1

Introduction

1.1 The Piezoelectric Effect

In the most general terms, a material is piezoelectric if it transforms electrical into mechanical energy, and vice versa, in a reversible or lossless process. This transformation is evident at a macroscopic scale in what are commonly known as the direct and converse piezoelectric effects. The direct piezoelectric effect refers to the ability of a material to transform mechanical deformations into electrical charge. Equivalently, application of mechanical stress to a piezoelectric specimen induces flow of electricity in the direct piezoelectric effect. The converse piezoelectric effect describes the process by which the application of an electrical potential difference across a specimen results in its deformation. The converse effect can also be viewed as how the application of an external electric field induces mechanical stress in the specimen.

While the brothers Pierre and Jacques Curie discovered piezoelectricity in 1880, much the early impetus motivating its study can be attributed to the demands for submarine countermeasures that evolved during World War I. An excellent and concise history, before, during, and after World War I, can be found in [43]. With the increasing military interest in detecting submarines by their acoustic signatures during World War I, early research often studied naval applications, and specifically sonar. Paul Langevin and Walter Cady had pivotal roles during these early years. Langevin constructed ultrasonic transducers with quartz and steel composites. Shortly thereafter, the use of piezoelectric quartz oscillators became prevalent in ultrasound applications and broadcasting. The research by W.G. Cady was crucial in determining how to employ quartz resonators to stabilize high frequency electrical circuits.

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A number of naturally occurring crystalline materials including Rochelle salt, quartz, topaz, tourmaline, and cane sugar exhibit piezoelectric effects. These materials were studied methodically in the early investigations of piezoelectricity. Following World War II, with its high demand for quartz plates, research and development of techniques to synthesize piezoelectric crystalline materials flourished. These efforts have resulted in a wide variety of synthetic piezoelectrics, and materials science research into specialized piezoelectrics continues to this day.

1.1.1 Ferroelectric Piezoelectrics

Perhaps one of the most important classes of piezoelectric materials that have become popular over the past few decades are the ferroelectric dielectrics. A ferroelectric can have coupling between the mechanical and electrical response that is several times a large as that in natural piezoelectrics. Ferroelectrics include materials such as barium titanate and lead zirconate titanate, and their unit cells are depicted in Figure 1.1. When the centers of positive and negative charge in a unit cell of a crystalline material do not coincide, the material is said to be polar or dielectric. An electric dipole moment $\boldsymbol{p}$ is a vector that points from the center of negative charge to the center of positive charge, and its magnitude is equal to $|\boldsymbol{p}| = q \cdot \delta$ where q is the magnitude of the charge at the centers and δ is the separation between the centers. The limiting volumetric density of dipole moments is the polarization vector $\boldsymbol{P}$. Intuitively we think of the polarization vector $\boldsymbol{P}$ as measuring the asymmetry of the internal electric field of the piezoelectric crystal lattice. Ferroelectrics exhibit spontaneous electric polarization that can be reversed by the

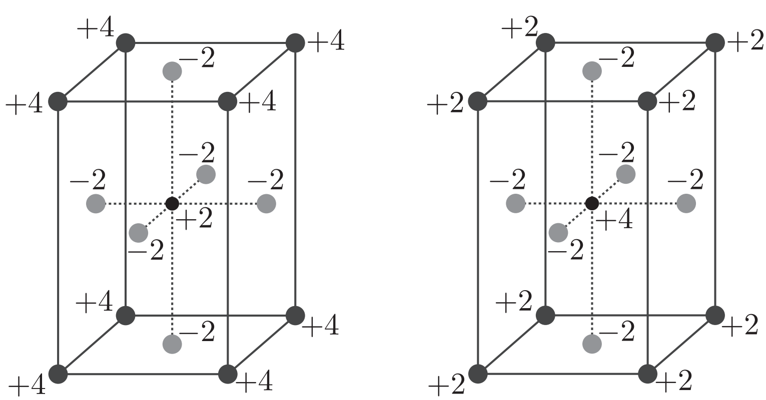

Figure 1.1 Barium titanate and lead zirconate titanate. (Left) Barium titanate $BaTiO_3$ with Ba^{+2} cation at the center, O^{-2} anions on the faces, and Ti^{+4} cations at the corners of the unit cell. (Right) Lead zircanate titanate $PbZr_{\alpha}Ti_{(1-\alpha)}O_3$ with Ti^{+4} or Zr^{+4} cation at the center, $O^{-}2$ anions on the faces, and Pb^{+2} cations at the corners of the unit cell.

application of an external electric field. In other words, the polarization of the material is evident during a spontaneous process, one that evolves to a state that is thermodynamically more stable. Understanding this process requires a discussion of the micromechanics of a ferroelectric.

