mathematics can be beautiful, creative, and applied to all sorts of different real-world situations. Because of the uneven border of the shape, students will need to come up with different creative ways to find the area. This lesson also provides students opportunities to discuss estimation. In studies of mathematics in the world, estimation has been found to be one of the most used concepts and one that is undertaught in schools. We are sure your students will enjoy making their own piece of mathematical art.

In the Investigate activity, students build off the Visualize activity as they imagine complex two dimensional shapes as the bases of buildings. Students are asked to use multilink cubes to construct the buildings, giving them an important opportunity to understand volume. In this activity, we also provide an opportunity for students to work with rational numbers. Students in sixth grade are learning to expand their number system, yet questions in traditional textbooks often ask the students only to work with whole numbers. We have provided problems that use rational numbers, fractions, to support students' growth in understanding of both volume and rational numbers. Students are asked to visualize fractions of multilink cubes as they work to determine volume and connect the idea of volume to the idea of area for the shapes that they worked with in the Visualize activity. Students often have trouble understanding the difference between area and volume because they have not had enough experience spending time connecting their numbers with visual two- and three-dimensional models. We hope that this set of activities will provide time for fun and challenge together, and that students will get an enjoyable opportunity to struggle and to use their creativity in finding different ways to see and solve problems.

Jo Boaler

How Big Is the Footprint?

Snapshot

Students develop methods for finding the area of irregular polygons by exploring ways to decompose two-dimensional figures and reason about partial square units.

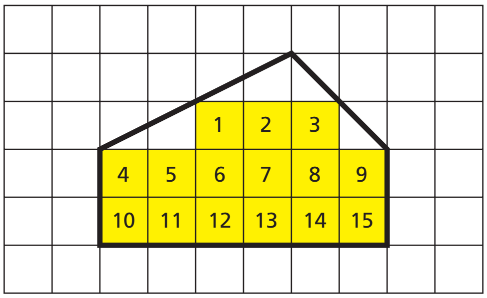
Connection to CCSS
6.G.1

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5–10 min	Show students the Quadrilateral in Question sheet on the document camera and ask, How might we find the area of this shape?	Quadrilateral in Question sheet, to display
Explore	30+ min	Small groups develop methods for finding the area of the quadrilateral. For each method, groups create a visual proof to share with the class.	• Quadrilateral in Question sheets, multiple copies per group • Optional: colors
Discuss	20+ min	Groups present their solutions to the area of the quadrilateral, and the class discusses how they decomposed the shape and accounted for the partial squares. Come to consensus about methods that make sense.	
Explore	30+ min	Small groups choose from the Polygon 1–4 sheets to find the area of another shape, drawing on the previous discussion. Groups develop visual proofs of their solutions to share.	• Polygon 1–4 sheets, for groups to choose from • Optional: colors
Discuss	20+ min	Shape by shape, groups present their solutions and discuss what methods make sense. Discuss what the various methods have in common and how you might select a strategy for finding the area of a shape.	

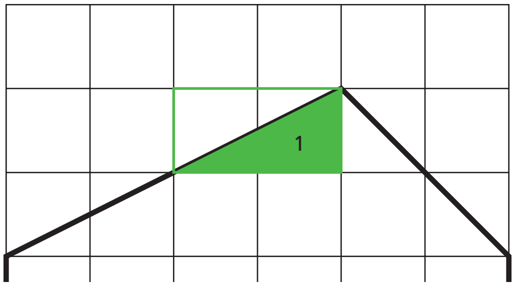
To the Teacher

Two ideas are central to this lesson, one conceptual and one mindset. At the heart of the conceptual work students are doing in this activity is making sense out of partial square units. A colleague of ours conducted a study in a sixth-grade classroom in which students engaged in an area task similar to this one (Ruef, 2016). Students developed many ways of addressing the partial units created by the angled side. Some students ignored them, believing that only whole units counted. In this method, students focus on stacks of square units, as shown in the figure here. While this does not lead to a fully accurate count of the area of the figure, it anticipates the way calculus approximates the area under a curve. If students in your class invent this way of thinking about area, it is worth naming that they have an idea that they will use in calculus to deal with the challenge of curves.

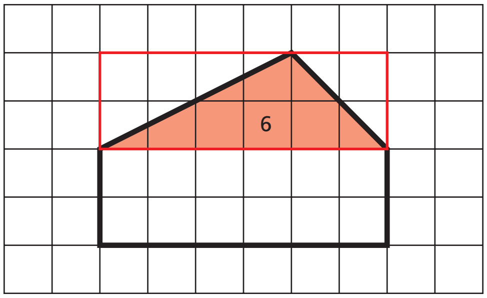


Some students count only whole squares when finding the area of an irregular shape.

Other students in Ruef's study developed various ways to create whole units out of the partial units, including ways that use the space *not* covered by the shape. For instance, some students visualized the partial units as half of larger rectangles, as in the methods shown in the next images. Both of these methods are accurate and have connections to thinking about slope, fractions, and decomposition of two-dimensional figures.



Some students make a rectangle and halve the area.



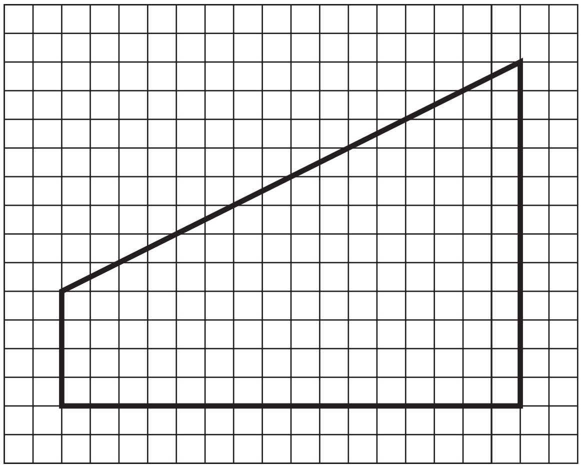
Some students make a large rectangle to cover the top of the larger shape that is a triangle and then halve the area.

The second central idea in this lesson is students' authority over the mathematics. This activity will challenge students to develop methods that they are uncertain about, or even, as in Ruef's study, to attempt incomplete or conflicting methods. It is crucial that students be the ones to determine whether a method makes sense, accounts for the full area, and is accurate. Ruef found that when placing the authority with students to make sense and come to consensus in this seemingly simple task, students took as long as three days to explore, debate, gather evidence, discuss, and come to agreement, and even then, they wanted their teacher to confirm that they were correct. The teacher resisted being positioned as the mathematical authority in the room, which made students responsible for deciding when *they* were convinced. We encourage you to take from this example the fortitude to resist students' requests that you decide who is correct and what makes sense. This is a prime activity in which to establish your students, at the beginning of the year, as the only ones who can decide whether and how a mathematical argument makes sense.

Activity

Launch

Launch the activity by showing the class the Quadrilateral in Question sheet in the document camera. Ask, How might you find the area of this shape? What do you notice that could help you? Give students a chance to turn and talk to a partner about these questions. Allow students to share some of their observations with the class. Pose the task for the day.



Quadrilateral in Question

Explore

Students work in small groups using the Quadrilateral in Question sheet to find its area. Ask students to find as many different ways as they can to make sense out of the area. You may want to provide groups with multiple copies of the Quadrilateral in Question sheet to represent each of their methods. Ask students to make a proof of their solutions on the sheet to share with the class to convince others that their solutions make sense. A proof can include drawings, numbers, arrows, calculations, and any other features that make reasoning clear and convincing, and colors can help students communicate about different parts of their solutions.

Discuss

Begin the discussion by asking each group to present their different solutions. If possible, try to make these solutions visible side by side so that the class can compare the solutions and methods. If there is disagreement about the area, discuss what led to these different results. Ask, Which methods do we think make the most sense? Why? It is likely that differences in method and solution will center on how students dealt with partial squares. Focus the class discussion on the following questions:

- How did you decompose the shape to find its area?
- How did you account for the partial squares?
- How do you know that you have accurately counted the area?
- How do the different methods prove each other? What do they have in common? What differences do you see?

Be sure the class can come to agreement about the area of this shape and methods that accurately count the partial squares. This is a crucial opportunity to promote argumentation, justification, and student authority over mathematics. Allow students to continue to work on developing consensus even if that requires sending groups back to work on making sense of the different methods, and reconvening the discussion afterward.

Explore

Ask students in their small groups to choose a shape from the Polygon sheet set. Provide copies of all the Polygon sheets. For each shape they choose, ask students to explore the following questions:

- What is the area of the shape? How do you know?
- How can you use what you learned in the first shape to help you find the area of other shapes?

Ask groups to develop visual proofs for each shape they work with. Again, you may want to provide groups with colors to support communicating about the different parts of their solutions.

Discuss

Begin the discussion by asking groups to share the areas they found for the different shapes, talking about each shape in turn. Come to consensus about the area and draw connections between the methods and reasoning students used for different shapes.

Discuss the following questions:

- What methods make the most sense for finding the area of irregular polygons?
- How can we decide what method to use for a given shape?

Come up with some class conjectures about the best methods to use based on your results from today.

