CHAPTER 1

# An Introduction to Data and Functions

## Overview

How can you describe patterns in data? In this chapter we explore how to use graphs to visualize the shape of single-variable data and to show changes in two-variable data. Functions, a fundamental concept in mathematics, are introduced and used to model change.

**After reading this chapter, you should be able to**

- describe patterns in single- and two-variable data
- construct a "60-second summary"
- define a function and represent it with words, tables, graphs, and equations
- identify properties of functions
- use the language of functions to describe and create graphs



## 1.1 Describing Single-Variable Data

This course starts with you. How would you describe yourself to others? Are you a 5-foot 6-inch, black, 26-year-old female studying biology? Or perhaps you are a 5-foot 10-inch, Chinese, 18-year-old male English major. In statistical terms, characteristics such as height, race, age, and major that vary from person to person are called *variables*. Information collected about a variable is called *data*.[1]

Some variables, such as age, height, or number of people in your household, can be represented by a number and a unit of measure (such as 18 years, 6 feet, or 3 people). These are called *quantitative variables*. For other variables, such as gender or college major, we use categories (such as male and female or biology and English) to classify information. These are called *categorical* (or *qualitative*) data. The dividing line between classifying a variable as categorical or quantitative is not always clear-cut. For example, you could ask individuals to list their years of education (making education a quantitative variable) or ask for their highest educational category, such as college or graduate school (making education a categorical variable).

Many of the controversies in the social sciences have centered on how particular variables are defined and measured. For nearly two centuries, the categories used by the U.S. Census Bureau to classify race and ethnicity have been the subject of debate.

[1] *Data* is the plural of the Latin word *datum* (meaning "something given")—hence one datum, two data.

# Visualizing Single-Variable Data

Humans are visual creatures. Converting data to an image can make it much easier to recognize patterns.

## Bar charts: How well educated are Americans?

Categorical data are usually displayed with a bar chart. The length of the bar for a single category tells you either the *frequency count* (the number of observations that fall into that category) or the *relative frequency* (the percentage of the total observations).

Since the relative size of the bars is the same using either frequency or relative frequency counts, the two scales are often put on different vertical axes of the same chart. For example as shown in **Figure 1.1**, a bar chart of the educational attainment of Americans in 2008, approximately 60 million Americans had a high school degree but never went to college, and this represents approximately 31% of Americans 25 years or older.

**Exploration**

Exploration 1.1 provides an opportunity to collect your own data and to think about issues related to classifying and interpreting data.

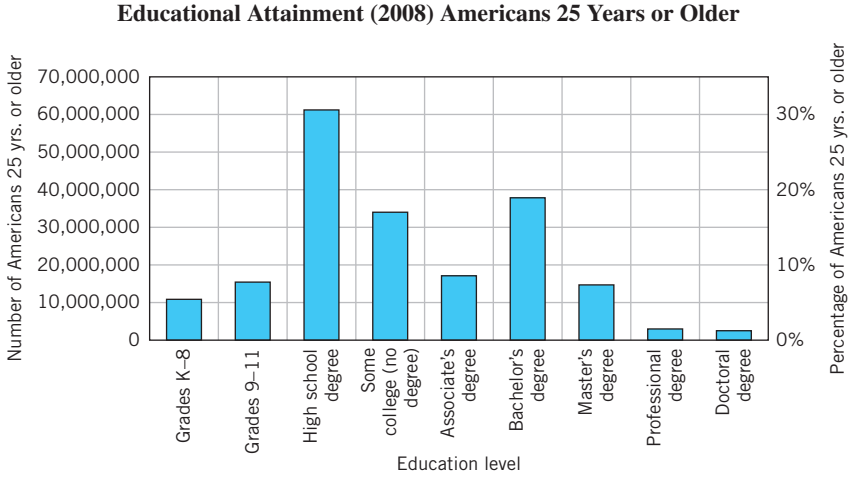



**FIGURE 1.1** **Bar chart showing the education levels for Americans age 25 or older.**

Source: U.S. Bureau of the Census, www.census.gov.

Always, remember to ask: Percentage of what? The bar chart in **Figure 1.1** represents the percentage of Americans with high school degrees relative to Americans 25 years or older, not the percentage of all Americans.

### EXAMPLE 1 | What Does the Bar Chart Tell Us?

In 2018, for the first time since 1940, when the U.S. Census started to collect data on educational attainment, one third of adults (25 years or older) in the U.S. had a bachelor's degree or higher.

**a.** How does this compare to a decade earlier?

**b.** What does this bar chart not tell us?

**Solution**

**a.** Using **Figure 1.1**, in 2008, those with bachelor's degrees but no further education number about 38 million or 19%.
Those with bachelor's degree or higher would be approximately

$$\underbrace{\text{Bachelor's} + \text{Master's} + \text{Professional} + \text{Doctoral}}_{\text{Total \% Bachelor's or higher}}$$

$$19\% + 7\% + 1\% + 1\% = 28\%$$

From 2008 to 2018, the percentage of Americans (25 years or older), with a bachelor's degree or higher, rose by approximately five percentage points.

**b.** The bar chart does not tell us the total size of the population or the total number (or percentage) of all Americans who had bachelor's degrees or higher. If we include all Americans, we would expect the percentage to be lower.

## An important aside: What a good graph should contain

When you encounter a graph in an article or you produce one for a class, there are three elements that should always be present:

1. An informative title that succinctly describes the graph
2. Clearly labeled axes (or a legend) including the units of measurement (e.g., indicating whether age is measured in months or years)
3. The source of the data cited in the data table, in the text, or on the graph

## Histograms: What is the distribution of ages in the U.S. population?

A histogram is a specialized form of a bar chart that is used to visualize single-variable quantitative data. The horizontal axis on a histogram is a subset of the real numbers with the unit (representing, for example, number of years) and the size of each interval marked. The intervals are usually evenly spaced to facilitate comparisons (e.g., placed every 10 years). The size of the interval can reveal or obscure patterns in the data. As with a bar chart, the vertical axis can be labeled with a frequency or a relative frequency count. For example, the histogram in **Figure 1.2** shows the distribution of ages in the United States as projected for 2010.

Applet

See course software "F1: Histograms" on the course website.

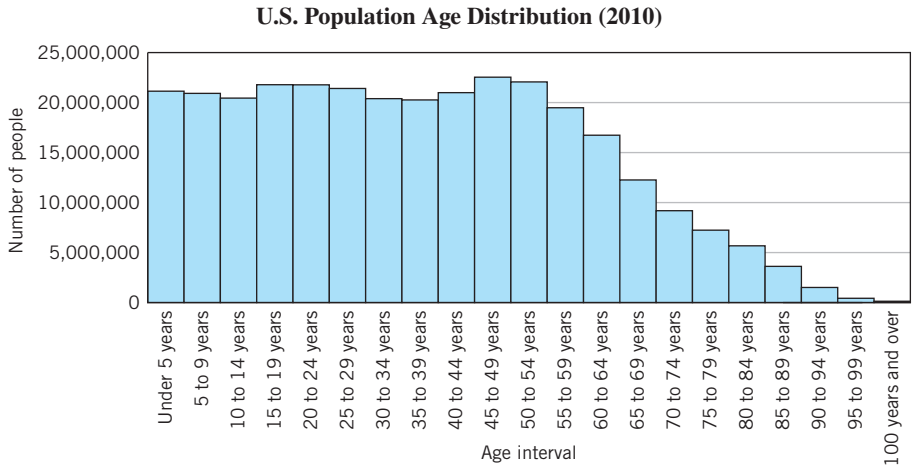



**FIGURE 1.2** **Histogram of the U.S. population in 5-year intervals.**

Source: U.S. Bureau of the Census, www.census.gov.

GCM

See the Graphing Calculator Manual (GCM) Chapter 1, "*Histograms, Scatter Plots, and Graphs of Functions,*" on the course website for instructions on using a graphing calculator in this chapter.

### EXAMPLE 2 | What Does the Histogram Tell Us?

**a.** What 5-year age interval contains the most Americans? Roughly how many are in that interval? See **Figure 1.2**.

**b.** Estimate the number of people under age 20.

**c.** Construct a topic sentence for a report about the U.S. population.

**Solution**

**a.** The interval from 45 to 49 years contains the largest number of Americans, about 23 million.

**b.** The sum of the frequency counts for the four intervals below age 20 is about 84 million.

**c.** According to estimates by the U.S. Census Bureau, in 2010 the number of Americans in each 5-year age interval will be fairly flat up to age 45, will peak between ages 45 and 54, and then fall in a gradual decline.

## Numerical Descriptors: What Is "Average" Anyway?

In 2010 the U.S. Bureau of the Census reported that the mean age for Americans was 37.8 and the median age was 36.9 years.

### The Mean and Median

The *mean* is the sum of a list of numbers divided by the number of terms in the list.

The *median* is the middle value of a list of numbers ordered from the smallest to the largest; half the numbers are less than or equal to the median and half are greater than or equal to the median.

If the number of observations is odd, then the median is the middle number in the list. If the number of observations is even, then the median is the mean of the two middle numbers in the list.

The mean age of 37.8 represents the sum of the ages of every American divided by the total number of Americans. The median age of 36.9 means that if you placed all the ages in order, 36.9 would lie right in the middle; that is, half of Americans are younger than or equal to 36.9 and half are 36.9 or older.

In the press you will most likely encounter the word "average" rather than the term "mean" or "median."[2] The term "average" is used very loosely. It usually represents the mean, but it could also represent the median or something much more vague, such as the "average" American household. For example, the media reported that:

> The *average* American home in 2008 had more television sets than people. . . . There were 2.86 TV sets in the typical home and 2.5 people.[3]
>
> The *average* American family owed more than $8,000 in credit debt in 2008 . . . and is averaging about 5.4 credit cards.[4]

**Reading**

See the reading "The Median Isn't the Message," on the course website, to find out how an understanding of the median gave renewed hope to the renowned scientist Stephen J. Gould when he was diagnosed with cancer.

### The significance of the mean and median

The median divides the number of entries in a data set into two equal halves and is unchanged by changes in values above and below it. For example, as long as the median income is larger than the poverty level, it will remain the same even if all poor people suddenly increase their incomes up to that level and everyone else's income remains the same.

The mean is the most commonly used statistic in the news media and, unlike the median, can be affected by a few extreme values called *outliers*. For example, suppose Bill Gates, founder of Microsoft and one of the richest people in the world, were to move into a town of 10,000 people, all of whom earned nothing. The median income would be $0, but the mean income would be in the millions. That's why income studies usually use the median.

**EXAMPLE 3** | "Million-Dollar Manhattan Apartment? Just About Average"

According to a report cited on *www.therealdeal.com*, in November 2009 the median price of purchasing an apartment in Manhattan was $850,000 and the mean price was $1.32 million. How could there be such a difference in price? Which value do you think better represents apartment prices in Manhattan?

**Solution**

Apartments that sold for exorbitant prices (in the millions) could raise the mean above the median. If you want to buy an apartment in Manhattan, the median price is probably more important because it tells you that half of the apartments cost $850,000 or less.

[2]The word "average" has an interesting derivation according to Klein's etymological dictionary. It comes from the Arabic word *awariyan*, which means "merchandise damaged by seawater." The idea being debated was that if your ships arrived with water-damaged merchandise, should you have to bear all the losses yourself or should they be spread around, or "averaged," among all the other merchants? The words *averia* in Spanish, *avaria* in Italian, and *avarie* in French still mean "damage."

[3]Source: The Nielson Co., 2009.

[4]Source: *www.creditcards.com*.

## An Introduction to Explore & Extend

Throughout the text, there are *Explore & Extend* activities that provide you with ideas for going deeper or previewing what comes next. While these activities are optional, it is hoped they will be engaging, and doing them will increase your understanding. Enjoy!

## Explore & Extend

### 1.1 Could This Be True?

Lake Wobegon is a mythical town where "all the women are strong, all the men are good-looking, and all the children are above average."—*Garrison Keillor*

Could this be true? If you consider only the children in Lake Wobegon, could all the children be "above average" if you use the median? What if you use the mean? If you consider all the children in the United States, could it be possible that all the children in Lake Wobegon are "above average"? Why or why not?

## An Introduction to Algebra Aerobics

In each section of the text there are "Algebra Aerobics" with answers in the back of the book. They are intended to give you practice in the algebraic skills introduced in the section and to review skills we assume you have learned in previous courses. These skills should provide a good foundation for doing the exercises at the end of each section.

## Algebra Aerobics 1.1

**1.** Fill in the following table. Round decimals to the nearest thousandth.

| Fraction | Decimal | Percent |
|---|---|---|
| $\frac{7}{12}$ | | |
| | 0.025 | |
| | | 2% |
| $\frac{1}{200}$ | | |
| | 0.35 | |
| | | 0.8% |

**2.** Calculate the following:

**a.** A survey reported that 80 people, or 16% of the group, were smokers. How many people were surveyed?

**b.** Of the 236 students who took a test, 16.5% received a B grade. How many students received a B grade?

**c.** Six of the 16 people present were from foreign countries. What percent were foreigners?

**3.** Given the list of numbers, find the mean and the median.

**a.** 9, 2, –2, 6, 5

**b.** –2, 2, 5, 6

**4.** The mean hourly wage of six convenience store employees is \$7.50. The hourly wages of five of the employees are \$4.75, \$5.50, \$5.75, \$8.00, and \$9.50. Find the hourly wage of the sixth.

**5.** When looking through the classified ads, you found that 16 jobs had a starting salary of \$20,000, 8 had a starting salary of \$32,000, and 1 had a starting salary of \$50,000. Find the mean and median starting salary for these jobs.

**6.** **a.** Fill in the table. Round your answers to the nearest whole number.

| Age | Frequency Count | Relative Frequency (%) |
|---|---|---|
| 1–20 | | 38 |
| 21–40 | 35 | |
| 41–60 | 28 | |
| 61–80 | | |
| Total | 137 | |

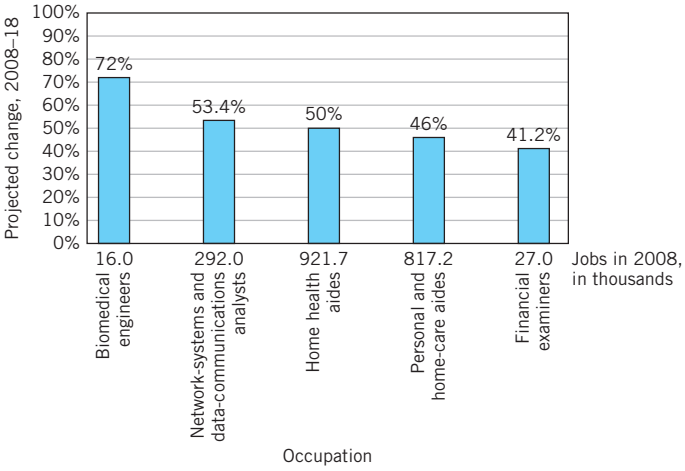
**b.** Calculate the percentage of the population who are over 40 years old.

**7.** In the following chart, which job type had the largest number of jobs in 2008? The smallest? Which job type was projected to increase the most? The least? What should you consider in interpreting these percentages?

**7.** (continued)

**Job Projections**

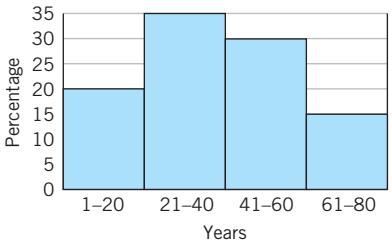
Occupations that were projected to add jobs the fastest from 2008 to 2018, according to the Labor Department.




Source: U.S. Dept of Labor.

**8.** From the histogram below, create a frequency distribution table. Assume that the total number of people represented by the histogram is 1352. (*Hint:* Estimate the relative frequencies from the graph and then calculate the frequency count in each interval.)

**Distribution of Ages (in years)**




**9.** Calculate the mean and median for the following data:

**a.** $475, $250, $300, $450, $275, $300, $6000, $400, $300

**b.** 0.4, 0.3, 0.3, 0.7, 1.2, 0.5, 0.9, 0.4

**10.** Explain why the mean may be a misleading numerical summary of the data in Problem 9(a).

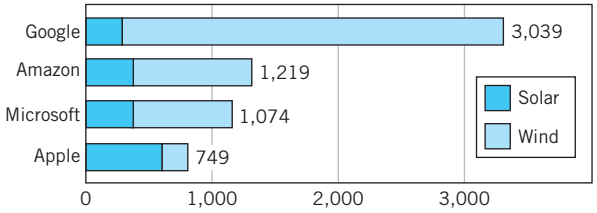
## Exercises for Section **1.1**

Course software required for Exercise 22 and recommended for Exercise 21. Internet required for Exercise 17(b).

**1.** The following graph shows the top global corporate buyers of renewable energy as of 2018. Describe the difference between Google and Apple in terms of the amount, type, and percent of renewable energy purchased by the five corporations.

**Top Global Corporate Buyers of Renewable Energy**

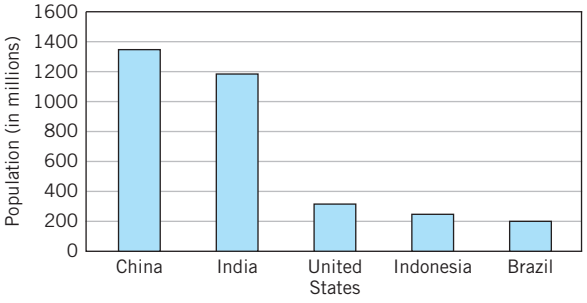
Cumulative renewable energy purchased, (in megawatts):




NOTE: As of March 2018; Source: Bloomberg New Energy Finance
Karl Gelles/USA TODAY

**2.** The accompanying bar chart shows the five countries with the largest populations in 2010.

**The Five Most Populous Countries (2010)**




Source: CIA Factbook, *www.cia.gov/cia*.

**a.** What country has the largest population, and approximately what is its population size?

**b.** The population of India is projected in the near future to exceed the population of China. Given the current data, what is the minimum number of additional persons needed to make India's population larger than China's?

**c.** What additional information would you need in order to calculate the percentage of the 2010 world population for each of these five countries?

**3.** The following bar graph was published in a 2009 report by the Organization for Economic Cooperation and Development (OECD).

**a.** Summarize what you think this graph tells you about unmet health care needs.

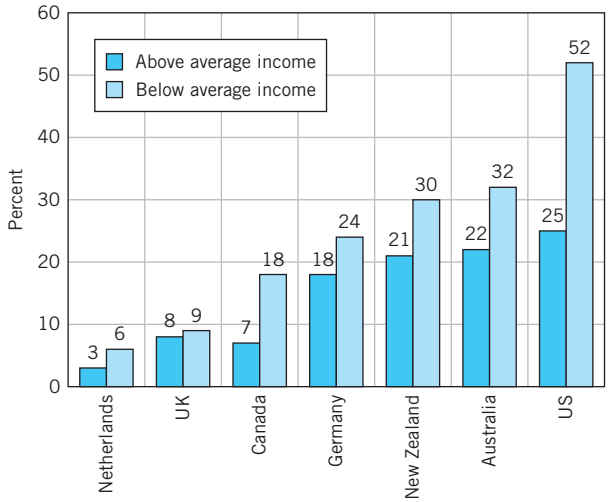
**b.** For which country was the disparity between below and above average incomes the largest? The smallest?

**c.** What additional information would you need in order to calculate the number of people with unmet health care needs due to costs in each country?

**3.** (continued)

**Health Care Gap**

Persons reporting an unmet health care need* due to costs in seven OECD countries, by income group, 2007



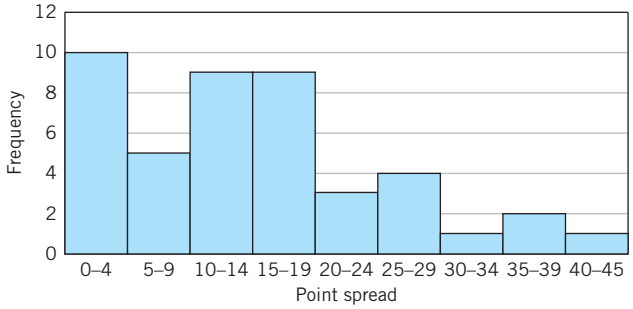

*Did not get medical care, missed medical test, treatment or follow-up, or did not fill prescription or missed doses.

Source: "Measuring disparities in health status and in access and use of health care in OECD countries", Michael de Looper and Gaetan Lafortune, OECD Health Working Paper No. 43, 2009, available at *www.oecd.org/health*.

**4.** The point spread in a football game is the difference between the winning team's score and the losing team's score. For example, in the 2010 Super Bowl game, the New Orleans Saints won with 31 points versus the Indianapolis Colts' 17 points. So the point spread was 14 points.

**a.** In the accompanying histogram, what is the interval with the most likely point spread in a Super Bowl? The least likely?

**Point Spreads in 44 Super Bowls**




Source: *www.docsports.com*.

**b.** What percentage of these 44 Super Bowl games had a point spread of 9 or less? Of 30 or more?

**5.** Given here is a table of salaries taken from a survey of some recent graduates (with bachelor degrees) from a well-known university in Pittsburgh.

| Salary (in thousands) | Number of Graduates Receiving Salary |
|---|---|
| 21–25 | 2 |
| 26–30 | 3 |
| 31–35 | 10 |
| 36–40 | 20 |
| 41–45 | 9 |
| 46–50 | 1 |

**a.** How many graduates were surveyed?

**b.** Is this quantitative or qualitative data? Explain.

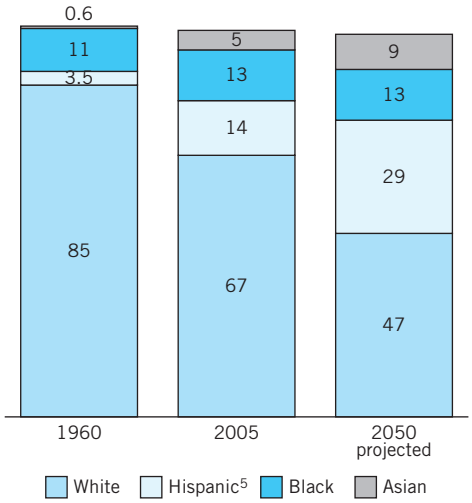
**c.** What is the relative frequency of people having a salary between $26,000 and $30,000?

**d.** Create a histogram of the data.

**6.** The accompanying bar chart shows the predictions of the Pew Research Center about the future racial and ethnic composition of American society. American Indian/Alaskan American comprised less than 1% of the population for each of the years on the chart and thus is not represented.

**U.S. Population 1960–2050**

Share of total, by racial and ethnic groups




Source: *Immigration to play lead role in future U.S. growth.* Pew Research Center, 2008.

**a.** The U.S. Bureau of the Census projected that there will be approximately 438 million people in the United States in the year 2050. Approximately how many people will be of Hispanic origin in 2050?

**b.** Describe three changes in the racial composition of the United States since 1960.

**c.** Write a topic sentence describing an overall trend.

**7. a.** Compute the mean and median for the list: 5, 18, 22, 46, 80, 105, 110.

**b.** Change one of the entries in the list in part (a) so that the median stays the same but the mean increases.

**8.** Suppose that a church congregation has 100 members, each of whom donates 10% of his or her income to the church. The church collected $250,000 last year from its members.

**a.** What was the mean contribution of its members?

**b.** What was the mean income of its members?

**c.** Can you predict from the given data, the median income of its members? Explain your answer.

[5]Hispanic used to be considered a racial classification. It is now considered an ethnic classification, since Hispanics can be black, white, or any other race. For this survey, Hispanics formed one category and did not overlap with other categories.

**9.** Suppose that annual salaries in a certain corporation are as follows:

| | |
|---|---|
| Level I (30 employees) | $18,000 |
| Level II (8 employees) | $36,000 |
| Level III (2 employees) | $80,000 |

Find the mean and median annual salary. Suppose that an advertisement is placed in the newspaper giving the mean annual salary of employees in this corporation as a way to attract applicants. Why would this be a misleading indicator of salary expectations?

**10.** Suppose the grades on your first four exams were 78%, 92%, 60%, and 85%. What would be the lowest possible average (using the mean) that your last two exams could have so that your grade in the class, based on the mean of the six exams, is at least 82%?

**11.** Read Stephen Jay Gould's article "The Median Isn't the Message" and explain how an understanding of statistics brought hope to a cancer victim.

Reading

**12.** **a.** On the first quiz (worth 25 points) given in a section of college algebra, one person received a score of 16, two people got 18, one got 21, three got 22, one got 23, and one got 25. What were the mean and median of the quiz scores for this group of students?

**b.** On the second quiz (again worth 25 points), the scores for eight students were 16, 17, 18, 20, 22, 23, 25, and 25.

