1

Basic Heat Transfer

1.1 Importance of Heat Transfer

The subject of heat transfer is of fundamental importance in many branches of engineering. A *mechanical engineer* may be interested in knowing the mechanisms of heat transfer involved in the operation of equipment, such as boilers, condensers, air preheaters and economizers, and in thermal power plants, in order to improve performance. Refrigeration and air-conditioning systems also involve heat-exchanging devices, which need careful design. *Electrical engineers* are keen to avoid material damage due to hot spots, developed by improper heat transfer design in electric motors, generators and transformers. An *electronic engineer* is interested in knowing the most efficient methods of heat dissipation from chips and other semiconductor devices so that they can operate within safe operating temperatures. A *computer hardware engineer* wants to know the cooling requirements of circuit boards, as the miniaturization of computing devices is advancing rapidly. *Chemical engineers* are interested in heat transfer processes in various chemical reactions. A *metallurgical engineer* may need to know the rate of heat transfer required for a particular heat treatment process, such as the rate of cooling in a casting process, as this has a profound influence on the quality of the final product. *Aeronautical engineers* are interested in knowing the heat transfer rate in electronic equipment that uses compact heat exchangers for minimizing weight, in rocket nozzles and in heat shields used in re-entry vehicles. An *agricultural engineer* would be interested in the drying of food grains, food processing and preservation. *Civil engineers* need to be aware of the thermal stresses developed in quick-setting concrete, and the effect of heat and mass transfer on buildings and building materials. Finally, an *environmental engineer* is concerned with the effect of heat on the dispersion of pollutants in air, diffusion of pollutants in soils, thermal pollution in lakes and seas and their impact on life (Incropera et al. [1]).

The study of heat transfer can offer economical and efficient solutions to critical problems encountered in many branches of engineering. For example, we could consider the development of heat pipes that can transport heat at a much greater rate than copper or even silver rods of the same dimensions, even at almost isothermal conditions. The development of modern gas turbine blades, in which the gas temperature exceeds the melting point of the material of the blade, is possible by providing efficient cooling systems, and this is another example of the success of heat transfer design methods. The design of computer chips, which encounter heat fluxes of the same order those occurring in re-entry vehicles, especially when the surface temperature of the chips is limited to less than 100 °C, is another success story for heat transfer analysis.

Although there are many successful heat transfer designs, further developments are still necessary in order to increase the lifespan and efficiency of the many devices discussed above, which can lead to many more inventions. Also, if we are to protect our environment, it is essential to understand the many heat transfer processes involved and to take appropriate action, where necessary.

1.2   Heat Transfer Modes

Heat transfer is the exchange of thermal energy between physical systems. The rate of heat transfer is dependent on the temperatures of the systems and the properties of the intervening medium through which the heat is transferred. The three fundamental modes of heat transfer are conduction, convection and radiation. Heat transfer, the flow of energy in the form of heat, is a process by which a system changes its internal energy, hence it is of vital use in applications of the *first law of thermodynamics*. Conduction is also known as diffusion, not to be confused with diffusion related to the mixing of constituents of a fluid.

The direction of heat transfer is from a region of high temperature to another region of lower temperature, and is governed by the *second law of thermodynamics*. Heat transfer changes the internal energy of the systems from which and to which the energy is transferred. Heat transfer will occur in a direction that increases the entropy of the collection of systems. Heat transfer is that section of engineering science that studies the energy transport between material bodies due to a temperature difference (Bejan [2], Holman [3], Incropera and Dewitt [4], Sukhatme [5]). The three modes of heat transfer are

- Conduction
- Convection
- Radiation.

The conduction mode of heat transport occurs either because of an exchange of energy from one molecule to another, without the actual motion of the molecules, or because of the motion of any free electrons that are present. Therefore, this form of heat transport depends heavily on the properties of the medium and takes place in solids, liquids and gases if a difference in temperature exists.

Molecules present in liquids and gases have freedom of motion, and by moving from a hot to a cold region, they carry energy with them. The transfer of heat from one region to another, due to such macroscopic motion in a liquid or gas, added to the energy transfer by conduction within the fluid, is called *heat transfer by convection*. Convection may be free, forced or mixed. When fluid motion occurs because of a density variation caused by temperature differences, the situation is said to be a free or natural convection. When the fluid motion is caused by an external force, such as pumping or blowing, the state is defined as being one of forced convection. A mixed convection state is one in which both natural and forced convections are present. Convection heat transfer also occurs in boiling and condensation processes.

All bodies emit thermal radiation at all temperatures. This is the only mode that does not require a material medium for heat transfer to occur. The nature of thermal radiation is such that a propagation of energy, carried by *electromagnetic waves*, is emitted from

the surface of the body. When these electromagnetic waves strike other body surfaces, a part is reflected, a part is transmitted and the remaining part is absorbed. All modes of heat transfer are generally present in varying degrees in a real physical problem. The important aspects in solving heat transfer problems are identifying the significant modes and deciding whether the heat transferred by other modes can be neglected.

1.3 Laws of Heat Transfer

It is important to quantify the amount of energy being transferred per unit time, and this requires the use of rate equations. For heat conduction, the rate equation is known as *Fourier's law*, which is expressed for one dimension as

$$q_x = -k\frac{dT}{dx} \tag{1.1}$$

where q_x is the heat flux in the x direction (W/m^2); k is the thermal conductivity (W/m · K), a property of material, and dT/dx is the temperature gradient (K/m).

For convective heat transfer, the rate equation is given by *Newton's law of cooling* as

$$q = h\left(T_w - T_a\right) \tag{1.2}$$

where q is the convective heat flux; (W/m^2); $(T_w - T_a)$ is the temperature difference between the wall and the fluid and h is the convection heat transfer coefficient, (W/m^{2}K).

The convection heat transfer coefficient frequently appears as a boundary condition in the solution of heat conduction through solids. We assume h to be known in many such problems. In the analysis of thermal systems, we can again assume an appropriate h if not available (e.g. heat exchangers, combustion chambers). However, if required, h can be determined via suitable experiments, although this is a difficult option.

The maximum flux that can be emitted by radiation from a black surface is given by the *Stefan–Boltzmann law*:

$$q = \sigma\, T_w^4 \tag{1.3}$$

where q is the radiative heat flux, (W/m^2); σ is the Stefan–Boltzmann constant (5.669×10^{-8}) in W/m^{2}K^4; and T_w is the surface temperature (K).

The heat flux emitted by a real surface is less than that of a black surface and is given by

$$q = \varepsilon\, \sigma\, T_w^4 \tag{1.4}$$

where ε is the radiative property of the surface and is referred to as the *emissivity*. The net radiant energy exchange between any two surfaces, 1 and 2, is given by

$$Q = F_\in F_G\, \sigma\, A_1\left(T_1^4 - T_2^4\right) \tag{1.5}$$

where F_ε is a factor that takes into account the nature of the two radiating surfaces; F_G is a factor that takes into account the geometric orientation of the two radiating surfaces and A_1 is the area of surface 1. When a heat transfer surface, at temperature T_1, is completely

enclosed by a much larger surface at temperature T_2, the net radiant exchange can be calculated by

$$Q = q A_1 = \in_1 \sigma A_1 \left(T_1^4 - T_2^4 \right) \tag{1.6}$$

With respect to the laws of thermodynamics, only the first law is of interest in heat transfer problems. The increase of energy in a system is equal to the difference between the energy transfer by heat to the system and the energy transfer by work done on the surroundings by the system, that is,

$$dE = dQ - dW \tag{1.7}$$

where Q is the total heat entering the system and W is the work done on the surroundings. Since we are interested in the rate of energy transfer in heat transfer processes, we can restate the first law of thermodynamics as follows:

The rate of increase of the energy of the system is equal to the difference between the rate at which energy enters the system and the rate at which the system does work on the surroundings, that is,

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} \tag{1.8}$$

where t is the time.

The important fluid properties associated with conduction phenomena are presented in Appendixes A2–A8.

1.4 Steady-State Heat Conduction

Heat conduction is the transfer of heat (internal energy) by microscopic collisions of particles and movement of electrons within a body. The microscopically colliding objects, which include molecules, atoms and electrons, transfer disorganized microscopic kinetic and potential energy, jointly known as internal energy. Conduction takes place in all phases of matter: solids, liquids, gases and plasmas. The rate at which energy is conducted as heat between two bodies is a function of the temperature difference (temperature gradient) between the two bodies and the properties of the conductive medium through which the heat is transferred. Thermal conduction is originally called diffusion.

Steady-state conduction is the form of conduction that happens when the temperature difference(s) driving the conduction are constant, so that (after an equilibration time), the spatial distribution of temperatures (temperature field) in the conducting object does not change any further. In steady-state conduction, the amount of heat entering any region of an object is equal to the amount of heat coming out (if this are not so, the temperature would be rising or falling, as thermal energy is tapped from or trapped in a region).

For example, a bar may be cold at one end and hot at the other, but after a state of steady-state conduction is reached, the spatial gradient of temperatures along the bar does not change any further, as time proceeds. Instead, the temperature at any given section of the rod remains constant, and this temperature varies linearly in space, along the direction of heat transfer.

1.4.1 One-Dimensional Heat Conduction

A one-dimensional approximation of the heat conduction equation is feasible for many physical problems, such as plane walls and fins (Bejan [2], Holman [3], Incropera and Dewitt [4], Ozisik [6]). In these problems, any major temperature variation is in one direction only and the variation in all other directions can be ignored. Other examples of one-dimensional heat transfer occur in cylindrical and spherical solids in which the temperature variation occurs only in the radial direction. In this section, such one-dimensional problems are considered for steady-state conditions, in which the temperature does not depend on time. Time-dependent and multidimensional problems will be discussed in later sections.

The steady-state heat conduction equation for a plane wall, shown in Figure 1.1, is

$$kA\frac{d^2 T}{dx^2} = 0 \tag{1.9}$$

where k is the thermal conductivity and A is the cross-sectional area perpendicular to the direction of heat flow. The problem is complete with the following description of the boundary conditions.

> At $x = 0$, $T = T_1$; and at $x = L$, $T = T_2$

The exact solution to Equation 1.9 is

$$kAT = C_1 x + C_2 \tag{1.10}$$

On applying the appropriate boundary conditions to Equation 1.10, we obtain
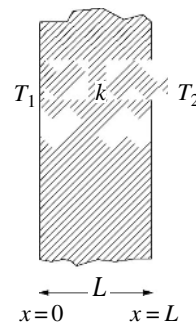
$$C_2 = kAT_1 \tag{1.11}$$

And

$$C_1 = -\frac{kA(T_1 - T_2)}{L} \tag{1.12}$$

Therefore, substituting constants C_1 and C_2 into Equation 1.10 results in

$$T = -\frac{(T_1 - T_2)x}{L} + T_1 \tag{1.13}$$

Figure 1.1 Heat conduction through a homogeneous wall.

The above equation indicates that the temperature distribution within the wall is linear. The heat flow, Q, can be written as

$$Q = -kA\frac{dT}{dx} = \frac{kA(T_1 - T_2)}{L} \tag{1.14}$$

Example 1.1   The wall of an industrial furnace is constructed from 0.3 m thick fireclay brick having a thermal conductivity 1.7 W/m · K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively, as shown in Figure 1.2. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?

Solution

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity and surface temperatures.

   Find: Wall heat loss.
   Schematic: Figure 1.2
   Assumptions:

Steady-state conditions
One-dimensional conduction through the wall
Constant thermal conductivity

   Analysis: Since heat transfer through the wall is by conduction, the heat flux be determined from Fourier's law. Using Equation 1.1 gives

$$q_x = k\frac{\Delta T}{L} = 1.7\frac{W}{m}\cdot K \times \frac{250\,K}{0.15\,m} = 2833\,W/m^2$$
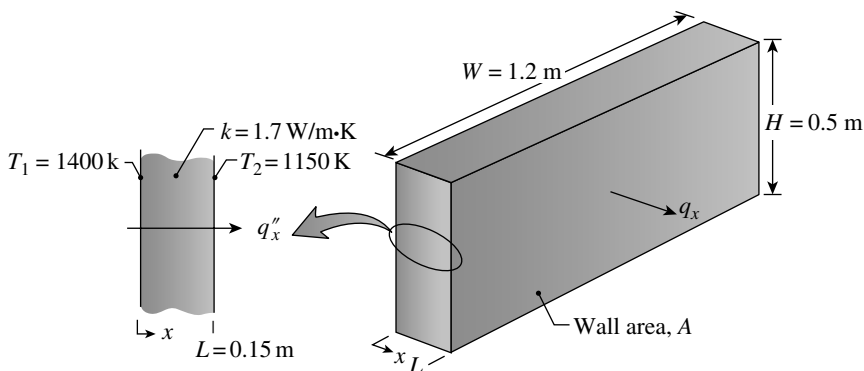

Figure 1.2 One-dimensional heat conduction slab.