The micromechanics of ferroelectric dielectrics is subtle and interesting. Above a critical temperature T_c, the Curie temperature, the crystal structure of a ferroelectric is usually symmetric, and a plot of the polarization versus applied electric charge is generally nonlinear and single-valued as shown in Figure 1.2.

However, with cooling below the Curie temperature T_c, a thermodynamic process drives a structural phase transition so that the final crystalline phase has a lower symmetry. At the lower temperature it can be shown [18] that the lower symmetry crystal phase has at least two energetically equivalent configurations or variants. Furthermore, with the application of an external electric field, it must be the case that it is possible switch among these crystalline variants in a reversible process. The ferroelectric material forms *domains* that consist of these energetically equivalent crystalline variants. Figure 1.3 depicts schematically the 180° and 180°/90° domains [31] that can appear in single crystal barium titanate $BaTiO_3$ [31]. Note in the figure that the polarization vectors are opposite from one domain to the next, and their average polarization over a macroscale can have zero effective polarization. Because of the presence of these domains, below the Curie

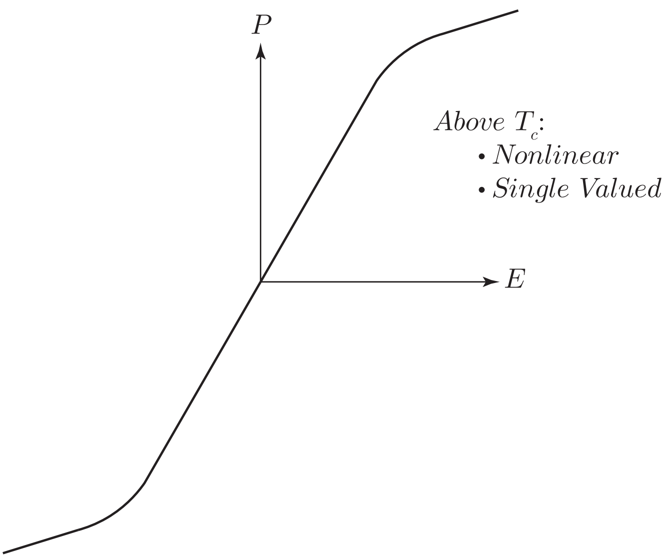

Figure 1.2 Polarization versus applied electrical field for ferroelectric above the Curie temperature T_c.

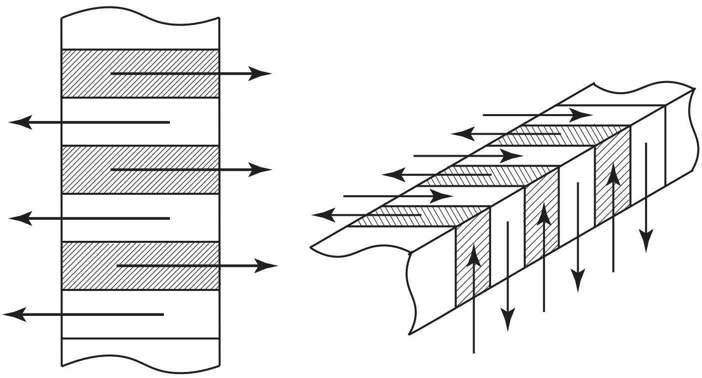

Figure 1.3 180° and 180°/90° domains in $BaTiO_3$, [31]. Source: Walter J. Merz, Domain Formation and Domain Wall Motion in Ferro-electric $BaTiO_3$ Single Crystals, em Physical Review, Volume 95, Number 3, August 1, 1954, pp. 690–698.