**i.** If the mean of the scores for the nine students was 21, then what was the missing score?

**ii.** If the median of the scores was 22, then what are possible scores for the missing ninth student?

**13.** Why is the mean age larger than the median age in the United States? What prediction would you make for your State? What predictions would you make for other countries? You can check your predictions with data from the U.S. Census Bureau at *www.census.gov*.

**14.** Up to and including Donald Trump, the ages of the last 16 presidents when they first took office were 55, 51, 54, 51, 60, 62, 43, 55, 56, 52, 69, 64, 46, 54, 47, and 70 years.

**a.** Find the mean and median ages of the past 16 presidents when they took office.

**b.** If the mean age of the past 17 presidents is 55.59, at what age did the missing president take office?

**c.** Beginning with age 40 and using 5-year intervals, find the frequency count for each age interval.

**d.** Create a frequency histogram using your results from part (c).

**15.** Herb Caen, a Pulitzer Prize–winning columnist for the *San Francisco Chronicle*, remarked that a person moving from state A to state B could raise the average IQ in both states. Is he right? Explain.

**16.** Why do you think most researchers use median rather than mean income when studying "typical" households?

**17.** According to the Federal Reserve Board, in 2007, the median net worth of American households was $120,300 and the mean net worth was $556,300.

**a.** How could there be such a wide discrepancy?

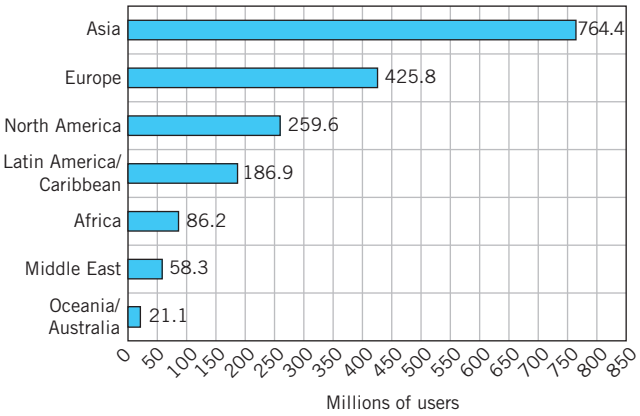
**b.** Household net worth started to decline after 2007. Using the Internet and a source that adjusts for inflation, compare the current net worth of U.S. households to 2007.

**18.** Read the *CHANCE News* article and explain why the author was concerned.

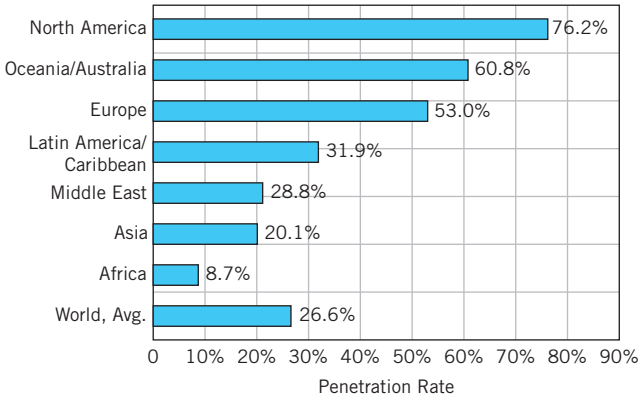
Reading

**19.** Use the following graphs to describe three interesting facts about Internet use worldwide in 2009.

**Internet Users in the World by Geographic Regions—2009**




**World Internet Penetration Rates by Geographic Regions—2009**

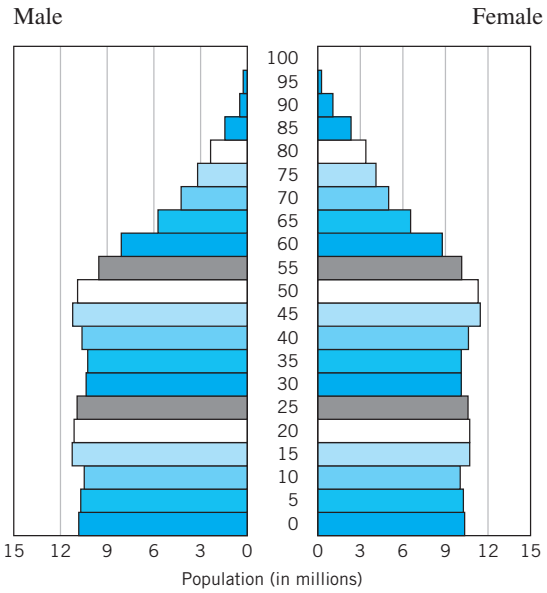



Source: Internet World Stats, *www.internetworldstats.com*. Estimated Internet users are 1,802,330,457 for December 31, 2009.

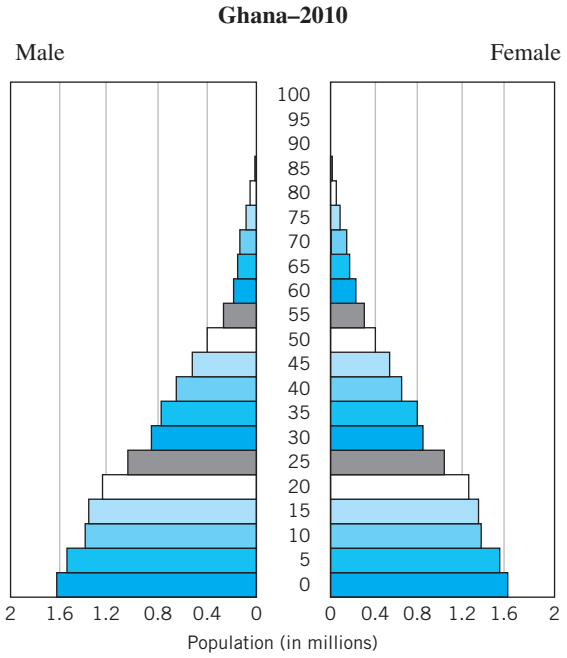


**20.** Population pyramid charts are used to depict the overall age structure of a society. The accompanying pyramids show the age structure in 2010 for Ghana, a developing country, and for the United States, an industrialized nation. Describe three major differences in the distribution of ages in the these two countries in 2010.

**United States–2010**

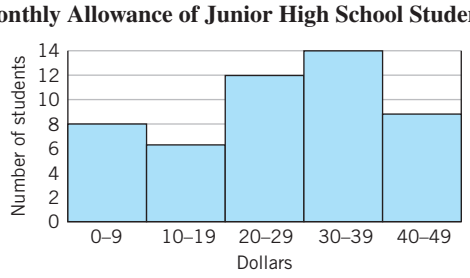


**20.** (continued)




Source: U.S. Bureau of Census, International Data Base.

**21.** Estimate the mean and median from the given histogram. (*Hint:* Replace each dollar interval with a dollar amount approximately in the middle of the interval.)

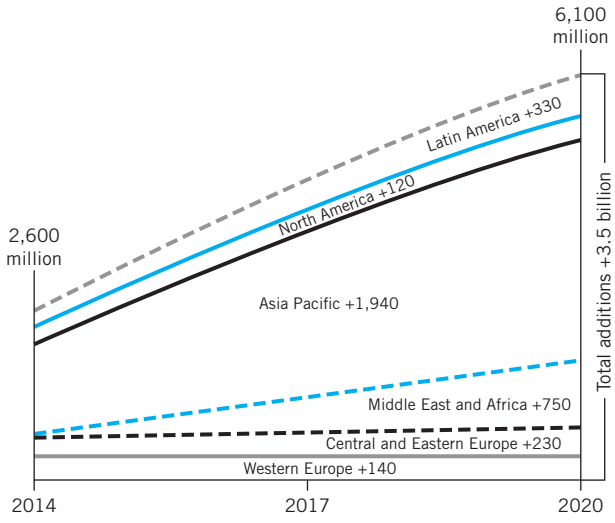



**22.** (Computer and course software required.) Open up the course software "F1: Histograms" in *FAM 1000 Census Graphs* in the course software. The 2009 U.S. Census data on 1000 randomly selected U.S. individuals and their families are imbedded in this program. You can use it to create histograms for education, age, and different measures of income. Try using different interval sizes to see what patterns emerge. Decide on one variable (say education) and compare the histograms of this variable for different groups of people. For example, you could compare education histograms for men and women or for people living in two different regions of the country. Pick a comparison that you think is interesting. Create a possible headline for these data. Describe three key features that support your headline.

**23.** According to the Pew Research Center, roughly three quarters (77%) of Americans owned a smartphone in January 2017. The following graph shows predictions for 2020 for smartphone subscriptions in different regions of the world.

**a.** According to Pew Research Center, what region of the world will account for the largest increase in smartphone subscriptions between 2014 and 2020? What region will account for the smallest increase? Explain your results.

**b.** In 2020, the world population is predicted to be about 7.8 billion people. What percent of the world population are predicted to be using smartphones in 2020?




# 1.2 Describing Relationships Between Two Variables

By looking at two-variable data, we can learn how change in one variable affects change in another. How does the weight of a child determine the amount of medication prescribed by a pediatrician? How does median age or income change over time? In this section we examine how to describe these changes with graphs, data tables, words, and equations.

**Exploration**

Exploration 1.1 Homework provides an opportunity to collect and analyze two-variable data.

## Visualizing Two-Variable Data

**EXAMPLE 1** | Scatter Plots

**Table 1.1** shows data for two variables, the year and the median age of the U.S. population. Plot the data in **Table 1.1** and then use your graph to describe the changes in the U.S. median age over time.

**TABLE 1.1** **Median Age of the U.S. Population, 1850–2050***

| Year | Median Age | Year | Median Age |
|---|---|---|---|
| 1850 | 18.9 | 1950 | 30.2 |
| 1860 | 19.4 | 1960 | 29.5 |
| 1870 | 20.2 | 1970 | 28.0 |
| 1880 | 20.9 | 1980 | 30.0 |
| 1890 | 22.0 | 1990 | 32.8 |
| 1900 | 22.9 | 2000 | 35.3 |
| 1910 | 24.1 | 2005 | 36.7 |
| 1920 | 25.3 | 2010 | 36.9 |
| 1930 | 26.4 | 2025 | 38.2 |
| 1940 | 29.0 | 2050 | 39.0 |

*Data for 2010–2050 are projected

Source: U.S., Bureau of the Census, www.census.gov.

Data
**MEDAGE**
There are Excel and Graph Link files for the median age data called MEDAGE on the course website.

**Solution**

In **Table 1.1** we can think of a year and its associated median age as an *ordered pair* of the form (year, median age). For example, the first two elements correspond to the ordered pair (1850, 18.9) and the second corresponds to (1860, 19.4). **Figure 1.3** shows a scatter plot of the data. The graph is called a *time series* because it shows changes over time. In newspapers and magazines, the time series is the most frequently used form of data graphic.[6]

**Median age of U.S. population over time: A time series**

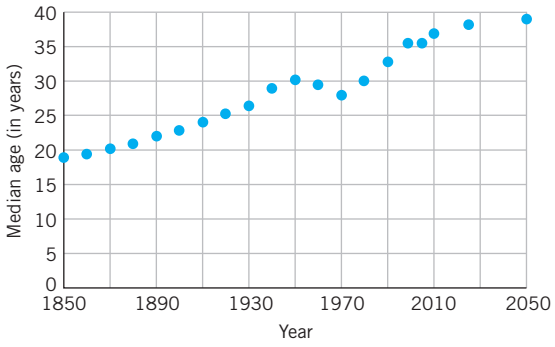



**FIGURE 1.3**

Our graph shows that the median age of the U.S. population grew quite steadily for one hundred years, from 1850 to 1950. Although the median age decreased between 1950 and 1970, since 1970 it has continued to increase. From 1850 to 2010, the median age nearly doubled, and projections for 2025 and 2050 indicate continued increases, though at a slower pace.

## Constructing a "60-Second Summary"

To communicate effectively, you need to describe your ideas succinctly and clearly. One tool for doing this is a "60-second summary"—a brief synthesis of your thoughts that could be presented in one minute. Quantitative summaries should be straightforward and concise. They often start with a topic sentence that summarizes the key idea, followed by supporting quantitative evidence.

[6]Edward Tufte in *The Visual Display of Information* (Cheshire, Conn.: Graphics Press, 2001, p. 28) reported on a study that found that more than 75% of all graphics published were time series.

After you have identified a topic you wish to write about or present orally, some recommended steps for constructing a 60-second summary are:

- Collect relevant information (possibly from multiple sources, including the Internet).
- Search for patterns, taking notes.
- Identify a key idea (out of possibly many) that could provide a topic sentence.
- Select evidence and arguments that support your key idea.
- Examine counterevidence and arguments and decide if they should be included.
- Construct a 60-second summary, starting with your topic sentence.

To help your ideas take shape, put them down on paper, then refine and modify. Quantitative reports should not be written in the first person. For example, you might say something like "The data suggest that . . ." rather than "I found that the data . . ."

### EXAMPLE 2 | A 60-Second Summary

The annual federal surplus (+) or deficit (–) since World War II is shown in **Table 1.2** and **Figure 1.4** (a scatter plot where the points have been connected). Construct a 60-second summary describing the changes over time.

**TABLE 1.2** **Fiscal year[7] Federal Budget: Surplus (+) or Deficit (–)**

| Year | Billions of Dollars | Year | Billions of Dollars |
|---|---|---|---|
| 1945 | –48 | 1995 | –164 |
| 1950 | –3 | 1996 | –107 |
| 1955 | –3 | 1997 | –22 |
| 1960 | –0 | 1998 | 69 |
| 1965 | –1 | 1999 | 126 |
| 1970 | –3 | 2000 | 236 |
| 1975 | –53 | 2001 | 128 |
| 1980 | –74 | 2002 | –158 |
| 1985 | –212 | 2003 | –378 |
| 1990 | –221 | 2004 | –413 |
| 1991 | –269 | 2005 | –318 |
| 1992 | –290 | 2006 | –248 |
| 1993 | –255 | 2007 | –161 |
| 1994 | –203 | 2008 | –459 |
| | | 2009 | –1,413 |

Source: U.S. Office of Management and Budget.

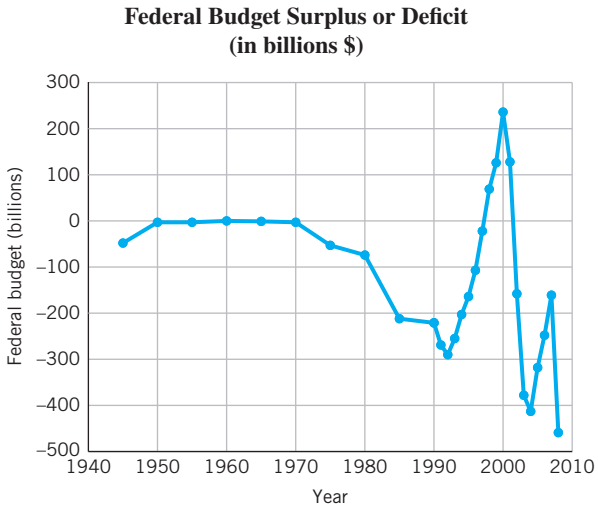



**FIGURE 1.4** Note that the data point (2009, –1,413) is not included on the graph.

#### Solution

Between 1945 and 2009 the annual U.S. federal deficit moved from a 30-year stable period, with as little as $0 deficit, to a period of oscillations, leading in 2009 to the largest deficit ever recorded. From the 1970s to 1992, the federal budget ran an annual deficit, which generally was getting larger until it reached almost $300 billion in 1992. From 1992 to 1997, the deficit steadily decreased, and from 1998 to 2001 there were relatively large surpluses. The maximum surplus for this time period occurred in 2000, when it reached $236 billion. But by 2002 the federal government was again running large deficits. In 2009 the deficit reached $1,413 billion, the largest recorded up to that time.

[7]The government's fiscal year runs from October 1 through September 30 and is designated by the year in which it ends. For example, the fiscal year for 2010 is from October 1, 2009 to September 30, 2010.

## Algebra Aerobics **1.2a**

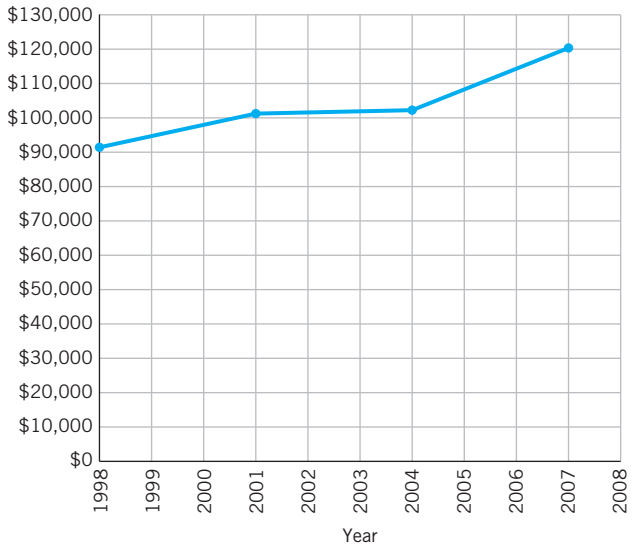
**1.** The net worth of a family at any given time is the difference between assets (what is *owned*) and liabilities (what is *owed*). The following table and graph show the median net worth of U.S. families, adjusted for inflation.

**Median Family Net Worth (adjusted for inflation using 2007 dollars)[8]**

| Year | Median Net Worth ($) |
|---|---|
| 1998 | 91,300 |
| 2001 | 101,200 |
| 2004 | 102,200 |
| 2007 | 120,300 |

Source: The Federal Reserve Board, *www.federalreserve.gov*.

**Median Family Net Worth (in 2007 dollars)**




**a.** Write a few sentences about the trend in U.S. median family net worth.

**b.** What additional information might be useful in describing the trend in median net worth?

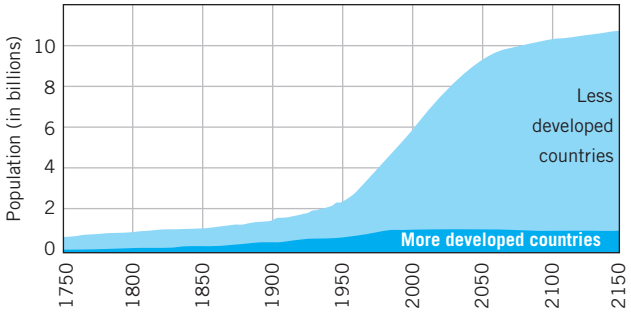
**2.** Use the World Population graph to estimate:

**a.** The year when the world population reached 4 billion.

**b.** The year that it is projected to reach 8 billion.

**c.** The number of years it will take to grow from 4 to 8 billion.

**World Population with Projections to 2150**




Source: Population Reference Bureau, *www.prb.org*.

**3.** Use the World Population graph to estimate the following projections for the year 2150.

**a.** The total world population.

**b.** The total populations of all the more developed countries.

**c.** The total populations of all the less developed countries.

**d.** Write a topic sentence about the estimated world population in 2150.

**4.** Use the World Population graph to answer the following:

**a.** The world population in 2000 was how many times greater than the world population in 1900? What was the difference in population size?

**b.** The world population in 2100 is projected to be how many times greater than the world population in 2000? What is the difference in population size?

**c.** Describe the difference in the growth in world population from 1900 to 2000 versus the projected growth from 2000 to 2100.

## Using Equations to Describe Change

Sometimes the relationship between two variables can also be described with an equation. An equation gives a rule on how change in the value of one variable affects change in the value of the other. If the variable $n$ represents the number of years of education beyond eighth grade and $E$ represents yearly median earnings (in dollars) for people living in the United States, then the following equation and graph in **Figure 1.5** model the relationship between $E$ and $n$:

$$E = 1140 + 6250\,n$$

[8]"Constant dollars" is a measure used by economists for comparing the value of money over time, eliminating the effects of inflation or deflation. It is based on the buying power of the dollar in a certain base year. For example, the net worth in 1998 of $91,300 "in constant 2007 dollars" was equal to the amount of goods and services that cost $91,300 in 2007. The actual net worth in 1998, measured in what is called "current dollars," was much lower.

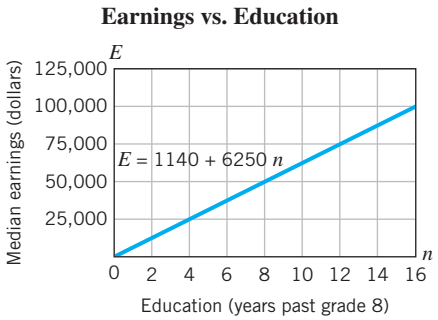



**FIGURE 1.5**

This equation provides a useful tool for describing how earnings and education are linked and for making predictions.[9] For example, to predict the median earnings, $E$, for those with a high school education, we replace $n$ with 4 (representing 4 years beyond eighth grade, or a high school education) in our equation to get

$$\begin{aligned} E &= 1140 + 6250 \cdot 4 \\ &= \$26{,}140 \end{aligned}$$

Our equation predicts that for those with a high school education, median earnings will be about $26,140.

An equation that is used to describe a real-world situation is called a *mathematical model*. Such models offer compact, often simplified descriptions of what may be a complex situation. The accuracy of the predictions made with such models can be questioned and disciplines outside of mathematics may be needed to help answer such questions. Yet these models are valuable guides in our quest to understand social and physical phenomena in our world.

## Describing the relationship between abstract variables

Variables can represent quantities that are not associated with real objects or events. The following equation or mathematical sentence defines a relationship between two quantities, which are named by the abstract variables $x$ and $y$:

$$y = x^2 + 2x - 3$$

*Solutions to the equation* are pairs of values for $x$ and $y$ that make the sentence true. By convention, we express these solutions as ordered pairs of the form $(x, y)$.

### EXAMPLE 3 | Identifying Solutions to an Equation

Given $y = x^2 + 2x - 3$

**a.** Are (1, 0) and (0, 1) solutions to $y = x^2 + 2x - 3$?

**b.** How many solutions are there?

**c.** Create a table with several solutions for this equation.

**d.** Use technology to graph the equation. State the relationship between points on the graph and solutions to the equation. Label one solution on your graph.

**Solution**

**a.** (1, 0) would be a solution to $y = x^2 + 2x - 3$, since $0 = 1^2 + 2(1) - 3$, whereas (0, 1) would not be a solution, since $1 \neq 0^2 + 2(0) - 3$.

**b.** There are infinitely many possible solutions to the equation $y = x^2 + 2x - 3$, since we could substitute any real number for $x$ and find a corresponding $y$.

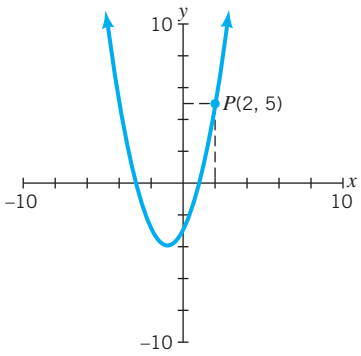
**c.** **Table 1.3** lists a few solutions.

**d.** The graph of the equation is shown in **Figure 1.6**. All the points on the graph represent solutions to the equation, and every solution is a point on the graph of the equation.

[9] In "Looking for Links Between Education and Earnings," in Section 2.11, we show how such equations are derived and how they are used to analyze the relationship between education and earnings.

**TABLE 1.3**

| $x$ | $y$ |
|---|---|
| –4 | 5 |
| –3 | 0 |
| –2 | –3 |
| –1 | –4 |
| 0 | –3 |
| 1 | 0 |
| 2 | 5 |
| 3 | 12 |




**Coordinates of point $P$ are:**
(*horizontal* coordinate, *vertical* coordinate)
$(x, y)$
$(2, 5)$

**FIGURE 1.6** **Graph of $y = x^2 + 2x - 3$ where one solution of infinitely many solutions is labeled.**

Note that sometimes an arrow is used to show that a graph extends indefinitely in the indicated direction. In **Figure 1.6**, the arrows show that both arms of the graph extend indefinitely upward.

### Solutions of an Equation

The *solutions* of an equation in two variables $x$ and $y$ are the ordered pairs $(x, y)$ that make the equation a true statement.

### Graph of an Equation

The *graph* of an equation in two variables displays the set of points that are solutions to the equation.

### EXAMPLE 4 | Solutions for Equations in One or Two Variables

Describe how the solutions for the following equations are similar and how they differ.

**a.** $3x + 5 = 11$

**b.** $x + 2 = x + 2$

**c.** $3 + x = y + 5$

**Solution**

The solutions are similar in the sense that each solution for each particular statement makes the statement true. They are different because:

**a.** There is only one solution ($x = 2$) of the single-variable equation $3x + 5 = 11$.

**b.** There are an infinite number of solutions for $x$ of the single-variable equation $x + 2 = x + 2$, since the left-side expression is the same as the right-side expression, any real number will make the statement a true statement.

**c.** There are infinitely many solutions, in the form of ordered pairs $(x, y)$, of the two-variable equation $3 + x = y + 5$.