Look-Fors

- **How are students making sense out of the partial square units?** This is the central mathematical concept of the lesson and one that students will likely struggle with. Encourage students to use color or to diagram the shape to make visible the ways they are counting. Ask students how they are dealing with the partial squares and ask them to describe their reasoning. If students argue that the partial squares do not count, push them to articulate why. If they are counting the partial square, push them to describe how they are doing so and how they know they have counted accurately. As you interact with students as they develop methods, be sure that all students are ready to share their reasoning with the class so that you can have a fruitful discussion about the contradictions in their methods.
- **How are students decomposing the shapes to find area?** Some students may decompose based on whole and partial squares, while others may decompose using larger polygons. For instance, some students may see this shape as a rectangle with a triangle on top. This is a useful way of thinking about nonrectangular polygons, as all can be decomposed into triangles and rectangles, so the methods students develop here thinking in terms of triangles and rectangles can be used generally. When these students share their thinking with the class, be sure to provide time for all students to see the polygon as the composition of other shapes.
- **How are students making sense of conflicting solutions?** Students may or may not be concerned by having multiple solutions presented simultaneously. This is a task with only one correct answer, though there are many ways to reason about that answer. Groups will undoubtedly have different methods and some different answers about a shape's area. Be sure to draw their attention to these by asking, Can this shape have different areas, or only one area? Why? If the class can agree that a shape can have only one area, then they need to consider which of the potential areas makes sense, and it is only based on sense making that a method can be deemed correct. You may want to examine some methods side by side to determine why they lead to different (or even the same) answer. Support the class in narrowing the field of solutions by deciding which they can agree do not make sense and why. If students still have multiple answers, they may need additional time to work on each method before returning to discuss them again.

- **How are students using what they learned from the first shape to help them find the areas of other polygons?** The first shape is intended to provide opportunities to reason about partial squares and the decomposition of two-dimensional figures. Both of these ideas are supportive of finding the areas of the polygons in the second round of exploration. If students get stuck, you may want to ask them to refer back to the class's previous discussion and the set of methods they agreed made sense. You might ask, How could you adapt these methods to use with this polygon? Why would those methods make sense? How is this shape like the Quadrilateral in Question? How could that help you think about methods that could make sense? In the closing discussion, ask the class to make connections between the methods used across shapes, to support them in seeing generalizable methods for finding the area of polygons.

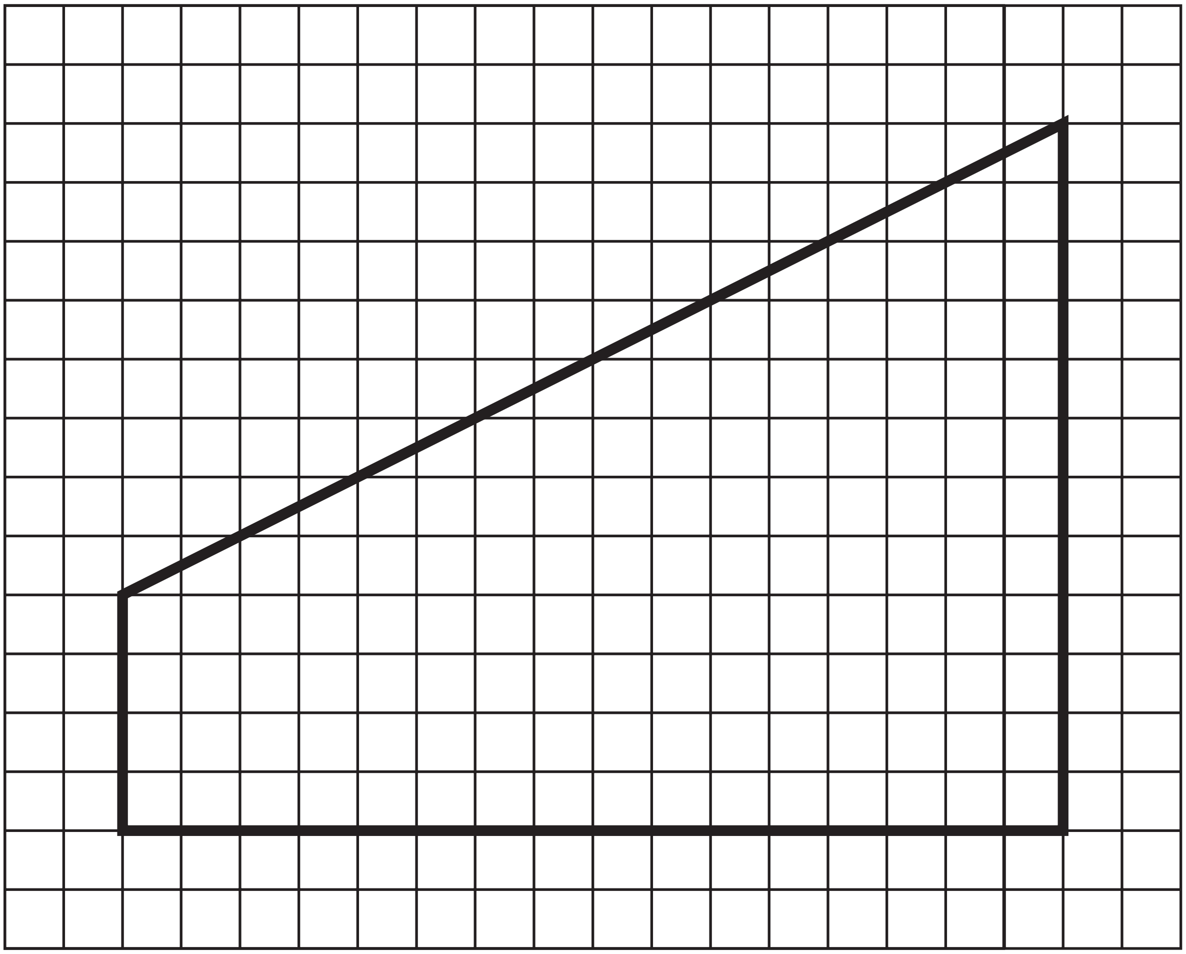
Reflect

What do you think are the best methods for finding the area of polygons? Why?

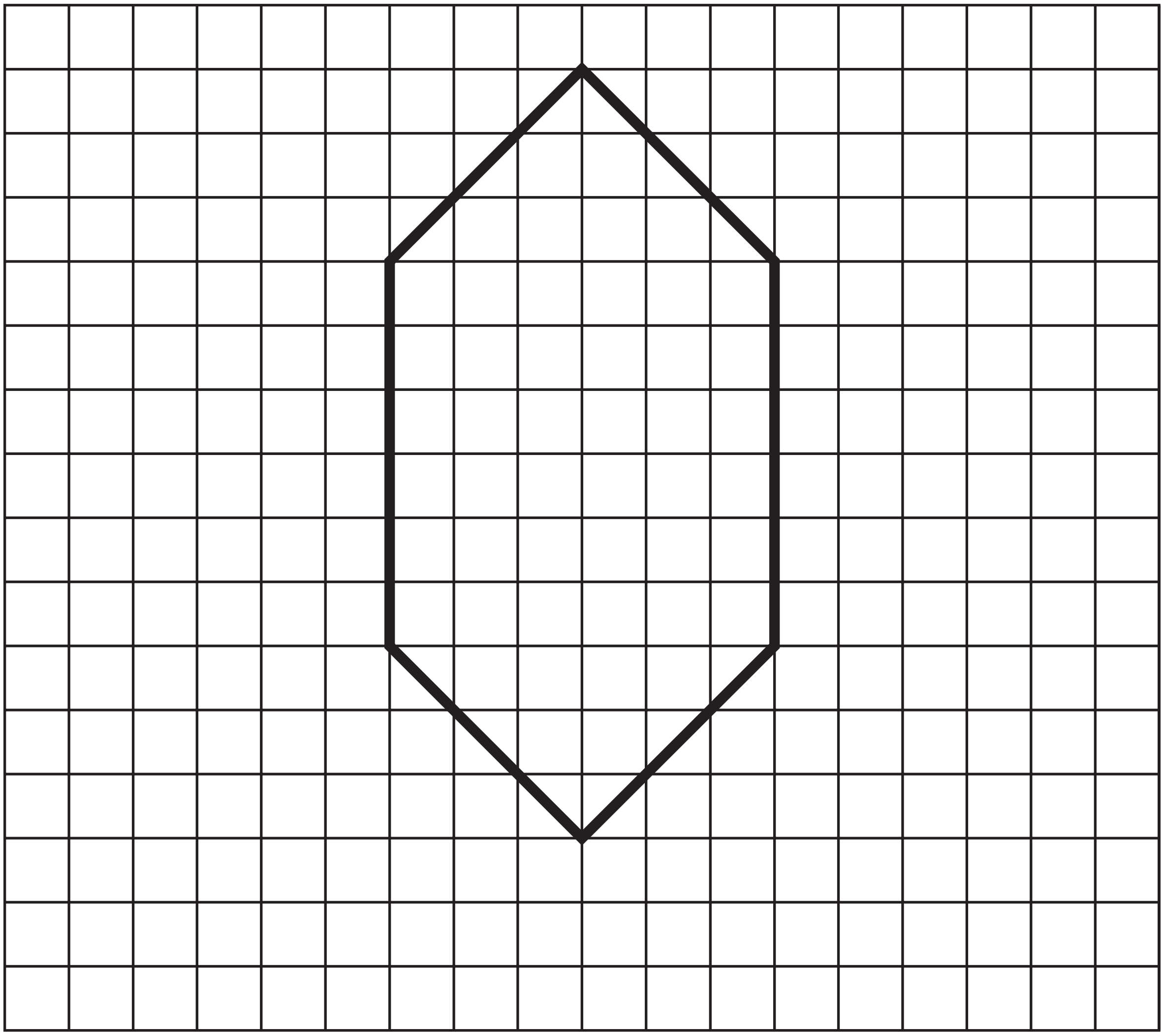
Reference

Ruef, J. (2016). *Building powerful voices: Co-constructing public sensemaking* (Doctoral dissertation). Stanford, CA: Stanford University.