The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area, $A = H \times W$ is then

$$(HW)\, q_x = (0.5\,\mathrm{m} \times 1.2\,\mathrm{m}\,)2833\,\mathrm{W/m^2} = 1700\,\mathrm{W}$$

Comments: Note the direction of heat flow and the distinction between heat flux and heat rate.

1.4.2 Three-Dimensional Heat Conduction Equation

The determination of temperature distribution in a medium (solid, liquid, gas or a combination of phases) is the main objective of a conduction analysis, that is, to know the temperature in the medium as a function of space at steady state and as a function of time during the transient state. Once this temperature distribution is known, the heat flux at any point within the medium, or on its surface, may be computed from Fourier's law, Equation 1.1. The knowledge of the temperature distribution within a solid can be used to determine the structural integrity via a determination of the thermal stresses and distortion. The optimization of the thickness of an insulating material and the compatibility of any special coatings or adhesives used on the material can be studied by knowing the temperature distribution and the appropriate heat transfer characteristics.

We shall now derive the conduction equation in Cartesian coordinates, as per Carslaw and Jaeger [7], by applying the energy conservation law to a differential control volume, as shown in Figure 1.3. The solution of the resulting differential equation, with prescribed boundary conditions, gives the temperature distribution in the medium.

Figure 1.3 A differential control volume for heat conduction analysis.
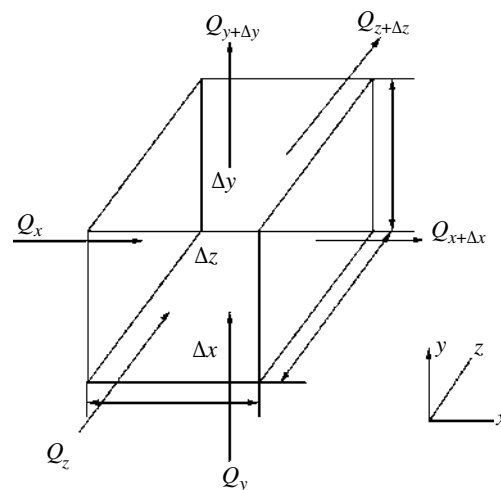
A Taylor series expansion results in

$$Q_{x+dx} = Q_x + \frac{\partial Q_x}{\partial x}\Delta x$$

$$Q_{y+dy} = Q_y + \frac{\partial Q_y}{\partial y}\Delta y \tag{1.15}$$

$$Q_{z+dz} = Q_z + \frac{\partial Q_z}{\partial z}\Delta z$$

Note that the second- and higher-order terms are neglected in the above equation. The heat generated in the control volume is $G \Delta x \Delta y \Delta z$ and the rate of change in energy storage is given as

$$\rho \Delta x \Delta y \Delta z \, c_p \frac{\partial T}{\partial t} \tag{1.16}$$

Now, with reference to Figure 1.3, we can write the energy balance as inlet energy + energy generated = energy stored + exit energy that is,

$$G \Delta x \Delta y \Delta z + Q_x + Q_y + Q_z = \rho \Delta x \Delta y \Delta z \frac{\partial T}{\partial x} + Q_{x+dx} + Q_{y+dy} + Q_{z+dz} \tag{1.17}$$

Substituting Equation 1.15 into the above equation, and rearranging, results in

$$-\frac{\partial Q_x}{\partial x}\Delta x - \frac{\partial Q_y}{\partial y}\Delta y - \frac{\partial Q_z}{\partial z}\Delta z + G \Delta x \Delta y \Delta z = \rho \Delta x \Delta y \Delta z \, c_p \frac{\partial T}{\partial t} \tag{1.18}$$

The total heat transfer Q in each direction can be expressed as

$$Q_x = \Delta y \Delta z \, q_x = -k_x \Delta y \Delta z \frac{\partial T}{\partial x}$$

$$Q_y = \Delta x \Delta z \, q_y = -k_y \Delta x \Delta z \frac{\partial T}{\partial x} \tag{1.19}$$

$$Q_z = \Delta x \Delta y \, q_z = -k_z \Delta x \Delta y \frac{\partial T}{\partial z}$$

Substituting Equation 1.19 into Equation 1.18 and dividing by the volume, $\Delta x \, \Delta y \, \Delta z$, we get

$$\frac{\partial}{\partial x}\left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y}\left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z}\left[k_z \frac{\partial T}{\partial z} \right] + G = \rho c_p \frac{\partial T}{\partial t} \tag{1.20}$$

Equation 1.20 is the transient heat conduction equation for a stationary system expressed in Cartesian coordinates. The thermal conductivity, k, in the above equation is a vector. In its most general form, the thermal conductivity can be expressed as a tensor, that is,

$$k = \begin{bmatrix} k_{xx} & k_{xy} & k_{xz} \\ k_{yx} & k_{yy} & k_{yz} \\ k_{zx} & k_{zy} & k_{zz} \end{bmatrix} \tag{1.21}$$

Equations 1.20 and 1.21 are valid for solving heat conduction problems in anisotropic materials with a directional variation in the thermal conductivities. In many situations, however, the thermal conductivity can be taken as a non-directional property, that is, it is isotropic. In such materials, the heat conduction equation is written as (constant thermal conductivity)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{G}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{1.22}$$

where $\alpha = k/\rho c_p$ is the *thermal diffusivity*, which is an important parameter in transient heat conduction analysis.

If the analysis is restricted only to steady-state heat conduction with no heat generation, the equation is reduced to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \tag{1.23}$$

For a one-dimensional case, the steady-state heat conduction equation is further reduced to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \tag{1.24}$$

The heat conduction equation for a cylindrical coordinate system is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left[k_r r\frac{\partial T}{\partial r}\right] + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left[k_\phi\frac{\partial T}{\partial \phi}\right] + \frac{\partial}{\partial z}\left[k_z\frac{\partial T}{\partial z}\right] + G = \rho c_p\frac{\partial T}{\partial t} \tag{1.25}$$

where the heat fluxes can be expressed as

$$\begin{aligned}
q_r &= -k_r\frac{\partial T}{\partial r}\\
q_\phi &= -\frac{k_\phi}{r}\frac{\partial T}{\partial \phi}\\
q_z &= -k_z\frac{\partial T}{\partial z}
\end{aligned} \tag{1.26}$$

The heat conduction equation for a spherical coordinate system is given by

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left[k_r r^2\frac{\partial T}{\partial r}\right] + \frac{1}{r^2 sin^2\theta}\frac{\partial}{\partial \phi}\left[k_\phi\frac{\partial T}{\partial \phi}\right] + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[k_\theta \sin\theta\frac{\partial T}{\partial \theta}\right] + G = \rho c_p\frac{\partial T}{\partial t} \tag{1.27}$$

where the heat fluxes can be expressed as

$$\begin{aligned}
q_r &= -k_r\frac{\partial T}{\partial r}\\
q_\phi &= -\frac{k_\phi}{r\sin\theta}\frac{\partial T}{\partial \phi}\\
q_\theta &= -\frac{k_\theta}{r}\frac{\partial T}{\partial \theta}
\end{aligned} \tag{1.28}$$

It should be noted that for both cylindrical and spherical coordinate systems, Equations 1.25 and 1.27 can be derived in a similar fashion as for Cartesian coordinates by considering the appropriate differential control volumes.

1.4.3 Boundary and Initial Conditions

The heat conduction equations discussed above will be complete for any problem only if the appropriate boundary and initial conditions are stated. With the necessary boundary and initial conditions, a solution to the heat conduction equations is possible. The boundary conditions for the conduction equation can be of two types or a combination of these two – the *Dirichlet* condition, in which the temperature on the boundaries is known and/or the *Neumann* condition, in which the heat flux is imposed (see Figure 1.4) as per Lewis et al. [8]:

Dirichlet condition

$$T = T_0 \text{ on } \Gamma_T \tag{1.29}$$

Neumann condition

$$q = -k \frac{\partial T}{\partial n} = C \text{ on } \Gamma_{qf} \tag{1.30}$$

In Equations 1.29 and 1.30, T_0 is the prescribed temperature; Γ is the boundary surface; n is the outward direction normal to the surface; and C is the constant flux given. The insulated, or adiabatic, condition can be obtained by substituting $C = 0$. The convective heat transfer boundary condition also falls into the *Neumann* category, and can be expressed as

$$-k \frac{\partial T}{\partial n} = h(T_w - T_a) \text{ on } \Gamma_{qc} \tag{1.31}$$

It should be observed that the heat conduction equation has second-order terms and hence requires two boundary conditions. Since time appears as a first-order term, only one initial value (i.e. at some instant of time all temperatures must be known) needs to be specified for the entire body, that is,

$$T = T_0 \text{ all over the domain } \Omega \text{ at } t = t_0 \tag{1.32}$$

where t_0 is a reference time.

The constant or variable temperature, conditions are generally easy to implement as temperature is a scalar. However, the implementation of surface fluxes is not as straightforward.
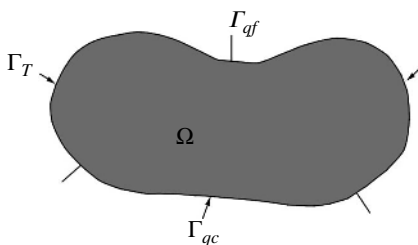

Figure 1.4 Boundary condition.

Equation 1.30 can be rewritten with the direction cosines of the outward normal as

$$k_x \frac{\partial T}{\partial x} \widetilde{\imath} + k_y \frac{\partial T}{\partial y} \widetilde{m} + k_z \frac{\partial T}{\partial z} \widetilde{n} = C \text{ on } \Gamma_{qf} \tag{1.33}$$

Similarly, Equation 1.31 can be rewritten as

$$k_x \frac{\partial T}{\partial x} \widetilde{\imath} + k_y \frac{\partial T}{\partial y} \widetilde{m} + k_z \frac{\partial T}{\partial z} \widetilde{n} = h(T - T_a) \text{ on } \Gamma_{qc} \tag{1.34}$$

where, $\widetilde{\imath}$, $\widetilde{m}$ and $\tilde{n}$ are the direction cosines of the appropriate outward surface normals.

The general energy equation for heat conduction, taking into account the spatial motion of the body is given by

$$\frac{\partial}{\partial x} \left[k_x \frac{\partial T}{\partial x} \right] + \frac{\partial}{\partial y} \left[k_y \frac{\partial T}{\partial y} \right] + \frac{\partial}{\partial z} \left[k_z \frac{\partial T}{\partial z} \right] + G = \rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right)$$

$$\tag{1.35}$$

where u, v and w are the components of the velocity in the three directions, x, y and z respectively.

1.5   Transient Heat Conduction Analysis

In the above, we have discussed steady-state heat conduction, in which the temperature in a solid body is assumed to be invariant with respect to time. However, many practical heat transfer applications are unsteady (transient) in nature and in such problems the temperature varies with respect to time. For instance, in many industrial plant components, such as boilers or refrigeration and air-conditioning equipment, the heat transfer process is transient during the initial stages of operation, so the analysis of transient heat conduction is very important.