### EXAMPLE 5 | Estimating Solutions From a Graph

The graph of the equation $x^2 + 4y^2 = 4$ is shown in **Figure 1.7**.

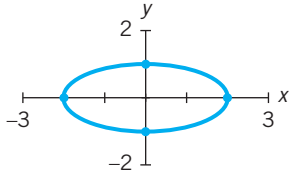



**FIGURE 1.7**

**a.** From the graph, estimate three solutions of the equation.

**b.** Check your solutions using the equation.

**Solution**

**a.** The coordinates (0, 1), (–2, 0), and (1, 0.8) appear to lie on the ellipse, which is the graph of the equation $x^2 + 4y^2 = 4$.

**b.** If substituting the ordered pair (0, 1) into the equation makes it a true statement, then (0, 1) is a solution.

| | |
|---|---|
| Given | $x^2 + 4y^2 = 4$ |
| substitute $x = 0$ and $y = 1$ | $(0)^2 + 4(1)^2 = 4$ |
| evaluate | $4 = 4$ |

We get a true statement, so (0, 1) is a solution to the equation.
For the ordered pair (−2, 0):

| | |
|---|---|
| Given | $x^2 + 4y^2 = 4$ |
| substitute $x = -2$ and $y = 0$ | $(-2)^2 + 4(0)^2 = 4$ |
| evaluate | $4 + 0 = 4$ |
| | $4 = 4$ |

Again we get a true statement, so (−2, 0) is a solution to the equation.
For the ordered pair (1, 0.8):

| | |
|---|---|
| Given | $x^2 + 4y^2 = 4$ |
| substitute $x = 1$ and $y = 0.8$ | $(1)^2 + 4(0.8)^2 = 4$ |
| evaluate | $1 + 4(0.64) = 4$ |
| | $3.56 \neq 4$ |

We get a false statement, so (1, 0.8) is not a solution, although it is close to a solution.

## Explore & Extend

### 1.2 Finding Patterns

In each data table (a) to (e), look for a pattern in terms of how $y$ changes when $x$ changes. Describe with words how to find $y$ in terms of $x$. Assume the pattern continues indefinitely and extend the data table to include a few negative values for $x$. Describe the pattern that you found with a formula.

**a.**

| $x$ | $y$ |
|---|---|
| 0 | 0.0 |
| 1 | 0.5 |
| 2 | 1.0 |
| 3 | 1.5 |
| 4 | 2.0 |

**b.**

| $x$ | $y$ |
|---|---|
| 0 | 5 |
| 1 | 8 |
| 2 | 11 |
| 3 | 14 |
| 4 | 17 |

**c.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |

**d.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 |
| 2 | 8 |
| 3 | 27 |
| 4 | 64 |

**e.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 2 |
| 2 | 12 |
| 3 | 36 |
| 4 | 80 |

Create a formula for $y$ in terms of $x$ and generate a data table. Show that data table to your class or a fellow student and see if they can describe your data table with words and a formula.

## Algebra Aerobics **1.2b**

**1. a.** Describe in your own words how to compute the value for $y$, given a value for $x$, using the following equation:

$$y = 3x^2 - x + 1$$

**b.** Which of the following ordered pairs represent solutions to the equation?

$$(0, 0), (0, 1), (1, 0), (-1, 2), (-2, 3), (-1, 0)$$

**c.** Use $x = 0, \pm 1, \pm 2, \pm 3$ to generate a small table of values that represent solutions to the equation.

**2.** Repeat the directions in Problem 1(a), (b), and (c) using the equation $y = (x - 1)^2$.

**3.** Given the equations $y_1 = 4 - 3x$ and $y_2 = -2x^2 - 3x + 5$, fill in the following table.

| $x$ | −4 | −2 | −1 | 0 | 1 | 2 | 4 |
|---|---|---|---|---|---|---|---|
| $y_1$ | | | | | | | |
| $y_2$ | | | | | | | |

**a.** Use the table to create two scatter plots, one for the ordered pairs $(x, y_1)$ and the other for $(x, y_2)$.

**b.** Draw a smooth curve through the points on each graph.

**c.** Is (1, 1) a solution for equation $y_1$? For $y_2$?

**d.** Is (−1, 6) a solution for equation $y_1$? For $y_2$?

**e.** Look at the graphs. Is the ordered pair (−3, 2) a solution for either equation? Verify your answer by substituting the values into each equation.

**4.** Given the equation $y = x^2 - 3x + 2$,

**a.** If $x = 3/2$, find $y$.

**b.** Find two points that are *not* solutions to this equation.

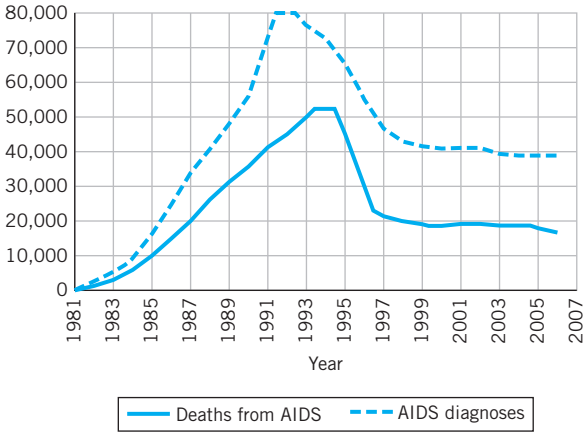
**c.** If available, use technology to graph the equation and then confirm your results for parts (a) and (b).

## Exercises for Section 1.2

Access to Internet required for Exercises 3(d) and 4(d) and recommended for Exercise 16.

**1.** In June 1981, the first cases of AIDS were reported in the United States. In 2008, it was estimated that more than one million people were living with HIV in the United States and that more than half a million died after developing AIDS. The following graph shows AIDS diagnoses and deaths from 1981 to 2007. Assume you work for a newspaper and are asked to report on the following data.

**AIDS Diagnoses and Deaths**




Source: *www.avert.org*. Avert is an international AIDS charity.

**a.** What are three important facts that emerge from this graph?

**b.** Construct a 60-second summary that could accompany the graph in the newspaper article.

**2.** The following graph shows changes in the Dow Jones Industrial Average, which is based on 30 stocks that trade on the New York Stock Exchange and is the best-known index of U.S. stocks.

**Dow Jones Industrial Average**


Note that the symbol on the lower left corner of the graph indicates that the vertical axis does not start at 0.

Source: Yahoo Finance. *http://finance.yahoo.com*.

**a.** What time period does the graph cover?

**b.** Estimate the lowest Dow Jones Industrial Average. During what month and year did it occur?

**c.** Estimate the highest value for the Dow Jones Industrial Average during that period. When did it occur?

**d.** Write a topic sentence describing the change in the Dow Jones Industrial Average over the given time period.

**3.** The accompanying table shows the number of personal and property crimes in the United States from 1980 to 2007.

| Year | Personal Crimes (in thousands) | Property Crimes (in thousands) |
|---|---|---|
| 1980 | 1,345 | 12,064 |
| 1985 | 1,328 | 11,103 |
| 1990 | 1,820 | 12,655 |
| 1995 | 1,799 | 12,064 |
| 2000 | 1,425 | 10,183 |
| 2005 | 1,391 | 10,175 |
| 2006 | 1,418 | 9,984 |
| 2007 | 1,408 | 9,843 |

Source: U.S. Bureau of the Census, www.census.gov.

**a.** Create a scatter plot of the personal crimes over time. Connect the points with line segments.

**b.** Approximately how many *times* more property crimes than personal crimes were committed in 1980? In 2007?

**c.** Write a topic sentence about property and personal crime from 1980 to 2007.

**d.** Do you think the trends you observed continued to present day? Use the Internet to check your prediction.

**4.** The following graph shows the amount of revenues from newspaper ad sales over time.

**a.** What are the ordered pairs associated with the highest and lowest revenues?

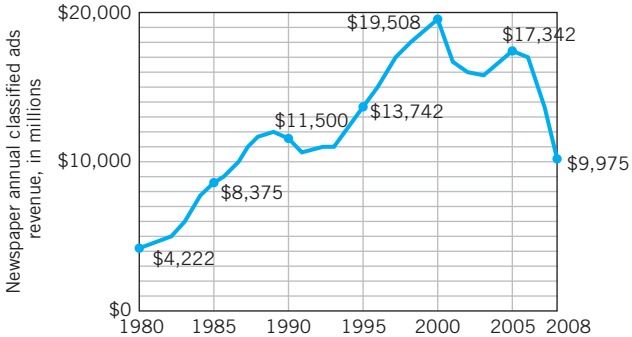
**b.** What are the units for the ordered pair (1995, 13742)? Interpret the meaning of this ordered pair.

**c.** Write a 60-second summary about these changes. Can you think of any reasons to explain changes in revenue over time?

**d.** What do you think happened to revenues since 2008? Use the Internet to check your predictions.

**Newspaper Classified Ads Revenue Has Plummeted in the Past Few Years**

Newspaper revenue from classied ads, 1980–2008




Source: Business Analysis and Research Newspaper Association of America, "Advertising Expenditures." Available at *http://www.naa.org*.

**5.** The National Center for Chronic Disease Prevention and Health Promotion published the following data on the chances that a man has had prostate cancer at different ages.

**Lifetime Risk of Developing Prostate Cancer**

| Age | Risk | Percent Risk |
|---|---|---|
| 45 | 1 in 25,000 | 0.004% |
| 50 | 1 in 476 | 0.21% |
| 55 | 1 in 120 | 0.83% |
| 60 | 1 in 43 | 2.3% |
| 65 | 1 in 21 | 4.8% |
| 70 | 1 in 13 | 7.7% |
| 75 | 1 in 9 | 11.1% |
| 80 | 1 in 6 | 16.7% |

**a.** What is the relationship between age and getting prostate cancer?

**b.** Make a scatter plot of the percent risk for men of the ages given.

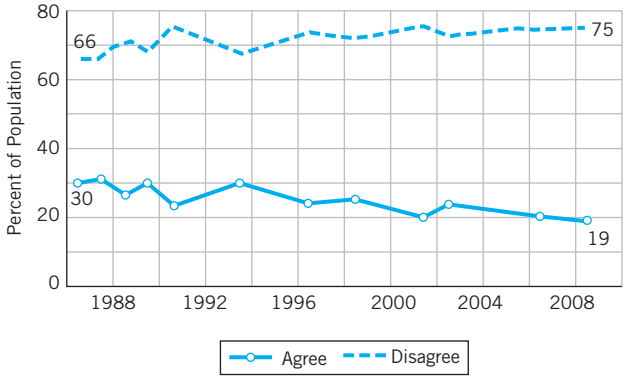
**c.** Use the "percent risk" data to find how much more likely men 50 years old will develop prostate cancer than men who are 45. How much more likely are men 55 years old to develop prostate cancer than men who are 50?

**d.** Looking at these data, when would you recommend annual prostate checkups to begin for men? Explain your answer in terms of the interests of the patient and of the insurance company.

**6.** Use the following graph to write a 60-second summary of the attitudes of Americans about the role of women in society.

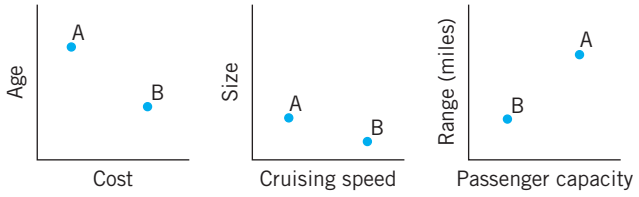
**The Changing Role of Women**

Women should return to their traditional role in society




Source: Pew Research Center for the People & the Press May 21, 2009 survey report "Trends in Political Values and Core Attitudes: 1987–2009."

**7.** The following three graphs describe two cars, A and B.




For parts (a)–(d), decide whether the statement is true or false. Explain your reasoning.

**a.** The newer car is more expensive.

**b.** The slower car is larger.

**c.** The larger car is newer.

**d.** The less expensive car carries more passengers.

**e.** State two other facts you can derive from the graphs.

**f.** Which car would you buy? Why?

**8. a.** Which (if any) of the following ordered pairs $(x, y)$ is a solution to the equation $y = x^2 - 2x + 1$? Show how you came to your conclusion.

$$(-2, 7) \quad (1, 0) \quad (2, 1)$$

**b.** Find one additional ordered pair that is a solution to the preceding equation. Show how you found your solution.

**9.** Consider the equation $R = 2 - 5T$.

**a.** Determine which, if any, of the following points $(T, R)$ satisfy this equation.

$$(0, 4) \quad (1, -3) \quad (2, 0)$$

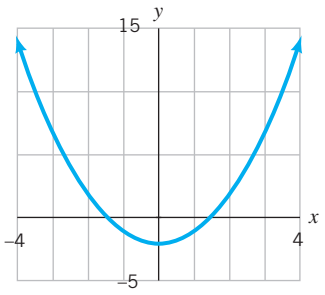
**b.** Find two additional ordered pairs that are solutions to the equation.

**c.** Make a scatter plot of the solution points found.

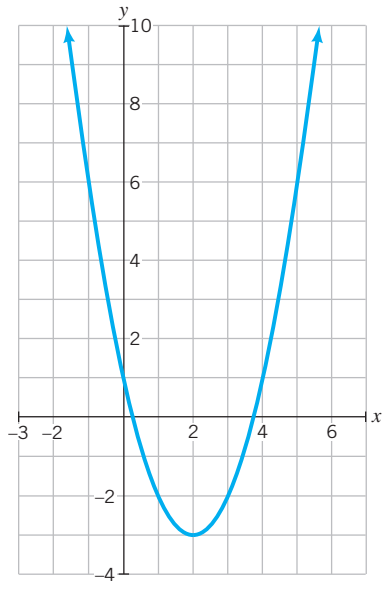
**d.** What does the scatter plot suggest about where more solutions could be found? Check your predictions.

**10.** Use the accompanying graph to estimate the missing values for $x$ or $y$ in the table.

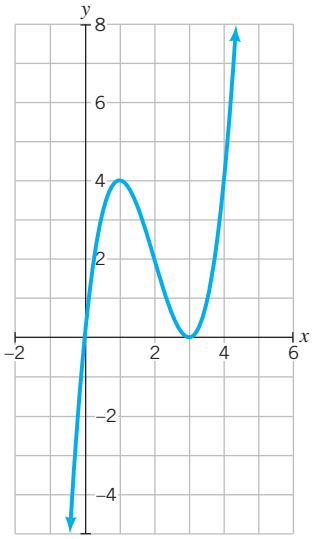
| $x$ | $y$ |
|---|---|
| −3 | |
| | 2 |
| −1 | |
| | −2 |
| | −1 |
| 2 | |
| | 7 |




**11.** The graph of the equation $(x - 2)^2 - 3 = y$ is shown. Estimate three solutions from the graph and then check your solutions using the equation.



**12.** The graph of the equation $x(x-3)^2 = y$ is shown. Estimate three solutions from the graph and then check your solutions using the equation.




**13.** For parts (a)–(d) use the following equation: $y = -2x^2$.

**a.** If $x = 0$, find the value of $y$.

**b.** If $x$ is greater than zero, what can you say about the value of $y$?

**c.** If $x$ is a negative number, what can you say about the value of $y$?

**d.** Can you find an ordered pair that represents a solution to the equation when $y$ is greater than zero? If so, find it; if not, explain why.

**14.** Find the ordered pairs that represent solutions to each of the following equations when $x = 0$, when $x = 3$, and when $x = -2$.

**a.** $y = 2x^2 + 5x$ **c.** $y = x^3 + x^2$

**b.** $y = -x^2 + 1$ **d.** $y = 3(x-2)(x-1)$

**15.** Given the four ordered pairs $(-1, 3)$, $(1, 0)$, $(2, 3)$, and $(1, 2)$, for each of the following equations, identify which points (if any) are solutions for that equation.

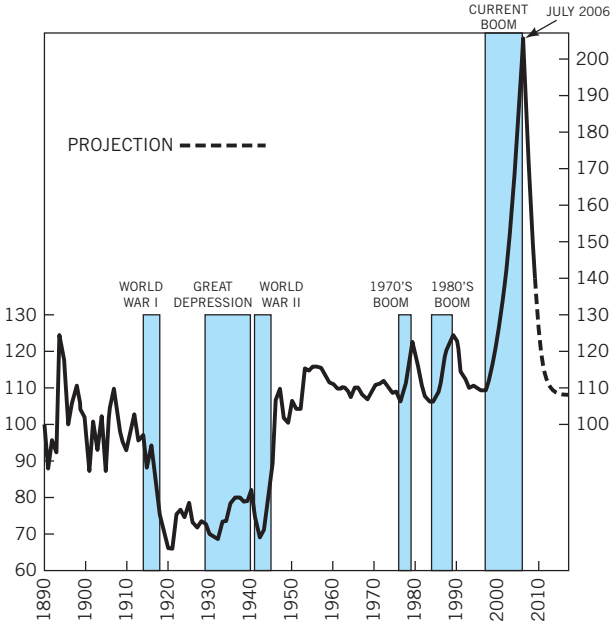
**a.** $y = 2x + 5$ **c.** $y = x^2 - x + 1$

**b.** $y = x^2 - 1$ **d.** $y = \dfrac{4}{x+1}$

**16.** Yale economist Robert J. Shiller and Wellesley economist Karl Case created the Case-Shiller index of American housing prices that tracks the value of housing back to 1890 in consistent terms, factoring out the effects of inflation. The 1890 benchmark is 100 on the chart. If a standard house sold in 1890 for $100,000 (inflation-adjusted to today's dollars), an equivalent house would have sold for $66,000 in 1920 (66 on the index scale) and $220,000 in 2006 (220 on the index scale).

Use the following graph, based on the Case-Shiller index, to write a 60-second summary about home values in the United States since 1890. You can use the Internet to find the current value of homes and see if these projections are accurate.

**A History of Home Values**




# 1.3 An Introduction to Functions

## What Is a Function?

When we speak informally of one quantity being a function of some other quantity, we mean that one depends on the other. For example, someone may say that what they wear is a function of where they are going, or what they weigh is a function of what they eat, or how well a car runs is a function of how well it is maintained.

In mathematics, the word "function" has a precise meaning. A function is a special type of relationship between two quantities. If the value of one quantity uniquely determines the value of a second quantity, then the second quantity is a *function* of the first.

For example, median age (as shown in Example 1) is a function of time, since each input of a year determines a unique (one and only one) output value of a median age.

A *function* is a rule that assigns to each input quantity exactly one output quantity. The output is a function of the input.

## Representing Functions: Words, Tables, Graphs, and Equations

A function rule can be described using words, data tables, graphs, or equations. The median age example uses data tables and graphs. When a function is described in these various ways, each description provides the same information but with a different emphasis.

### EXAMPLE 1 | Sales Tax

Twelve states have a sales tax of 6%; that is, for each dollar spent in a store in these states, the law says that you must pay a tax of 6 cents, or \$0.06. Represent the sales tax as a function of purchase price using words, an equation, table, and graph.[10]

**Solution**

#### Using Words

The sales tax is 6% of the purchase price, which means you multiply the purchase price by six hundredths to get the sales tax.

#### Using an Equation

We can write this relationship as an equation where $T$ represents the amount of sales tax and $P$ represents the price of the purchase (both measured in dollars):

$$\text{amount of sales tax} = 0.06 \bullet \text{price of purchase}$$
$$T = 0.06P$$

Our function rule says: "Take the given value of $P$ and multiply it by 0.06; the result is the corresponding value of $T$." The equation represents $T$ as a function of $P$, since for each value of $P$ the equation determines a unique (one and only one) value of $T$. The purchase price, $P$, is restricted to dollar amounts greater than or equal to zero.

#### Using a Table

We can use this formula to make a table of values for $T$ determined by the different values of $P$. See **Table 1.4**. Such tables are sometimes seen beside cash registers.

**TABLE 1.4**

| *P (purchase price in \$)* | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *T (sales tax in \$)* | 0.00 | 0.06 | 0.12 | 0.18 | 0.24 | 0.30 | 0.36 | 0.42 | 0.48 | 0.54 | 0.60 |

#### Using a Graph

The points in **Table 1.4** were used to create a graph of the function. See **Figure 1.8**. The table shows the sales tax only for selected purchase prices, but we could have used any positive dollar amount for $P$. We connected the points on the scatter plot to suggest the many possible intermediate values for price. For example, if $P = \$2.50$, then $T = \$0.15$.

**Graph of 6% Sales Tax**


**FIGURE 1.8**

[10]In 2018, a sales tax of 6% was the most common rate for a sales tax in the United States. See *taxadmin.org* for a listing of the sales tax rates for all of the states.

## Input and Output: Independent and Dependent Variables

Since a function is a rule that assigns to each input a unique output, we think of the output as being dependent on the input. We call the input of a function the *independent variable* and the output the *dependent variable*. When a set of ordered pairs represents a function, then each ordered pair is written in the form

**(input, output)**

or equivalently, **(independent variable, dependent variable)**

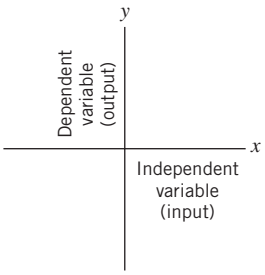



**FIGURE 1.9**

If $x$ is the independent and $y$ the dependent variable, then the ordered pairs would be of the form

$$(x, y)$$

The mathematical convention is for the first variable, or input of a function, to be represented on the horizontal axis and the second variable, or output, on the vertical axis. See **Figure 1.9**.

Sometimes the choice of the independent variable is arbitrary or not obvious. For example, economists argue as to whether wealth is a function of education or education is a function of wealth. As seen in the next example, there may be more than one correct choice.

**EXAMPLE 2** | Identifying Independent and Dependent Variables

In the sales tax example, the equation $T = 0.06P$ gives the sales tax, $T$, as a function of purchase price, $P$. In this case $T$ is the dependent variable, or output, and $P$ is the independent variable, or input. But for this equation we can see that $P$ is also a function of $T$; that is, each value of $T$ corresponds to one and only one value of $P$. It is easier to see the relationship if we solve for $P$ in terms of $T$, to get

$$P = \frac{T}{0.06}$$

Now we are thinking of the purchase price, $P$, as the dependent variable, or output, and the sales tax, $T$, as the independent variable, or input. So, if you tell me how much tax you paid, I can find the purchase price.

## When Is a Relationship Not a Function?

Not all relationships define functions. A function is a special type of relationship, one where for each input, the rule specifies one and only one output. Examine the following examples.

| Function | | Not a Function | | Function | |
|---|---|---|---|---|---|
| Input | Output | Input | Output | Input | Output |
| 1 → | 6 | 1 → | 6 | 1 → | 6 |
| 2 → | 7 | | 7 | 2 → | |
| 3 → | 8 | 2 → | 8 | 3 → | |
| | | 3 → | 9 | | |
| Each input has only one output. | | The input of 1 gives *two different outputs*, 6 and 7, so this relationship is *not* a function. | | Each input has only one output. Note that a function may have identical outputs for different inputs. | |

**EXAMPLE 3** | Does the Table Represent a Function?

Consider the data in **Table 1.5** on the rate of unemployment in the United States. Which variable/s could be used as the input of a function? What about the output?

**TABLE 1.5**

**Unemployment in the United States**

| Year, $Y$ | Unemployment Rate, $U$ |
|---|---|
| 1975 | 8.5 |
| 1980 | 7.1 |
| 1985 | 7.2 |
| 1990 | 5.6 |
| 1995 | 5.6 |
| 2000 | 4.0 |
| 2005 | 5.1 |
| 2008 | 5.8 |
| 2009 | 9.3 |
| 2010 | 9.6 |

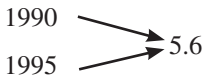
Source: Bureau of Labor Statistics, 2010 *http://www.bls.gov*.

**Solution**

### $U$ is a Function of $Y$

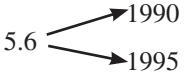
The year, $Y$, can be used as the input of a function where the unemployment rate, $U$, is the output. For each year there is one and only one output; therefore, $U$ is a function of $Y$.

Note that different inputs (such as 1990 and 1995) can have the same output (5.6) and still satisfy the conditions of a function.




### $Y$ is NOT a Function of $U$

The unemployment rate, $U$, can NOT be used as the input of a function where the year, $Y$, is the output. When $U = 5.6$, there are two corresponding values for $Y$, 1990 and 1995, and this would violate the condition for a function of a unique (one and only one) output for each input. Therefore $Y$ is NOT a function of $U$.




Remember that each input can have only one output to satisfy the conditions of a function.

### EXAMPLE 4 | The Input and Output of a Function on a Graph

How would the axes be labeled for each graph of the following functions?