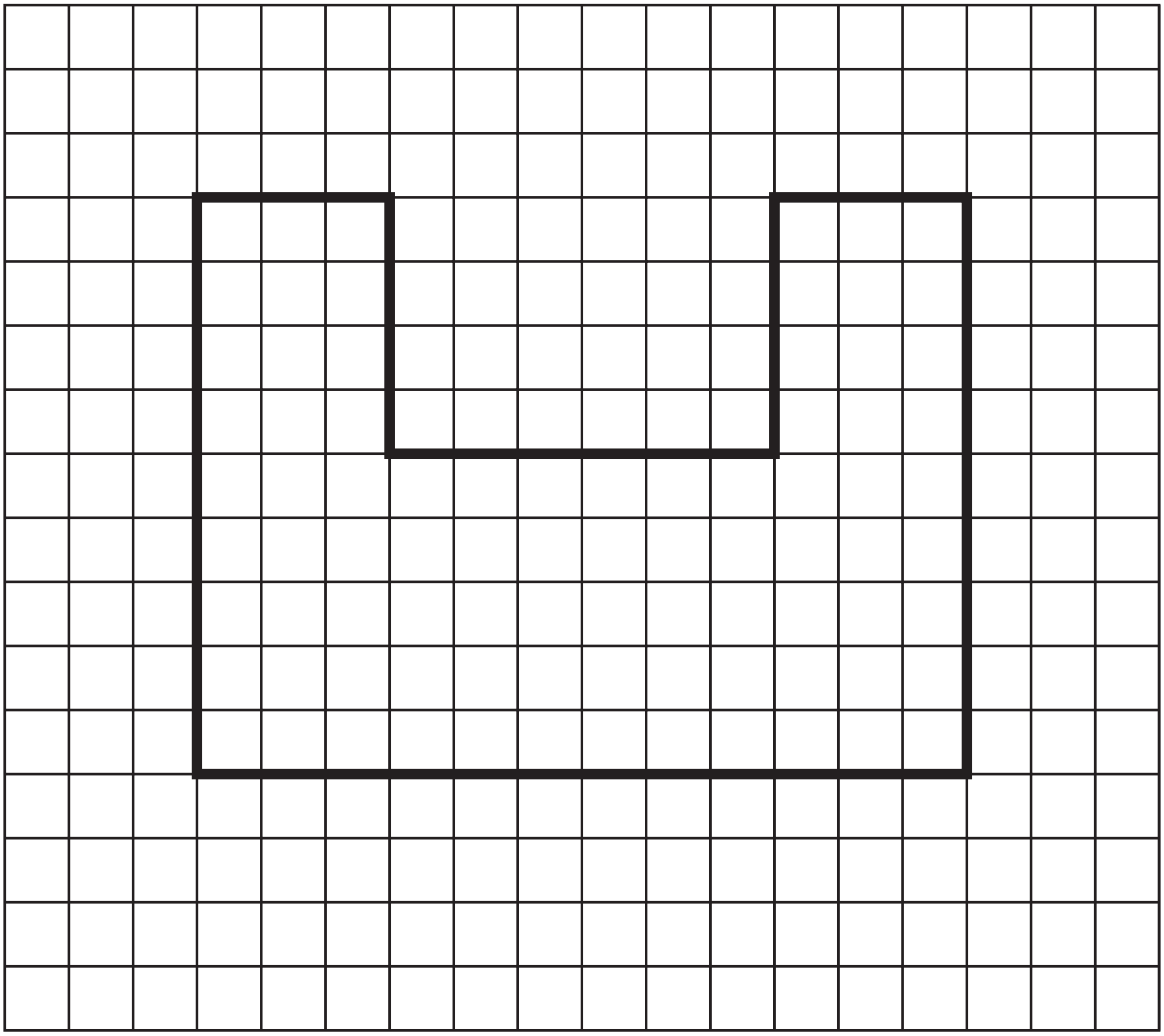
Quadrilateral in Question



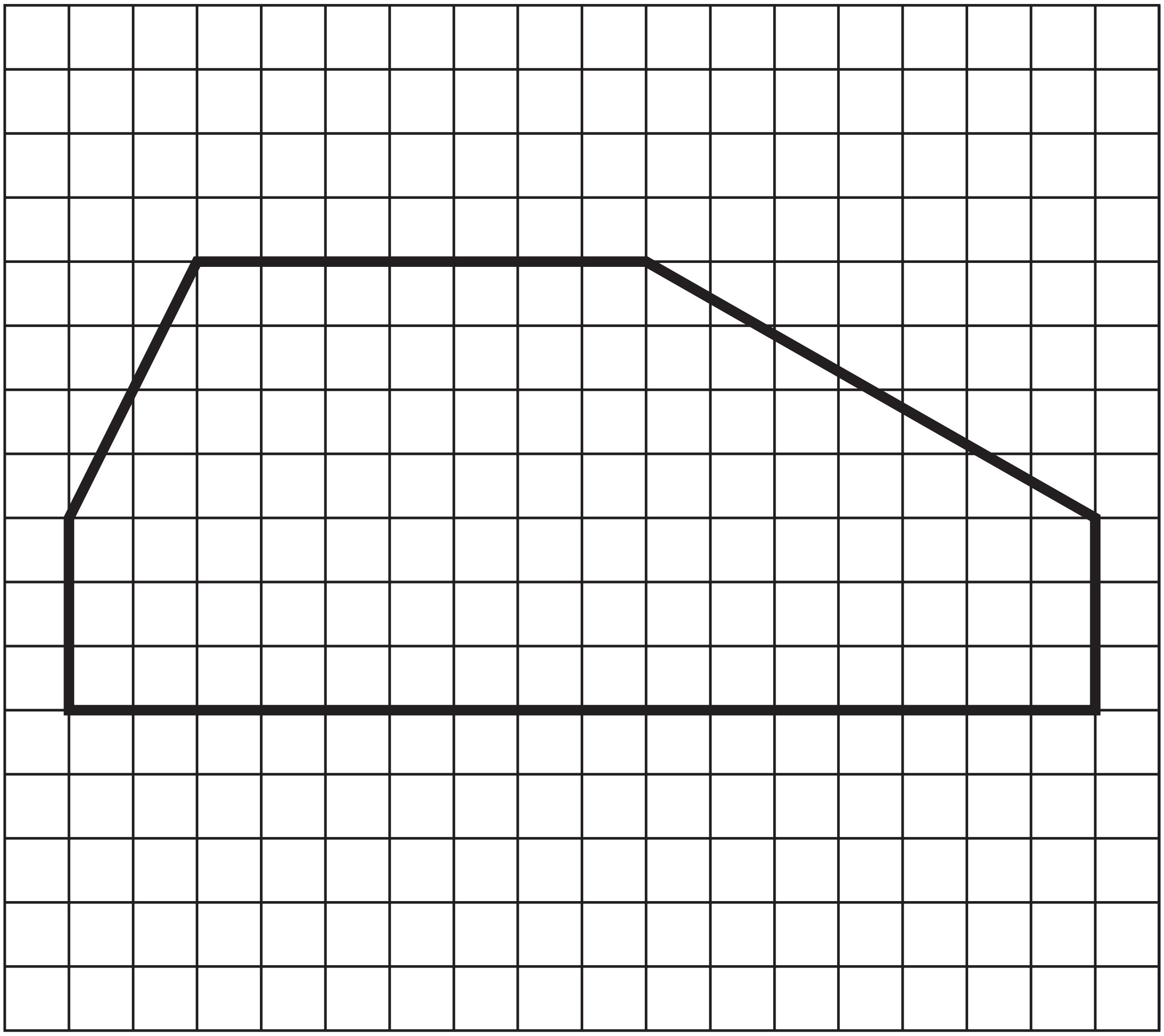
Polygon 1



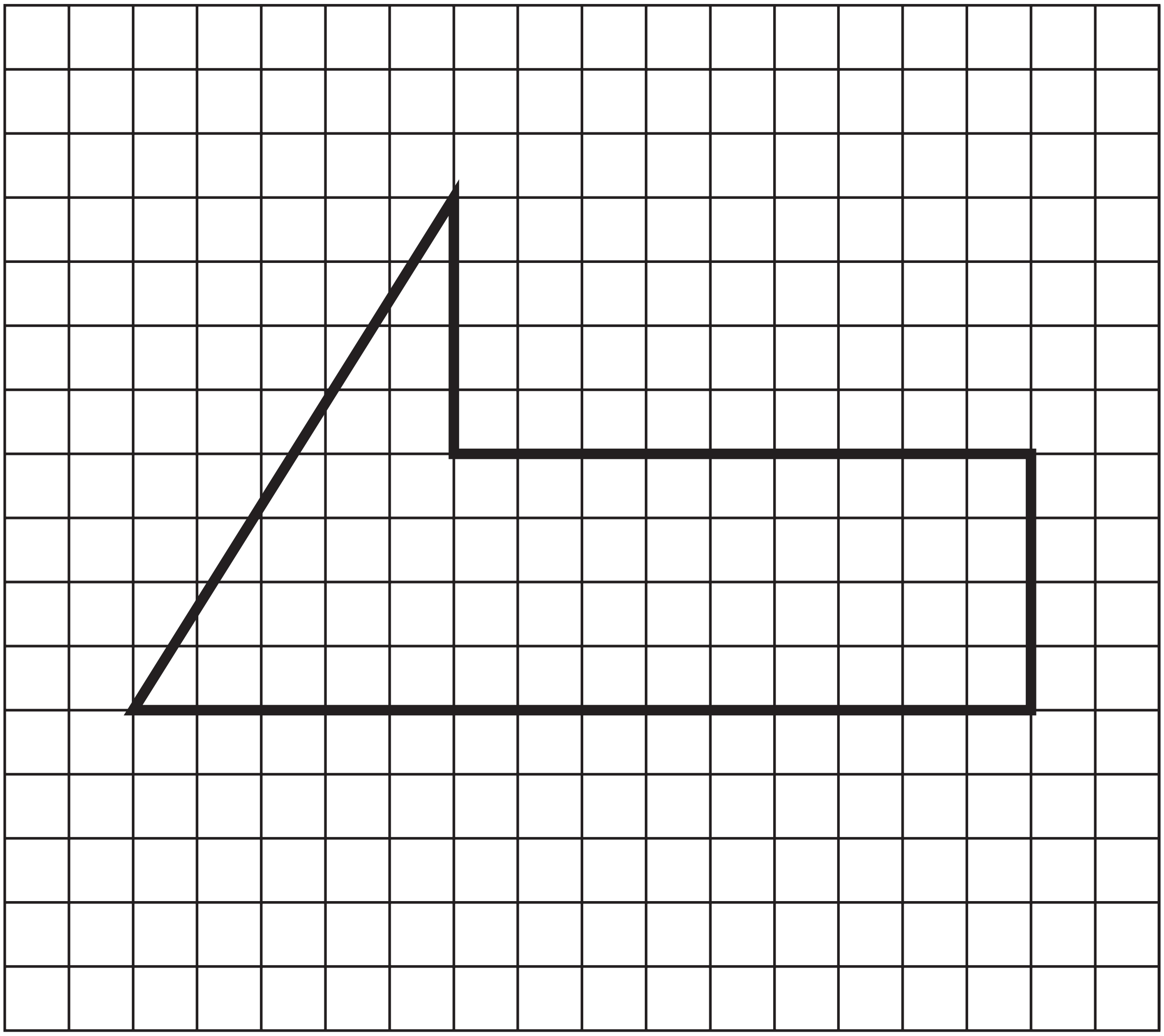
Polygon 2



Polygon 3



Polygon 4



Shards of a Shape

Snapshot

Students play with different ways to decompose a complex polygon to find its area.

Connection to CCSS
6.G.1

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5–10 min	Show students the Shards of a Shape sheet and ask them what they notice. Focus attention on the way the figure is decomposed into simpler polygons. Ask students how they might decompose the same figure to find its area.	Shards of a Shape sheet, to display
Play	25–30 min	Students work in partnerships or small groups to figure out how to decompose the figure to find its area. Students make a poster to show visual proof of the ways they are finding the area of the different parts of the figure.	• Shards of a Shape Outline sheet, multiple copies per group • Posters, one per group • Make available: colors, rulers or other straightedges, and tape
Discuss	20+ min	Conduct a gallery walk of students' posters and compare the ways groups have decomposed the image. Invite students to share their strategies for finding area. Ask, What are the most useful ways we have come up with?	

Activity	Time	Description/Prompt	Materials
Explore	20–30 min	Groups continue to work on finding the complete area of the figure using the most useful strategies shared in the discussion.	Make available: Copies of Shards of a Shape Outline sheet, colors, rulers or other straightedges, and tape
Discuss	15+ min	The class discusses the area of the figure and comes to consensus on what area makes sense given the evidence provided by different groups.	

To the Teacher

This activity represents a leap in complexity from the shapes students explored in the Visualize activity. We encourage you to focus the first part of the lesson purely on decomposing the figure. Give students the space to play with and explore different ways they might decompose the figure to find its area. At this stage, finding the area itself is not the goal. Rather students should focus on thinking about how to partition the figure into shapes that they might be able to use to find the area of the whole. There are an infinite number of ways to do this, and some decompositions are more useful, efficient, and elegant than others. Students may need many copies of the figure as they iterate to find ways to decompose that they think will be useful.

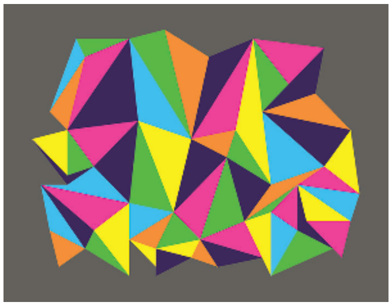
As students move to finding the area of the decomposed parts and the whole figure, they will need strategies for finding the area of such complex pieces. You may want to provide patty paper for students to use to trace, rotate, and flip as they work, if that helps them think through their decomposition.

Finally, we have not provided the answer to this task, precisely because it is critical that students be in charge of determining what area makes sense for this figure. You may find yourself in the position that you yourself do not know who or what is correct. Embrace this ambiguity and entrust students with the authority to use evidence to determine what makes sense.

Activity

Launch

Launch the activity by showing the class the Shards of a Shape sheet on a document camera. Ask, What do you notice about this shape? Give students a chance to turn and talk to a partner about what they notice. Ask students to share some of their observations with the class. Students may notice how the different polygons fit together to make a larger shape, or, conversely, they may see the larger shape as being cut into simpler polygons. Draw students' attention to the way the figure is decomposed into triangles and quadrilaterals. Students may notice that this decomposition makes you see things (like pyramids, mountains, or textures) that you wouldn't see otherwise.