1.5.1   Lumped Heat Capacity System

In this section, we consider the transient analysis of a body in which the temperature is assumed to be constant at any point within and on the surface of the body at any given instant of time. It is also assumed that the temperature of the whole body changes uniformly with time. Such an analysis is called a *lumped heat capacity* method and is a simple and approximate procedure in which no spatial variation in temperature is allowed. The change in temperature in such systems varies only with respect to time. It is therefore obvious that the lumped heat capacity analysis is limited to small-sized bodies and/or high thermal conductivity materials. Consider a body at an initial temperature T_0, immersed in a liquid maintained at a constant temperature T_a, as shown in Figure 1.5. At any instant in time, the convection heat loss from the surface of the body is at the expense of the internal energy of the body. Therefore, the internal energy of the body at any time will be equal to the heat convected to the surrounding medium, that is,
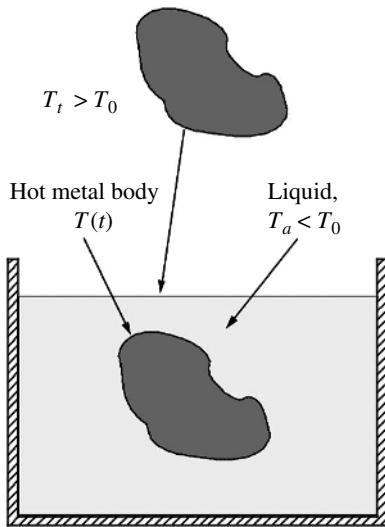
$$-\rho c_p V \frac{dT}{dt} = hA(T(t) - T_a) \tag{1.36}$$

Figure 1.5 Lumped heat capacity system: A hot metal body is immersed in a liquid maintained at a constant temperature.

where ρ is the density, c_p is the specific heat and V is the volume of the hot metal body; A is the surface area of the body; h is the heat transfer coefficient between the body surface and the surrounding medium; t is the time; and $T(t)$ is the instantaneous temperature of the body.

Equation 1.36 is a first-order differential equation in time, which requires an initial condition to obtain a solution. As mentioned previously, the initial temperature of the body at time $t = 0$, is T_0. Applying the variable separation concept to Equation 1.36, we get

$$\frac{dT}{T(t) - T_a} = -\frac{hA}{\rho c_p V} dt \tag{1.37}$$

Integrating between temperatures T_0 and $T(t)$, we obtain

$$\int_{T_0}^{T(t)} \frac{dT}{T(t) - T_a} = -\int_0^t \frac{hA}{\rho c_p V} dt \tag{1.38}$$

Note that the temperature changes from T_0 to $T(t)$ as the time changes from 0 to t.

Integration of the above equation results in a transient temperature distribution as follows:

$$\ln\left(\frac{T - T_a}{T_0 - T_a}\right) = \frac{-hAt}{\rho c_p V} \tag{1.39}$$

or

$$\frac{T - T_a}{T_0 - T_a} = e^{\left[-\frac{hA}{\rho c_p V}\right]t} \tag{1.40}$$

The quantity $\rho c_p V/hA$ is referred to as the time constant of the system because it has the dimensions of time. When $t = \rho c_p V/hA$, it can be observed that the temperature difference $(T(t) - T_a)$ has a value of 36.78% of the initial temperature difference $(T_0 - T_a)$.

The lumped heat capacity analysis gives results within an accuracy of 5% when

$$\frac{h(V/A)}{k_s} < 0.1 \tag{1.41}$$

where k_s is the thermal conductivity of the solid. It should be observed that *(V/A)* represents a characteristic dimension of the body. The above non-dimensional parameter can thus be rewritten as hL/k_s, which is known as the *Biot number*. The Biot number represents a ratio between conduction resistances within the body to convection resistance at the surface of the hot body.

1.6 Heat Convection

Having discussed the conduction mode of heat transfer, we now look at the heat transfer mechanism due to a fluid motion. This method of heat transfer, which is caused by fluid motion, is referred to as *heat convection*. The study of fluid motion (fluid dynamics) is an important subject that has wide application in many engineering disciplines. Several industries use computer-based fluid dynamics analysis (computational fluid dynamics or CFD) tools for both design and analysis. For instance, aerospace applications, turbo-machines, weather forecasting, electronic cooling arrangements and flow in heat exchangers are just a few examples. There has been a vast increase in the use of CFD tools in engineering industries in the past three decades, mainly because of the ever-increasing computing power.

It is obvious that flow with a higher incoming velocity will transport heat at a higher rate. The flow rate is often characterized by a quantity called the *Reynolds number*, which is defined as

$$Re = \frac{\rho_a u_a L}{\mu_a} \tag{1.42}$$

where u_a is the average inlet velocity, L is a characteristic dimension, for example, the width or height of the channel, ρ_a is a reference (inlet) density and μ_a is a reference (inlet) dynamic viscosity of the fluid. If the Reynolds number is small and below a certain critical value, the flow is laminar, and if it is above this critical number, then the flow becomes turbulent. The critical Reynolds number for pipe and channel flows, based on the diameter or height, is approximately 2000.

If the flow is forced into the channel by means of an external device, for example, a pump, then the convection process is referred to as *forced convection*, and the Reynolds number is normally high (Jaluria [9]; Massarotti et al. [10]; Minkowycz et al. [11]). In such situations, the fluid motion created by the density (or temperature) difference (buoyancy-driven motion) is negligibly small compared to the forced motion of the fluid. However, at low and moderate Reynolds numbers, the motion created by the local density (or temperature) differences in the fluid is comparable to that of the forced flow. A situation in which the forced and density-difference driven motions are equally important is called *mixed convection* transport (Aung and Worku [12]). If the forced flow is suddenly stopped and the fluid is stagnant inside the channel, then the fluid motion will be entirely influenced by the local density (or temperature) differences until an equilibrium state is

reached, that is, no local differences in density or temperature are present. Such a flow is often referred to as *natural, free* or *buoyancy-driven convection* (Nithiarasu *et al.* [13]). Also, the flow of fluid may be forced by external processes, or sometimes (in gravitational fields) by buoyancy forces caused when thermal energy expands the fluid (for example in a fire plume), thus influencing its own transfer. The latter process is often called "natural convection." All convective processes also move heat partly by diffusion. Another form of convection is forced convection. In this case, the fluid is forced to flow by the use of a pump, fan or other mechanical means.

The convection heat transfer mode is composed of two mechanisms. In addition to energy transfer due to *random molecular motion (diffusion)*, energy is also transferred by the *bulk* or *macroscopic motion* of the fluid. This fluid motion is associated with the fact that, at any instant, large numbers of molecules are moving collectively or as aggregates. Such motion, in the presence of a temperature gradient, contributes to the heat transfer. Because the molecules in the aggregate retain random motion, the total heat transfer is then due to a superposition of energy transport by the random motion of the molecules and by the bulk motion of the fluid. It is customary to use the term *convection* when referring to this cumulative transport and the term *advection* when referring to transport due to bulk fluid motion.

1.6.1 Flat Plate in Parallel Flow

Despite its simplicity, parallel flow over a flat plate (Figure 1.6) occurs in numerous engineering applications. As discussed earlier, laminar boundary layer development begins at the leading edge ($x = 0$) and transition to turbulence may occur at a downstream location (x_c) for which a critical Reynolds number $Re_{x,c}$ is achieved. We begin by considering conditions in the laminar boundary layer.

1.6.1.1 Laminar Flow Over an Isothermal Plate

The major convection parameters may be obtained by solving the appropriate form of the boundary layer equations. Assuming *steady, incompressible, laminar* flow with *constant fluid properties* and *negligible viscous dissipation* and recognizing that $dp/dx = 0$, the boundary layer equations reduce to
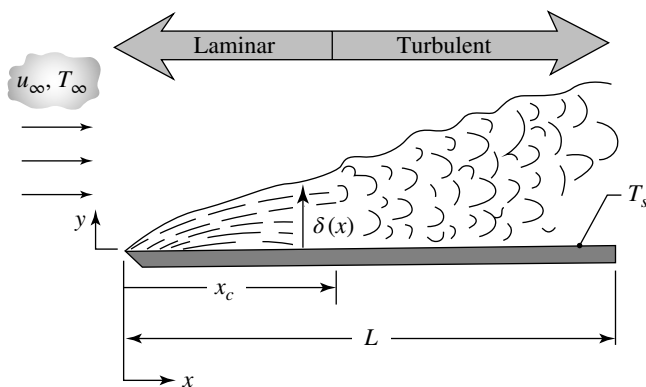

Figure 1.6 The flat plate in parallel flow (Incropera et al. [1]).

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1.43}$$

Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{1.44}$$

Energy:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{1.45}$$

Species:

$$u\frac{\partial \rho_A}{\partial x} + v\frac{\partial \rho_A}{\partial y} = D_{AB}\frac{\partial^2 \rho_A}{\partial y^2} \tag{1.46}$$

Solution of these equations is simplified by the fact that for constant properties, conditions in the velocity (hydrodynamic) boundary layer are independent of temperature and species concentration. For the solution procedure to solve the hydrodynamic problem, see *Fundamentals of Heat and Mass Transfer* by Incropera et al [4] and Blasius [14]. The local Nusselt number is of the form

$$Nu_x \equiv \frac{h_x x}{k} = 0.332\,Re_x^{1/2}\,Pr^{1/3} \qquad Pr \geq 0.6 \tag{1.47}$$

Integrating and substituting from Equation 1.47, it follows that $\bar{h}_x = 2h_x$, Hence

$$\overline{Nu_x} \equiv \frac{\bar{h}_x x}{k} = 0.664\,Re_x^{1/2}\,Pr^{1/3} \qquad Pr \geq 0.6 \tag{1.48}$$

Employing the heat and mass transfer analogy, it follows that

$$\overline{Sh}_x \equiv \frac{\bar{h}_{m,x} x}{D} = 0.664\,Re_x^{1/2}\,Sc^{1/3} \qquad Sc \geq 0.6 \tag{1.49}$$

If the flow is laminar over the entire surface, the subscript x may be replaced by L.

We see that, for laminar flow over a flat plate, the average friction and convection coefficients from the leading edge to a point x on the surface are twice the local coefficients at that point. We also note that, in using these expressions, the effect of variable properties can be treated by evaluating all properties at the film temperature, $T_f = (T_s + T_\infty)/2$. For fluids of small Prandtl number, namely, liquid metals, Equation 1.47 does not apply. However, for this case, the thermal boundary layer development is much more rapid than that of the velocity boundary layer $(\delta_t \gg \delta)$, and it is reasonable to assume uniform velocity $(u = u_\infty)$ throughout the thermal boundary layer. From a solution to the thermal boundary layer equation based on this assumption [5], it can then be shown that

$$Nu_x = 0.565\,Pe_x^{1/2} \qquad Pr \leq 0.05, Pe_x \geq 100 \tag{1.50}$$

where $Pe_x \equiv Re_x\,Pr$ is the Peclet number. Despite the corrosive and reactive nature of liquid metals, their unique properties (low melting point and low vapour pressure, as well as high thermal capacity and high conductivity) render them attractive as coolants in applications requiring high heat transfer rates.

A single correlating equation, which applies for all Prandtl numbers, has been recommended by Ozisik [6]. For laminar flow over an isothermal plate, the local convection coefficient may be obtained from

$$Nu_x = \frac{0.3387R\,e_x^{1/2}Pr^{1/3}}{\left[1 + (0.0468/Pr)^{2/3}\right]^{1/4}} \qquad Pe_x \geq 100 \tag{1.51}$$

with $\overline{Nu}_x = 2Nu_x$.

1.6.1.2 Turbulent Flow over an Isothermal Plate

From experiment [2], it is known that, for turbulent flows with Reynolds numbers up to approximately 10^8, the local friction coefficient is correlated to within 15% accuracy by an expression of the form

$$C_{f,x} = 0.0592Re_x^{-1/5} \qquad Re_{x,c} \leq Re_x \leq 10^8 \tag{1.52}$$

Moreover, it is known that, to a reasonable approximation, the velocity boundary layer thickness may be expressed as

$$\delta = 0.37xRe_x^{-1/5} \tag{1.53}$$

Comparing these results with those for the laminar boundary layer, Equations 1.48 and 1.51, we see that turbulent boundary layer growth is much more rapid (δ varies as $x^{4/5}$, in contrast to $x^{1/2}$ for laminar flow) and that the decay in the friction coefficient is more gradual ($x^{-1/5}$ vs $x^{-1/2}$). For turbulent flow, boundary layer development is influenced strongly by random fluctuations in the fluid and not by molecular diffusion. Hence relative boundary layer growth does not depend on the value of Pr or Sc, and Equation 1.53 may be used to obtain the thermal and concentration, as well as the velocity, boundary layer thicknesses. That is, for turbulent flow, $\delta \approx \delta_t \approx \delta_c$.