**a.** Density of water is a function of temperature.

**b.** Radiation intensity is a function of wavelength.

**c.** A quantity $Q$ is a function of time $t$.

**Solution**

**Figure 1.10** shows how to label the input and output of these functions on a graph.

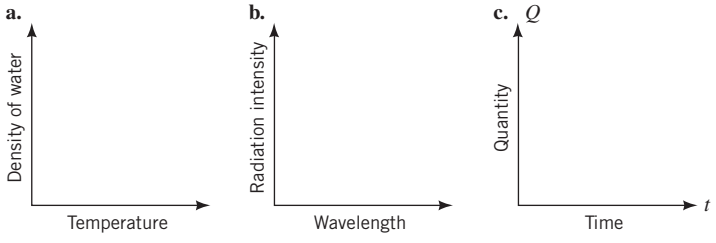



**FIGURE 1.10** **Various labels for axes, depending on the context.**

## How to tell if a graph represents a function: The vertical line test

For a graph to represent a function, each value of the input on the horizontal axis must be associated with one and only one value of the output on the vertical axis. If you can draw a

vertical line that intersects a graph in more than one point, then at least one input is associated with two or more outputs, and the graph does not represent a function.

**Vertical Line Test**

If there is a vertical line that intersects a graph more than once, the graph does not represent a function.

The graph in **Figure 1.11** represents $y$ as a function of $x$. For each value of $x$, there is only one corresponding value of $y$. No vertical line intersects the curve in more than one point. The graph in **Figure 1.12** does *not* represent a function. One can draw a vertical line (an infinite number, in fact) that intersects the graph in more than one point. **Figure 1.12** shows a vertical line that intersects the graph at both (4, 2) and (4, −2). That means that the value $x = 4$ does not determine one and only one value of $y$. It corresponds to $y$ values of both 2 and −2.

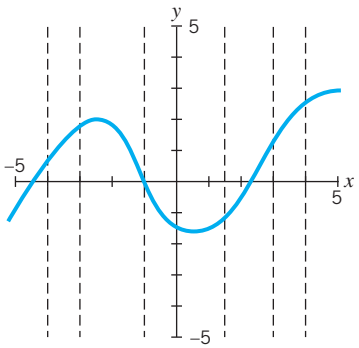



**FIGURE 1.11** **The graph represents $y$ as a function of $x$ since there is no vertical line that intersects the curve at more than one point.**

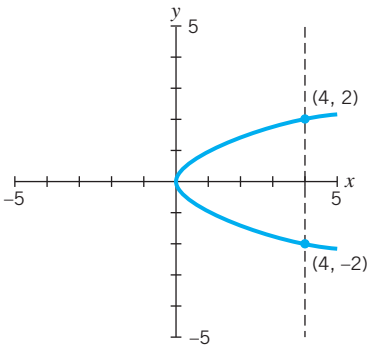



**FIGURE 1.12** **The graph does not represent $y$ as a function of $x$ since there is at least one vertical line that intersects this curve at more than one point.**

## Explore & Extend

### 1.3 Is It a Function?

Consider the set of data on the National Collegiate Athletic Association (NCAA) championship basketball games in the table. Which variables could be used as the input of a function? What about the output? What must be true of the scatter plots of the functions? Of the scatter plots that are not functions?

| Year | Number of Wins | University of Arizona Tournament Seed |
|---|---|---|
| 2000 | 27 | 1 |
| 2001 | 28 | 2 |
| 2002 | 24 | 3 |
| 2003 | 28 | 1 |
| 2004 | 20 | 9 |
| 2005 | 30 | 3 |
| 2006 | 20 | 8 |
| 2007 | 20 | 8 |
| 2008 | 19 | 10 |
| 2009 | 21 | 12 |

Note: A tournament seed is an assigned number between 1 and 16 that ranks a team's potential opponents at every future stage of the NCAA competition.

## Algebra Aerobics 1.3

**1.** Which of the following tables represent functions? Justify your answer.

**TABLE A**

| Input | Output |
|---|---|
| 1 | 5 |
| 2 | 8 |
| 3 | 8 |
| 4 | 10 |

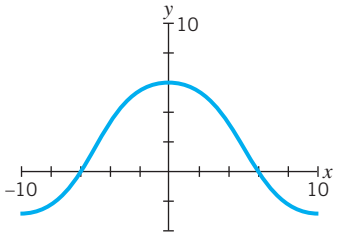
**TABLE B**

| Input | Output |
|---|---|
| 1 | 5 |
| 2 | 7 |
| 2 | 8 |
| 4 | 10 |

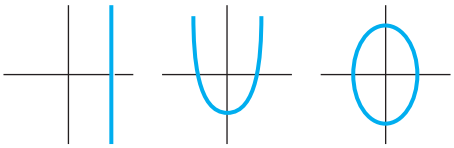
**2.** Does the following table represent a function? If so, why? If not, how could you change the values in the table so that it represents a function?

| Input | Output |
|---|---|
| 1 | 5 |
| 1 | 7 |
| 3 | 8 |
| 4 | 10 |

**3.** Refer to the following graph. Is $y$ a function of $x$?

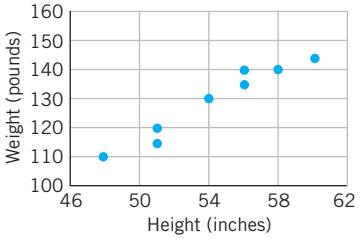



**4.** Which of the graphs represent functions and which do not? Why?




**5.** Consider the scatter plot in the graph below. Is weight a function of height? Is height a function of weight? Explain your answer.

**Graph of Weight versus Height**




**6.** Consider the following table.

**a.** Is $D$ a function of $Y$?

**b.** Is $Y$ a function of $D$?

| $Y$ | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
|---|---|---|---|---|---|---|
| $D$ | \$2.50 | \$2.70 | \$2.40 | −\$0.50 | \$0.70 | \$2.70 |

**7. a.** Write an equation for computing a 15% tip in a restaurant. Does your equation represent a function? If so, what are your choices for the independent and dependent variables?

**b.** How much would the equation suggest you tip for an \$8 meal?

**c.** Compute a 15% tip on a total check of \$26.42.

## Exercises for Section 1.3

A graphing program is required for Exercise 12.

**1.** The following table gives the high temperature in Santa Cruz, California, for each of five days in January, 2010.

**a.** Is the temperature a function of the date?

**b.** Is the date a function of the temperature?

| Date | High Temperature Santa Cruz, California |
|---|---|
| Jan. 1 | 62°F |
| Jan. 2 | 65°F |
| Jan. 3 | 64°F |
| Jan. 4 | 66°F |
| Jan. 5 | 64°F |

Source: *www.weather.com*.

**2.** Which of the following tables describe functions? Explain your answer.

**a.**

| Input value | −2 | −1 | 0 | 1 | 2 |
|---|---|---|---|---|---|
| Output value | −8 | −1 | 0 | 1 | 8 |

**b.**

| Input value | 0 | 1 | 2 | 1 | 0 |
|---|---|---|---|---|---|
| Output value | −4 | −2 | 0 | 2 | 4 |

**c.**

| Input value | 10 | 7 | 4 | 7 | 10 |
|---|---|---|---|---|---|
| Output value | 3 | 6 | 9 | 12 | 15 |

**d.**

| Input value | 0 | 3 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| Output value | 3 | 3 | 3 | 3 | 3 |

**3.** Determine whether each set of points represents a function. (*Hint:* It may be helpful to plot the points.)

**a.** (2, 6), (–4, 6), (1, –3), (4, –3)

**b.** (2, –2), (3, –2), (4, –2), (6, –2)

**c.** (2, –3), (2, 3), (–2, –3), (–2, 3)

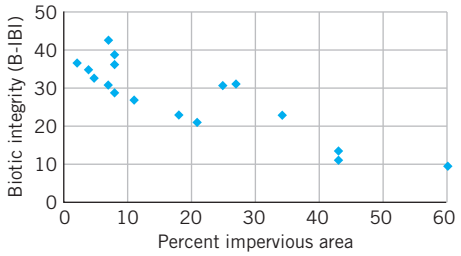
**d.** (–1, 2), (–1, 0), (–1, –1), (–1, –2)

**4.** Consider the accompanying table, listing the weights ($W$) and heights ($H$) of five individuals. Based on this table, is height a function of weight? Is weight a function of height? Justify your answers.

| Weight $W$ (lb) | Height $H$ (in) |
|---|---|
| 120 | 54 |
| 120 | 55 |
| 125 | 58 |
| 130 | 60 |
| 135 | 56 |

**5.** The accompanying graph uses a measure of urbanization as the percent of impervious area (area covered by materials such as asphalt, concrete, and so on that prevent the absorption of water) and shows the relationship to biotic integrity (the capability of supporting and maintaining a natural habitat).

**Urbanization Effects on Streams**




Source: Seattle Central College, Quantitative Environment Learning Project.

Use the graph to determine if the level of biotic integrity is a function of the percent of imperious area. Explain your answer.

**6.** Consider the accompanying graph describing the sales of organic products from 2005 to 2008. Use the vertical line test to determine if the percent change in sales of organic products is a function of time. Explain your answer.

**Organic Product Sales**


Source: The Nielsen Company.

**7.** **a.** Find an equation that represents the relationship between $x$ and $y$ in each of the accompanying tables.

**i.**

| $x$ | $y$ |
|---|---|
| 0 | 5 |
| 1 | 6 |
| 2 | 7 |
| 3 | 8 |
| 4 | 9 |

**ii.**

| $x$ | $y$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 5 |
| 3 | 10 |
| 4 | 17 |

**iii.**

| $x$ | $y$ |
|---|---|
| 0 | 3 |
| 1 | 3 |
| 2 | 3 |
| 3 | 3 |
| 4 | 3 |

**b.** Which of your equations represents $y$ as function of $x$? Justify your answer.

**8.** For each of the accompanying tables find a function that takes the $x$ values and produces the given $y$ values.

**a.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |

**b.**

| $x$ | $y$ |
|---|---|
| 0 | –2 |
| 1 | 1 |
| 2 | 4 |
| 3 | 7 |
| 4 | 10 |

**c.**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | –1 |
| 2 | –4 |
| 3 | –9 |
| 4 | –16 |

**9.** The basement of a large department store features discounted merchandise. Their policy is to reduce the previous month's price of the item by 10% each month for 5 months, and then give the unsold items to charity.

**a.** Let $S_1$ be the sale price for the first month and $P$ the original price. Express $S_1$ as a function of $P$. What is the price of a \$100 garment on sale for the first month?

**b.** Let $S_2$ be the sale price for the second month and $P$ the original price. Express $S_2$ as a function of $P$. What is the price of a \$100 garment on sale for the second month?

**c.** Let $S_3$ be the sale price for the third month and $P$ the original price. Express $S_3$ as a function of $P$. What is the price of a \$100 garment on sale for the third month?

**d.** Let $S_5$ be the sale price for the fifth month and $P$ the original price. Express $S_5$ as a function of $P$. What is the final price of a \$100 garment on sale for the fifth month? By what total percentage has the garment now been reduced from its original price?

**10.** Write a formula to express each of the following sentences:

**a.** The sale price is 20% off the original price. Use $S$ for sale price and $P$ for original price to express $S$ as a function of $P$.

**b.** The time in Paris is 6 hours ahead of New York. Use $P$ for Paris time and $N$ for New York time to express $P$ as a function of $N$. (Represent your answer in terms of a 12-hour clock.) How would you adjust your formula if $P$ comes out greater than 12?

**c.** For temperatures above 0°F the wind chill effect can be estimated by subtracting two-thirds of the wind speed (in miles per hour) from the outdoor temperature. Use $C$ for the effective wind chill temperature, $W$ for wind speed, and $T$ for the actual outdoor temperature to write an equation expressing $C$ in terms of $W$ and $T$.

**11.** Determine whether $y$ is a function of $x$ in each of the following equations. If the equation does not define a function, find a value of $x$ that is associated with two different $y$ values.

**a.** $y = x^2 + 1$ **c.** $y = 5$

**b.** $y = 3x - 2$ **d.** $y^2 = x$

**12.** (Graphing program required.) For each equation, write an equivalent equation that expresses $z$ in terms of $t$. Use technology to sketch the graph of each equation. Is $z$ a function of $t$? Why or why not?

**a.** $3t - 5z = 10$ **c.** $2(t - 4) - (z + 1) = 0$

**b.** $12t^2 - 4z = 0$

**13.** If we let $D$ stand for ampicillin dosage expressed in milligrams and $W$ stand for a child's weight in kilograms, then the equation

$$D = 50W$$

gives a rule for finding the safe maximum daily drug dosage of ampicillin (used to treat respiratory infections) for children who weigh less than 10 kilograms (about 22 pounds).[11]

**a.** What are logical choices for the independent and dependent variables?

**b.** Does the equation represent a function? Why?

**c.** Generate a small table and graph of the function.

# 1.4 The Language of Functions

Not all equations represent functions. (See **Figure 1.12** where the graph of the equation does not pass the vertical line test.) But functions have important qualities, so it is useful to have a way to indicate when a relationship is a function.

## Function Notation

When a quantity $y$ is a function of $x$, we can write

$$y \text{ is a function of } x$$

or in abbreviated form, we say:

$$y \text{ equals "} f \text{ of } x\text{"}$$

or using function notation,

$$y = f(x)$$

The expression $y = f(x)$ means that the rule $f$ is applied to the input value $x$ to give the output value, $f(x)$:

$$\textbf{output} = f\textbf{(input)}$$

or

$$\textbf{dependent variable} = f\textbf{(independent variable)}$$

The letter $f$ is often used to denote the function, but we could use any letter, not just $f$.

### Understanding the symbols

Suppose we create a function where the input is multiplied by three and 1 is added to the product. We could write this function as

$$y = 3x + 1 \qquad (1)$$

or with function notation as

$$R(x) = 3x + 1 \quad \text{where } y = R(x) \qquad (2)$$

Equations (1) and (2) represent the same function, but with function notation we name the function—in this case $R$—and identify the input, $x$, and output, $3x + 1$.

Function notation can provide considerable economy in writing and reading, by using $R(x)$ instead of the full expression to represent the function.

[11] Information extracted from Anna M. Curren and Laurie D. Muntlay, *Math for Meds: Dosages and Solutions*, 6th ed., San Diego: W. I. Publications, 1990.

**The Language of Functions**

If $y$ is a function, $f$, of $x$, then

$$y = f(x)$$

where
$f$ is the name of the function,
$y$ is the output or *dependent* variable,
$x$ is the input or *independent* variable.

$f$(input) = output
$f$(independent) = dependent

## Finding Output Values: Evaluating a Function

Function notation is particularly useful when a function is being evaluated at a specific input value. Suppose we want to find the value of the previous function $R(x)$ when our input value is 10. Using Equation (1) we would say, "find the value of $y$ when $x = 10$." With function notation, we simply write $R(10)$. To evaluate the function $R$ at 10 means calculating the value of the output when the value of the input is 10:

$$\begin{aligned} R(\text{input}) &= \text{output} \\ R(x) &= 3x + 1 \end{aligned}$$

Substitute 10 for $x$

$$\begin{aligned} R(10) &= 3(10) + 1 \\ &= 31 \end{aligned}$$

So, applying the function rule $R$ to the input value of 10 gives an output value of 31.

**Common Error**

The expresssion $f(x)$ does not mean "$f$ times $x$." It means the function $f$ evaluated at $x$.

**EXAMPLE 1** | Using Function Notation with Equations

Evaluate $f(5)$, $f(0)$, and $f(-2)$ for the function $f(x) = 2x^2 + 3$.

**Solution**

To evaluate $f(5)$, we replace every $x$ in the formula with 5.

Given $f(x) = 2x^2 + 3$

substitute 5 for $x$

$$\begin{aligned} f(5) &= 2(5)^2 + 3 \\ &= 2(5)(5) + 3 \\ &= 53 \end{aligned}$$

Similarly,

if $x = 0$, $f(0) = 2(0)^2 + 3 = 3$

if $x = -2$,

$$\begin{aligned} f(-2) &= 2(-2)^2 + 3 \\ &= (2 \cdot 4) + 3 \\ &= 11 \end{aligned}$$

## Finding Input Values: Solving Equations

In Example 1, we found output values when we knew the input. Sometimes the situation is reversed and we know the output and want to find the corresponding input.

**EXAMPLE 2** | What is the Value of the Input?

**a.** Use the sales tax function $T = 0.06P$ to find the purchase price $P$ when the sales tax $T$ on a purchase is \$8.40.

**b.** Given $f(x) = 5x - 2.5$, find $x$ when $f(x) = 21$.

**Solution**

**a.** We want to find $P$ when $T = \$8.40$, so substitute \$8.40 for $T$ and then solve the equation:

| | |
|---|---|
| | $T = 0.06P$ |
| substitute \$8.40 for $T$ | $\$8.40 = 0.06P$ |
| divide both sides by 0.06 | $\$140 = P$ |

The purchase price is \$140 when the sales tax is \$8.40.

**b.** To find an input value for $x$ that results in $f(x) = 21$, solve the equation

| | |
|---|---|
| | $21 = 5x - 2.5$ |
| add 2.5 to each side | $23.5 = 5x$ |
| divide by 5 | $4.7 = x$ |

When the input is $x = 4.7$, then $f(x) = 21$ or equivalently $f(4.7) = 21$.

## Finding Input and Output Values From Tables and Graphs

**EXAMPLE 3** | Using Function Notation with Data Tables

Use **Table 1.6** to fill in the missing values:

**a.** $S(0) = ?$

**b.** $S(-1) = ?$

**c.** $S(?) = 4$

**TABLE 1.6**

| Input, $x$ | $-1$ | $-2$ | 0 | 1 | 2 |
|---|---|---|---|---|---|
| Output, $S(x)$ | 1 | 4 | 0 | 1 | 4 |

**Solution**

**a.** $S(0)$ means to evaluate $S$ when the input $x = 0$. The table says that the corresponding output is also 0, so $S(0) = 0$.

**b.** $S(-1) = 1$.

**c.** $S(?) = 4$ means to find the input when the output is 4. When the output is 4, the input is $-2$ or 2, so $S(-2) = 4$ and $S(2) = 4$.

**EXAMPLE 4** | Using Function Notation with Graphs

Use the graph in **Figure 1.13** to estimate the missing values:

**a.** $f(0) = ?$

**b.** $f(-5) = ?$

**c.** $f(?) = 0$

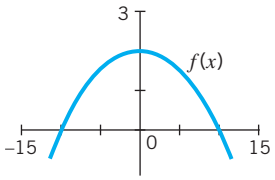



**FIGURE 1.13** **Graph of a function.**

**Solution**

Remember that by convention the horizontal axis represents the input (or independent variable) and the vertical axis represents the output (or dependent variable).

**a.** $f(0) = 2$

**b.** $f(-5) = 1.5$

**c.** $f(10) = 0$ and $f(-10) = 0$

## Rewriting Equations Using Function Notation

In order to use function notation, an equation needs to be in the form

$$\text{output} = \text{some rule applied to input}$$

or equivalently

$$\text{dependent variable} = \text{some rule applied to independent variable}$$

Translating an equation into this format is called putting the equation in *function form*. Many graphing calculators and computer graphing programs accept only equations in function form.

To put an equation into function form, we first need to identify the independent and the dependent variables. The choice is sometimes obvious, at other times arbitrary. If we use the mathematical convention that $x$ represents the input or independent variable and $y$ the output or dependent variable, when we put equations into function form, we want to solve for $y$ or

$$y = \text{some rule applied to } x$$

### EXAMPLE 5 | Does the Equation Represent a Function?

Analyze the equation $4x - 3y = 6$. Decide whether or not the equation represents $y$ as a function of $x$. If it does, write the relationship using function notation.

**Solution**

First, put the equation into function form. Since $y$ is the output, then

| | |
|---|---|
| given the equation | $4x - 3y = 6$ |
| subtract $4x$ from both sides | $-3y = 6 - 4x$ |
| divide both sides by $-3$ | $\frac{-3y}{-3} = \frac{6 - 4x}{-3}$ |
| simplify | $y = \frac{6}{-3} + \frac{-4x}{-3}$ |
| simplify and rearrange terms | $y = \frac{4}{3}x - 2$ |

We now have an expression for $y$ in terms of $x$.

Using technology or by hand, we can generate a table and graph of the equation. See **Table 1.7** and **Figure 1.14**. Since the graph passes the vertical line test, $y$ is a function of $x$.

**TABLE 1.7**

| $x$ | $y$ |
|---|---|
| −3 | −6 |
| 0 | −2 |
| 3 | 2 |
| 6 | 6 |

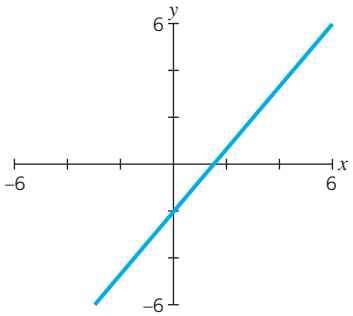



**FIGURE 1.14** **Graph of $y = \frac{4}{3}x - 2$.**

If we name our function $f$, then using function notation, we have

$$y = f(x) \qquad \text{where } f(x) = \frac{4}{3}x - 2$$

### EXAMPLE 6 | An Equation Where the Input Gives More Than One Output

Analyze the equation $y^2 - x = 0$. Generate a graph of the equation. Decide whether or not the equation represents a function. If the equation represents a function, write the relationship using function notation. Assume $y$ is the output.

**Solution**

Put the equation in function form:

| | |
|---|---|
| Given the equation | $y^2 - x = 0$ |
| add $x$ to both sides | $y^2 = x$ |

To solve this equation, we take the *square root* of both sides of the equation and we get

$$y = \pm\sqrt{x}$$

This gives us two solutions for any value of $x > 0$ as shown in **Table 1.8**. For example, if $x = 4$, then $y$ can either be 2 or $-2$ since both $2^2 = 4$ and $(-2)^2 = 4$.

**TABLE 1.8**

| $x$ | $y$ |
|---|---|
| 0 | 0 |
| 1 | 1 or $-1$ |
| 2 | $\sqrt{2}$ or $-\sqrt{2}$ |
| 4 | 2 or $-2$ |
| 9 | 3 or $-3$ |

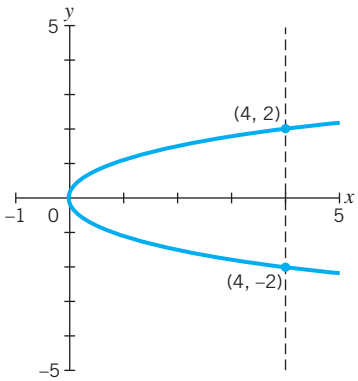



**FIGURE 1.15** **Graph of the equation $y^2 = x$ is not a function.**

The graph of the equation in **Figure 1.15** does not pass the vertical line test. In particular, the solutions (4, –2) and (4, 2) lie on the same vertical line. So $y$ is not a function of $x$ and we cannot use function notation to represent this relationship.

## Algebra Aerobics **1.4a**

**1.** Given $g(x) = 3x$, evaluate $g(0)$, $g(-1)$, $g(1)$, $g(20)$, and $g(100)$.

**2.** Consider the function $f(x) = x^2 - 5x + 6$. Find $f(0)$, $f(1)$, and $f(-3)$.

**3.** Given the function $f(x) = \dfrac{2}{x-1}$, evaluate $f(0)$, $f(-1)$, and $f(-3)$.

**4.** Determine the value of $t$ for which each of the functions has a value of 3.

$$r(t) = 5 - 2t \qquad p(t) = 3t - 9 \qquad m(t) = 5t - 12$$

In Problems 5–7 solve for $y$ in terms of $x$. Determine if $y$ is a function of $x$. If it is, rewrite using $f(x)$ notation.

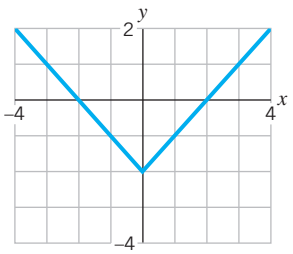
**5.** $2(x - 1) - 3(y + 5) = 10$

**6.** $x^2 + 2x - y + 4 = 0$

**7.** $7x - 2y = 5$

**8.** From the accompanying graph, estimate $f(-4)$, $f(-1)$, $f(0)$, and $f(3)$. Find two approximate values for $x$ such that $f(x) = 0$.

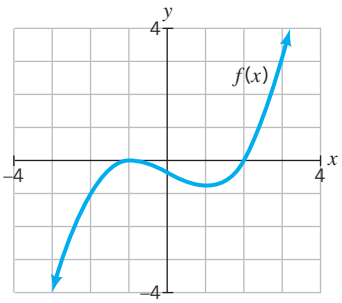
**Graph of $f(x)$.**




**9.** From the table, find $f(0)$ and $f(20)$. Find two values of $x$ for which $f(x) = 10$. Explain why $f(x)$ is a function.

| $x$ | $f(x)$ |
|---|---|
| 0 | 20 |
| 10 | 10 |
| 20 | 0 |
| 30 | 10 |
| 40 | 20 |

**10.** From the accompanying graph, estimate $f(-3)$, $f(0)$, $f(1)$, and $f(3)$. Find two approximate values of $x$ for which $f(x) = 0$ and $f(x) = 1.5$.