Shards of a Shape

Tell students that today they are going to be playing with finding the area of this complex polygon. Point out that while this image shows a really interesting decomposition, it's not as useful for finding area as the ways students were decomposing shapes in the Visualize activity. Pose the question, How would you decompose this shape to find its area?

Play

Working with partners or in small groups, students use the Shards of a Shape Outline sheet, in which just the outline of the figure is shown on dot paper. Invite students to try the following and consider these questions:

- Study the image. What do you notice? What strategies could you use to decompose?
- Try more than one way to decompose the figure into smaller shapes that could help you find the figure's area. Which way to decompose the shape looks like it will be the most useful?

- Find the area of the shape. Color-code the different parts of your decomposed shape to help others see how you found the area. Put your shape on a poster so that you have more room to show how you are finding the area of the figure. Make proofs of your solution.

It is unlikely that all groups will finish finding the area of the entire figure before you call the class together to discuss their work in progress. Simply ask students to show on their chart as much of their thinking as it currently stands before posting it for the discussion. They will have the chance to return to it afterward.

Discuss

Display all the posters of students' decomposed figures. Do a gallery walk and ask students to think about the following questions as they look at the class's work in progress:

- What do the different ways of decomposing have in common? Why do you think we used this shared strategy?
- How are our strategies different? Which ideas are most interesting and why?

After students have had a chance to examine one another's work, discuss the following as a class:

- What differences and similarities did you notice about the ways we decomposed the figure?
- (Zoom in on specific parts of the image that students tackled differently.) Why did the groups decompose this portion of the figure differently? Which ways do you think will be the most useful for finding area? Why?
- What strategies did groups use for finding the area of different parts of the figure? (Invite students to explain some of the different ways they tackled area, with particular attention to partial units.)
- What are the most useful ways to decompose and find area we have come up with? Why? What questions do we still have?

Explore

Students return to working on finding the area of the Shards of a Shape figure. Invite students to continue using the poster they have started or to get a fresh copy of the Shards of a Shape Outline sheet to start over. Students should be free to use any of the strategies shared in the previous discussion, combining or adapting them as they see fit to find an area they feel confident in. Groups should be prepared with visual proof of their area to share with and convince the class.

Discuss

Post all of the groups' work for the class to see. Invite groups to share the area they found for the figure. Ask, What is the area? How do we know? The goal of this discussion is to come to agreement on the area. Students will undoubtedly have different areas. As you discuss what students have found, ask the class to examine these differing areas to figure out why they are different and what they believe is the most accurate answer. This debate is crucial for students to make sense of the task and use evidence to convince one another. Identifying errors and convincing others of what led to those errors are critical skills of argumentation.