Using Equation 1.52 with the modified Reynolds (or Chilton and Colburn [15]) analogy, gives equations

$$\frac{C_f}{2} = StPr^{2/3} = j_H \quad 0.6 < Pr < 60 \tag{1.54}$$

$$\frac{C_f}{2} = St_m Sc^{2/3} = j_m \qquad 0.6 < Sc < 3000 \tag{1.55}$$

where j_H and j_m are the Colburn j factors for heat and mass transfer respectively. Then the local Nusselt number for turbulent flow is

$$Nu_x = St\,Re_x\,Pr = 0.0296\,Re_x^{4/5}\,Pr^{1/3} \qquad 0.6 \leq Pr \leq 60 \tag{1.56}$$

and the local Sherwood number is

$$Sh_x = St_m\,Re_x Sc = 0.0296\,Re_x^{4/5}\,Sc^{1/3} \quad 0.6 \le Sc \le 3000 \tag{1.57}$$

Enhanced mixing causes the turbulent boundary layer to grow more rapidly than the laminar boundary layer and to have larger friction and convection coefficients.

Expressions for the average coefficients may now be determined. However, since the turbulent boundary layer is generally preceded by a laminar boundary layer, we first consider mixed boundary layer conditions.

1.6.1.3   Boundary Layer Development Over Heated Plate

We are especially interested in convection heat transfer, which occurs between a fluid in motion and a bounding surface when the two are at different temperatures. Consider fluid flow over the heated surface of Figure 1.7. A consequence of the fluid–surface interaction is the development of a region in the fluid through which the velocity varies from zero at the surface to a finite value u_∞ associated with the flow. This region of the fluid is known as the *hydrodynamic*, or *velocity, boundary layer*. Moreover, if the surface and flow temperatures differ, there will be a region of the fluid through which the temperature varies from T_s at $Y = 0$ to T_∞ in the outer flow. This region, called the thermal boundary layer, may be smaller, larger or the same size as that through which the velocity varies. In any case, if $T_s > T_\infty$, convection heat transfer will occur from the surface to the outer flow.

The convection heat transfer mode is sustained both by random molecular motion and by the bulk motion of the fluid within the boundary layer. The contribution due to random molecular motion (diffusion) dominates near the surface where the fluid velocity is low. In fact, at the interface between the surface and the fluid ($y = 0$), the fluid velocity is zero and heat is transferred by this mechanism only. The contribution due to bulk fluid motion originates from the fact that the boundary layer grows as the flow progresses in the x direction. In effect, the heat that is conducted into this layer is swept downstream and is eventually transferred to the fluid outside the boundary layer. Appreciation of boundary layer phenomena is essential to understanding convection heat transfer. It is for this reason that the discipline of fluid
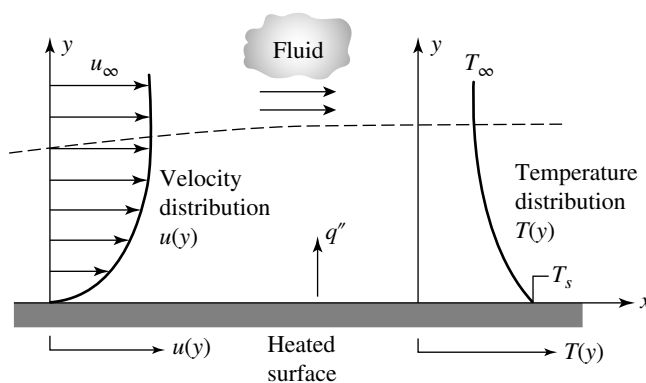

Figure 1.7 Boundary layer development in convection heat transfer.

Table 1.1 Typical values of the convection heat transfer coefficient.

Process	$h(\text{W/m}^2 \cdot \text{K})$
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–45,000

mechanics will play a vital role in our later analysis of convection. Regardless of the particular nature of the convection heat transfer process, the appropriate rate equation is of the form

$$q'' = h(T_s - T_\infty) \tag{1.58}$$

where q'', the convective heat flux (W/m^2), is proportional to the difference between the surface and fluid temperatures, T_s and T_∞, respectively. This expression is known as Newton's law of cooling, and the parameter h $(\text{W/m}^2 \cdot \text{K})$ is termed the *convection heat transfer coefficient*. It depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion and an assortment of fluid thermodynamic and transport properties. Any study of convection ultimately reduces to a study of the means by which h may be determined. In the solution of such problems we presume h to be known, using typical values given in Table 1.1. When Equation 1.58 is used, the convection heat flux is presumed to be positive if heat transferred from the surface $(T_s > T_\infty)$ and negative if heat is transferred to the surface $(T_\infty > T_s)$. However, if $T_\infty > T_s$, there is nothing to preclude us from expressing Newton's law of cooling as

$$q'' = h(T_\infty - T_s) \tag{1.59}$$

in which case heat transfer is positive if it is to the surface.

1.6.2   Internal Flow

Having acquired the means to compute convection transfer rates for external flow, we now consider the convection transfer problem for internal flow. In contrast, an internal flow, such as flow in a pipe, is one for which the fluid is confined by a surface. Hence the boundary layer is unable to develop without eventually being constrained. The internal flow configuration represents a convenient geometry for heating and cooling fluids used in chemical processing, environmental control and energy conversion technologies.

Our objectives are to develop an appreciation for the physical phenomena associated with internal flow and to obtain convection coefficients for flow conditions of practical importance. We begin by considering velocity (hydrodynamic) effects pertinent to internal flows, focusing on certain unique features of boundary layer development. Thermal boundary layer effects are considered next, and an overall energy balance is applied to determine fluid temperature variations in the flow direction. Finally, correlations for estimating the convection heat transfer coefficient are presented for a variety of internal flow conditions including the flow of supercritical fluids through tubes.

1.6.2.1 Hydrodynamic Considerations

When considering external flow, it is necessary to ask only whether the flow is laminar or turbulent. However, for an internal flow we must also be concerned with the existence of entrance and fully developed regions.

1.6.2.2 Flow Conditions

Consider laminar flow in a circular tube of radius r_o (Figure 1.8), where fluid enters the tube with a uniform velocity. We know that when the fluid makes contact with the surface, viscous effects become important, and a boundary layer develops with increasing x. This development occurs at the expense of a shrinking inviscid flow region and concludes with boundary layer merger at the centreline. Following this merger, viscous effects extend over the entire cross-section, and the velocity profile no longer changes with increasing x. The flow is then said to be fully developed, and the distance from the entrance at which this condition is achieved is termed the hydrodynamics entry length, $x_{fd,h}$. As shown in Figure 1.8, the fully developed velocity profile is parabolic for laminar flow in a circular tube. For turbulent flow, the profile is flatter, due to turbulent mixing in the radial direction.

When dealing with internal flows, it is important to be cognizant of the extent of the entry region, which depends on whether the flow is laminar or turbulent. The Reynolds number for flow in a circular tube is defined as

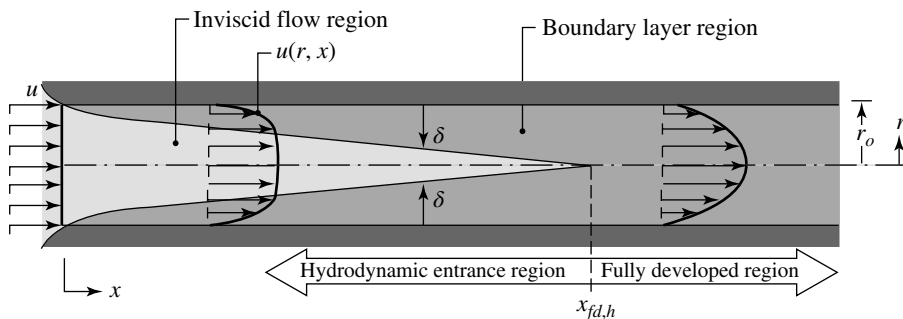$$Re_D = \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} \tag{1.60}$$

Figure 1.8 Laminar, hydrodynamic boundary layer development in a circular tube.

where u_m is the mean fluid velocity over the tube cross-section and D is the tube diameter. In a fully developed flow, the critical Reynolds number corresponding to the onset of turbulence is

$$Re_{D,C} \approx 2300 \tag{1.61}$$

although much larger Reynolds numbers ($Re_D \approx 10{,}000$) are needed to achieve fully turbulent conditions. The transition to turbulence is likely to begin in the developing boundary layer of the entrance region.

For laminar flow ($Re_D \approx 2300$), the hydrodynamic entry length may be obtained from an expression of the form (Incropera et al.[1])

$$\left(\frac{x_{fd,h}}{D} \right)_{lam} \approx 0.05\, Re_D \tag{1.62}$$

This expression is based on the presumption that fluid enters the tube from a rounded converging nozzle and is hence characterized by a nearly uniform velocity profile at the entrance (Figure 1.8). Although there is no satisfactory general expression for the entry length in turbulent flow, we know that it is approximately independent of Reynolds number and that, as a first approximation by Bejan [2],

$$10 \leq \left(\frac{x_{fd,h}}{D} \right)_{turb} \leq 60 \tag{1.63}$$

For the purposes of this text, we shall assume fully developed turbulent flow for $(x/D) > 10$.

1.6.2.3 Mean Velocity

The velocity varies over the cross-section and there is no well-defined free stream, it is necessary to work with a mean velocity u_m when dealing with internal flows. This velocity is defined such that, when multiplied by the fluid density ρ and the cross-sectional area of the tube A_c, it provides the rate of mass flow through the tube. Hence

$$\dot{m} = \rho u_m A_c \tag{1.64}$$

For steady, incompressible flow in a tube of uniform cross-sectional area, $\dot{m}$ and u_m are constants independent of x. From Equations 1.60 and 1.64 it is evident that, for flow in a circular tube ($A_c = \pi D^2/4$), the Reynolds number reduces to

$$Re_D = \frac{4\dot{m}}{\pi D \mu} \tag{1.65}$$

Since the mass flow rate may also be expressed as the integral of the mass flux (ρu) over the cross-section

$$\dot{m} = \int \rho u(r,x) dA_c \tag{1.66}$$

It follows that, for incompressible flow in a circular tube,

$$u_m = \frac{\int \rho u(r,x) dA_c}{\rho A_c} = \frac{2\pi\rho}{\rho \pi\, r_0^2} \int_0^{r_o} u(r,x) r\, dr = \frac{2}{r_0^2} \int_0^{r_o} u(r,x) r\, dr \tag{1.67}$$

The above expression may be used to determine u_m at any axial location x, from knowledge of the velocity profile $u(r)$ at that location.

1.6.2.4 Velocity Profile in the Fully Developed Region

The form of the velocity profile may readily be determined for the laminar flow of an incompressible, constant property fluid in the fully developed region of a circular tube. An important feature of hydrodynamic conditions in the fully developed region is that both the radial velocity component v and the gradient of the axial velocity component $(\partial u/\partial x)$ are everywhere zero.

$$v = 0 \text{ and } \left(\frac{\partial u}{\partial x}\right) = 0 \tag{1.68}$$

Hence the axial velocity component depends only on r, $u(x,r) = u(r)$.