## Domain and Range

A function is often defined only for certain values of the input (or independent variable) and corresponding values of the output (or dependent variable).

**Domain and Range of a Function**

The *domain* of a function is the set of possible values of the input.
The *range* is the set of corresponding values of the output.

Many times, especially under real-world conditions, there are restrictions on the domain and the range. The restrictions are often implied, as in Example 7 below. As we study each family of functions, we return to the question of whether there are restrictions on the domain and range.

**EXAMPLE 7** | Finding a Reasonable Domain and Range

In the sales tax example at the beginning of this section, we used the equation

$$T = 0.06P$$

to represent the sales tax, $T$, as a function of the purchase price, $P$ (where all units are in dollars). What are the domain and range of this function?

**Solution**

Since negative values for $P$ are meaningless, $P$ is restricted to dollar amounts greater than or equal to zero. In theory there is no upper limit on prices, so we assume $P$ has no maximum amount. In this example,

$$\text{the domain is all dollar values of } P \text{ greater than or equal to } 0$$

We can express this more compactly as

$$\text{domain} = \text{dollar values of } P \geq 0$$

What are the corresponding values for the tax $T$? The values for $T$ in our model cannot be negative. As long as there is no maximum value for $P$, there will be no maximum value for $T$. So,

$$\text{the range is all dollar values of } T \text{ greater than or equal to } 0$$

or

$$\text{range} = \text{dollar values of } T \geq 0$$

(Note: The symbol ≥ means "greater than or equal to and the symbol ≤ means "less than or equal to")

## Representing the domain and range with interval notation

Interval notation is often used to represent the domain and range of a function.

### Interval Notation

A *closed interval* $[a, b]$ indicates all real numbers $x$ for which $a \le x \le b$.
Closed intervals include their endpoints.

An *open interval* $(a, b)$ indicates all real numbers $x$ for which $a < x < b$.
Open intervals exclude their endpoints.

*Half-open (or equivalently half-closed)* intervals are represented by
$[a, b)$ which indicates all real numbers $x$ for which $a \le x < b$ or
$(a, b]$ which indicates all real numbers $x$ for which $a < x \le b$.

For example, if the domain is values of $x$ greater than or equal to 50 and less than or equal to 100, then

$$\begin{aligned} \text{domain} &= \text{all } x \text{ values with } 50 \le x \le 100 \\ &= [50, 100] \end{aligned}$$

If the domain is values of $x$ greater than 50 and less than 100, then

$$\begin{aligned} \text{domain} &= \text{all } x \text{ values with } 50 < x < 100 \\ &= \text{interval } (50, 100) \end{aligned}$$

If we want to exclude 50 but include 100 as part of the domain, we would represent the interval as (50, 100]. This interval can be displayed on the real number line as:


In general, a hollow dot indicates exclusion and a solid dot inclusion. *Note:* Since the notation $(a, b)$ can also mean the coordinates of a point, we will say the *interval* $(a, b)$ when we want to refer to an interval.

## Visualizing the domain and range

Recall that by convention, the input of a function is represented on the horizontal axis and the output on the vertical axis. **Figure 1.16** illustrates the set of inputs or domain = [–4, 7] and the outputs or range of $f(x) = [-2, 6]$.

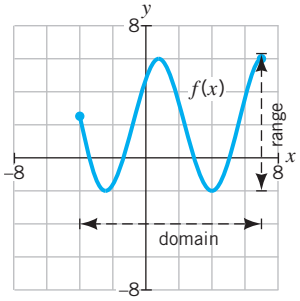



**FIGURE 1.16**

### EXAMPLE 8 | Finding the Domain and Range From a Graph

The graph in **Figure 1.17** shows the water level of the tides in Pensacola, Florida, over a 24-hour period. Are the Pensacola tides a function of the time of day? If so, identify the independent and dependent variables. Use interval notation to describe the domain and range of this function.

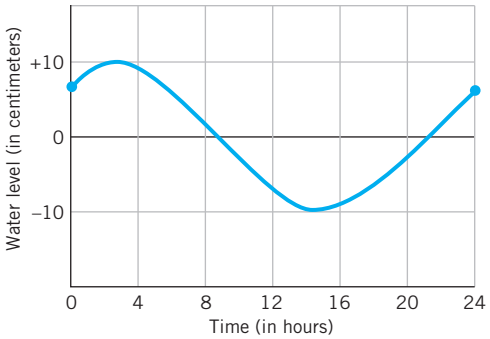



**FIGURE 1.17** **Diurnal tides in a 24-hour period in Pensacola, Florida.**

Source: Adapted from Fig. 8.2 in *Oceanography: An Introduction to the Planet Oceanus*, by Paul R. Pinet. Copyright © 1992 by West Publishing Company, St. Paul, MN. All rights reserved.

### Solution

The Pensacola tides are a function of the time of day since the graph passes the vertical line test. The independent variable is time, and the dependent variable is water level. The domain is from 0 to 24 hours, and the range is from about −10 to +10 centimeters. Using interval notation:

$$\text{domain} = [0, 24]$$
$$\text{range} = [-10, 10]$$

## EXAMPLE 9 | Summarizing Music Cassette Sales

The following graph in **Figure 1.18** shows music cassette tape sales, $S(t)$ over time, $t$.

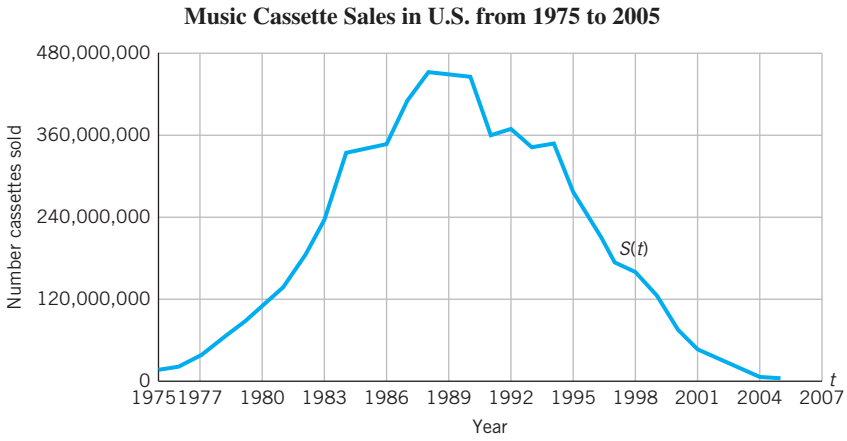



**FIGURE 1.18**

Source: Support Team, www.swivel.com.

Find the following:

**a.** _____ is a function of _____.

**b.** Input (with units): _____ Output (with units): _____

**c.** Estimate: Domain: _____ Range: _____

**d.** Estimate: $S(1983) \approx$ _____

$t$ such that $S(t) \approx 120,000,000$

What are the practical implications of these statements?

**e.** Estimate the highest and lowest values of $S(t)$.

What are the practical implications of these statements?

**Solution**

**a.** Cassette sales $S(t)$ is a function of time, $t$. For each input $t$, there is one and only one output, $S(t)$.

**b.** Input: time in years Output: number of cassettes sold

**c.** Domain: 1975 to 2005 Range: approx. 1 million to 450 million

**d.** $S(1983) \approx 240{,}000{,}000$, which means that in 1983 there were approximately 240 million cassettes sold.

$S(1980) \approx 120{,}000{,}000$ and $S(1999) \approx 120{,}000{,}000$, which means that in both 1980 and in 1999 there were approximately 120 million cassettes sold.

**e.** The highest value of $S(t)$ is approximately 450 million in 1988. The lowest value is approximately 1 million in 2005. Cassette sales soared from the mid-1970s up until 1988, reaching 450 million. But with CD sales and the Internet market, cassette sales declined rapidly to about 1 million in 2005.

## Algebra Aerobics **1.4b**

**1.** Express each of the following using interval notation.

**a.** $x > 2$

**b.** $4 \le x < 20$

**2.** Express the given interval as an inequality.

**a.** $[-3, 10)$

**b.** $(-2.5, 6.8]$

**3.** Express each of the following statements in interval notation.

**a.** Harry's GPA is at least 2.5 but at most 3.6.

**b.** A good hitter has a batting average of at least 0.333.

**c.** Starting annual salary at a position is anything from \$35,000 to \$50,000 depending upon experience.

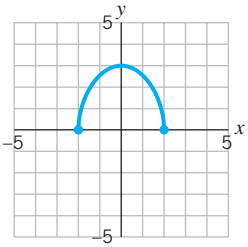
**4.** You invest \$1,500 in a 10-year bond that gives you 4% interest on \$1,500 each year for 10 years.

**a.** Construct a function that represents the amount of interest, $I(n)$, earned after $n$ years. What is the domain and range in this context? What is the input and output?

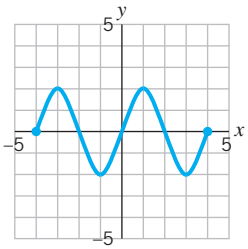
**b.** Find $I(6)$.

**c.** Find $n$ if $I(n) = 240$.

**5.** Determine the domain and the range of the functions whose graphs are given.




**GRAPH A**




**GRAPH B**

## Exercises for Section **1.4**

**1.** Given $T(x) = x^2 - 3x + 2$, evaluate $T(0)$, $T(-1)$, $T(1)$, and $T(-5)$.

**2.** Given $f(x) = \dfrac{x}{x-1}$, evaluate $f(0)$, $f(-1)$, $f(1)$, $f(20)$, and $f(100)$.

**3.** Assume that for persons who earn less than \$20,000 a year, income tax is 16% of their income.

**a.** Generate a formula that describes income tax in terms of income for people earning less than \$20,000 a year.

**b.** What are you treating as the independent variable? The dependent variable?

**c.** Does your formula represent a function? Explain.

**d.** If it is a function, what is the domain? The range?

**4.** Suppose that the price of gasoline is \$3.09 per gallon.

**a.** Generate a formula that describes the cost, $C$, of buying gas as a function of the number of gallons of gasoline, $G$, purchased.

**b.** What is the independent variable? The dependent variable?

**c.** Does your formula represent a function? Explain.

**4.** (continued)

**d.** If it is a function, what is the domain? The range?

**e.** Generate a small table of values and a graph.

**5.** The cost of driving a car to work is estimated to be \$2.00 in tolls plus 32 cents per mile. Write an equation for computing the total cost $C$ of driving $M$ miles to work. Does your equation represent a function? What is the independent variable? What is the dependent variable? Generate a table of values and then graph the equation.

**6.** For each equation, write the equivalent equation that expresses $y$ in terms of $x$. If the equation represents a function, use function notation to express the relationship.

**a.** $3x + 5x - y = 3y$ **c.** $x(x - 1) + y = 2x - 5$

**b.** $3x(5 - x) = x - y$ **d.** $2(y - 1) = y + 5x(x + 1)$

**7.** If $f(x) = x^2 - x + 2$, find:

**a.** $f(2)$ **b.** $f(-1)$ **c.** $f(0)$ **d.** $f(-5)$

**8.** If $g(x) = 2x + 3$, evaluate $g(0)$, $g(1)$, and $g(-1)$.

**9.** Look at the accompanying table.

**a.** Find $p(-4)$, $p(5)$, and $p(1)$.

**b.** For what value(s) of $n$ does $p(n) = 2$?

| $n$ | –4 | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|
| $p(n)$ | 0.063 | 0.125 | 0.25 | 0.5 | 1 | 2 | 4 | 8 | 16 | 32 |

**10.** The data in the table show the median weight for various ages of Caucasian men. (1 kg ≈ 2.2 lbs).

| Age (years) | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 | 65 | 70 | 75 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Weight (kg) | 72 | 76 | 78 | 80 | 82 | 83 | 83.5 | 83.5 | 82.5 | 82 | 80 | 78 |

Source: Steven Halls, MD.

**a.** Which variable(s) could be used as the input of a function? Explain your answer.

**b.** What is the input (with units)? Output (with units)?

**c.** What is the domain? The range?

**d.** If $f$ represents this function, complete the following for one pair of values:

$f($_____$) =$ _____.

**11.** The data in the table show the depth and corresponding velocity of the Columbia River in the state of Washington.

| Depth (ft) | Velocity (ft/sec) | Depth (ft) | Velocity (ft/sec) |
|---|---|---|---|
| 0.7 | 1.55 | 7.3 | 0.91 |
| 2.0 | 1.11 | 8.6 | 0.59 |
| 2.6 | 1.42 | 9.9 | 0.59 |
| 3.3 | 1.39 | 10.6 | 0.41 |
| 4.6 | 1.39 | 11.2 | 0.22 |
| 5.9 | 1.14 | | |

Source: Savini, J., and Bodhaine, G. L. (1971), Analysis of current meter data at Columbia River gauging stations, Washington and Oregon; USGS Water Supply Paper 1869-F.

**a.** Which variable(s) could be used as the input of a function? Explain your answer.

**b.** What is the input (with units)? Output (with units)?

**c.** What is the domain? The range?

**d.** If $g$ represents this function, complete the following for one pair of values:

$g($_____$) =$ _____.

**12.** Pennsylvania has a flat-rate personal income tax. It can be represented as $T(I) = 0.0307I$, where $I$ is the income in dollars and $T(I)$ is the tax owed in dollars.

**a.** What is the income tax rate in Pennsylvania?

**b.** What is the input? The output?

**c.** Estimate $T(35{,}000)$. What is its meaning?

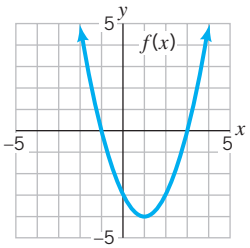
**d.** Estimate $I$ if $T(I) = 1535$. What is its meaning?

**13.** From the accompanying graph of $y = f(x)$:

**a.** Find $f(-2)$, $f(-1)$, $f(0)$, and $f(1)$.

**b.** Find two values of $x$ for which $f(x) = -3$.

**c.** Estimate the range of $f$. Assume that the arms of the graph extend upward indefinitely.




**14.** Given $f(x) = 1 - 0.5x$ and $g(x) = x^2 + 1$, evaluate:

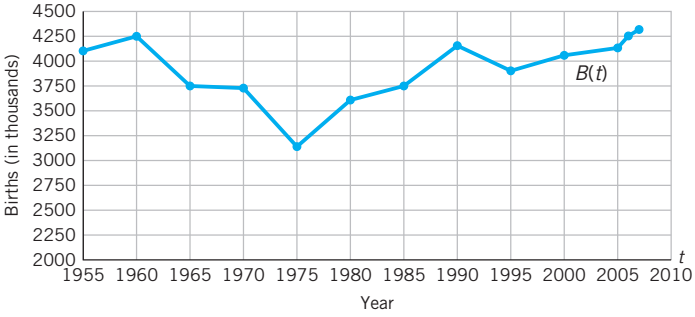
**a.** $f(0)$, $g(0)$ **c.** $f(2)$, $g(1)$

**b.** $f(-2)$, $g(-3)$ **d.** $f(3)$, $g(3)$

**15.** If $f(x) = (2x - 1)^2$, evaluate $f(0)$, $f(1)$, and $f(-2)$.

**16.** The following graph shows live births, $B(t)$, over time, $t$, for the years 1955–2007.

**Live Births in the USA 1955–2007**




Source: U.S. Statistical Abstract, 2010.

Determine the following:

**a.** _____ is a function of _____.

**b.** Input (with units): _____ Output (with units): _____

**c.** Estimate Domain: _____ Range: _____

What is the practical meaning of these values?

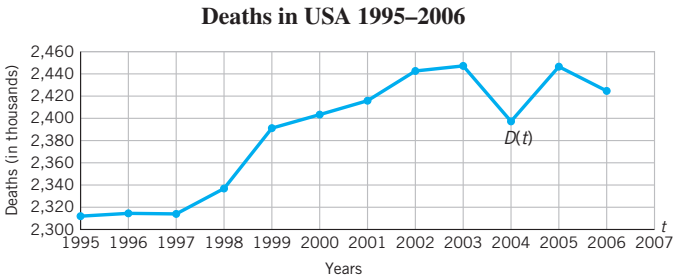
**d.** Estimate $B(2000) \approx$ _____

What is the practical meaning of this value?

**e.** Estimate $t$ if $B(t) = 3750$

What is the practical meaning of these values?

**17.** The following graph shows U.S. deaths, $D(t)$, over time, $t$, for the years 1995–2006 in 1-year intervals.




Source: U.S. Statistical Abstract, 2010.

Determine the following:

**a.** _____ is a function of year _____.

**b.** Input (with units): _____
Output (with units): _____

**c.** Domain: _____ Range: _____
What is the practical meaning of these values?

**d.** Estimate $D(2000) \approx$ _____
What is the practical meaning of this value?

**e.** Estimate $t$ if $D(t) = 2390$
What is the practical meaning of this value?

**f.** Using Exercise 16, find $B(2000) - D(2000)$ and interpret its meaning.

# 1.5 Visualizing Functions

In this section we return to the question: How does change in one variable affect change in another variable? Graphs are one of the easiest ways to recognize change. We start with four basic questions:

## Is There a Maximum or Minimum Value?

If a function has a *maximum* (or *minimum*) value, then it appears as the highest point (or lowest point) on its graph.

**EXAMPLE 1** | Estimating the Maximum and Minimum From a Graph

Determine if each function in **Figure 1.19** has a maximum or minimum, then estimate its value.

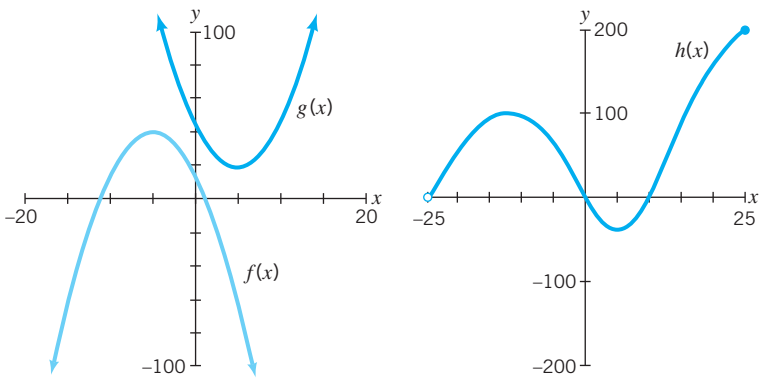



**FIGURE 1.19** **Graphs of $f(x)$, $g(x)$, and $h(x)$.**

**Solution**

The function $f(x)$ in **Figure 1.19** appears to have a maximum value of 40 when $x = -5$ but has no minimum value since both arms of the function extend indefinitely downward.

The function $g(x)$ appears to have a minimum value of 20 when $x = 5$, but no maximum value since both arms of the function extend indefinitely upward.

The function $h(x)$ appears to have a maximum value of 200, which occurs when $x = 25$, and a minimum value of $-50$, when $x = 5$.

## When Is the Output of the Function Positive, Negative, or Zero?

From a graph we can estimate over what intervals a function $f(x) > 0$, $f(x) < 0$ and when $f(x) = 0$.

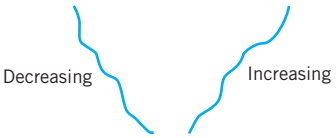
### EXAMPLE 2 | Estimating the Output of a Function From a Graph

In **Figure 1.19**, estimate the $x$ interval when $h(x)$ is positive, negative, or zero. Use interval notation when appropriate.

**Solution**

$h(x) > 0$ over $x$ intervals $(-25, 0)$ and $(10, 25]$
$h(x) < 0$ over $x$ interval $(0, 10)$
$h(x) = 0$ when $x = 0$ and 10

## Is the Function Increasing or Decreasing?




A function $f$ is *decreasing* over a specified interval if the values of $f(x)$ decrease as $x$ increases over the interval. A function $f$ is *increasing* over a specified interval if the values of $f(x)$ increase as $x$ increases over the interval.

The graph of an increasing function rises as we move from left to right. The graph of a decreasing function falls as we move from left to right.

### EXAMPLE 3 | Increasing and Decreasing Production

**Figure 1.20** shows over 100 years of annual natural gas production in the United States. Create a 60-second summary about gas production from 1900 to 2009 in the U.S.

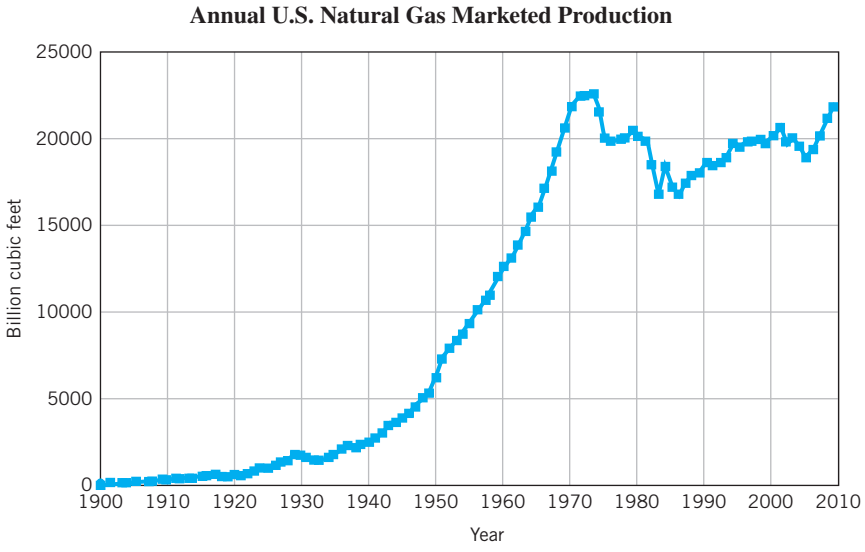



**FIGURE 1.20**

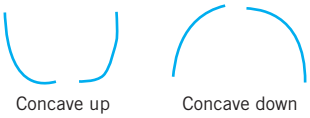
Source: U.S. Energy Information Administration.

**Solution**

Natural gas production in the United States overall showed a steady increase from 1900 to the early 1970s, increasing from a relatively small amount in 1900 to a peak of over 22,000 billion cubic feet in 1973. For the next 10 years production generally decreased, with a few exceptions, to about 16,000 billion cubic feet in the early 1980s, reaching a low not seen since the mid-1960s. From the mid-1980s to the beginning of the twenty-first century there again was a steady increase in production. In the twenty-first century production at first decreased and then from 2006 to 2009 increased steadily, reaching 21,000 billion cubic feet, coming close to its all-time high in 1973.

## Is the Graph Concave Up or Concave Down?

What does the concavity of a graph mean? The graph of a function is *concave up* if it bends upward and it is *concave down* if it bends downward.




Concavity is independent of whether the function is increasing or decreasing.


**EXAMPLE 4** | Graphs are Not Necessarily Pictures of Events[12]

The graph in **Figure 1.21** shows the speed of a roller coaster car as a function of time.

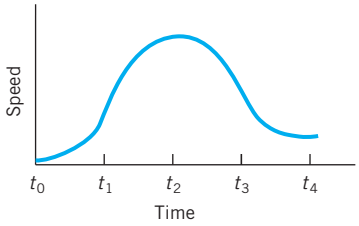



**FIGURE 1.21**

**a.** Describe how the speed of the roller coaster car changes over time. Describe the changes in the graph as the speed changes over time.

**b.** Draw a picture of a possible track for this roller coaster.

**Solution**

**a.** The speed of the roller coaster car increases from $t_0$ to $t_2$, reaching a maximum for this part of the ride at $t_2$. The speed decreases from $t_2$ to $t_4$.

The graph of speed versus time is concave up and increasing from $t_0$ to $t_1$ and then concave down and increasing from $t_1$ to $t_2$. From $t_2$ to $t_3$ the graph is concave down and decreasing, and from $t_3$ to $t_4$ it is concave up and decreasing.

**b.** A picture for a possible track of the roller coaster is shown in **Figure 1.22**. Notice how the track is an upside-down picture of the graph of speed versus time. (When the roller coaster car goes down the speed increases, and when the roller coaster car goes up, the speed decreases.)

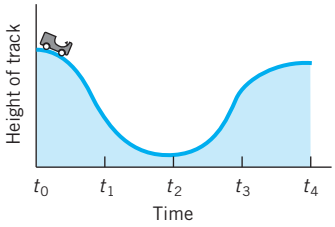



**FIGURE 1.22**

[12]Example 4 is adapted from Shell Centre for Mathematical Education, *The Language of Functions and Graphs*. Manchester, England: University of Nottingham, 1985.

## Getting the Big Idea

We now have the basic vocabulary for describing a function's behavior. Think of a function and its graph as telling a story. We want to decipher not just the individual words, but the basic plot. In each case we should ask: Is there an overall pattern? Are there significant deviations from that pattern?