Look-Fors

- **How are students decomposing the figure?** There are an infinite number of ways to decompose this polygon. Some ways will lead to more challenges when attempting to find area, while others will reduce the complexity of finding area. Pay attention to how students are using the vertices and sides of the figure, and whether students are decomposing into shapes that have right angles. Using right triangles will make finding area much more manageable than using the triangles shown in the original Shards of a Shape image. If students are struggling to come up with a decomposition strategy, focus their attention on just part of the image. You might ask them to point to a part of the image that looks most familiar, or where they can see a shape hidden inside. Then ask, How could you find the area of this part? What shape do you see? Ask students to mark it on their outline and show evidence of the area, before repeating this process building outward from the part they just figured out.
- **What strategies are students using to find area? How are they dealing with partial units?** Draw on the strategies that students developed in the Visualize activity and notice as you look at students' work which of these strategies

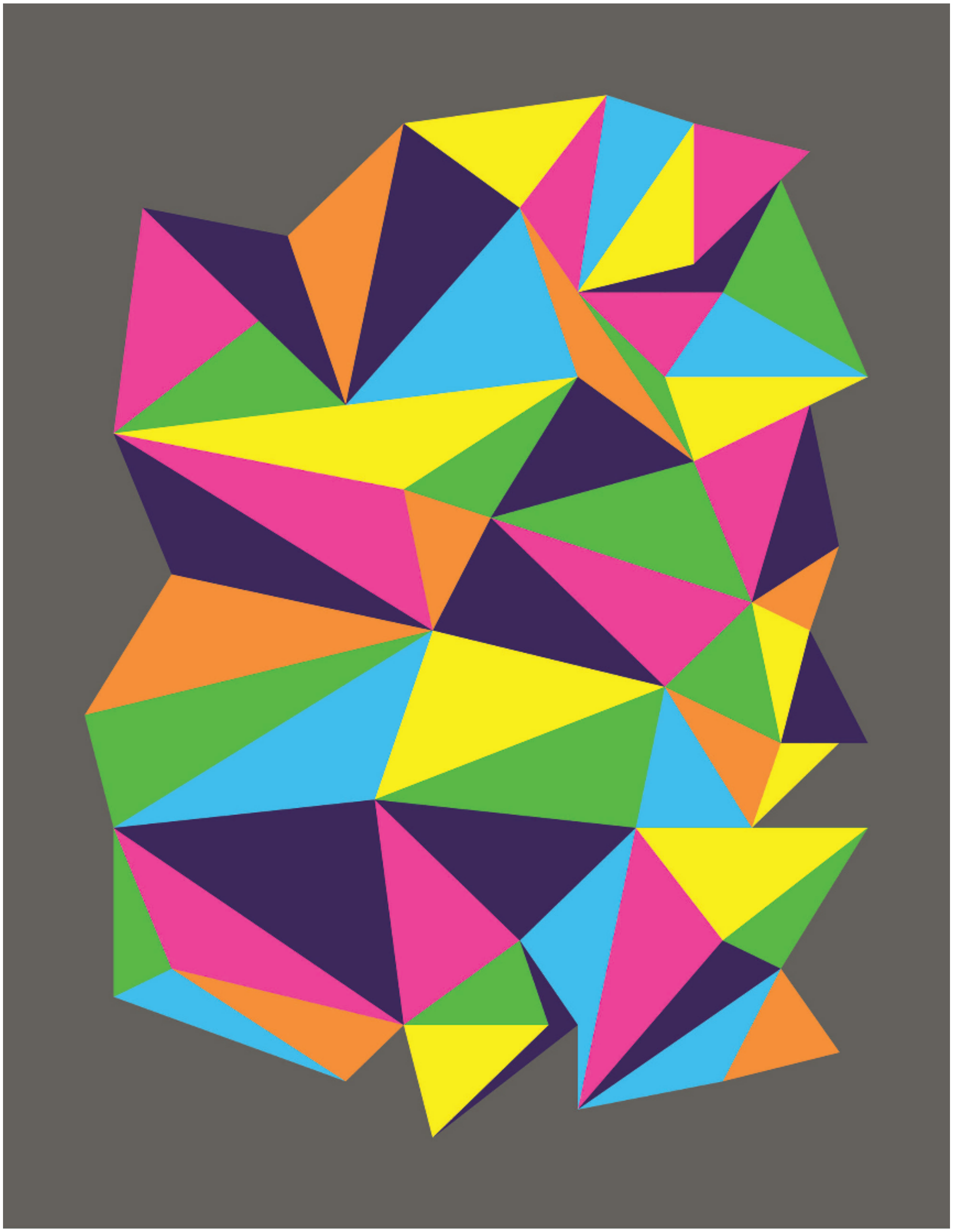
students are using. Are there any strategies students are ignoring? Are students attending to partial units with meaning? You may want to make explicit connections between this shape and the work from the Visualize activity by pointing out the posters students made on that day. You might ask, How are you counting the partial squares? Which strategy from the other day are you using here? How do you know you've counted the area accurately? Push students to make connections and justify.

- **Are students accounting for the entire shape?** In decomposing such a complex shape, it is possible students will simply miss some of the parts. This is particularly likely if students work from several different starting points, say by lopping a triangle off on one side, then pulling a rectangle out of the middle, then moving to another triangle on the far side. Encourage students to look at the entire figure and color-code the different parts to help them keep track of all the shapes they have created through decomposition. Students may want to start over with a new sheet if they get too tangled up in the many shapes they have created. You may also want to encourage students to explore whether cutting the figure into more or fewer shapes makes finding the area (and keeping track) easier.
- **Are students using evidence to determine what makes sense?** In the discussion of the area of the figure, it is critical that agreement on the area is based on evidence rather than on whose voice has the most power or authority in the classroom. Discussions like these can sometime hinge not on *what* is being said but on *who* is saying it. Use this opportunity to enforce the norm that arguments must be supported with evidence from mathematics, not just loud, confident voices. You might ask the class, Are you convinced? Why or why not? Is there another possible answer? Why? Which parts of the area can we agree on? Why are the other parts difficult for us to agree on? What do we need to figure out? How will we do that? Use these kinds of facilitation questions to hold the class to a high standard of evidence, even if it means the discussion must continue on another day.

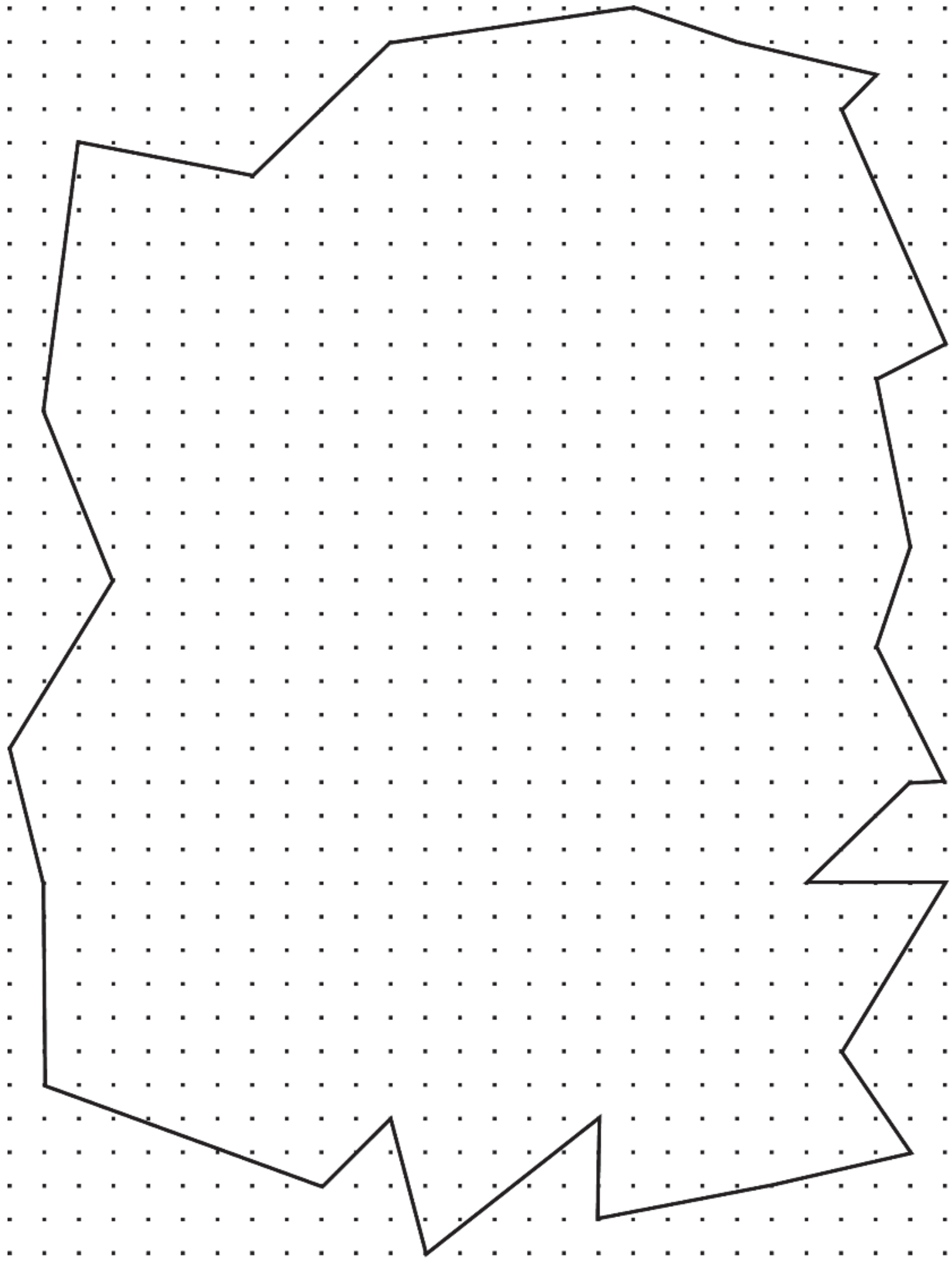
Reflect

What strategy for decomposing to find area did you find most useful? Why?