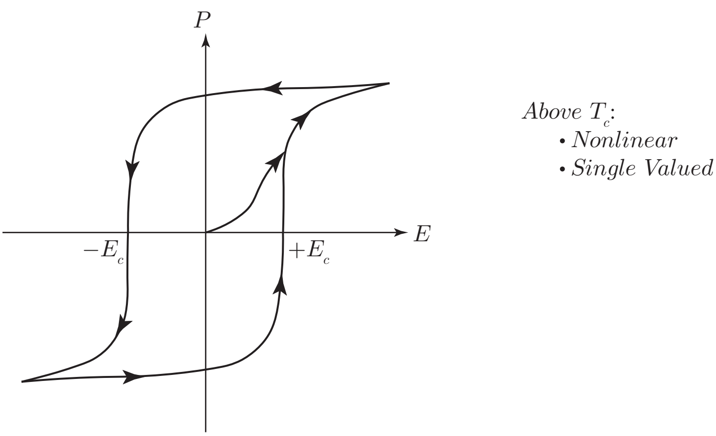

Figure 1.4 Polarization versus electrical field hysteresis below the Curie temperature T_c.

temperature T_c the polarization versus applied electric field takes the form of a hysteresis loop as shown in Figure 1.4. Initially, the domains cancel their effects over the macroscopic specimen and $P = 0$ at $E = 0$. The polarization P increases as in Figure 1.2 for a range of electric field E. When a critical value E_c, the coercive electric field strength, is reached, the domains abruptly switch so that they are approximately well-aligned with the external electric field. With all domains having approximately aligned polarization vectors, the polarization again follows a nonlinear single valued curve until saturation is achieved. When the electric field is reversed, and reaches the opposite coercive electric field strength $-E_c$, the domains switch again so their polarization vectors are approximately aligned with the second variant. The result of this cyclic process is that after the transient response there is a nonzero polarization, the spontaneous polarization, for an electric field strength $E = 0$. At a macroscopic scale, then, the effective or average polarization can switch with the application of the external electric field.

1.1.2 One Dimensional Direct and Converse Piezoelectric Effect

In view of these observations, at a fundamental level, the micromechanics of piezoelectricity is understood in terms of crystalline asymmetry. While the most general theory of linear piezoelectricity of material continua in three dimensions is discussed in Chapter 5, intuition can be built by considering a one dimensional example. Figure 1.5 depicts the direct piezoelectric effect graphically, while the converse effect is shown in Figure 1.6. For the specimens shown, the mechanical variables are the stress T and strain S, and the electrical variables include the electric field E, electric displacement D, voltage $V = \phi^+ - \phi^-$, and the electrical potential ϕ. In Figure 1.5 we suppose that the top and bottom of the specimen are free to displace. A thin film electrode, one that does not alter the mechanical properties of the specimen, is applied to the top and bottom surfaces by a deposition or sputtering process. An ideal current meter, over which the potential difference is approximately zero, is attached to the top and bottom electroded surfaces. A positive stress T is applied as shown. As we discuss in Chapter 5 the constitutive laws that couple the electrical and mechanical variables can take many forms. In this one dimensional example we choose to express the dependency among the electrical and mechanical variables as

$$\begin{Bmatrix} S \\ D \end{Bmatrix} = \begin{bmatrix} s & d \\ d & \varepsilon \end{bmatrix} \begin{Bmatrix} T \\ E \end{Bmatrix}. \tag{1.1}$$

where s is the mechanical compliance constant, d is the piezoelectric coupling constant, and ε is the permittivity constant. See Chapter 5 for a precise definition of these constants when interpreted as elements of tensors suitable for piezoelectric continua in three dimensions. Since the current meter is ideal, the electrical potential ϕ at the top electrode is equal to the potential at the bottom electrode, and the electric field $\boldsymbol{E} = -\nabla\phi \approx -(\phi^+ - \phi^-)/t \approx 0$. From Equation 1.1 it follows that the strain $S = sT$ is positive, and the top and bottoms of the specimens displace by approximately $u \approx S \cdot t/2$, and the specimen stretches by a total amount $\approx S \cdot t$. The magnitude of the charge on the electrodes can be obtained from the boundary conditions $|q| = DA = dTA$. Thus, we see that either the application of a positive stress T, or equivalently of a positive strain $S = sT$, induces charges having magnitude $|q| = dTA$ on the faces of the specimen.