### EXAMPLE 5 | Generating a Rough Graph

Generate a rough graph of each of the following situations.

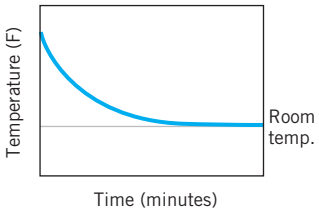
**a.** A cup of hot coffee cooling.

**b.** U.S. venture capital (money provided by investment companies to business start-ups) increased modestly but steadily in the mid-1990s, soared during the "dotcom bubble" (in the late 1990s), with a high in 2000, and then suffered a drastic decrease in 2001 back to pre-dotcom levels. After a few years of growth another sharp decline started in 2008.

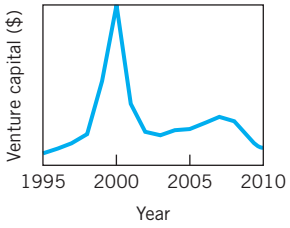
**c.** A simple predator-prey model: initially as the number of lions (the predators) increases, the number of gazelles (their prey) decreases. When there are not enough gazelles to feed all the lions, the number of lions decreases and the number of gazelles starts to increase.

**Solution**

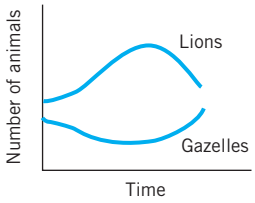
**a. A Cup of Hot Coffee Cooling over Time**




**b. U.S. Venture Capital 1995–2010**




**c. Predator-Prey Model of Lions vs. Gazelles**




### EXAMPLE 6 | Looking for Patterns

You are a TV journalist. Summarize for your viewers the essence of the graph in **Figure 1.23**. The poverty rate is the percentage of the population living in poverty.

Note: The vertical scale is used in two different ways on this graph.

**Number in Poverty and Poverty Rate: 1959 to 2016**

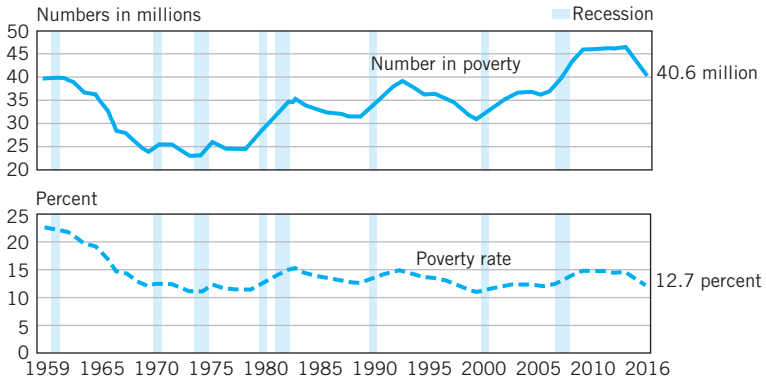



**FIGURE 1.23**

Note: The data for 2013 and beyond reflect the implementation of the redesigned income questions. The data points are placed at the midpoints of the respective years.

Source: U.S. Census Bureau, *Current Population Survey*, 1960 to 2017. Annual Social and Economic Supplements.

**Solution**

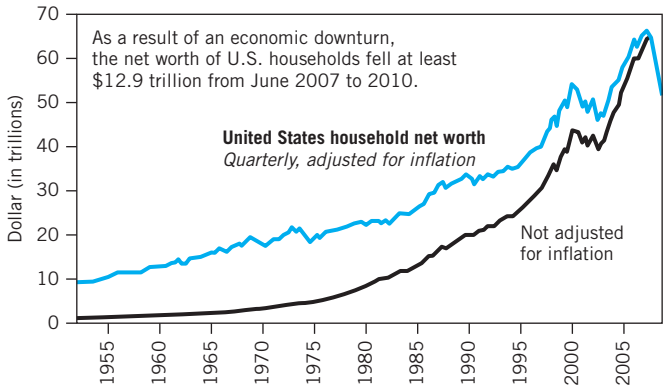
In 2016, the number of people in poverty was about the same as it was in 1959 with approximately 40 million people living in poverty. From 1959 to 2016, the number of people in poverty and the poverty rate increased during periods of recession, with a record high poverty rate in 1959 and the number of people in poverty hitting record highs after the 2008 recession.

### EXAMPLE 7 | Looking for Deviations From a Pattern

The following graph illustrates household net worth in the United States. Household net worth encompasses all of a household's assets (such as housing, stocks, personal property) minus their total debts (such as mortgages, other unpaid loans, credit card debt).

**a.** Describe how household net worth has changed since 1955.

**b.** What title would you give to the graph for a newspaper article about household net worth in the United States?




Source: *Federal Reserve Board.*

**Solution**

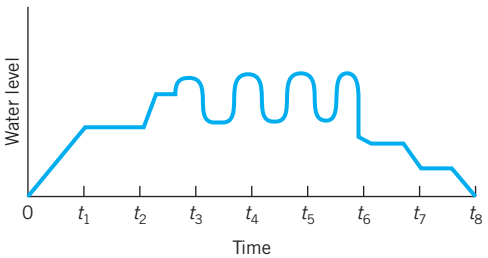
**a.** From 1955 to 2000, household net worth in the United States (both adjusted and not adjusted for inflation) generally increased, with only a few relatively slight decreases at the end of the 1960s and from 1973 to 1975. The more dramatic decreases were in the twenty-first century. After falling for a few years at the beginning of the twenty-first century, household net worth began a steep climb to an all-time high of about $65 trillion dollars in 2007, which was followed by its steepest decline, losing almost $13 trillion. At end of the first decade of the twenty-first century, household net worth had declined to slightly over $50 trillion, and if adjusted for inflation, was close to where it started at the beginning of the century.

**b.** A similar graph appeared in *The New York Times* entitled "A Sudden Decline." Another title could be "A Dramatic Loss after Growth for Years."

## Explore & Extend

### 1.5 Bridget in the Bathtub

Bridget, the 6-year-old daughter of a professor at the University of Pittsburgh, loved playing with her rubber duckie in the bath at night. Her mother drew the accompanying graph for her math class. It shows the water level (measured directly over the drain) in Bridget's tub as a function of time.

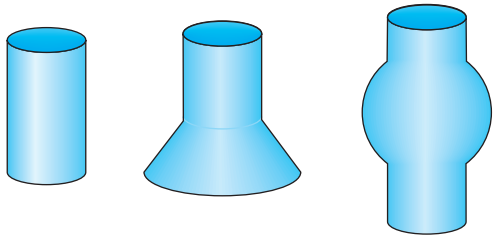


Pick out the time period during which:

- The tub is being filled
- Bridget is entering the tub
- She is playing with her rubber duckie
- She leaves the tub
- The tub is being drained

**Water in Containers: Sketch the Flow**

Assume that water is pouring at a constant rate into each of the containers shown. The height of water in the container is a function of the volume of liquid. Sketch a graph of this function for each container.



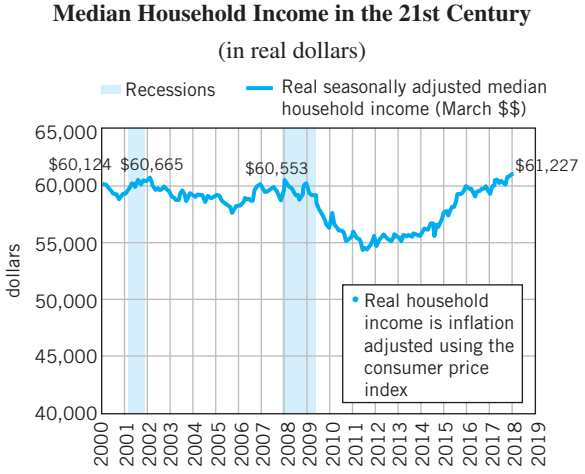
(*a*) (*b*) (*c*)

The rest of this text examines patterns in functions and their graphs. Families of functions—linear, exponential, logarithmic, power, polynomial, quadratic, and rational—provide useful mathematical models for describing the world around us.

## Algebra Aerobics 1.5

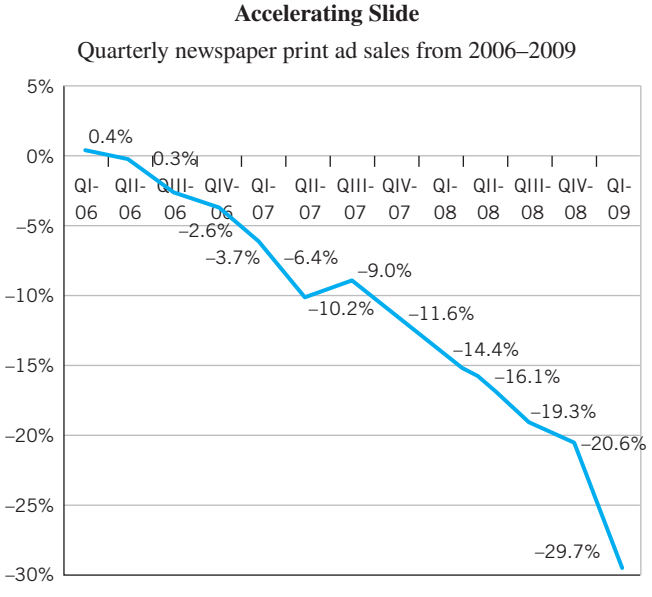
**1.** Create a topic sentence for each of the following graphs for a newspaper article.

**a.**




Source: www.dshort.com

**b.**




Source: Newspaper Association of America.

**2.** Use the accompanying graph showing mean contract mortgage rates, $D(t)$, over time, $t$, to estimate the following:

**a.** Domain: Range:

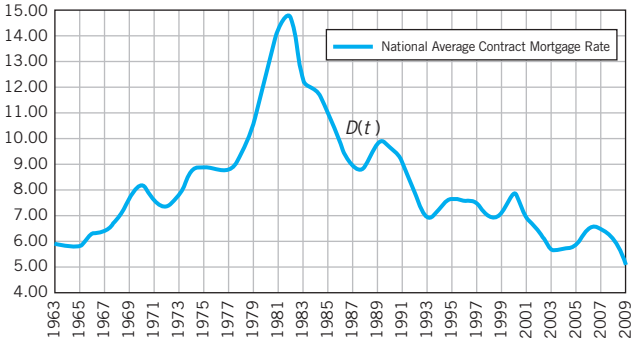
**b.** $D(2008) \approx$

**c.** $t$ if $D(t) \approx 7.00$

**d.** The maximum value for mortgage rates during the time period represented on the graph. In what year does the maximum occur? What are the approximate coordinates at the maximum point?

**e.** The minimum value for mortgage rates. In what year does this occur? What are the coordinates of this point?

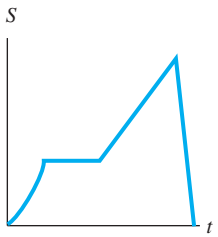
**Mean Contract Mortgage Rates in United States 1963 to 2009**



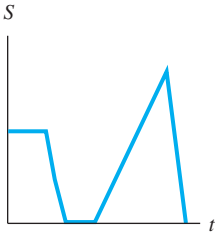

Source: *www.Mortgage-X.com*.
Note: The contract mortgage rate is based on conventional fixed and adjustable rate mortgages for a given time period.

**3.** Choose which of the graphs is the "best" graph to describe the following situation. Speed ($S$) is on the vertical axis and time ($t$) is on the horizontal axis.

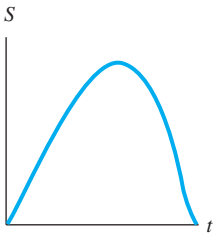
A child in a playground tentatively climbs the steps of a large slide, first at a steady pace, then gradually slowing down until she reaches the top, where she stops to rest before sliding down.



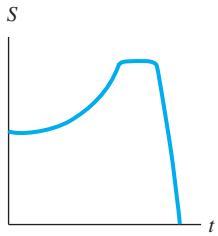

Graph *A*




Graph *B*




Graph *C*




Graph *D*

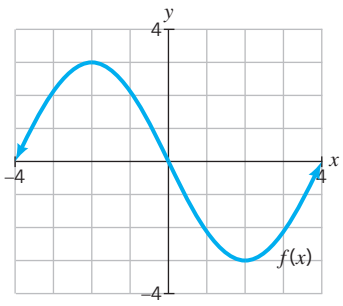
**4.** Generate a rough sketch of the following situation. U.S. AIDS cases increased dramatically, reaching an all-time high for a relatively short period, and then consistently decreased, until a recent small increase.

**5.** Sketch a graph for each of the following characteristics, and then indicate with arrows where the functions are increasing and where they are decreasing.

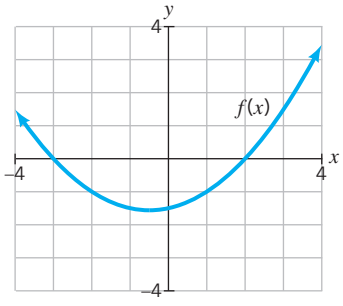
**a.** Concave up with a minimum point at $(-2, 1)$.

**b.** Concave down with a maximum point at $(3, -2)$.

**6.** Estimate the interval for $x$ when $f(x) = 0$, when $f(x) < 0$, and when $f(x) > 0$ for each function in Graphs *A* and *B*.




Graph *A*




Graph *B*

## Exercises for Section **1.5**

Graphing program required for Exercise 27.

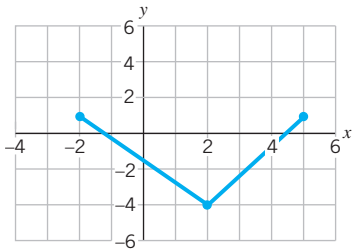
**1.** Identify the graph (*A* or *B*) that

**a.** Increases for $1 < x < 3$

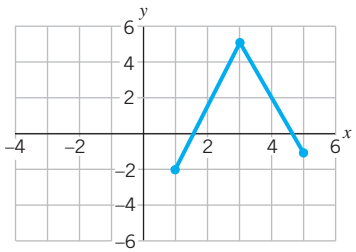
**b.** Increases for $2 < x < 5$

**c.** Decreases for $-2 < x < 2$

**d.** Decreases for $3 < x < 5$



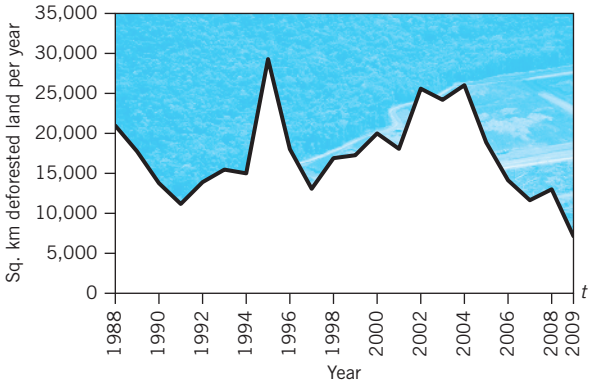

Graph *A*




Graph *B*

**2.** The following graph shows the number of square kilometers of deforestation, $D(t)$, in the Brazilian Amazon during the years, $t$, from 1988 to 2009.

**Deforestation in the Brazilian Amazon, 1988–2009**




Source: ©luoman/iStockphoto

Find the following:

**a.** _____ is a function of _____.

**b.** Input (with units): _____
Output (with units): _____

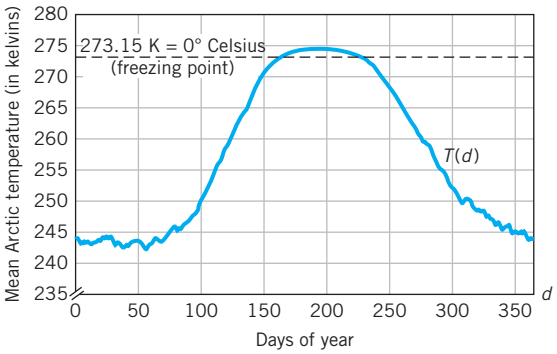
**c.** Estimate the Domain: _____ Range: _____

**d.** Estimate $D(1990) \approx$ $D(2008) \approx$
Estimate $t$ so that $D(t) \approx 22{,}000$

**e.** From the graph, estimate the maximum amount of land deforested in a particular year. In what year did that occur? In what year was the least amount of land deforested?

**3.** The following graph shows the daily mean Arctic temperature: $T(d)$, in kelvins, over $d$ days of the year. (*Note:* Kelvin is a unit of temperature; 273.15 K = 0° Celsius and 1 kelvin = −272.15° Celsius.)

**Mean Arctic Temperature**




Source: Danish Meteorological Institute.

Find the following:

**a.** _____ is a function of _____.

**b.** Input (with units): _____
Output (with units): _____

**c.** Estimate Domain: _____ Range: _____

**d.** Estimate $T(100) \approx$ _____

Estimate the interval for $d$ over which $T(d) > 273.15$ (the melting point of ice).

What are the practical implications of these statements?

**e.** Estimate the maximum and minimum values of $T(d)$.

What are the practical implications of these statements?

**f.** Describe the overall shape of the graph of $T(d)$.

**4.** For each of the following functions,

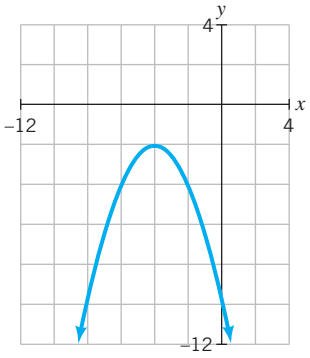
**a.** Over which interval is the function decreasing?

**b.** Over which interval is the function increasing?

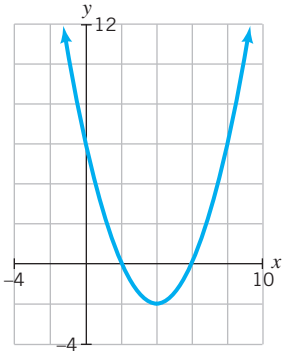
**c.** Does the function appear to have a minimum? If so, where?

**d.** Does the function appear to have a maximum? If so, where?

**e.** Describe the concavity.

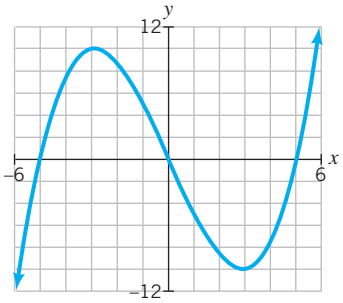



Graph *A*




Graph *B*

**5.** For the following function,




**a.** Over which interval(s) is the function positive?

**b.** Over which interval(s) is the function negative?

**c.** Over which interval(s) is the function decreasing?

**d.** Over which interval(s) is the function increasing?

**e.** Does the function appear to have a minimum? If so, where?

**f.** Does the function appear to have a maximum? If so, where?

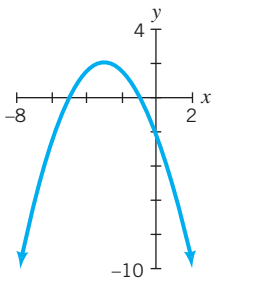
**6.** Choose which graph(s) A, B, C, D, match the description: As $x$ increases, the graph is:

**a.** Increasing and concave up

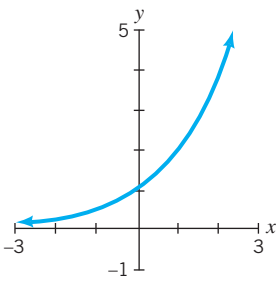
**b.** Increasing and concave down

**c.** Concave up and appears to have a minimum value at (–3, 2)

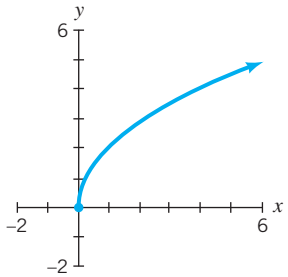
**d.** Concave down and appears to have a maximum value at (–3, 2)



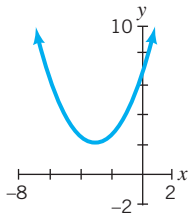

Graph *A*




Graph *C*




Graph *B*




Graph *D*

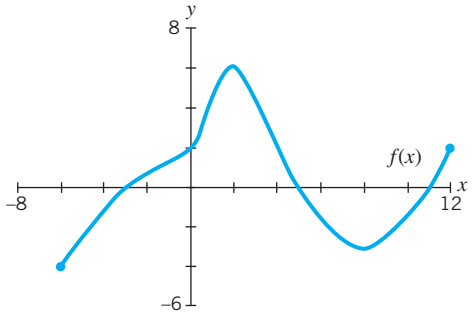
**7.** Examine each of the graphs in Exercise 6. Assume each graph describes a function $f(x)$. The arrows indicate that the graph extends indefinitely in the direction shown.

**a.** For each function estimate the domain and range.

**b.** For each function estimate the $x$ interval(s) where $f(x) > 0$.

**c.** For each function estimate the $x$ interval(s) where $f(x) < 0$.

**8.** Look at the graph of $y = f(x)$ in the accompanying figure.




**a.** Find $f(-6)$, $f(2)$, and $f(12)$.

**b.** Find $f(0)$.

**c.** For what values of $x$ is $f(x) = 0$?

**d.** Is $f(8) > 0$ or is $f(8) < 0$?

**e.** How many times would the line $y = 1$ intersect the graph of $f(x)$?

**f.** What are the domain and range of $f(x)$?

**g.** What is the maximum? The minimum?

**9.** Use the graph of Exercise 8 to answer the following questions about $f(x)$.

**a.** Over which interval(s) is $f(x) < 0$?

**b.** Over which interval(s) is $f(x) > 0$?

**c.** Over which interval(s) is $f(x)$ increasing?

**d.** Over which interval(s) is $f(x)$ decreasing?

**e.** How would you describe the concavity of $f(x)$ over the interval (0, 5) for $x$? Over (5, 8) for $x$?

**f.** Find a value for $x$ when $f(x) = 4$.

**g.** $f(-8) = ?$

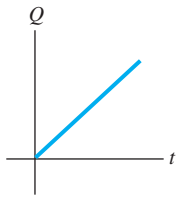
**10.** Match each graph with the best description of the function. Assume that the horizontal axis represents time, $t$.

**i.** The height of a ball thrown straight up is a function of time.

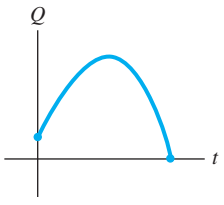
**ii.** The distance a truck travels at a constant speed is a function of time.

**iii.** The number of daylight hours is a function of the day of the year.

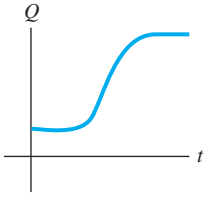
**iv.** The temperature of a pie baking in an oven is a function of time.



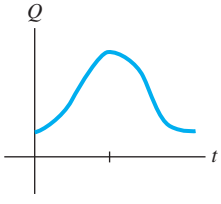

Graph *A*




Graph *C*



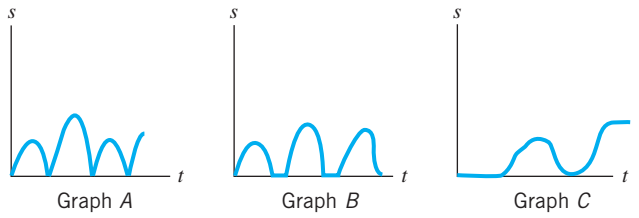

Graph *B*



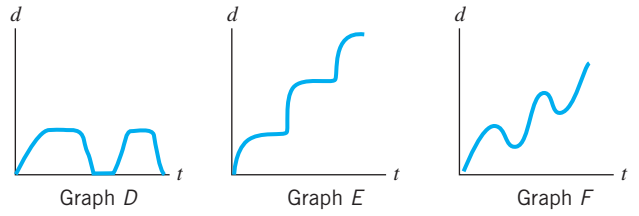

Graph *D*

**11.** Choose the best graph to describe the situation.

**a.** A student in a large urban area takes a local bus whose route ends at the college. Time, $t$, is on the horizontal axis and speed, $s$, is on the vertical axis.

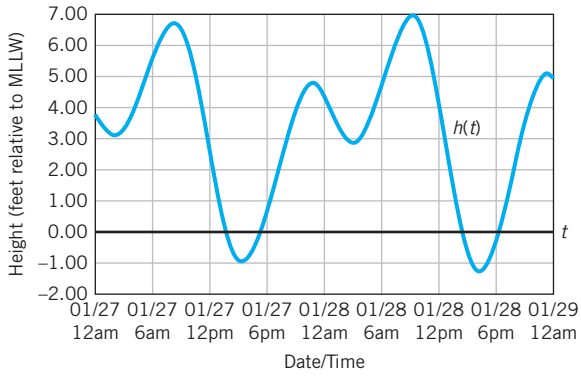



**b.** What graph depicts the total distance the student traveled in the bus? Time, $t$, is on the horizontal axis and distance, $d$, is on the vertical axis.