Shards of a Shape



Shards of a Shape Outline



Rising from the Footprint

Snapshot

Students explore finding the volume of solids that are not rectangular by constructing buildings on shapes explored in the Visualize activity.

Connection to CCSS
6.G.2

Agenda

Activity	Time	Description/Prompt	Materials
Launch	5–10 min	Show students the Shanghai Skyline image and draw attention to the different shapes and heights of the buildings. Show the U-Shaped Building Footprint sheet and ask students to imagine that this is the footprint of a building $6\frac{1}{2}$ units tall. Ask, *What will it look like? How might you find its volume?*	• Shanghai Skyline, to display • U-Shaped Building Footprint sheet, to display
Explore	20–30 min	Students work in partnerships or small groups to develop methods for finding the volume of the U-shaped building, and others that are not rectangular solids.	• U-Shaped Building Footprint sheet, one per partnership • Make available: snap cubes and copies of the Building Footprint A–D sheets
Discuss	15–20 min	Discuss the different methods students have developed to find the volume of the U-shaped building and come to consensus on its volume.	

Activity	Time	Description/Prompt	Materials
Explore	30+ min	Students return to work in their groups to find the volumes of other buildings, posting their solutions clustered by building. Then groups of students curate each building's solutions and determine what can be shared and discussed about the solutions for that building.	• Building Footprint A–D sheets, one per partnership for students to choose from • Snap cubes • Display space for solutions for each building
Discuss	15–20 min	Each curating group shares something about the solutions for their building, and the class discusses any questions these groups pose. Tell students these figures are all prisms. Ask, How do you find the volume of a prism?	

To the Teacher

In this activity, we build on the foundation laid in the Visualize activity. In that work, students decomposed figures to find their area, and in this activity, we use similar shapes as footprints for buildings that rise from them. We have provided these similar figures here on grid paper, this time using square units that are the same size as most snap cubes so that students can build up, layer by layer, if they wish. The footprints with partial units pose a challenge, and students will need to figure out how they might build and find the volume using whole cubes when the footprints are not made of whole units. For buildings with a whole number of units in the footprint, we have made the heights fractional, pushing students to think about the volume with half cubes.

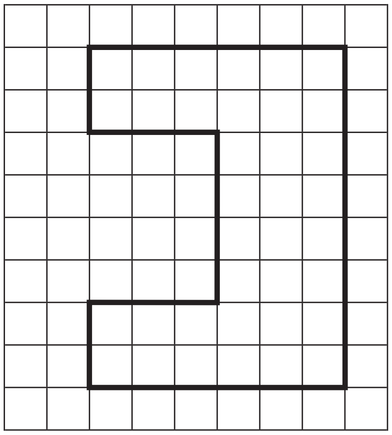
In each of these cases, we think that using whole snap cubes is a valuable experience for constructing volume and provides a concrete model for discussing what is happening with partial units either in the base or the height. The challenge will necessitate students to visualize how what they can construct is different from the figure being described in the activity.

Activity

Launch

Launch the activity by showing the Shanghai Skyline image (see next page). Point out that the buildings in any city or town come in different shapes and heights. You might ask students what shapes they see in this skyline or what differences they notice among the buildings. Then show the class the U-Shaped Building Footprint sheet and remind students that they explored the area of a shape similar to this in the Visualize activity.

Shanghai Skyline



U-Shaped Building Footprint

Ask students to imagine that this is the footprint of a building in a city. This building is going to be $6\frac{1}{2}$ units high. Ask, What does the building look like? Ask students to turn and talk to a partner and describe what they see in their minds. Invite students to offer descriptions of the building and what they see in their minds. Encourage descriptive language and gestures that help students communicate what they imagine. Perhaps this building actually reminds them of one they have seen.

Tell students that today their challenge is to find the volume of this building and the others that are similar to what they have already seen before. Ask, How can you reason about the volume of buildings that are not rectangular solids?

Explore

Students work with a partner or in a small group to develop methods for finding the volume of buildings that are not rectangular solids, starting with the U-shaped building. Provide partners with a copy of the U-Shaped Building Footprint sheet and make snap cubes available. Ask students, What is the building's volume? Develop a method for finding the volume of this building.

Once partners have found the volume of the U-shaped building, invite them to choose another footprint and find the volume of the building. Once all groups have found the volume of the U-shaped building, gather the class together for a discussion.

Discuss

Discuss the following questions:

- What is the volume of the U-shaped building? How do you know?
- What strategies did you use?
- How did you deal with the fractional height? How did you count partial cubes?

Come to consensus about the volume of the U-shaped building and ways of reasoning about volume with partial cubes.

Explore

Students return to their groups to investigate the volumes of the other buildings. For each volume students find, ask them to represent their thinking on the page as clearly as they can so that other groups can understand what they have done. For each building, create a display space in the classroom and have students post solutions on the wall or bulletin board clustered by building.

Once students have had the chance to work on several buildings, assign a group of students to curate each building's solutions. In their group, students can reorder or regroup the solutions. Ask students to look at what the solutions have in common or where they are different. In their curating group they need to decide, What is interesting for us to talk about for this building's volume and the class's solutions? The group should choose something to show and a question for the class to discuss.

Discuss

Invite each curating group to share something from the solutions they curated and pose a question that the class can discuss. Be sure to draw attention to places where groups disagreed or where they agreed but used very different strategies.

At the close of the discussion, tell students that each of these buildings is a *prism.* Pose the question, How do we find the volume of prisms?

Look-Fors

- **Are students drawing on their experience finding the areas of these footprints?** If you still have posters or student work available from the Visualize activity, it can serve as a useful reference for students. It is a conceptual stretch, however, for students to think of volume and area as related, particularly since teachers work hard to ensure that students don't get area and perimeter confused. The relationship between area and volume will require that students shift from thinking in square units used to cover the two-dimensional shape to cubic units used to cover and build up from the footprint. Don't underestimate the shift in conceptual thinking that this requires, and the relationship between area and volume will not be immediately obvious to all students. Encourage students to build using the snap cubes and ask them questions about how what they are doing is and is not related to the work they did to find area. You might ask, How is this like finding area? How is it different? Can area help you find volume? How?