In contrast, Figure 1.6 illustrates how the application of an electric potential across a piezoelectric specimen generates a deformation. In this case, we assume

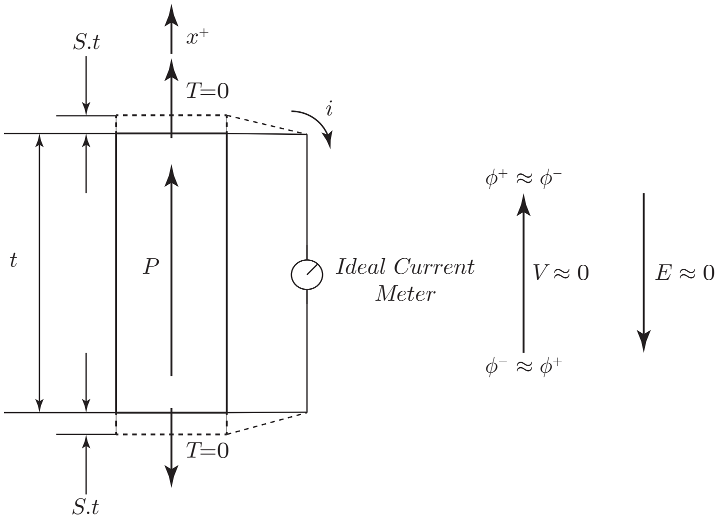

Figure 1.5 The direct piezoelectric effect.

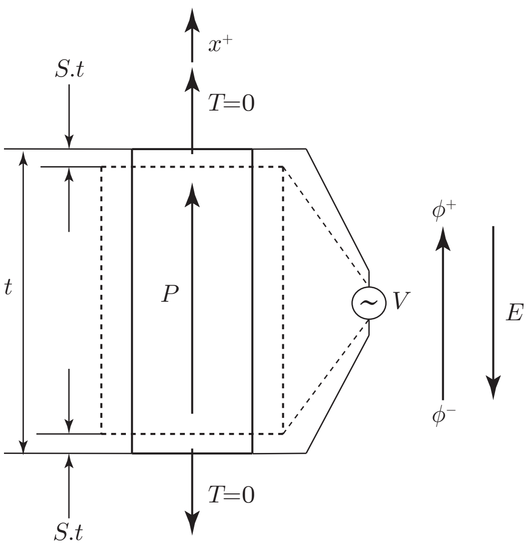

Figure 1.6 The converse piezoelectric effect.

that the potential at the top electrode is ϕ^+, the potential at the lower electrode is ϕ^-, and the voltage source has a voltage $V = \phi^+ - \phi^-$. Since the stress $T = 0$ and $\boldsymbol{E} = -\nabla\phi$, the electrical field E is approximately given by $E = -V/t$, and the strain in the specimen at its top and bottom is $S = dE = -dV/2t$. In this case, the specimen compresses in total approximately by $-dV$.

1.2 Applications

While the earliest applications of the piezoelectric effect were mostly related to ultrasound generation or sound measurements, the commercially viable applications of piezoelectricity have grown remarkably in diversity since that time. A quick survey of companies such as PCB® or PI Ceramics® that offer a broad range of piezoelectric devices can quickly convey the breadth of applications. For example, PI Ceramics notes that a piezoelectric device finds use in "…*high-end technology markets, such as medical technology, mechanical and automotive engineering or semiconductor technology, but is also present in everyday life, for example as a generator of ultrasonic vibrations in a cleaning bath for glasses and jewelry or in medical tooth cleaning*." The companies PCB® [36] and PI Ceramics® [37] include descriptions of broad applications areas. Even within each of these areas, the specific inventory of applications can be vast. Accelerometers, for example, are but one specific class of piezoelectrically-based sensor that are used to measure vibration, shock, and acceleration for testing, control, online estimation, and system monitoring. The company PCB® [36] notes on `http://www.pcb.com/TestMeasurement/Accelerometers` that their products in this niche of sensors support applications in "…*balancing, bearing, analysis, biomechanics, building vibration monitoring, biomechanics, bridge monitoring, component durability testing, crash testing, drop testing, fatigue testing, gearbox monitoring, environmental testing, ground vibration testing, impact measurements, impulse response measurements, machinery vibration, modal analysis, package testing, product qualification, quality control, seismic monitoring, structural testing, structure-borne noise, vibration analysis, vibration isolation, and vibration stress screening.*"