The radial dependence of the axial velocity may be obtained by solving the appropriate form of the x-momentum equation. This form is determined by first recognizing that, for the conditions of Equation 1.68, the net momentum flux is everywhere zero in the fully developed region. Hence the momentum conservation requirement reduces to a simple balance between shear and pressure forces in the flow. For the annular differential elements of Figure 1.9, this force balance may be expressed as

$$\tau_r\,(2\pi r\,dx) - \left\{\tau_r(2\pi r\,dx) + \frac{d}{dr}\,[\tau_r(2\pi r\,dx)]dx\right\}$$
$$+ p(2\pi r\,dr) - \left\{p(2\pi r\,dr) + \frac{d}{dx}\,[p(2\pi r\,dr)dx]\right\} = 0 \tag{1.69}$$

which reduces to

$$-\frac{d}{dr}(r\tau_r) = r\frac{dp}{dx} \tag{1.70}$$

with $y = r_o - r$, Newton's law of velocity, the equation

$$\tau_{xy} = \tau_{yx} = \mu\left[\frac{\partial u}{\partial y} + \frac{\partial r}{\partial x}\right]$$
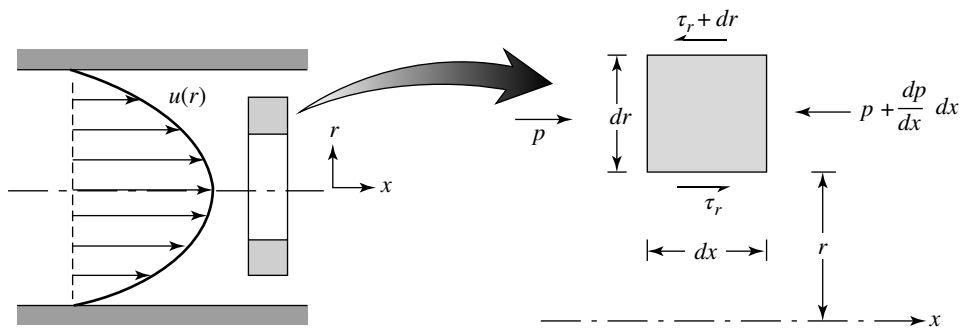

Figure 1.9 Force balance on a differential element for laminar, fully developed flow in a circular tube.

assumes the form

$$\tau_r = -\mu \frac{du}{dr} \tag{1.71}$$

and Equation 1.70 becomes

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dp}{dx} \tag{1.72}$$

Since the axial pressure gradient is independent of r, Equation 1.72 may be solved by integrating twice to obtain

$$r\frac{du}{dr} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{2} + C_1 \tag{1.73}$$

and

$$u(r) = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{4} + C_1 \ln r + C_2 \tag{1.74}$$

The integration constant may be determined by invoking the boundary conditions

$$u(r_o) = 0 \quad \text{and} \quad \frac{\partial u}{\partial r}\Big|_{r=0} = 0 \tag{1.75}$$

which respectively, impose the requirements of zero slip at the tube surface and radial symmetry about the centreline. It is a simple matter to evaluate the constants, and it follows that

$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right)r_o^2\left[1 - \left(\frac{r}{r_o}\right)^2\right] \tag{1.76}$$

Hence the fully developed velocity profile is parabolic. Note that the pressure gradient must always be negative.

The foregoing result may be used to determine the mean velocity of the flow. Substituting Equation 1.76 into Equation 1.68 and integrating, we obtain

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx} \tag{1.77}$$

Substituting this result into Equation 1.69, the velocity profile is then

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right] \tag{1.78}$$

Since u_m can be computed from knowledge of the mass flow rate, Equation 1.77 can be used to determine the pressure gradient.

1.6.3 Forced Convection Relationships

The summary of forced convection relationships (from *Handbook of Heat Transfer* by Rohsenow et al. [16]) are presented here for easy reference.

- Laminar flow entrance length:

$$\frac{X/D}{Re_D} \approx 10^{-2} \tag{1.79}$$

- Skin friction coefficient definition:

$$C_{f,x} = \frac{\tau_W}{\frac{1}{2}\rho U^2} \tag{1.80}$$

- Laminar fully developed (Hagen–Poiseuille) flow between parallel plates with spacing *D:*

$$u(y) = \frac{3}{2}U\left[1 - \left(\frac{y}{D/2}\right)^2\right] \tag{1.81}$$

with

$$U = \frac{D^2}{12\mu}\left(-\frac{dP}{dx}\right) \tag{1.82}$$

- Laminar fully developed (Hagen–Poiseuille) flow in a tube with diameter *D*:

$$u = 2U\left[1 - \left(\frac{r}{r_0}\right)^2\right] \tag{1.83}$$

with

$$U = \frac{r_0^2}{8\mu}\left(-\frac{dP}{dx}\right) \tag{1.84}$$

- Hydraulic radius and diameter:

$$r_h = \frac{A}{p} \quad \text{hydraulic radius} \tag{1.85}$$

$$D_h = \frac{4A}{p} \quad \text{hydraulic diameter} \tag{1.86}$$

- Friction factor:

$$f = \begin{cases} \dfrac{\tau_w}{\frac{1}{2}\rho U^2} & \tag{1.87} \\[2ex] \dfrac{24}{Re_{Dh}} & D_h = 2D \ \text{ parallel plates } (D = \text{spacing}) \tag{1.88} \\[2ex] \dfrac{16}{Re_{Dh}} & D_h = D \ \text{ round tube } (D = \text{diameter}) \tag{1.89} \end{cases}$$

$$\Delta P = f \frac{4L}{D_h} \left(\frac{1}{2} \rho U^2 \right) \tag{1.90}$$

- Nusselt number:

$$N_u = \frac{hD}{k} = D \frac{\partial T / \partial r \big|_{r=r_0}}{T_0 - T_m} \tag{1.91}$$

- Laminar thermal entrance length:

$$X_T \approx 10^{-2} Pr.D_h.Re_{Dh} \tag{1.92}$$

- Thermally developing Hagen–Poiseuille flow ($Pr = \infty$):
  - Round tube, isothermal wall:

$$Nu_x = \begin{cases} 1.077 \, x_*^{-1/3} - 0.70 & x_* \le 0.01 \\ 3.657 + 6.874 \left(10^3 x_* \right)^{-0.488} e^{-57.2 x_*} & x_* > 0.01 \end{cases} \tag{1.93}$$

$$Nu_{0-x} = \begin{cases} 1.615 x_*^{-1/3} - 0.70 & x_* \le 0.05 \\ 1.615 x_*^{-1/3} - 0.20 & 0.005 < x_* \le 0.03 \\ 3.657 + 0.0499/x_* & x_* > 0.03 \end{cases} \tag{1.94}$$

  - Round tube, uniform heat flux:

$$Nu_x = \begin{cases} 3.302 \, x_*^{-1/3} - 1.0 & x_* \le 0.00005 \\ 1.302 x_*^{-1/3} - 0.5 & 0.0005 < x_* \le 0.0015 \\ 4.364 + 8.68 (10^3 x_*)^{-0.506} e^{-41 x_*} & x_* > 0.001 \end{cases} \tag{1.95}$$

$$Nu_{0-x} = \begin{cases} 1.953 \, x_*^{-1/3} & x_* \le 0.03 \\ 4.364 + 0.0722/x_* & x_* > 0.03 \end{cases} \tag{1.96}$$

  - Parallel plates, isothermal surfaces:

$$Nu_{0-x} = \begin{cases} 1.233 \, x_*^{-1/3} + 0.40 & x_* \le 0.001 \\ 7.541 + 6.874 \left(10^3 x_* \right)^{-0.488} e^{-245 x_*} & x_* > 0.01 \end{cases} \tag{1.97}$$

$$Nu_{0-x} = \begin{cases} 1.849 \, x_*^{-1/3} & x_* \le 0.00005 \\ 1.849 x_*^{-1/3} & 0.0005 < x_* \le 0.006 \\ 7.541 + 0.0235/x_* & x_* > 0.006 \end{cases} \tag{1.98}$$

○ Parallel plates, uniform heat flux:

$$
Nu_x = \begin{cases}
1.490\, x_*^{-1/3} & x_* \le 0.0002 \\
1.490\, x_*^{-1/3} - 0.40 & 0.0002 < x_* \le 0.0001 \\
8.235 + 8.68\,(10^3 x_*)^{-0.506}\, e^{-164 x_*} & x_* > 0.001
\end{cases}
\tag{1.99}
$$

$$
Nu_{0-x} = \begin{cases}
2.236\, x_*^{-1/3} & x_* \le 0.001 \\
2.236\, x_*^{-1/3} + 0.90 & 0.001 < x_* \le 0.01 \\
8.235 + 0.0364/x_* & x_* > 0.01
\end{cases}
\tag{1.100}
$$

- Thermally and hydraulically developing flow:
  ○ Round tube, isothermal wall:

$$
Nu_x = 7.55 + \frac{0.024\, x_*^{-1.14}\left(0.0179\,\mathrm{Pr}^{0.17}\, x_*^{-0.64} - 0.14\right)}{\left(1 + 0.0358\,\mathrm{Pr}^{0.17}\, x_*^{-0.64}\right)^2}
\tag{1.101}
$$

$$
Nu_{0-x} = 7.55 + \frac{0.024\, x_*^{-1.14}}{1 + 0.0358\,\mathrm{Pr}^{0.17}\, x_*^{-0.64}}
\tag{1.102}
$$

$$
\frac{\Delta P}{\frac{1}{2}\rho U^2} = 13.74\,(x_+)^{1/2} + \frac{1.25 + 64 x_+ - 13.74(x_+)^{1/2}}{1 + 0.00021(x_+)^{-2}}
\tag{1.103}
$$

$$
x_+ = \frac{x/D}{\mathrm{Re}_D}
\tag{1.104}
$$

○ Round tube, uniform heat flux:

$$
\frac{Nu_x}{4.364\left[1 + (Gz/29.6)^2\right]^{1/6}} = \left\{1 + \left[\frac{Gz/19.04}{\left[1 + (Pr/0.0207)^{1/2}\right]\left[1 + (Gz/29.6)^2\right]^{1/3}}\right]^{3/2}\right\}^{1/3}
\tag{1.105}
$$

- Optimal channel sizes:
  ○ Laminar flow, parallel plates:

$$
\frac{D_{opt}}{L} \approx 2.7\, Be^{-1/4} \qquad Be = \frac{\Delta P L^2}{\mu \alpha}
\tag{1.106}
$$

$$
\frac{q_{\max}}{HLW} \approx 0.60\, \frac{k}{L^2}(T_{\max} - T_0)\, Be^{1/2}
\tag{1.107}
$$

- Staggered plates:

$$\frac{D_{opt}}{L} \approx 5.4 Pr^{-1/4} \left(Re_L \frac{L}{b} \right)^{-1/2} \tag{1.108}$$

for the range

$$P_r = 0.72 \quad 10^2 \le Re_L \le 10^4 \quad 0.5 \le \frac{Nb}{L} \le 1.3$$

- Bundle of cylinders in cross flow:

$$\frac{S_{opt}}{D} \simeq 1.59 \frac{(H/D)^{0.52}}{\widetilde{P}^{0.13} . Pr^{0.24}} \quad \widetilde{P} = \frac{\Delta P D^2}{\mu v} \tag{1.109}$$

for the range

$$0.72 \le Pr \le 50 \quad 10^4 \le \widetilde{P} \le 10^8 \quad 25 \le \frac{H}{D} \le 200$$

$$\frac{S_{opt}}{D} \simeq 1.70 \frac{(H/D)^{0.52}}{R e_D^{0.26} . Pr^{0.24}} \tag{1.110}$$

$$\frac{T_D - T_\infty}{q D/kLW} \simeq \frac{4.5}{R e_D^{0.90} . Pr^{0.64}} \tag{1.111}$$

with

$$Re_D = U_\infty \frac{D}{\nu} \quad 140 \le Re_D \le 14{,}000$$

- Array of pin fins with impinging flow:

$$\frac{S_{opt}}{L} \simeq 0.81 Pr^{-0.25} . Re_L^{-0.32} \tag{1.112}$$

for the range

$$0.06 \le \frac{D}{L} \le 0.14 \quad 0.28 \le \frac{H}{L} \le 0.56 \quad 0.72 \le Pr \le 7$$
$$10 \le Re_D \le 700 \quad 90 \le Re_L \le 6000$$