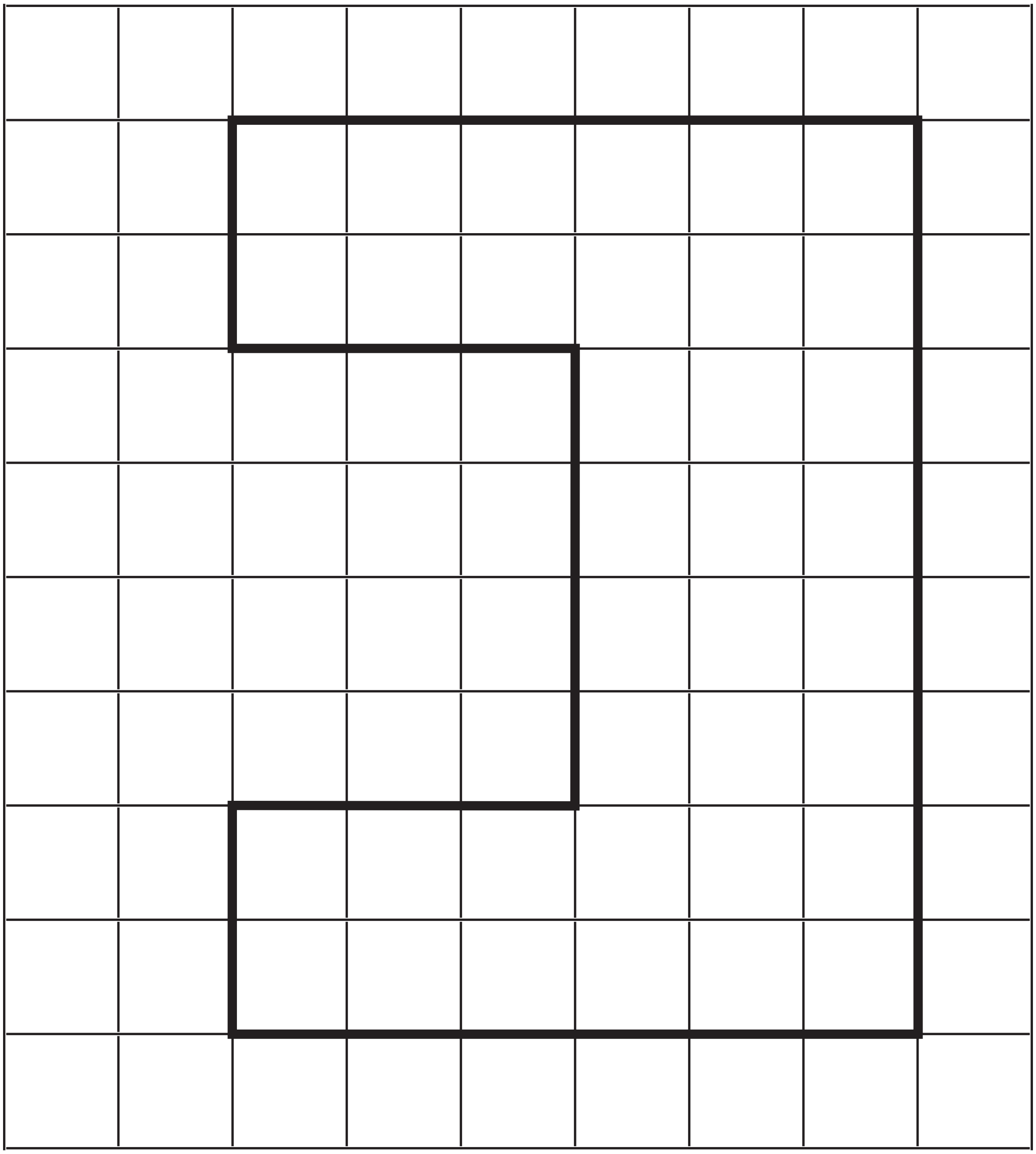
- **How are students reasoning about partial cubes?** All of the figures we have created in this investigation require that students reason about partial cubes when they cannot build with partial cubes. Students may tackle this challenge by building figures too large and imagining shaving parts off, or by doing the opposite, building figures too small and imagining adding missing parts back on. How are students doing this? How are they mentally accounting for what is being added or subtracted? This requires both the conceptual understanding that such slicing is needed and the spatial relations to imagine what cannot be made concrete well enough to count the parts. Again, this is quite challenging work, and we encourage you to take that challenge seriously when students struggle. Ask them questions to support visualizing the building, such as, What kind of block(s) would you need to build this building so that it was accurate? How big would those blocks be in relationship to the cubes we have? How do you know? How could we count the volume of just those pieces?
- **Are students thinking about volume as layers of cubes?** A key part of understanding volume is thinking in layers of cubic units. Students may have had experiences with volume in the past that focused only on multiplying dimensions of rectangular solids. If this is the case, students will struggle with imagining the volume of nonrectangular solids, which require thinking conceptually rather than formulaically. You might ask students to construct the building from the ground up and ask them, What would the volume be if the building was only one unit high? Two units high? Three units high? And so on. Such questions may help students both attend to the pattern that is created by building in layers and reason about half-unit heights as well.

Reflect

How do we find the volume of prisms?

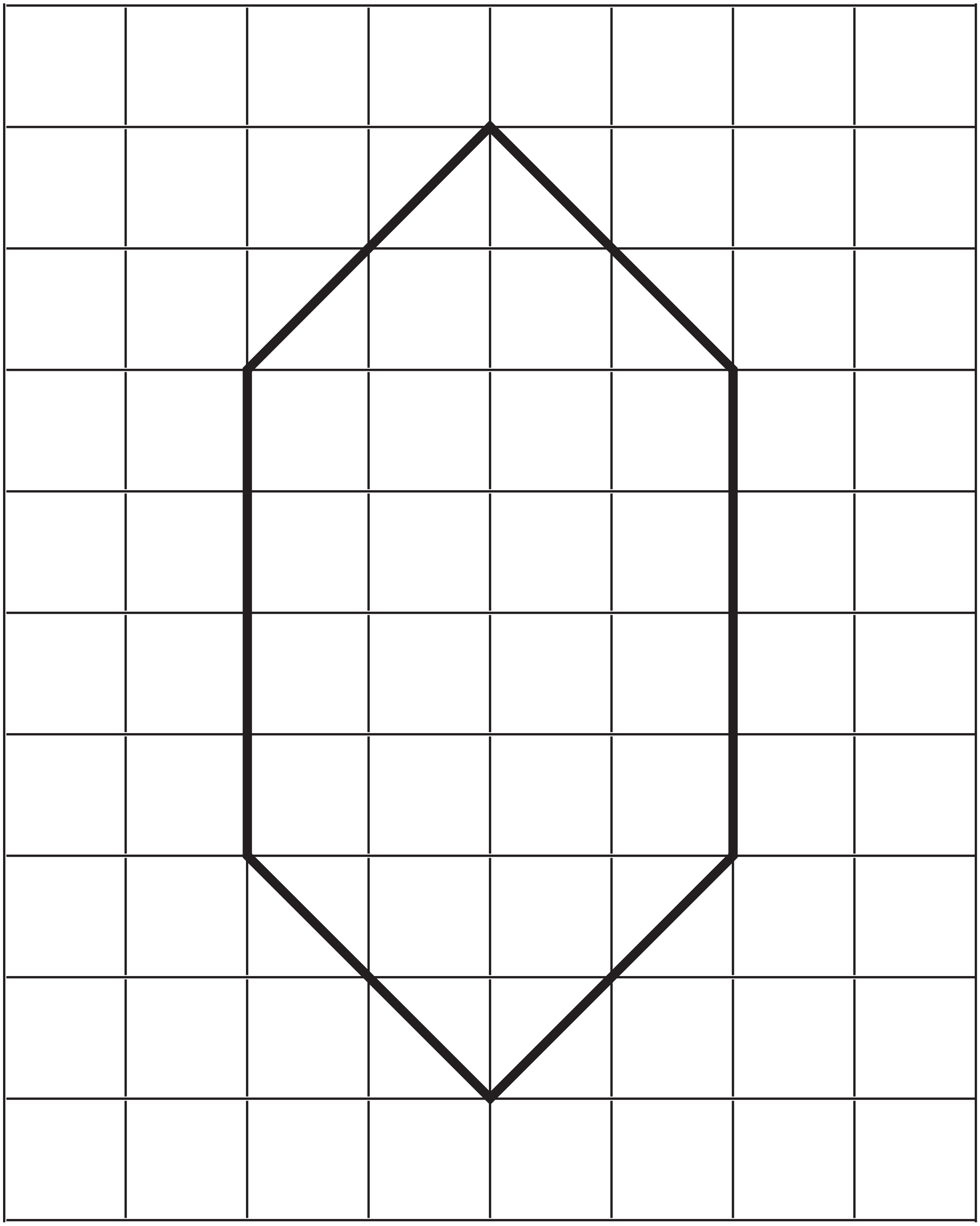
Shanghai Skyline

U-Shaped Building Footprint



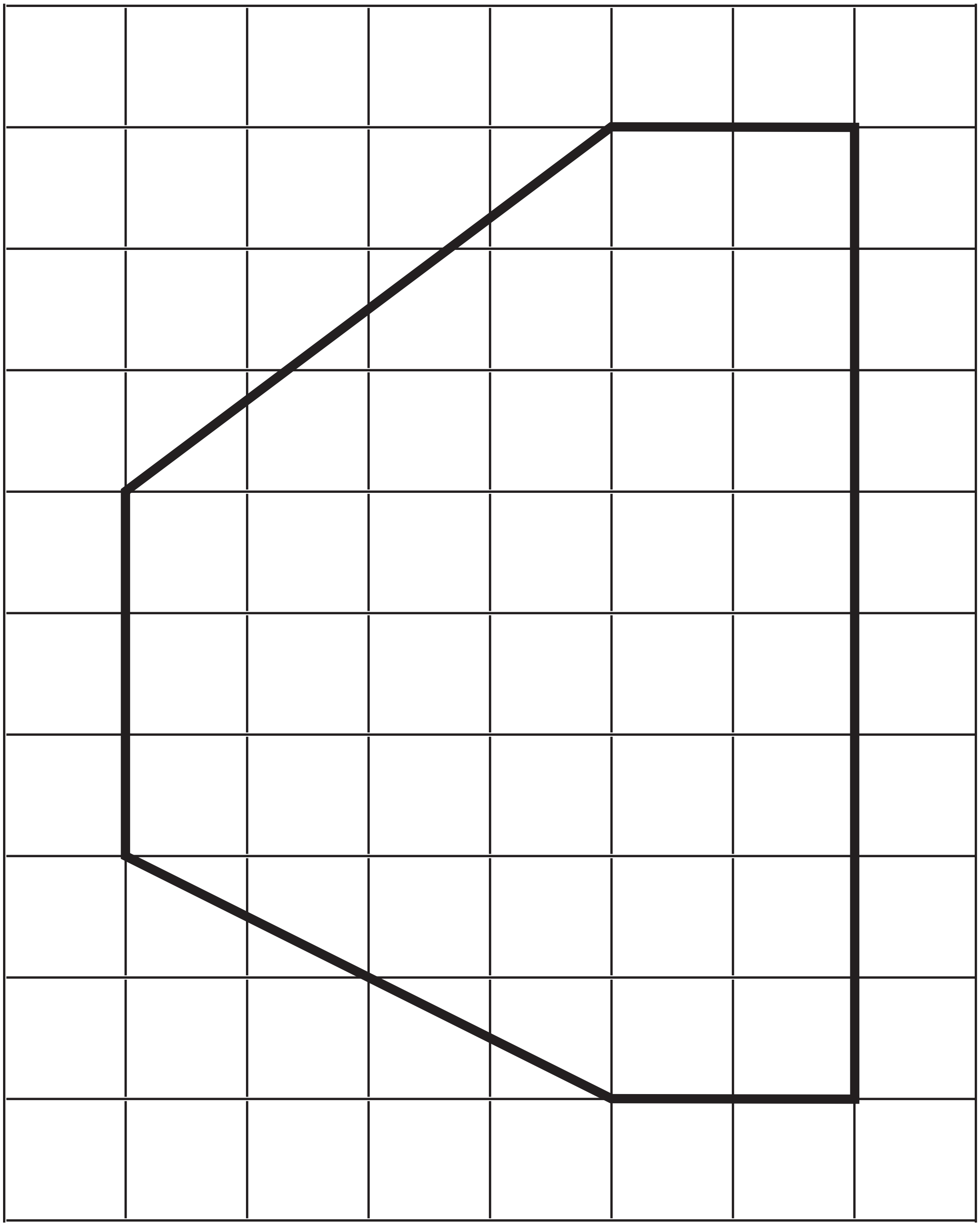
This building is $6\frac{1}{2}$ units high.