Despite the growth of the applications of piezoelectric technology catalogued above, it is common that they are often grouped into three general categories: energy applications, sensors, and actuators or motors. We discuss these next.

1.2.1 Energy Applications

While small piezoelectric specimens typically to do not output large currents or power, they can generate very high voltage differences under application of stress. It is for this reason that piezoelectric igniters are commonly used in gas broilers, gas stoves and ranges, gas fireplaces, or other appliances. The igniters usually operate by releasing a spring loaded switch that impacts the piezoelectric specimen, thereby generating a large, transient potential difference [36]. Such a voltage is high enough that it induces current flow across a gap that ignites the gas. In addition, the use of piezoelectric transducers in energy harvesting applications is a rapidly growing field. See for example [14] and the references therein for a good, comprehensive technical treatment of this topic. As noted before, while small piezoelectric transducers *usually* are not appropriate as large supplies of power, they are well suited to applications that require local, modular, or isolated energy sources for microscale electromechanical (MEMs) devices. Often, classes of sensors require small sources of energy to perform their associated measurements, and piezoelectric transducers have proven to be a viable route to support such sensors. There are numerous studies of energy reclamation from a wide range of sources [14]. Examples of composite piezoelectric structures that are connected to linear, ideal, passive electrical networks are studied in Chapters 6, 7, and 8. These electromechanical models are also suitable for the study of nonlinear switching strategies for energy harvesting. The emerging field of MEMs or NEMs (nanoelectromechanical systems) robotics requires microscale energy supplies to enable their mobility, and piezoelectric transducers are often the choice to develop self-contained MEMs robots.

1.2.2 Sensors

Among all the various categories of applications of piezoelectric devices and systems, it is perhaps the piezoelectric sensors that have had the most profound impact on measurement technology infrastructure. As noted above, there are a wide and growing collection of piezoelectric transducers that serve as sensors. These include pressure transducers, load cells or force transducers, torque transducers, acoustic microphones, accelerometers, and vibration sensors. The underlying physical basis of nearly all these sensors is the same: the forces applied on opposite sides of a piezoelectric specimen generate a source of charge that is highly sensitive to the applied force, and the measurement of the charge or current provides an estimate of such forces over extremely high frequencies. Often the bandwidth of such sensors extends to the hundreds of kilohertz range. The diversity of piezoelectrically-based, commercial microphones available from PCB® is illustrated in Figure 1.7. This figure illustrates that the operating range of these sensors is as high as O(100) kHz, and their sensitivity varies from O(1) mV/Pa to O(10) mV/Pa. The physics underlying a wide range of piezoelectric sensors is studied in Chapters 3, 4, 5, 6, 7, and 8.

1.2.3 Actuators or Motors

Piezoelectric actuators or motors are available commercially at companies such as PCB® or PI Ceramics®, and they have been the topic of research in academia and national laboratories for decades. The most popular commercially available actuators are the linear or stack actuators and the bender actuators. Stack actuators