**12.** The accompanying graph shows the changes in the height of tides in San Francisco Bay, $h(t)$, over time, $t$, between January 27, 2010, and January 29, 2010.

**Height of Tides in San Francisco**




Source: National Oceanic and Atmospheric Association. *www.tidesandcurrents.hoaa.gov*.
Note: MLLW (mean lower low water level) is the mean lowest tide.

Estimate the following:

**a.** The $t$ interval(s) when $h(t) > 0$. Explain what this means about the tide level.

**b.** The $t$ interval(s) when $h(t) < 0$. Explain what this means about the tide level.

**c.** The value(s) of $t$ when $h(t) = 0$.

**d.** One interval when $h(t)$ is decreasing. Explain what this means about the tide level.

**e.** The value of $h(t)$ when $t$ is 6 p.m., 1/27.

**f.** The $t$ interval(s) when the graph is concave down.

**g.** The maximum and minimum values of $h(t)$. Explain these results in terms of tide levels.

**13.** The accompanying graph shows the unemployment rate for those people 16 years and older from January 1999 to January 2010.

**Unemployment Rate in the United States**


Source: Bureau of Labor Statistics, *www.bls.gov*.

**a.** Estimate the maximum and the minimum unemployment rate between January 1999 and January 2010.

**b.** Estimate the time frame when the unemployment rate was experiencing an overall decrease.

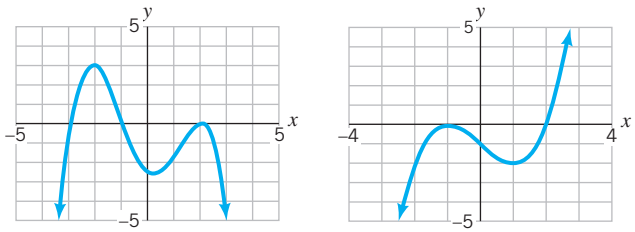
**c.** When does the unemployment rate seem to be rising the fastest?

**d.** Write a 60-second summary about the unemployment rate from January 1999 to January 2010.

**14.** **a.** In the accompanying graphs, estimate the coordinates of the maximum and minimum points (if any) of the function.

**b.** Specify the interval(s) over which each function is increasing.

**c.** Estimate when $f(x) = 0$, $f(x) > 0$, and $f(x) < 0$.



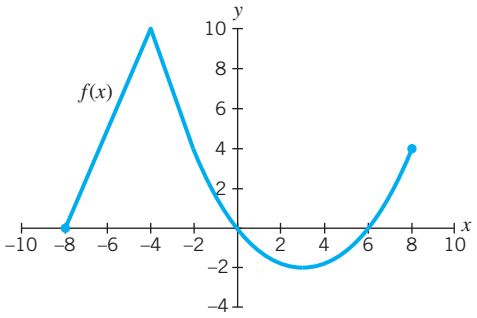

**15.** Use the graph to estimate:

**a.** When $f(x) = 0$, when $f(x) > 0$, and when $f(x) < 0$

**b.** The coordinates of the maximum and minimum for $f(x)$

**c.** When $f(x)$ is increasing and decreasing

**d.** The domain and range of $f(x)$



**16.** Sketch a plausible graph for each of the following and label the axes.

**a.** The amount of snow in your backyard each day from December 1 to March 1.

**b.** The temperature during a 24-hour period in your home town during one day in July.

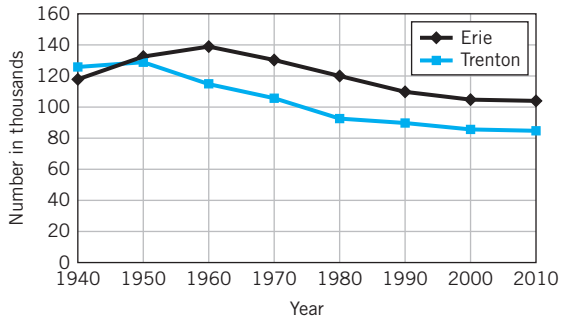
**c.** The amount of water inside your fishing boat if your boat leaks a little and your fishing partner bails out water every once in a while.

**d.** The total hours of daylight each day of the year.

**e.** The temperature of an ice-cold drink left to stand.

**17.** Examine the accompanying graph, which shows the populations of two towns.

**Population Trends: Erie, PA, and Trenton, NJ**




Source: *www.demographics.com.*

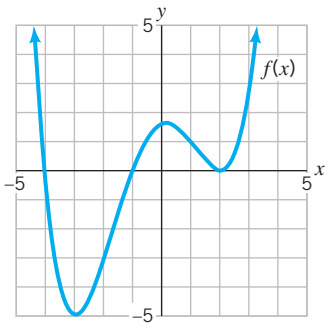
**a.** What is the range of population size for Erie? For Trenton?

**b.** During what years did the population of Erie increase?

**c.** During what years are both populations relatively stable?

**d.** When were the populations approximately equal?

**18.** The graph of the following function $f(x)$ has a domain of $(-\infty, +\infty)$.




**a.** When is $f(x) = 0$?

**b.** Over what $x$ interval(s) is $f(x) > 0$?

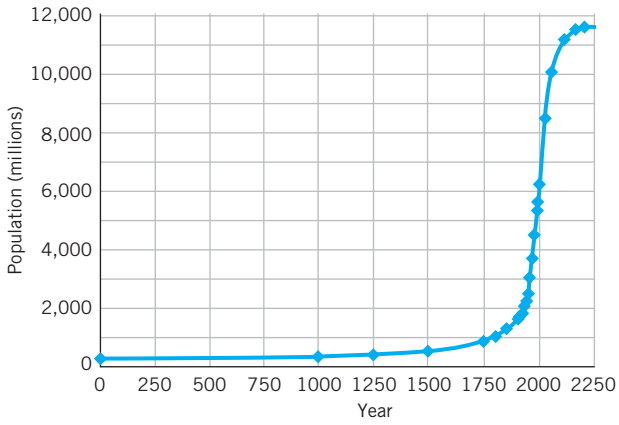
**c.** Over what $x$ interval(s) is $f(x) < 0$?

**d.** Estimate the maximum and minimum value for $f(x)$ over the domain of the function.

**e.** Over what $x$ interval(s) is $f(x)$ concave up and increasing? Concave up and decreasing?

**19.** The accompanying graph shows the world population over time and future predictions.

**Historical World Population with Predictions**




Source: World Online.

**a.** Over what interval does the world population show dramatic growth?

**b.** Does the dramatic growth slow down?

**c.** Write a topic sentence for a report for the United Nations.

**20.** Make a graph showing what you expect would be the relative ups and downs throughout the year of sales (in dollars) of

**a.** Turkeys

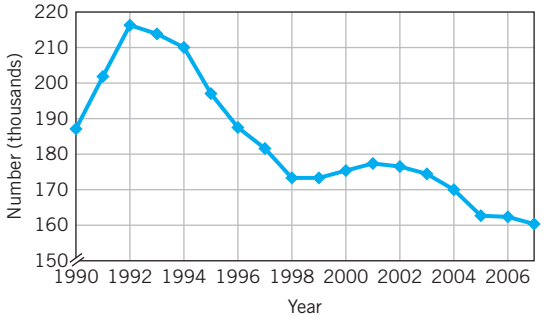
**b.** Candy

**c.** Bathing suits in your state

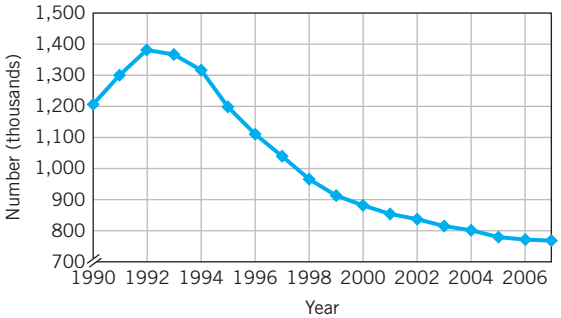
**d.** Textbooks at your school bookstore

**21.** Examine the graphs of military reserve enlisted personnel over the years 1990 to 2007, then answer the following questions.

**Female Enlisted Reserve Personnel**




**Male Enlisted Reserve Personnel**




Source: U.S. Dept. of Defense, *Official Guard and Reserve Manpower Strengths and Statistics*, annual.

**21.** (continued)

**a.** In what year(s) did female and male enlisted personnel reach a maximum?

**b.** What was the maximum and minimum for both men and women over the time interval [1990, 2007]?

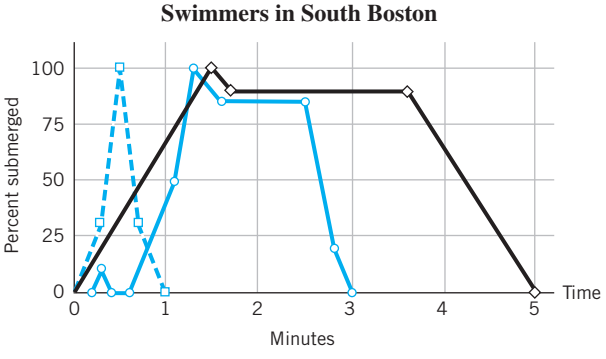
**c.** Describe the trends in number of female and male reserve enlisted personnel.

**d.** Given the current state of world affairs, what would you expect would happen to the enlistment numbers for 2007 to the present?

**22.** Generate a rough sketch of a graph of internal pressure vs. time for the following situation: When a soda is removed from the fridge, the internal pressure is slightly above the surrounding air pressure. With the can unopened, the internal pressure soon more than doubles, stabilizing at a level three times the surrounding air pressure.

**23.** A student breaks her ankle and is taken to a doctor, who puts a cast on her leg and tells her to keep the foot off the ground altogether. After 2 weeks she is given crutches and can begin to walk around more freely, but at the beginning of the third week she falls and is resigned to keeping stationary again for a while. After 6 weeks from her first fall, she is given a walking cast in which she can begin to put her foot on the ground again. She is now able to limp around using crutches. Her walking speed slowly progresses. At 12 weeks she hits a plateau and, seeing no increase in her mobility, starts physical therapy. She rapidly improves. At 16 weeks the cast is removed and she can walk freely. Make a graph of the student's mobility level during her recovery.

**24.** Every January 1 a hardy group called the L Street Brownies celebrates the New Year by going for a swim at the L Street Beach in South Boston. The water is always very cold, and swimmers adopt a variety of strategies for getting into it. The graph shows the progress of three different friends who join in the event, with percentage of body submerged on the vertical axis and time on the horizontal axis. Match the graphs to the descriptions of how each of the friends manages to get completely submerged in the icy ocean.




**a.** Ali has done this before and confidently walks in until his head is underwater; then he puts his head out and swims around a few minutes; then he walks out.

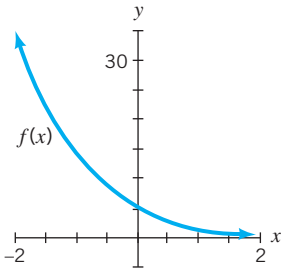
**b.** Ben dashes in until the water is up to his knee, trips over a hidden rock, and falls in completely; he stands up and, since he is now totally wet, runs back out of the water.

**c.** Cat puts one foot in, takes it out again, and shivers. She makes up her mind to get it over with, runs until she is up to her waist, dives in, swims back as close to the water's edge as she can get, stands up, and steps out of the water.

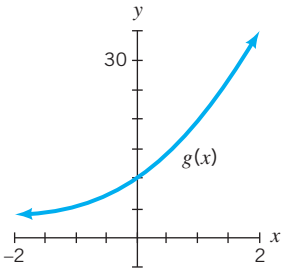
**25.** Examine the graphs of the functions $y = f(x)$ and $y = g(x)$. Complete the following:

**a.** As $x$ approaches $+\infty$, $f(x)$ approaches _____.
As $x$ approaches $-\infty$, $f(x)$ approaches _____.

**b.** As $x$ approaches $+\infty$, $g(x)$ approaches _____.
As $x$ approaches $-\infty$, $g(x)$ approaches _____.

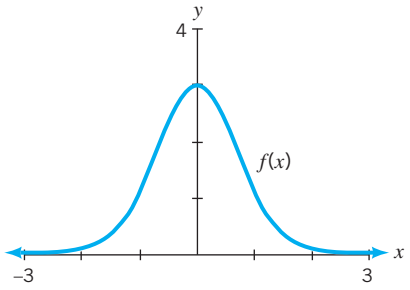



Graph *A*




Graph *B*

**26.** Describe the behavior of $f(x)$ in the accompanying figure over the interval $(-\infty, +\infty)$ for $x$, using such words as "increases," "decreases," "concavity," "maximum/minimum," and "approaches infinity."




**27.** (Graphing program required.) This exercise is to be done with a partner. Name the partners person #1 and person #2.

**a.** Person #1, using technology, graphs the function $f(x) = 0.5(x - 3)(x + 2)^2$, but does not show the graph to person #2.

**b.** Person #1 describes to person #2 the behavior of the graph of $f(x)$ so that he/she can sketch it on a piece of paper.

**c.** Switch roles; now person #2, using technology, graphs $g(x) = -0.5(x - 3)(x + 2)^2$, but does not show the graph to person #1.

**d.** Person #2 describes to person #1 the behavior of the graph of $g(x)$ so that he/she can sketch it on a piece of paper.

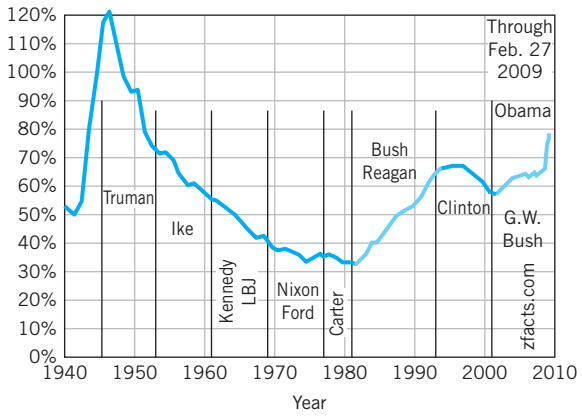
**e.** Compare the accuracy of the graphs and compare the shapes of the two graphs. What do $f(x)$ and $g(x)$ have in common? How do they differ?

**28.** In Section 1.2 we examined the annual federal budget *surplus* or *deficit*. The federal *debt* takes into account the cumulative effect of all the deficits and surpluses for each year together with any interest or payback of principal. The accompanying graph shows the accumulated gross federal debt as a percent of the gross domestic product from 1940 to 2010.

The gross domestic product (GDP) is a measure of the total output of goods and services produced in a given country.

**The National Debt as a Percent of Gross Domestic Product**

(Data through 2007 is from Bush's whitehouse.gov)




Source: *www.zfacts.com*.

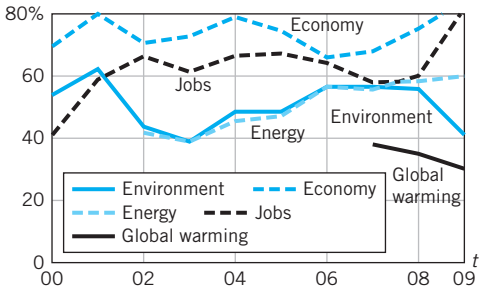
**a.** What are three interesting facts that emerge from this graph?

**b.** Construct a 60-second summary that could accompany the graph.

**29.** The accompanying graph shows the issues that people in the U.S. rated as "top priority" from 2000 to 2009. Let $f(t)$ represent the percent choosing the environment as a top priority where $t$ = year.

**Environment Less of a Priority**

*Percent rating the issue a "top priority"*




Source: Pew Research Center.

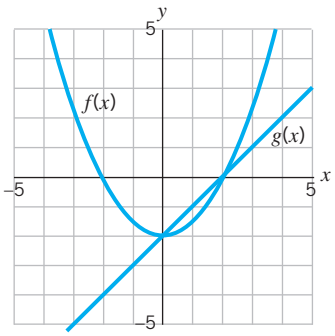
**a.** For what intervals of $t$ did $f(t)$ increase? Decrease? Explain the implications of your answers.

**b.** Estimate the maximum values of $f(t)$ and minimum values of $f(t)$. Explain the implications of your answers.

**c.** Construct a 60-second summary that could accompany this graph.

**30.** For the functions $f(x)$ and $g(x)$ shown on the accompanying graph, find the values of $x$ that make the following true.

**a.** $f(x) = 0$ **b.** $g(x) = 0$ **c.** $f(x) = g(x)$



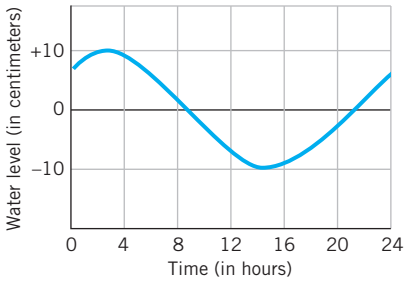

# Chapter Summary

## Single-Variable Data

Single-variable data can be represented with graphs such as bar charts and histograms and with numerical descriptors such as mean and median.

## Two-Variable Data

Graphs of two-variable data show how change in one variable affects change in the other. The accompanying graph is called a *time series* because it shows changes over time.




## Equations in Two Variables

The *solutions of an equation* in two variables $x$ and $y$ are the ordered pairs $(x, y)$ that make the equation a true statement. For example, $(3, -5)$ is one solution of the equation $2x + y = 1$.

The *graph of an equation* in two variables displays the set of points that are solutions to the equation.

## Functions

A variable $y$ is a *function* of a variable $x$ if each value of $x$ determines a unique (one and only one) value of $y$. Functions can be represented with words, graphs, equations, and tables.

When a set of ordered pairs represents a function, then each ordered pair is written in the form

(independent variable, dependent variable)
(input, output)
$(x, y)$

By convention, on the graph of a function, the input (or independent variable) is represented on the horizontal axis and the output (or dependent variable) on the vertical axis.

The *domain* of a function is the set of all possible values of the input.

The *range* is the set of corresponding values of the output.

A graph does not represent a function if it fails the *vertical line test*. If you can draw a vertical line that crosses the graph two or more times, the graph does not represent a function.

## Function Notation

The expression $f(x)$ means the rule $f$ is applied to the input value $x$ to give the output value, $f(x)$:

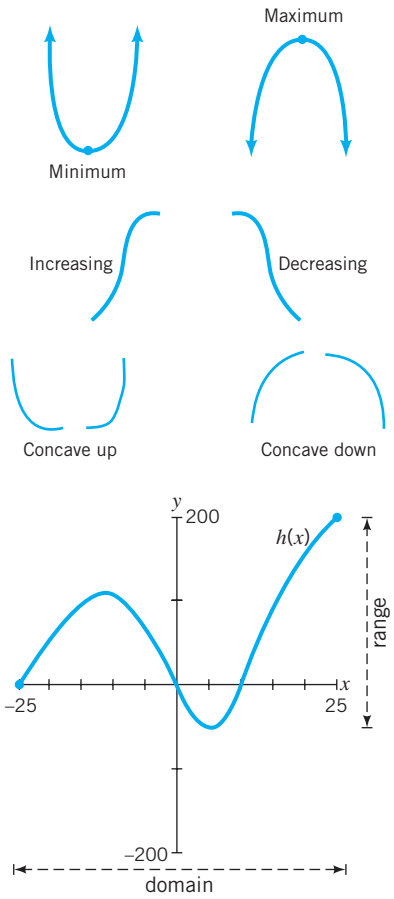
$$f(\text{input}) = \text{output}$$

For example, $f(x) = 2x + 5$ tells us $f$ is the name of a function where the input is $x$, the output is $f(x)$, and the rule is to multiply the input by 2 and add 5. To evaluate this function when the input is 4, we write

$$f(4) = (2 \cdot 4) + 5 \Rightarrow f(4) = 13$$

## Visualizing Functions

Graphs of functions may demonstrate the following properties:




# Check Your Understanding

**I. Is each of the statements in Problems 1–23 true or false?**

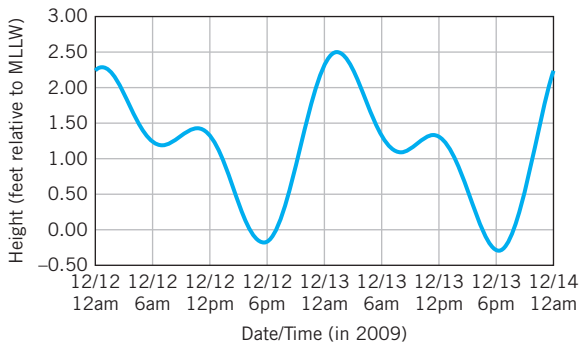
**1.** Histograms and bar charts are used to graph single-variable data.

**2.** Means and medians, both measures of central tendency, can be used interchangeably.

**3.** A scatter plot is a plot of data points $(x, y)$ for two-variable data.

Problems 4 and 5 refer to the accompanying figure.

**Height of Tides in Kahului, Hawaii**



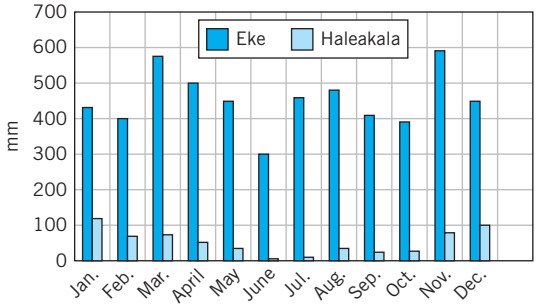

Source: NOAA, Tides and Currents. *http://tidesandcurrents.noaa.gov*.
Note: MLLW (mean lower low tide) is the average lowest tide.

**4.** Tides are the rise and fall of the sea level. The graph shows the changes in the height of the tide in Kahului, Hawaii, on December 12 and 13, 2009.

**5.** The maximum height of the tide, called high tide, occurred at about 1 a.m. on December 13, 2009.

**6.** The accompanying figure of the annual rainfall cycles at the Eke and Haleakala weather stations on Maui Island, Hawaii, show that the least rainfall at both stations occurs in June.

**Annual Rainfall Cycles, at Two Sites on Maui Island, Hawaii**




Source: Rainfall Atlas of Hawaii. *Water Resources Research Center*, University of Hawaii, Manoa.

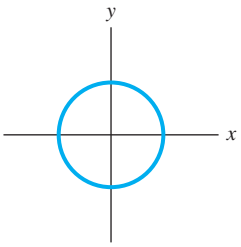
**7.** The independent variable of a function is also called the output of the function.

**8.** The graph of the equation describing the sales revenue $R$ ($ million) as a function of the amount spent on advertising $A$ ($ thousand) is the set of ordered pairs $(R, A)$ that satisfy the equation.

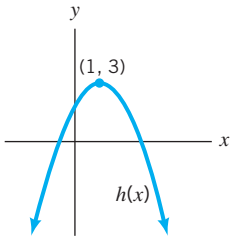
**9.** If sales revenue $R$ is a function of advertising $A$, then $R$ is the dependent variable and $A$ is the independent variable.

**10.** If $M = F(q)$, then $F$ is a function of $q$.

**11.** In the accompanying graph $y$ is a function of $x$.




Problems 12–14 refer to the accompanying graph of $h(x)$.




**12.** The function $h(x)$ has a maximum value at $(1, 3)$.

**13.** The function $h(x)$ is concave down.

**14.** The function $h(x)$ increases to the right of $x = 1$ and increases to the left of $x = 1$.

Problems 15 and 16 refer to the following table.

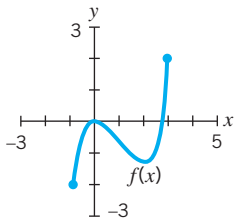
| $R$ | −1 | 2 | 0 | 3 |
|---|---|---|---|---|
| $S$ | 10 | 8 | 6 | 4 |

**15.** The variable $S$ is a function of $R$.

**16.** The variable $S$ is decreasing over the domain for $R$.

**17.** A set of ordered pairs of the form $(M, C)$ could represent $C$ as a function of $M$.

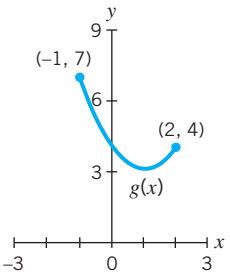
Problems 18 and 19 refer to the accompanying graph of $f(x)$.