- Turbulent duct flow:

$$\frac{D_{opt}/L}{\left(1 + t/D_{opt}\right)^{4/11}} = 0.071 Pr^{-5/11} . Be^{-1/11} \tag{1.113}$$

$$\frac{q_{\max}}{HLW} \le 0.57 \frac{k}{L^2} (T_{\max} - T_0) Pr^{4/99} \left(1 + \frac{t}{D_{opt}} \right)^{-67/99} . Be^{47/99} \tag{1.114}$$

with

$$Be = \frac{\Delta P L^2}{\mu \alpha}$$

for the range $10^4 \le Re_{Dh} \le 10^6$ $10^6 \le Re_L \le 10^8$ $10^{11} \le Be \le 10^{16}$

- Turbulent flow and entrance lengths:

$$\frac{X}{D} \simeq 10 \simeq \frac{X_T}{D} \tag{1.115}$$

- Turbulent flow friction factor:

$$f \simeq 0.046 Re_D^{-1/5} \quad 2 \times 10^4 \le Re_D \le 10^5 \tag{1.116}$$

- Turbulent flow heat transfer:

$$St.Pr^{2/3} \simeq \frac{f}{2} \tag{1.117}$$

for $Pr \ge 0.5$

$$Nu_D = \frac{hD}{k} = 0.023 Re_D^{4/5}.Pr^{1/3} \tag{1.118}$$

for $Pr \ge 0.50$

$$2 \times 10^4 \le Re_D \le 10^6$$

$$Nu_D = 0.023 Re_D^{4/5}.Pr^n \tag{1.119}$$

where n = 0.4 for heating the fluid and n = 0.3 for cooling the fluid in the range

$$\frac{L}{D} > 60 \quad 0.7 \le Pr \le 120 \quad 2500 \le Re_D \le 1.24 \times 10^5$$

$$Nu_D = 0.027 Re_D^{4/5}.Pr^{1/3} \left(\frac{\mu}{\mu_0}\right)^{0.14} \tag{1.120}$$

in the range

$$0.7 \le Pr \ge 16{,}700 \quad Re_D \ge 10^4$$

Here

$\mu_0 = \mu(T_0)$ (T_0 is the wall temperature)
$\mu = \mu(T_m)$ (T_m is the bulk temperature)

$$Nu_D = \frac{(f/2)Re_D.Pr}{1.07 + 900/Re_D - 0.63/(1 + 10Pr) + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)} \tag{1.121}$$

$$Nu_D = \frac{(f/2)Re_D.Pr}{1.07 + 12.7(f/2)^{1/2}(Pr^{2/3} - 1)} \tag{1.122}$$

where

$$0.5 \le Pr \le 10^6 \quad 4000 \le Re_D \le 5 \times 10^6$$

and f from Figure 5.13 from reference [16].

$$Nu_D = \frac{(f/2)(\text{Re}_D - 10^3)\,\text{Pr}}{1 + 12.7(f/2)^{1/2}\left(\text{Pr}^{2/3} - 1\right)} \tag{1.123}$$

where

$$0.5 \leq \text{Pr} \leq 10^6 \qquad 2300 \leq Re_D \leq 5 \times 10^6$$

and f from Figure 5.13 from reference [16].

$$Nu_D = 0.0214\left(Re_D^{0.8} - 100\right)Pr^{0.4} \tag{1.124}$$

where

$$0.5 \leq \text{Pr} \leq 1.5 \qquad 10^4 \leq \text{Re}_D \leq 5 \times 10^6$$

$$Nu_D = 0.012\left(Re_D^{0.87} - 280\right)Pr^{0.4} \tag{1.125}$$

where

$$1.5 \leq \text{Pr} \leq 500 \qquad 3 \times 10^3 \leq \text{Re}_D \leq 10^6$$

$$Nu_D = \begin{cases} 6.3 + 0.0167\,Re_D^{0.85} \cdot Pr^{\,0.93} \ \ q''_0 = constant & (1.126) \\ 4.8 + 0.0156\,Re_D^{0.85} \cdot Pr^{\,0.93} \ \ T_0 = constant & (1.127) \end{cases}$$

where, for Equations (1.154) and (1.155),

$$0.004 \leq Pr \leq 0.1 \qquad 10^4 \leq Re_D \leq 10^6$$

- Total heat transfer rate:

$$q = hA_w \Delta T_{lm} \tag{1.128}$$

- Isothermal wall:

$$\Delta T_{lm} = \frac{\Delta T_{in} - \Delta T_{out}}{In(\Delta T_{in}/\Delta T_{out})} \tag{1.129}$$

$$q = \dot{m}c_p \Delta T_{in}\left(1 - e^{-hA_w/\dot{m}\,c_p}\right) \tag{1.130}$$

- Uniform heat flux:

$$\Delta T_{lm} = \Delta T_{in} = \Delta T_{out} \tag{1.131}$$

1.7 Radiation

Thermal radiation is energy *emitted* by matter that is at a non-zero temperature. Although we will focus on radiation from solid surfaces, emission may also occur from liquids and gases. Regardless of the form of matter, the emission may be attributed to changes in the electron configurations of the constituent atoms or molecules. The energy of the radiation field is transported by electromagnetic waves (or alternatively, photons).
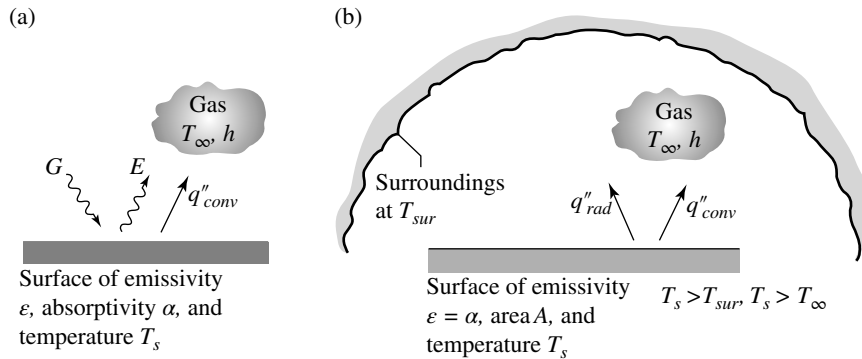
Figure 1.10 Radiation exchange: (a) at a surface and (b) between a surface and large surroundings.

While the transfer of energy by conduction or convection requires the presence of a material medium, radiation does not. In fact, radiation transfer occurs most efficiently in a vacuum.

Consider radiation transfer processes for the surface of Figure 1.10. Radiation that is emitted by the surface originates from the thermal energy of matter bounded by the surface, and the rate at which energy is released per unit area (W/m^2) is termed the surface emissive power E. There is an upper limit to the emissive power, which is prescribed by the *Stefan–Boltzmann law*

$$E_b = \sigma T_s^4 \tag{1.132}$$

where T_s is the absolute temperature (K) of the surface and σ is the *Stefan–Boltzmann constant* ($\sigma = 5.67 \times 10^{-8} \dfrac{\text{W}}{\text{m}^2} \cdot \text{K}^4$). Such a surface is called an ideal radiator or *blackbody*.

The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by

$$E = \varepsilon \sigma T_s^4 \tag{1.133}$$

where ε is a radiative property of the surface, termed the *emissivity*. With values in the range $0 \le \varepsilon \le 1$, this property provides a measure of how efficiently a surface emits energy relative to a blackbody. It depends strongly on the surface material and finish, and representative values are provided in Appendix A7.

Radiation may also be *incident* on a surface from its surroundings. The radiation may originate from a special source, such as the sun, or from other surfaces to which the surface of interest is exposed. Irrespective of the source(s), we designate the rate at which all such radiation is incident on a unit area of the surface as the *irradiation G* (Figure 1.10).

A portion, or all, of the irradiation may be absorbed by the surface, thereby increasing the thermal energy of the material. The rate at which radiant energy is absorbed per unit surface area may be evaluated from knowledge of a surface radiative property termed the absorptivity α. That is,

$$G_{abs} = \alpha G \tag{1.134}$$

where $0 \leq \alpha \leq 1$. If $\alpha < 1$ and the surface is *opaque*, portions of the irradiation are *reflected*. If the surface is semi-transparent, portions of the irradiation may also be *transmitted*. However, while absorbed and emitted radiation increase and reduce, respectively, the thermal energy of matter, reflected and transmitted radiation have no effect on this energy. Note that the value of α depends on the nature of the irradiation, as well as on the surface itself. For example, the absorptivity of a surface may differ from its absorptivity to radiation emitted by the walls of a furnace.

In many engineering problems (a notable exception being problems involving solar radiation or radiation from other very high temperature sources), liquids can be considered opaque, and gases can be considered transparent, to radiation heat transfer (Siegel and Howell [17] and Incropera et al. [1]). Solids can be opaque (as is the case for metals) or *semi-transparent* (as is the case for thin sheets of some polymers and some semiconducting materials). A special case that occurs frequently involves radiation exchange between a small surface at T_s and a much larger, isothermal surface that completely surrounds the smaller one (Figure 1.11). The surroundings could, for example, be the walls of a room or a furnace, whose temperature T_{sur} differs from that of an enclosed surface ($T_{sur} \neq T_s$). The irradiation may be approximated by emission from a blackbody at T_{sur}, in which case $G = \sigma T_{sur}^4$. If the surface is assumed to be one for which $\alpha = \varepsilon$ (a grey surface), the net rate of radiation heat transfer *from* the surface, expressed per unit area of the surface, is

$$q''_{rad} = \frac{q}{A} = \varepsilon E_b(T_s) - \alpha G = \varepsilon \sigma \left(T_s^4 - T_{sur}^4 \right) \tag{1.135}$$

This expression provides the difference between thermal energy that is released due to radiation emission and that which is gained due to radiation absorption.

There are many applications for which it is convenient to express the net radiation heat exchange emission and that which is gained due to radiation absorption.

$$q_{rad} = h_r A (T_s - T_{sur}) \tag{1.136}$$

where, from Equation 1.135, the radiation heat transfer coefficient h_r is

$$h_r \equiv \varepsilon \sigma (T_s + T_{sur}) \left(T_s^2 + T_{sur}^2 \right) \tag{1.137}$$

Here we have modelled the radiation mode in a manner similar to convection. In this sense we have linearized the radiation rate equation, making the heat rate proportional to a temperature difference rather than to the difference between two temperatures to the fourth power. Note, however, that h_r depends strongly on temperature, while the temperature dependence of the convection heat transfer coefficient h is generally weak.

The surfaces of Figure 1.11 may also simultaneously transfer heat by convection to an adjoining gas. For the conditions of Figure 1.11, the total rate of heat transfer from the surface is then

$$q = q_{conv} + q_{rad} = hA (T_s - T_\infty) + \varepsilon A \sigma \left(T_s^4 - T_{sur}^4 \right) \tag{1.138}$$

1.7.1   Radiation – Fundamental Concepts

A representation of the exchanges of energy between the source (the Sun) are, the Earth's surface, the Earth's atmosphere, and the ultimate sink outer space is called the 'greenhouse effect'. The ability of the atmosphere to capture and recycle energy emitted by
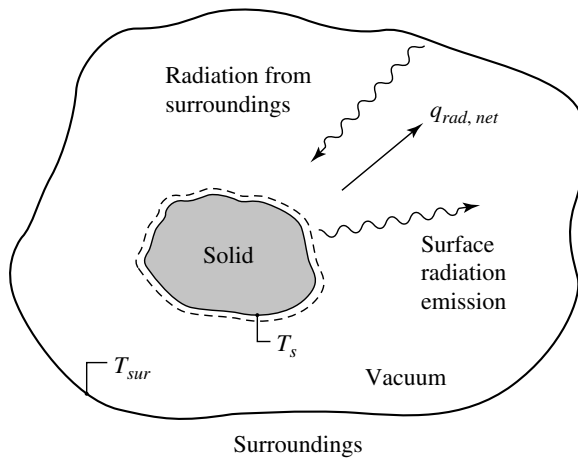
Figure 1.11 Radiation cooling of a heated solid.

the Earth surface is the defining characteristic of the greenhouse effect. The greenhouse effect is a process by which thermal radiation from a planetary surface is absorbed by atmospheric greenhouse gases, and is re-radiated in all directions. Since part of this re-radiation is back towards the surface and the lower atmosphere, it results in an elevation of the average surface temperature above what it would be in the absence of those gases.