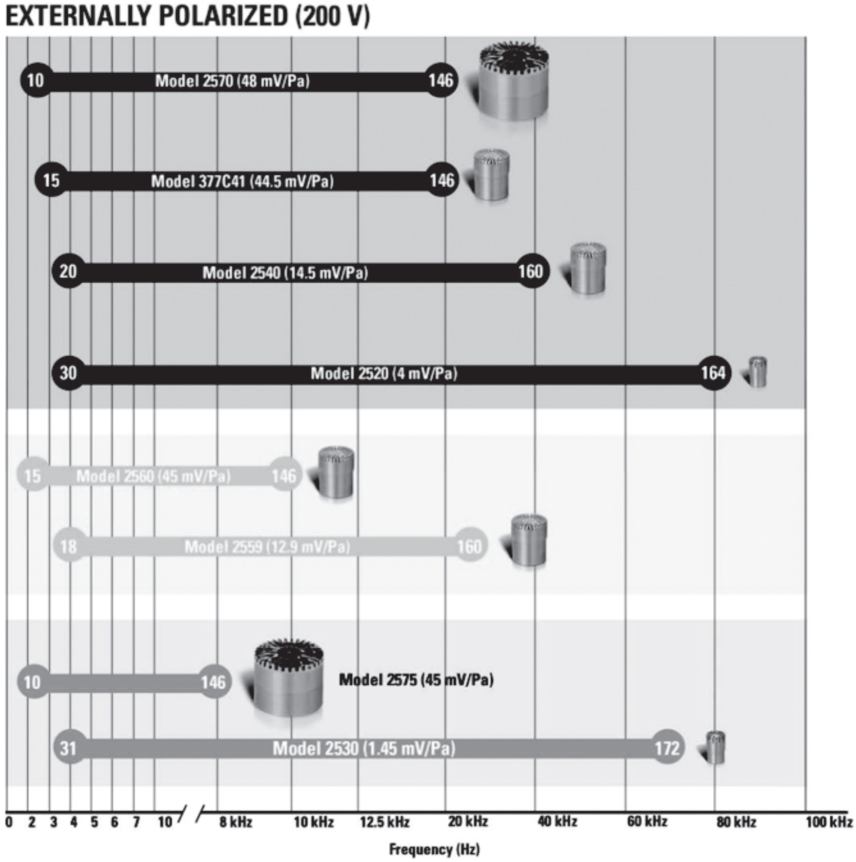

Figure 1.7 Piezoelectrically based microphones from PCB®, Source: HOW TO SELECT THE BEST TEST & MEASUREMENT PRECISION MICROPHONE, PCB Piezotronics, Inc. Retrieved from: http://www.pcb.com/Microphones Preamplifiers Acoustic Accessories/select.

developed by PI Ceramic® are depicted in Figure 1.8. Commercially available stack actuators usually have a small stroke, on the order O(10) to O(100) μm. These prototypical piezoelectric composites are studied in Chapters 5, 6, 7, and 8. Figure 1.9 depicts bender actuators available from PI Ceramics®. The bending actuators are intended for applications that require a larger stroke and operate at a lower frequency than the linear actuators. The stroke of the benders depicted in Figure 1.9 varies approximately from O(100) to O(1000) μm. Composite bending actuators are studied in Chapters 5, 6, 7, and 8.

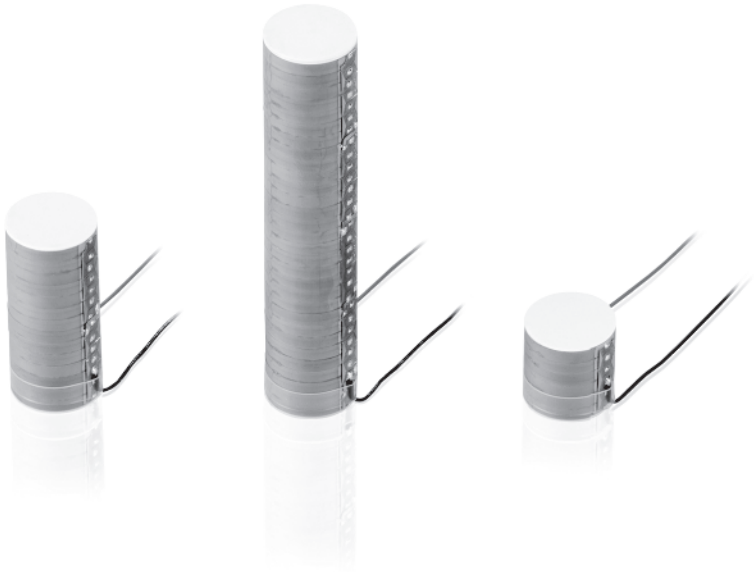

Figure 1.8 Piezoelectric stack actuators available from PI ceramic®, Source: GmbH & Co. KG owner.