Building Footprint A



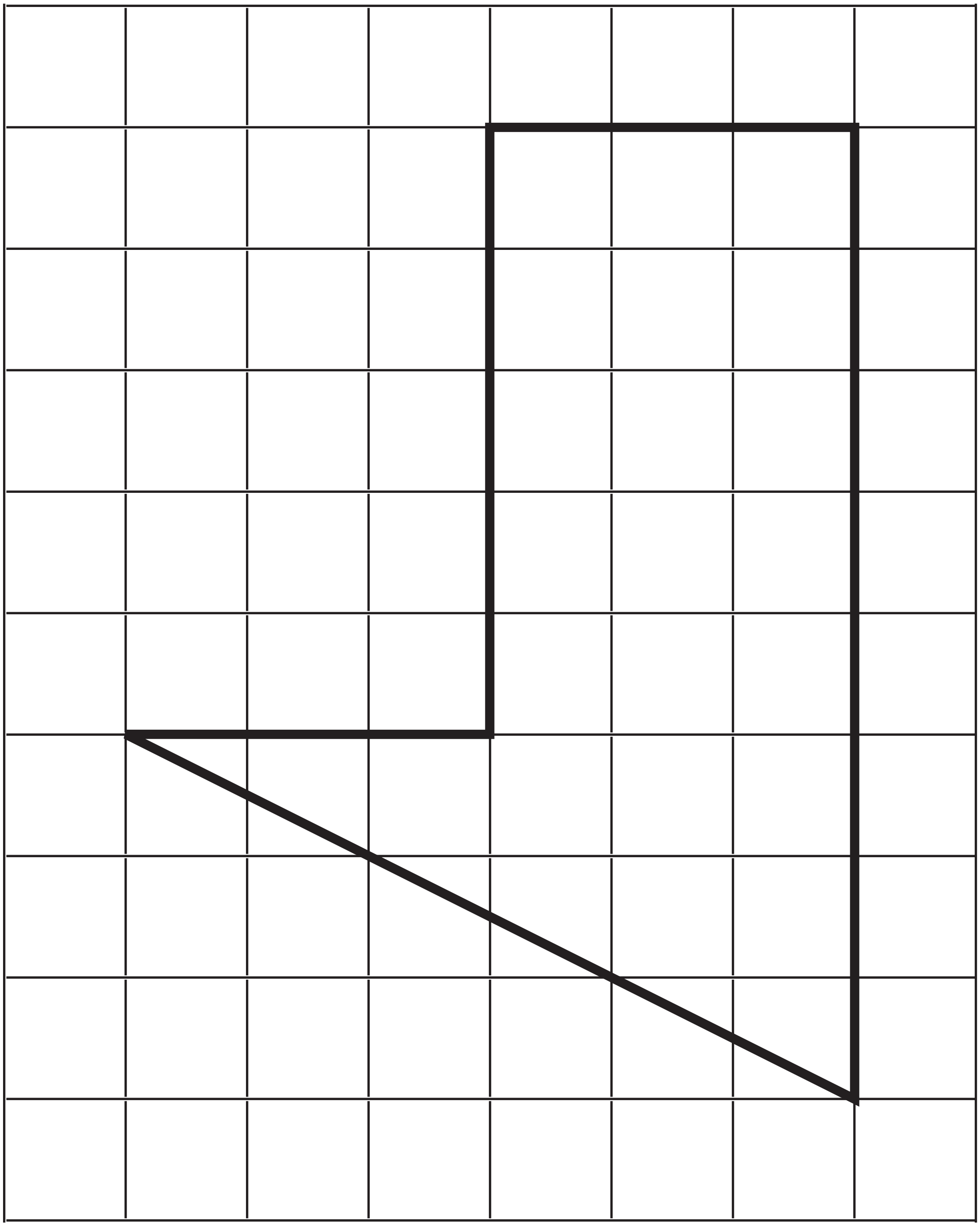
This building is 7 units high.

Building Footprint B



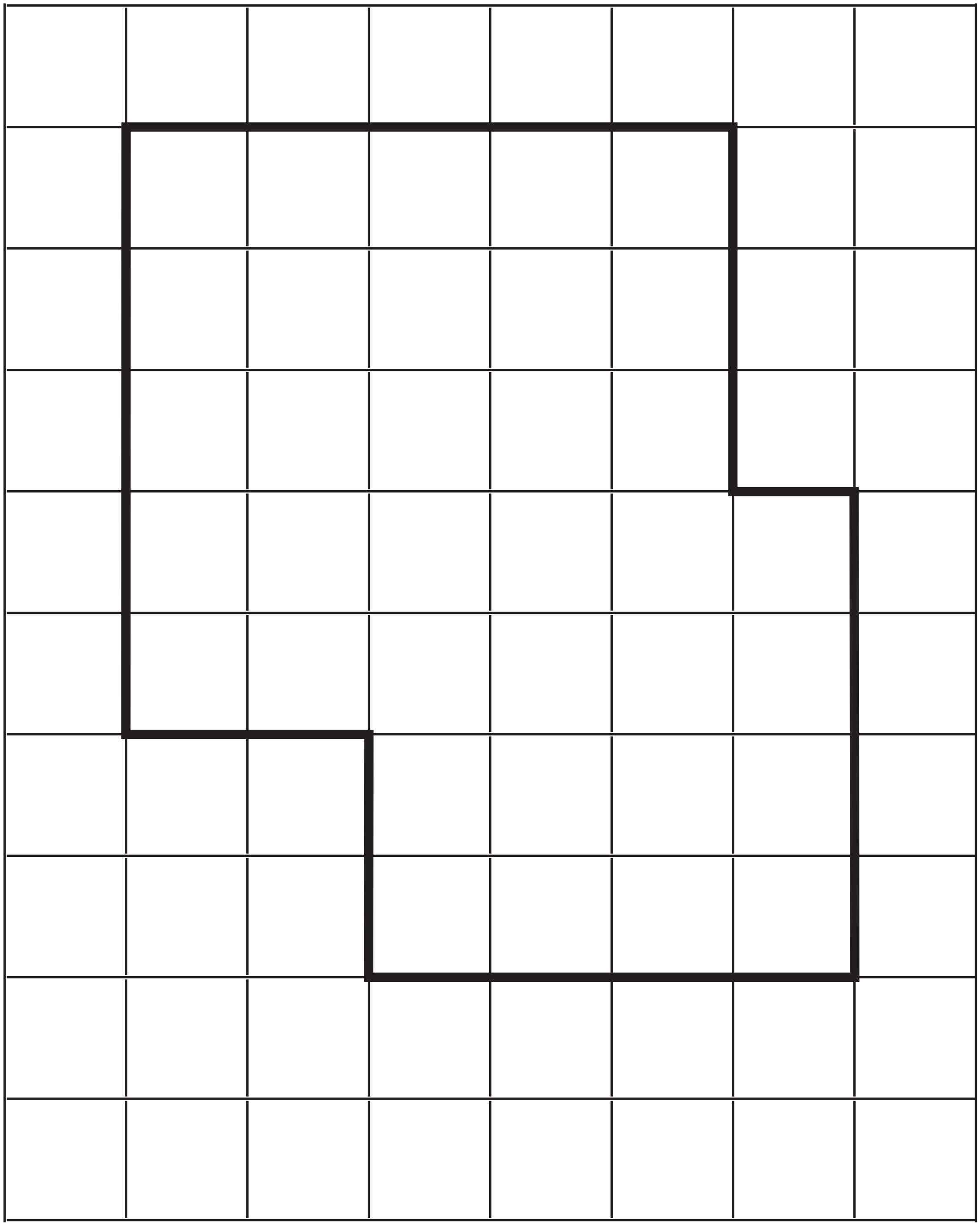
This building is 6 units high.

Building Footprint C



This building is 8 units high.

Building Footprint D



This building is $5\frac{1}{2}$ units high.