Consider a solid that is initially at a higher temperature T_s than that of its surroundings T_{sur}, but around which there exists a vacuum (Figure 1.11). The presence of the vacuum precludes energy loss from the surface of the solid by conduction or convection. However, our intuition tells us that the solid will cool and eventually achieve thermal equilibrium with its surroundings. This cooling is associated with a reduction in the internal energy stored by the solid and is a direct consequence of the emission of thermal radiation from the surface. In turn, the surface will intercept and absorb radiation originating from the surroundings. However, if $T_s > T_{sur}$ the net heat transfer rate by radiation $q_{rad,\ net}$ is from the surface, and the surface will cool until T_s reaches T_{sur}.

We associated thermal radiation with the rate at which energy is emitted by matter as a result of its finite temperature. At this moment, thermal radiations is being emitted by all the matter that surrounds you: by the furniture and walls of the room, if you are indoors, or by the ground, the buildings and the atmosphere and sun if you are outdoors. The mechanism of emission is related to energy released as a result of oscillations or transitions of the many electrons that constitute matter. These oscillations are, in turn, sustained by the internal energy, and therefore the temperature, of the matter. Hence we associated the emission of thermal radiation with thermally excited conditions within the matter.

All forms of matter emit radiation. For gases and for semi-transparent solids, such as glass and salt crystals at elevated temperatures, emission is a volumetric phenomenon, as illustrated in Figure 1.12. That is, radiation emerging from a finite volume of matter is the integrated effect of local emission through the volume. However, in this text we concentrate on situations for which radiation is a surface phenomenon. In most solids and
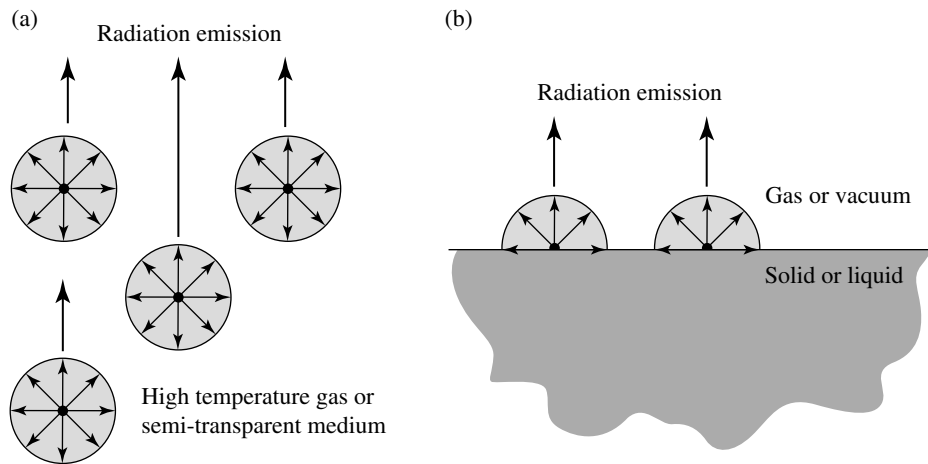
(a)

Radiation emission

(b)

Radiation emission

Gas or vacuum

Solid or liquid

High temperature gas or
semi-transparent medium

Figure 1.12 The emission process: (a) as a volumetric phenomenon, (b) as a surface phenomenon.

liquids, radiation emitted from interior molecules is strongly absorbed by adjoining molecules. Accordingly, radiation that is emitted from a solid or a liquid originates from molecules that are within a distance of approximately 1 μm from the exposed surface. It is for this reason that emission from a solid or a liquid into an adjoining gas or a vacuum can be viewed as a surface phenomenon, except in situations involving nano-scale or micro-scale devices.

We know that radiation is due to emission by matter and that its subsequent transport does not require the presence of any matter. But what is the nature of this transport? One theory views radiation as the propagation of a collection of particles termed photons or quanta. Alternatively, radiation may be viewed as the propagation of electromagnetic waves. In any case, we wish to attribute to radiation the standard wave properties of frequency ν and wavelength λ. For radiation propagating in a particular medium, the two properties are related by

$$\lambda = \frac{c}{\nu} \tag{1.139}$$

where c is the speed of light in the medium. For propagation in a vacuum, $c_0 = 2.998 \times 10^8 \text{ m/s}$. The unit of wavelength is commonly the micrometer (μm), where $1\mu\text{m} = 10^{-6}\text{m}$.

The complete electromagnetic spectrum is delineated in Figure 1.13. The short wavelength gamma rays, X-rays, and ultraviolet (UV) radiation are primarily of interest to the high-energy physicist and the nuclear engineer, while the long wavelength microwaves and radio waves ($\lambda > 10^5\mu\text{m}$) are of concern to the electrical engineer. It is the intermediate portion of the spectrum, which extends from approximately 0.1 to 100 μm and includes a portion of the UV and all of the visible and infrared (IR), that is termed thermal radiation because it is both caused by and affects the thermal state or temperature of matter. For this reason, thermal radiation is pertinent to heat transfer.

Thermal radiation emitted by a surface encompasses a range of wavelengths. As shown in Figure 1.14a, the magnitude of the radiation varies with wavelength, and the term
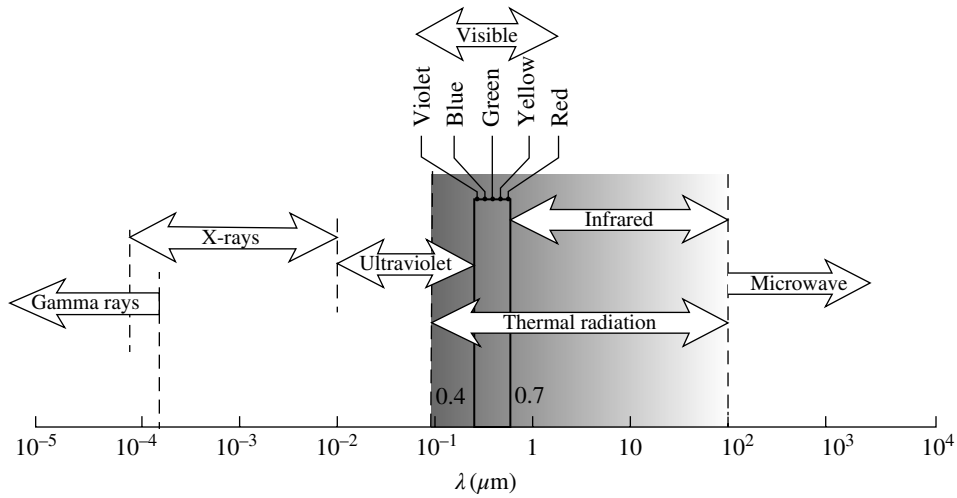
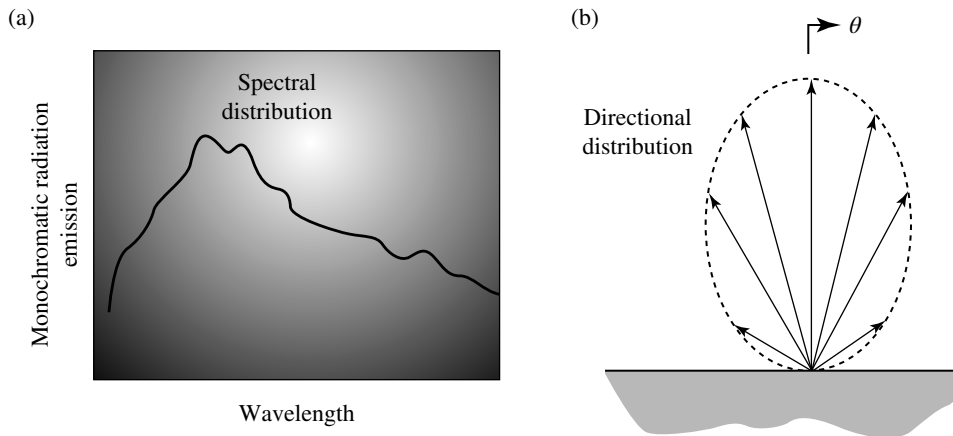**Figure 1.13** Spectrum of electromagnetic radiation.

Figure 1.14 Radiation emitted by a surface: (a) spectral distribution, (b) directional distribution.

spectral is used to refer to the nature of this dependence. As we will find, both the magnitude of the radiation at any wavelength and the spectral distribution vary with the nature and temperature of the emitting surface.

The spectral nature of thermal radiation is one of two features that complicate its description. The second feature relates to its directionality. As shown in Figure 1.14b, a surface may emit preferentially in certain directions, creating a directional distribution of the emitted radiation. To quantify the emission, absorption, reflection and transmission concepts, we must be able to consider both spectral and directional effects.

Radiative cooling is the process by which a body loses heat by radiation. Outgoing energy is an important effect in the Earth's energy budget. In the case of the Earth-atmosphere system, it refers to the process by which long-wave (infrared) radiation is

emitted to balance the absorption of short-wave (visible) energy from the Sun. Convective transport of heat and evaporative transport of latent heat both remove heat from the surface and redistribute it in the atmosphere.

Example 1.2 An un-insulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, respectively. What are the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 W/m² · K, what is the rate of heat loss from the surface per unit length of pipe?

Known: Un-insulated pipe of prescribed diameter, emissivity and surface temperature in a room with fixed wall and air temperatures
   Find:

Surface emissive power and irradiation
Pipe heat loss per unit length, q'

   Schematic: Figure 1.15
   Assumptions:

Steady-state conditions
Radiation exchange between the pipe and the room is between a small surface and a
   much larger enclosure
Surface emissivity and absorptivity are equal

   Analysis:
   The surface emissive power may be evaluated from Equation 1.133, while the irradiation corresponds to $G = \varepsilon \sigma T_{sur}^4$, hence

$$E = \varepsilon \sigma T_s^4 = 0.8 \left(5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2} \cdot \text{K}^4 \right) (473\,\text{K})^4 = 2270\,\text{W}/\text{m}^2$$

$$G = \sigma T_{sur}^4 = 5.67 \times 10^{-8} \frac{\text{W}}{\text{m}^2} \cdot \text{K}^4 (298\,\text{K})^4 = 447\,\text{W}/\text{m}^2$$
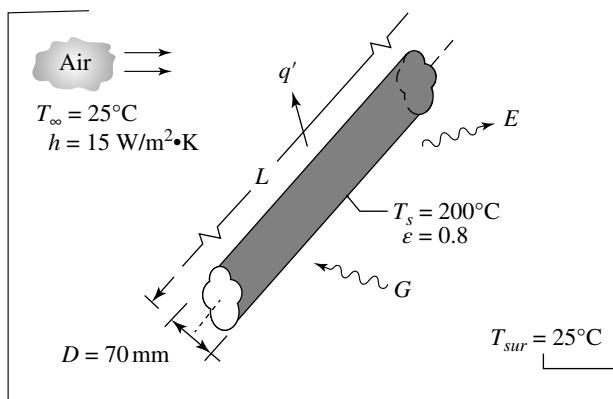

Figure 1.15 Schematic view for Example 1.2.

Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q = q_{conv} + q_{rad}$ and from Equation 1.138, with $A = \pi DL$,

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma\left(T_s^4 - T_{sur}^4\right)$$

The heat loss per unit length of pipe is then

$$\text{q}' = \frac{q}{L} = 15\,\text{W}/\text{m}^2 \cdot \text{K}(\pi \times 0.07\,\text{m})\,(200 - 25)^\circ\text{C}$$
$$+ 0.8(\pi \times 0.07\,\text{m})\,5.67 \times 10^{-8}\frac{\text{W}}{\text{m}^2}\cdot\text{K}^4\left(473^4 - 298^4\right)^\circ\text{C}$$
$$\text{q}' = 577\,\text{W}/\text{m} + 421\,\text{W}/\text{m} = 998\,\text{W}/\text{m}$$

Comments:

Note that temperature may be expressed in units of °C or K when evaluating the temperature difference for a convection (or conduction) heat transfer rate. However, temperature must be expressed in kelvins (K) when evaluating a radiation transfer rate.