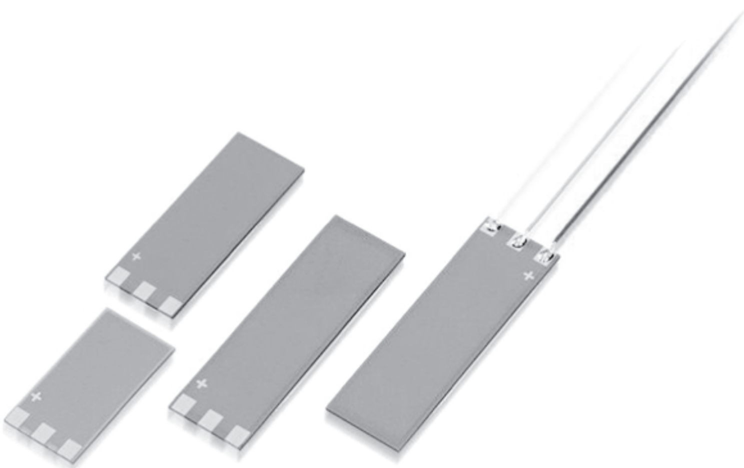

Figure 1.9 Piezoelectric bender actuators available from PI ceramic®, Source: GmbH & Co. KG owner.

1.3 Outline of the Book

The linear and bending actuators shown in Figure 1.8 and 1.9 are considered in examples throughout this text: they are perhaps the most simple macroscale actuators that are commercially available. In fact, they can also be used to understand the basic physics of piezoelectric sensors too. They illustrate the operating principles of and modeling techniques for piezoelectric composites that are evident in much more complex designs.

Unfortunately, the theoretical underpinnings of piezoelectric mechanics embraces a wide collection of fields of study that must be synthesized. We begin in Chapter 2. Section 2.1 reviews the fundamentals of vectors, bases, and frames of reference. This section is vital in developing an understanding of how physical vectors such as velocity, acceleration, stress vectors, electric field vectors, and electric displacement vectors are represented. The section culminates in a presentation of rotation matrices and their essential role in constructing change of bases for different representations of vectors. Section 2.2 then extends these results by introducing multilinear operators, or tensors, that act between vector spaces. Since vectors are first order tensors, they are a special case of the tensors presented in Section 2.2. In addition, another collection of physical variables critical to linear piezoelectricity are understood as tensors. These include the stress tensor, linear strain tensor, permittivity tensor, piezoelectric coupling tensor, stiffness tensor, and compliance tensor. The section concludes with a discussion of the role of rotation matrices in the representation and change of basis formula for n^{th} order tensors. Section 2.3 discusses symmetry properties and geometric properties of tensors and crystals. The discussion begins with an overview of the geometry of crystals in Section 2.3.1. The 14 Bravais lattices and seven crystal systems are defined, as are the 32 crystallographic point groups. This section concludes with examples of symmetry transformations for typical crystal classes, and a discussion of tensor invariance associated with symmetry operations.

Chapter 3 reviews the basics of continuum mechanics that are needed to build a coherent framework for linear piezoelectricity. The definition of the stress tensor in three dimensions is presented in Section 3.1.1, Cauchy's formula is presented in Section 3.1.2, and the equations of equilibrium are discussed in Section 3.1.3. Section 3.2 is dedicated to the study of the linear strain tensor, the mechanical field variable that is complementary to the stress tensor. The general definition of the linear strain tensor in three dimensions is presented, as well as the kinematic relationships for common structures such as the axial rod, Bernoulli–Euler beam, and Kirchoff plate. Strain energy is discussed in Section 3.3. The strain energy density function is introduced, and its role in determining additional symmetry properties of the material stiffness tensor is given. A review of the constitutive laws for linearly elastic materials is summarized in Section 3.4, and the structure of the constitutive relationships for triclinic, orthotropic, and transversely isotropic materials are summarized. The chapter ends with an introduction of the initial-boundary value problem of linear elasticity in Section 3.5.