**18.** The function $f(x)$ has both a maximum and a minimum over the interval $[-1, 3]$.

**19.** The function $f(x)$ decreases over the interval $(0, 2)$.

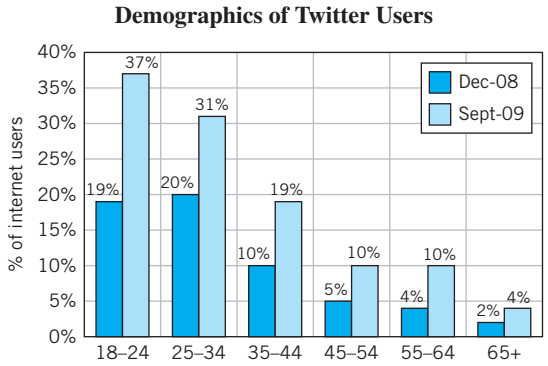
Problems 20 and 21 refer to the accompanying figure.




**20.** The domain of the function $g(x)$ is the interval $[-1, 2]$.

**21.** The range of the function $g(x)$ is the interval $[4, 7]$.

Problems 22 and 23 refer to the accompanying figure.




Source: Pew Research Center.

**22.** The histogram describes the number of Twitter users by age.

**23.** Internet users who are 18 to 34 years old are more likely to use Twitter than older users.

**II. In Problems 24–29, give an example as specified.**

**24.** A relationship between two variables $w$ and $z$ described as a table where $w$ is a function of $z$ but $z$ is not a function of $w$.

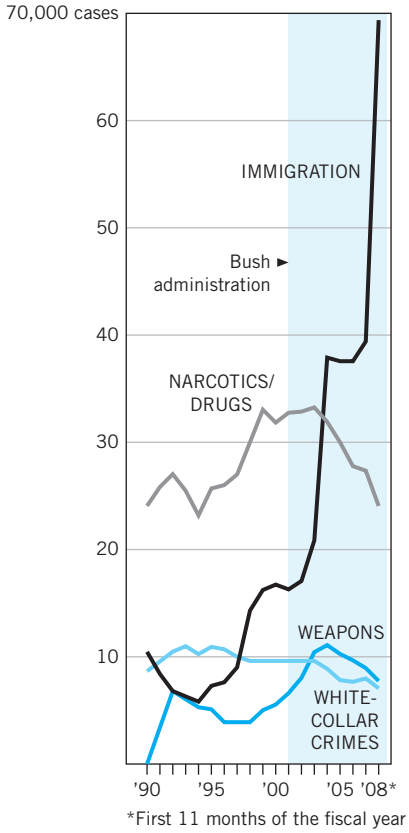
**25.** A relationship between two variables $w$ and $z$ represented by a graph where $w$ is a function of $z$.

**26.** A graph of a function that is concave up and has a minimum value at the point $(-2, 0)$.

**27.** A graph of a function where the domain is the set of real numbers and that has no maximum or minimum values.

**28.** A graph of a function for which $f(-2) = 0$.

**29.** A topic sentence describing federal prosecution of crimes as shown in the following graph.




Source: Transactional Records Access Clearinghouse, Syracuse University.

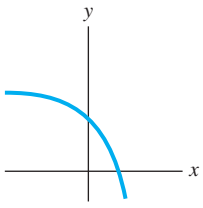
**III. Is each of the statements in Problems 30–40 true or false? If a statement is true, explain how you know. If a statement is false, give a counterexample or explain why it is false.**

**30.** A function can have either a maximum or a minimum but not both.

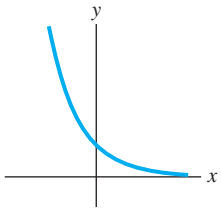
**31.** A function is any relationship between two quantities.

**32.** If $q > 0$, then $f(q) > 0$.

**33.** The graph in the accompanying figure is decreasing and concave down.



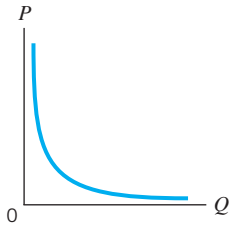

**34.** The graph in the accompanying figure is increasing and concave up.




**35.** All functions have at least one minimum value.

**36.** Sometimes the choice as to which variable will be the independent variable for a function is arbitrary.

**37.** The graph in the accompanying figure represents $P$ as a function of $Q$.




Problems 38 to 40 refer to the accompanying table.

**Hybrid Bicycle Sales by Year 2006–2008 as Percentage of Total Bicycle Sales**

| Year | 2006 | 2007 | 2008 |
|---|---|---|---|
| Percent of Sales (%) | 15.0 | 16.5 | 19.0 |

Source: U.S. Department of Commerce.
Note: A hybrid bike blends the best characteristics of both road and mountain bikes.

**38.** If hybrid bicycle sales are a function of time, the input is time in years and the output is the sales of hybrid bicycles as a percent.

**39.** If percent of hybrid sales, $S$, is represented by the function $S = f(t)$ where $t$ is time in years, then $f(19.0) = 2008$.

**40.** Hybrid bicycle sales, in percent of total bicycle sales, increased from 2006 to 2008.

# Chapter 1 Review: Putting It All Together

1. A man weighed 160 pounds at age 20. Now, at age 60, he weighs 200 pounds.
   - **a.** What percent of his age 20 weight is his weight gain?
   - **b.** If he went on a diet and got back to 160 pounds, what percent of his age 60 weight would be his weight loss?
   - **c.** Explain why the weight gain is a different percentage than the weight loss even though it is the same number of pounds.
2. Data from the *World Health Organization Report* 2009 shows vast differences in average spending per person on health for different countries. Data on health spending for selected countries is shown in the following table.

**Health Care Spending in 2007 Selected Countries**

| Country | As % of Gross Domestic Product (GDP) | Expenditure/Capita (in U.S. dollars) |
|---|---|---|
| China | 4.5 | 112.00 |
| India | 3.6 | 35.00 |
| Democratic People's Republic of Korea (North) | 3.5 | 0.30 |
| Norway | 8.9 | 7,376.00 |
| United States | 15.8 | 7,287.00 |

Source: *Health expenditure series*. World Health Organization, February 2009.

   - **a.** China had the largest population in the world in 2007 with 1.3 billion people. Estimate how much China spent on health care (in U.S. dollars) in 2007.
   - **b.** India and North Korea spent close to the same percentage of their gross domestic product on health care, but a very different amount per person (per capita) in 2007. How can this be?
   - **c.** Norway and the United States spent close to the same amount per person but very different percentages of their gross domestic product on health care. How can this be?
   - **d.** If Norway spent about the same percent of its GDP on health care as the United States, about how much would Norway be spending per person?
3. The *Sacramento Bee* newspaper had the following statement on January 20, 2010:

   > The average teacher salary last year was $66,995, an increase of 1.8 percent from 2008, according to new state figures. Districts laying off less-experienced, lower-paid teachers accounted for almost all of that increase—cumulative school payroll in California was flat from 2008 to 2009. Teacher pay varies widely by district.

   Explain how laying off lower-paid teachers could account for an increase in average salaries. Do you think the "average" used in the statement was the mean or the median? Explain your answer.
4. In 2006 there were 143,648 thousand metric tons of commercial fish caught worldwide. The chart lists the world's 10 leading commercial fishing nations.

**The 10 Leading Commercial Fishing Nations in 2006**

| | Fish Caught (in thousands of metric tons) |
|---|---|
| China | 51,521 |
| Peru | 7,046 |
| India | 6,979 |
| Indonesia | 6,052 |
| United States | 5,325 |
| Chile | 4,971 |
| Japan | 4,921 |
| Thailand | 4,162 |
| Vietnam | 3,618 |
| Russia | 3,390 |

Source: U.S. Bureau of Census, International Statistics, www.census.gov.

   - **a.** What are the mean and median for this set of data?
   - **b.** China caught what percent of the commercial fish among the leading 10 nations? Of the world?
   - **c.** Will the mean and median for the 10 leading nations change if China substantially increases the amount of commercial fish it catches while other nations remain at the same level?
5. The following table shows the five leading causes of death around the world in 2007.

**The Five Leading Causes of Death Worldwide**

| Cause | % of All Deaths |
|---|---|
| Heart disease | 12.2 |
| Stroke | 9.7 |
| Lower respiratory infections (e.g., pneumonia, emphysema, bronchitis) | 7.1 |
| Chronic Obstructive Pulmonary Disease | 5.1 |
| Diarrhoeal diseases | 3.7 |

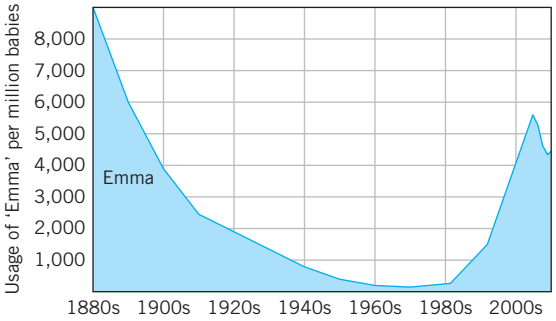
Source: U.S. Bureau of Census, International Statistics, www.census.gov.

   - **a.** What was the leading cause of death worldwide in 2007?
   - **b.** Create a bar chart using these data.
   - **c.** What percentage of worldwide deaths is not accounted for in the table? List at least two other diseases or conditions that can cause death. Could either of them account for more that 5% of all deaths worldwide?
6. Given $g(x) = 2x^2 + 3x - 1$, evaluate $g(0)$ and $g(-1)$.
7. In a certain state, car buyers pay an excise tax of 2.5%. This means that someone who buys a car must pay the state a one-time fee of 2.5% of the car's value.

   Use $P$ to represent the price of the car and $E$ to represent excise tax, then express $E$ as a function of $P$.

**8.** A waitress makes $2.50 an hour in wages and receives tips totaling 18% of the price of each meal she serves. Let $H$ represent the hours she works, $P$ represent the total cost of the meals she served, and $E$ represent what she earns in a week before taxes. Write an equation expressing $E$ in terms of $H$ and $P$.

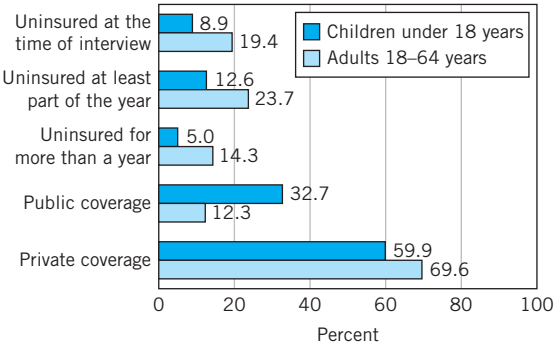
**9.** The following graph shows the number of babies per million babies that were named Emma in the United States between 1880 and 2008. Create a title for the graph and explain in a few sentences what is happening over time to the name Emma.




Source: *www.babynamewizard.com*.

**10.** The graph shows, by age group, the percentage of persons with and without health insurance in the United States in 2007.

**Health Insurance in the United States, 2007**




Source: COC/NCHS/National Health Interview Surveys, 2007.

**a.** Compare the percentages of children and adults aged 18–64 who were covered by a public insurance plan in 2007.

**b.** Is the following statement true or false? Justify your answer. "In 2007, children were less likely than adults aged 18–64 to be uninsured for more than one year."

**c.** Write a topic sentence for a newspaper article about health insurance in the United States in 2007.

**11. a.** Determine whether each set of points represents a function.

**i.** (6, −1), (5, −1), (8, 3), (−3, 2)

**ii.** (5, 3), (7, −1), (5, −2), (4, −3)

**iii.** (7, −3), (8, −3), (−2, −3), (9, −3)

**iv.** (4, 0), (4, −3), (4, 7), (4 −1)

**b.** Create a table of values for the variables $A$ and $B$, where $B$ is a function of $A$ but $A$ is not a function of $B$.

**c.** Create two graphs, one that represents a function and one that does not represent a function.

**12.** Find an equation that represents the relationship between $x$ and $y$ in each of the following tables. Specify in each case if $y$ is a function of $x$.

**a.**

| $x$ | $y$ |
|---|---|
| 0 | 2 |
| 1 | 4 |
| 2 | 6 |
| 3 | 8 |
| 4 | 10 |

**b.**

| $x$ | $y$ |
|---|---|
| 0 | −1 |
| 1 | 0 |
| 2 | 3 |
| 3 | 8 |
| 4 | 15 |

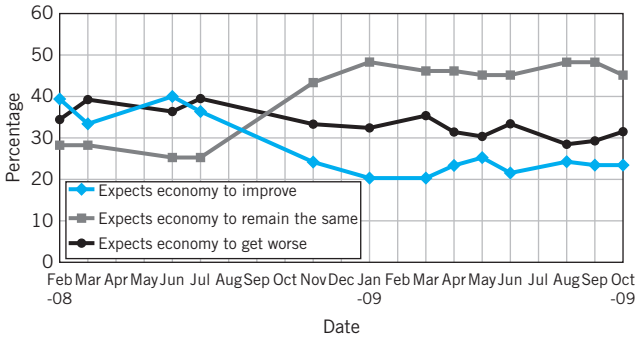
**c.**

| $x$ | $y$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 9 |
| 3 | 28 |
| 4 | 65 |

**13.** The following graph shows changes in U.S. opinion regarding expectations about the economy.

**a.** At what date was the percentage of people who expected the economy to improve approximately the same as those who expected the economy to get worse? Estimate that percentage.

**Poll Responses in 2008 and 2009 to "What do You Expect the Economy to be in 6 Months?"**




Source: *www.HarrisInteractive.com*.

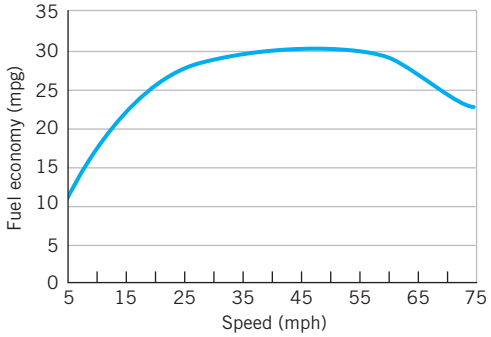
**b.** What type of reversal in opinion happened over the time period covered by the graph?

**c.** Write a topic sentence for a newspaper article about the shift in opinion over this time period.

**14.** The following graph shows fuel economy in relation to speed.

**a.** Does the graph represent a function? If not, why not? If so, what are the domain and range?

**b.** Describe in words how speed relates to fuel economy.




(mpg = miles per gallon of gas, mph = miles per hour)
Source: *www.fueleconomy.gov*.

**15.** In January 2010 the exchange rate for the Chinese yuan and U.S. dollar was about 7 to 1; that is, you could exchange 7 yuan for 1 U.S. dollar.

**15.** (continued)

**a.** Let $Y$ = number of Chinese yuan and $D$ = number of U.S. dollars. Complete the following table.

| $D$ | 1 | 2 | 3 | 4 | 20 | 100 |
|---|---|---|---|---|---|---|
| $Y$ | | | | | | |

**b.** Find an equation that expresses $Y$ as a function of $D$. What is the independent variable? The dependent variable?

**c.** Complete the following table:

| $Y$ | 7 | 14 | 21 | 35 | 70 | 700 |
|---|---|---|---|---|---|---|
| $D$ | | | | | | |

**d.** Find an equation that expresses $D$ as a function of $Y$. What is the independent variable? The dependent variable?

**16.** The average number of calories burned with exercising varies by the weight of the person. The following table gives the approximate calories burned per hour from disco dancing.

**Calories Burned per Hour from Disco Dancing**

| Weight | 100 lb. person | 125 lb. person | 150 lb. person | 175 lb. person | 200 lb. person |
|---|---|---|---|---|---|
| calories burned per hour | 264 | 330 | 396 | 462 | 528 |

Source: *http://www.fitresource.com/Fitness.*

Let $t$ be the number of hours spent exercising and $C$ the total number of calories burned.

**a.** Write an equation expressing $C_1$ in terms of $t$ for a 125 lb. person disco dancing and an equation for $C_2$ for a 175 lb. person disco dancing.

**b.** Do your equations in part (a) represent functions? If not, why not?

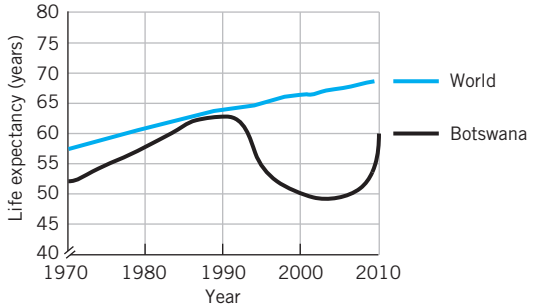
**c.** For the equations in part (a) that represent functions, what is the independent variable? The dependent variable? What is a reasonable domain? What is a reasonable range?

**d.** Generate small tables of values and graphs for $C_1$ and $C_2$. How do the two graphs compare?

**e.** How do you think these graphs would compare to the corresponding graphs for a 200 lb. person? Explain.

**17.** The following graph shows the life expectancy in Botswana compared with world life expectancy.

**Life Expectancy, World and Botswana, 1970–2010**




Source: World Bank, 2010.

**a.** Does the graph for Botswana represent a function? If not, why not? If so, what are the domain and range?

**b.** Over the given time period, when did life expectancy in Botswana reach its maximum? Estimate this maximum value.

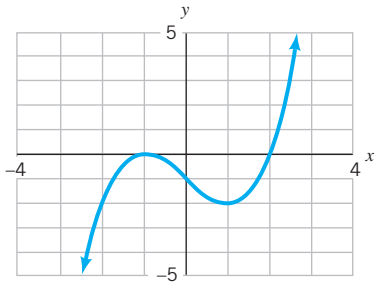
**c.** Over the given time period, when did life expectancy in Botswana reach its minimum? Estimate this minimum value. What might have caused the sudden drop in life expectancy?

**d.** Over what interval(s) was life expectancy in Botswana increasing? Decreasing?

**e.** Over what interval(s) is Botswana's graph concave down? Concave up?

**f.** Write a short description of life expectancy in Botswana compared with world life expectancy.

**18.** Here is the graph of $f(x)$.




**a.** Determine the values for $f(-1), f(0), f(2)$.

**b.** For what value(s) of $x$ does $f(x) = -2$?

**c.** Specify the interval(s) over which the function is

**i.** Increasing

**ii.** Decreasing

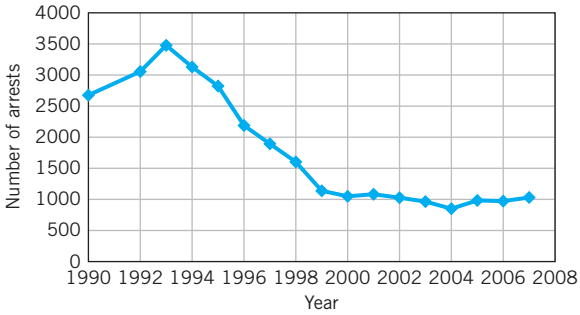
**iii.** Concave up

**iv.** Concave down

**v.** $f(x) = 0, f(x) > 0, f(x) < 0$.

**19.** Here is a graph of data collected by the FBI.

**Juvenile Arrests for Murder, 1990 to 2007**




Source: U.S. Bureau of the Census, *Statistical Abstract of the United States: 2009.*

If we let $N(x)$ = number of juvenile arrests for murder in year $x$,

**a.** Estimate $N(1993)$. What are the coordinates of this point? What does this point represent?

**b.** Estimate the coordinates of the minimum point. What does this point represent?

**c.** Over what interval is the function increasing? Over what interval is the function decreasing?

**d.** What is the domain of $N(x)$? What is the range?

**e.** Write a brief summary about juvenile arrests between 1990 and 2007.

**20.** From the graph of $y = f(x)$, estimate the following:

**a.** Domain and range of $f(x)$

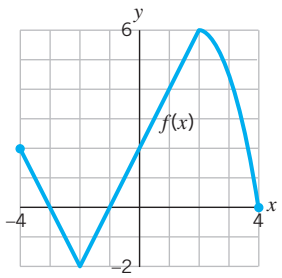
**b.** When $f(x) = 0$

**c.** Interval(s) where $f(x) > 0$

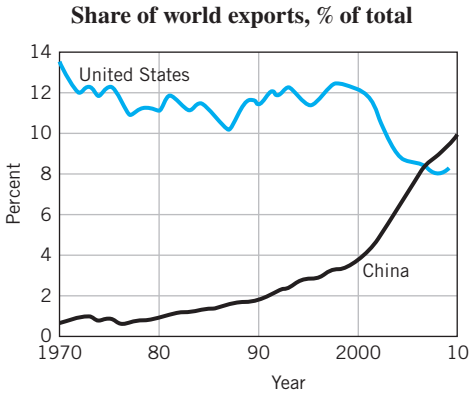
**d.** Interval(s) where $f(x) < 0$

**e.** Interval(s) where $f(x)$ is increasing

**f.** Interval(s) where $f(x)$ is decreasing




**21.** The accompanying graph shows the percent of total world exports for the United States and for China from 1970 to 2010. Write a 60-second summary comparing the difference between China and the United States in the share of total world exports for these countries between 1970 and 2010.



# EXPLORATION 1.1

## Collecting, Representing, and Analyzing Data

### Objectives

- explore issues related to collecting and analyzing data.
- learn techniques for organizing and graphing data using technology.
- describe and analyze single-variable data using histograms and numerical descriptors.
- describe and analyze two-variable data using scatter plots and line graphs.

### Material/Equipment

- class questionnaire (See Appendix B)
- measuring tapes in inches
- optional measuring devices: small eye chart, flexibility tester, measuring device for blood pressure
- computer with spreadsheet program or graphing calculator with statistical plotting features
- data from class questionnaire or other small data set formatted either as spreadsheet or Graph Link file
- overhead projector or projection panel for computer or graphing calculator
- transparencies for printing or drawing graphs for overhead projector (optional)

### Related Readings

(On the web at *www.wiley.com/college/kimeclark*)
"U.S. Government Definitions of Census Terms"
"Health Measurements"


### Related Software

"F1: Histograms," in *FAM 1000 Census Graphs*


### Procedure

This exploration may take two class periods. It can easily be modified to meet time constraints.

### Day One

*In a Small Group or with a Partner*

1. Pick (or your instructor will assign you) one of the undefined variables on the questionnaire. Spend about 15 minutes coming up with a workable definition and a strategy for measuring that variable. Be sure there is a way in which a number or single letter can be used to record each individual's response on the questionnaire.
2. Consult the reading "Health Measurements" if you decide to collect health data.

*Class Discussion*

After all of the definitions are recorded on the board, discuss your definition with the class. Is it clear? Does everyone in the class fall into one of the categories of your definition? Can anyone think of someone who might not fit into any of the categories? Modify the definition until all can agree on some wording. As a class, decide on the final version of the questionnaire and record it in your class notebook.

*In a Small Group or with a Partner*

Help each other when necessary to take measurements and fill out the entire questionnaire. Questionnaires remain anonymous, and you can leave blank any question you can't or don't want to answer. Hand in your questionnaire to your instructor by the end of class.

## Exploration-Linked Homework

Read "U.S. Government Definitions of Census Terms" for a glimpse into the federal government's definitions of the variables you defined in class. How do the "class" definitions and the "official" ones differ?

## Day Two

*Class Demonstration*

1. You'll need an electronic version of the data set from which to choose one variable for the whole class to study (e.g., age from the class data). If you haven't used a spreadsheet or graphing calculator before, you'll need a basic technical introduction. (*Note:* If you are using a TI-83 or TI-84 graphing calculator, there are basic instructions in the Graphing Calculator Manual on *www.wiley.com/college/kimeclark*.)
2. Select an interval size and then construct a frequency histogram and a relative frequency histogram. Construct another histogram using a different interval size. Are any new patterns revealed?
3. Calculate the mean and median using spreadsheet or graphing calculator functions.

*In a Small Group or with a Partner*

Choose another variable from your data. Pick an interval size, and then generate both a frequency histogram and a relative frequency histogram. Calculate the mean and median. If possible, make copies of the histograms for both your partner and yourself.

## Discussion/Analysis

With your partner(s), analyze and jot down patterns that emerge from the data. How could you describe your results? What are some limitations of the data? What other questions are raised and how might they be resolved? Prepare a summary to give to the class. If possible, use an overhead projector with a transparency of your histogram or a projector linked to your graphing calculator. If not, use a paper copy of your histogram.

## Exploration-Linked Homework

Read Section 1.2. For this assignment you can either create a two-variable data comparison from your class data or find published data and graphs to analyze.

*Creating a two-variable data comparison:*

1. Examine the list of categories from the class survey. Choose two variables that may be related.
2. Create a scatter plot of the variables you choose. Describe any trends or patterns in the data. What new information do you get from a scatter plot that is not evident from the data list?
3. Are there any deviations from the pattern(s) you observed?

*Finding a two-variable comparison:*

1. Find a published table or graph that shows a relationship between two variables. If you have access to the Internet, the following websites can be used to find data and graphs:

   *www.census.gov* *www.newyorktimes.com* *www.wallstreetjournal.com*

2. What are the two variables? How does change in one variable relate to change in the other variable? Are there any deviations from the pattern(s) you observed?

   Write a 60-second summary describing your results. Include questions raised by the data and your analysis.