We turn to a discussion of continuum electrodynamics in Chapter 4. Charge and current are introduced in Section 4.1, and the static electric and magnetic fields are discussed in Section 4.2. Maxwell's equations are introduced in Equation 4.10 in SI units. Section 4.3.1 relates the polarization and electric displacement vectors, and relates them to bound and mobile charge, respectively. Magnetization and magnetic field intensity are defined in Section 4.3.2, as are the free, bound, and polarization current densities, respectively. Section 4.3.3 discusses the form of Maxwell's equations in Gaussian units, which prove to be convenient for the derivation of the equations of piezoelectricity.

Chapter 5 presents the theory of linear piezoelectricity, starting with some one dimensional examples in Section 5.1. Section 5.2 gives the detailed account of how the equations of linear piezoelectricity are derived from Maxwell's equations, and Section 5.2.2 summarizes the initial-boundary value problem of linear piezoelectricity. Section 5.3 surveys the role of thermodynamics in the construction of various equivalent constitutive laws and their associated thermodynamic invariants. The structure of the constitutive laws generated by crystalline materials having different symmetry operations is described in Section 5.4.

Chapter 6 focuses on the use of Newton's method to derive the governing equations for linearly piezoelectric composite structures. The axial actuator model, which is a prototype for the linear actuators of the type depicted in Figure 1.8, is treated in Sections 6.1 and 6.2. Section 6.3 presents an analysis of the beam actuator as shown in the introduction in Figure 1.9. Section 6.4 uses Newton's method to derive the governing equations for a simplified model of piezoelectric composite plate bending.

Chapter 7 introduces powerful variational methods for deriving the governing equations of piezoelectric structures. The chapter begins with a review of variational calculus in Chapter 7.1. Hamilton's principle for mechanical systems is introduced in Section 7.2, and its generalization for linear piezoelectricity is presented in Section 7.3. The strength of these variational techniques is illustrated in Section 7.4, which shows how variational methods for electromechanical systems that consist of piezoelectric structures and attached ideal circuits can be modeled. Various authors have discussed variational methods for electromechanical systems over the years, and Section 7.5 discusses the relationships among some alternative forms of these principles. Section 7.6 illustrates how the electromechanical variational principle can be applied using Lagrangian densities $\mathfrak{L}$ instead of Lagrangian functions $\mathcal{L}$.

Chapter 8, the final chapter of this book gives a detailed description of approximation methods for linearly piezoelectric composite structures. The chapter begins in Section 8.1 with a discussion of the differences between classical, strong, and weak forms of the governing equations. As discussed in Section 8.1, the approximation strategies in the text are derived from the weak form of the governing equations. Section 8.2 gives a quick overview of modeling damping and dissipation, with the primary emphasis on viscous damping that is so popular in engineering vibrations texts. Galerkin approximations are introduced in Section 8.3, and example applications for the linear and bending actuators are summarized. Two classes of bases are described for use in the Galerkin approximations. Modal or eigenfunction bases are used in Section 8.3.1, and finite element functions are employed in Section 8.3.2. The chapter finishes with a collection of examples that summarize how transient and steady state solutions are obtained from the Galerkin approximations. Particular emphasis is placed on the derivation of complex frequency response equations and FRFs for the piezoelectric composite structures.

The Appendix contains three sections that provide supplementary material for the discussions throughout the text. Section S.1 gives a streamlined summary of the basic background for vibrations theory for single degree of freedom (SDOF) systems, distributed parameter systems (DPS), and multi-DOF (MDOF) systems. A supplementary account of tensor analysis is given in Section S.2. For those students seeking to understand the simplified version of tensor analysis covered in Chapter 2, this section shows how the simplified account fits in the general theory. Finally, Section S.3 discusses details regarding distributional and weak derivatives beyond the brief account in Chapter 8. A rigorous definition of a weak derivative is given, and the Sobolev spaces $W^{k,p}(\Omega)$ of all functions whose weak derivatives of order less than or equal to p are elements of the Lebesgue space $L^p(\Omega)$ is defined.