The net rate of radiation heat transfer from the pipe may be expressed as

$$q'_{rad} = \pi D(E - \alpha G)$$
$$q'_{rad} = \pi \times 0.07\,\text{m}\,(2270 - 0.8 \times 447)\,\text{W}/\text{m}^2 = 421\,\text{W}/\text{m}$$

In this situation the radiation and convection heat transfer rates are comparable because T_s is large compared to T_{sur}, and the coefficient associated with free convection is small. For more moderate values of T_s and the larger values of h associated with forced convection, the effect of radiation may often be neglected. The radiation heat transfer coefficient may be computed from Equation 1.137, and for the conditions of this problem its value is

$$h_r = 11\frac{\text{W}}{\text{m}^2}\cdot\text{K}.$$

1.8 Boiling Heat Transfer

Boiling is the liquid-to-vapour phase change process that occurs at a solid–liquid interface when the surface is heated above the saturation temperature of the liquid. The formation and rise of the bubbles and the liquid entrainment coupled with the large amount of heat absorbed during liquid–vapour phase change at essentially constant temperature are responsible for the very high heat transfer coefficients associated with nucleate boiling. Boiling is classified as pool boiling or flow boiling, depending on the presence of bulk fluid motion. Boiling is called pool boiling in the absence of bulk fluid flow and flow boiling (or forced convection boiling) in the presence of it. In pool boiling, the fluid is stagnant, and any motion of the fluid is due to natural convection currents, and the motion of the bubbles is due to influence of buoyancy. In flow boiling, the fluid has velocity relative to heating surface and is forced to move in a heated pipe or over a surface by external means such as a pump. Therefore, flow boiling is always accompanied by other convection effects.

There are three different boiling heat transfer mechanisms. The first is nucleate boiling, in which heat is transferred by means of vapour bubbles nucleating, growing and

finally detaching from the surface. In nucleate boiling of water, the boiling point of a substance is the temperature at which the vapour pressure of the liquid equals the pressure surrounding the liquid Kandlikar et al. [18] and the liquid evaporates, resulting in an abrupt change in vapour volume. The saturation temperature is the temperature for a corresponding saturation pressure at which a liquid boils into its vapour phase. The liquid can be said to be saturated with thermal energy. Any addition of thermal energy results in a phase transition. At standard atmospheric pressure and low temperatures, no boiling occurs and the heat transfer rate is controlled by the usual single-phase mechanisms. As the surface temperature is increased, local boiling occurs and vapour bubbles nucleate, grow into the surrounding cooler fluid and collapse. This is sub-cooled nucleate boiling, and is a very efficient heat transfer mechanism. At high bubble generation rates, the bubbles begin to interfere, and the heat flux no longer increases rapidly with surface temperature (this is the departure from nucleate boiling, or DNB).

At similar standard atmospheric pressure and high temperatures, the hydrodynamically quieter regime of film boiling is reached. Heat fluxes across the stable vapour layers are low, but rise slowly with temperature. Any contact between fluid and the surface that may be seen probably leads to the extremely rapid nucleation of a fresh vapour layer ("spontaneous nucleation"). At higher temperatures still, a maximum in the heat flux is reached (the critical heat flux, or CHF). The Leiden frost effect demonstrates how nucleate boiling slows heat transfer due to gas bubbles on the heater's surface. As mentioned, gas-phase thermal conductivity is much lower than liquid-phase thermal conductivity, so the outcome is a kind of "gas thermal barrier."

The second boiling heat transfer mechanism is convective boiling, in which heat is conducted through the liquid, which evaporates at the liquid–vapour interface and there isn't any bubble formation.

Finally, there is film boiling in which the heat is transferred by conduction and radiation through a film of vapour that covers the heated surface, and the liquid vaporizes at the vapour liquid interface. Nucleate boiling and film boiling may occur in both pool boiling and flow boiling, whereas forced convective boiling occurs only in flow boiling. Pool boiling and flow boiling further be classified into sub-cooled boiling and saturated boiling. Boiling is called sub-cooled when the temperature of the liquid is below the saturation temperature and it is called saturated when the liquid temperature is equal to saturation temperature of the liquid.

1.8.1   Flow Boiling

In flow boiling, the fluid is forced to move by an external source, as it undergoes a phase change process from liquid to vapour. The boiling in this case exhibits the combined effects of convection and pool boiling. The different stages encountered in flow boiling in a heated tube are illustrated in Figure 1.16, together with the variation of the heat transfer coefficient, wall temperature, flow patterns and liquid temperature along the tube. Initially, the liquid is sub-cooled, and heat transfer to the liquid is by single-phase forced convection. Then bubbles start forming on the inner surfaces of the tube, and the detached bubbles are drifted into the mainstream. This gives the fluid flow a bubbly appearance, and thus the name *bubbly flow regime*. As the fluid is heated further, the bubbles grow in size and eventually coalesce into slugs of vapour. Up to half of the volume in the tube in this slug flow regime is occupied by vapour. After a while the core of the flow consists of vapour only, and the liquid is confined only in the annular space
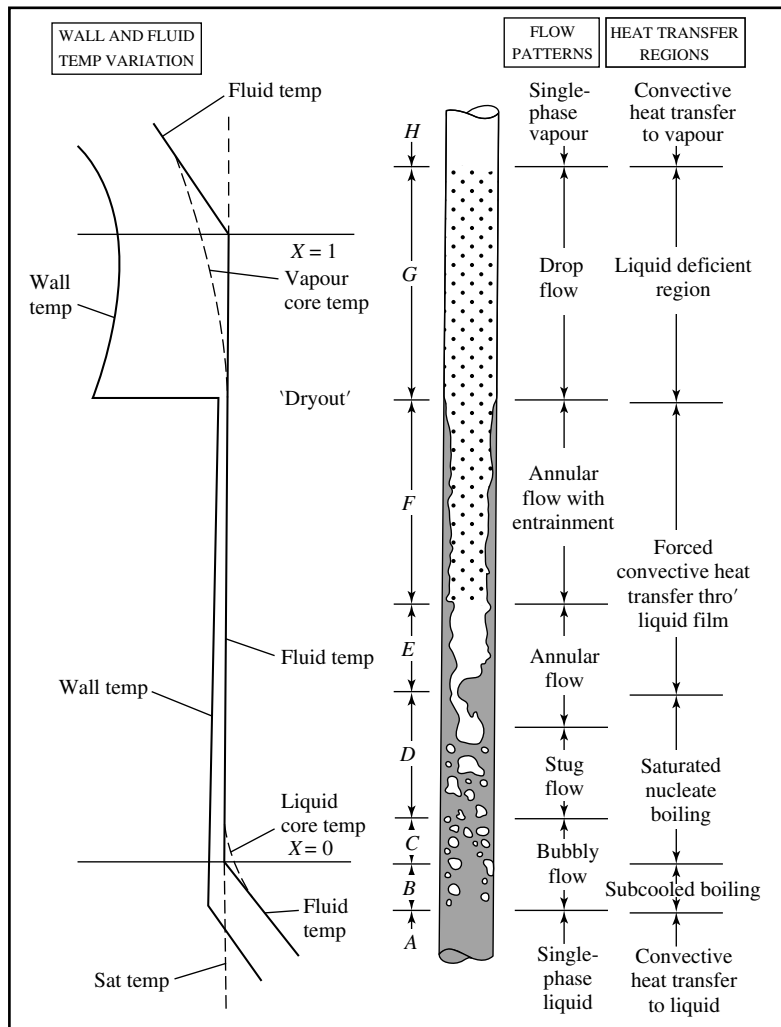
Figure 1.16 Flow boiling in a uniformly heated circular tube. (Collier and Thome [19])

between the vapour core and the tube walls. This is the annular-flow regime, and very high heat transfer coefficients are realized in this regime. As the heating continues, this annular liquid layer gets thinner and thinner, and eventually dry spots start to appear on the inner surfaces of the tube. The appearance of dry spots is accompanied by a sharp decrease in the heat transfer coefficient. This transition regime continues until the inner surface of the tube is completely dry. Any liquid at this moment is in the form of droplets suspended in the vapour core, which resembles a mist, and we have a *mist-flow regime* until all the liquid droplets are vaporized. At the end of the mist-flow regime there is saturated vapour, which becomes superheated with any further heat transfer.

With regard to details of flow boiling, refer to Chapter 5 for experimental evaluation of flow boiling correlations on different types of compact heat exchanger fins and refer to Chapter 6 for worked example for estimation of flow boiling heat transfer coefficient and pressure drop.

1.9 Condensation

Condensation occurs when a vapour is cooled and changes its phase to a liquid. During condensation, the latent heat of vaporization must be released. The amount of the heat is the same as that absorbed during vaporization at the same fluid pressure (Butterworth [20]).

There are several types of condensation:

- homogeneous condensation, as during a formation of fog
- condensation in direct contact with sub-cooled liquid
- condensation on direct contact with a cooling wall of a heat exchanger – the most common mode used in industry
- film condensation, when a liquid film is formed on the sub-cooled surface – usually occurs when the liquid wets the surface
- drop-wise condensation, when liquid drops are formed on the sub-cooled surface – usually occurs when the liquid does not wet the surface. Drop-wise condensation is difficult to sustain consistently, so industrial equipment is normally designed to operate in film condensation mode.

If a vapour is exposed to a surface at a temperature below saturation temperature then condensation occurs on the surface in the form of a liquid film or individual droplets. It is usually done by bringing the vapour into contact with a solid surface whose temperature is below the saturation temperature of the vapour. This is considered to be a form of convection heat transfer, since it involves fluid motion such as the flow of condensate to the bottom. The temperature remains constant under equilibrium conditions during a phase-change process at a fixed pressure. In this process, large amounts of heat (due to the large latent heat of vaporization released or absorbed) can be transferred during condensation, particularly at constant temperature (Collier and Thome [19]). However, it is necessary to maintain some difference between the surface temperature and the saturation temperature for an effective heat transfer mechanism. Heat transfer coefficients associated with condensation are generally much higher than those in other forms of convection processes that involve a single phase.

Two distinct forms of condensation are shown in Figure 1.17. They are film condensation and drop-wise condensation. In film condensation, the condensate wets the
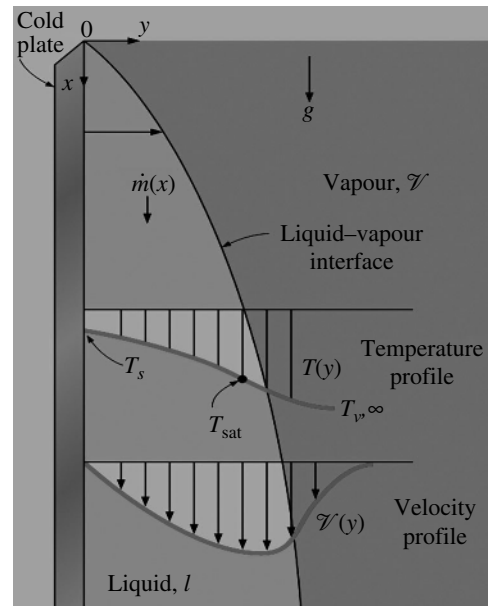

(a) Film condensation   (b) Drop-wise condensation

80°C   80°C

Droplets
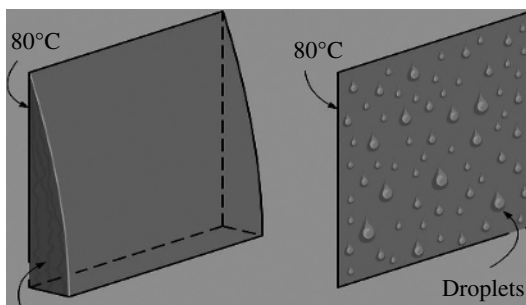
Liquid film

Figure 1.17 Types of condensation. (Collier and Thome [19])

Figure 1.18 Typical velocity and temperature profile of the condensate. (Collier and Thome [19])

surface and forms a liquid film on the surface that slides down under the influence of gravity. The liquid film thickness increases in the flow direction as more vapour condenses on the film. In Drop-wise condensation, the condensed vapour forms droplets on the surface instead of a continuous film, and the surface is covered by large numbers of droplets of varying diameters.

1.9.1   Film Condensation

In film condensation, the surface is covered by a liquid film of increasing thickness, and this "liquid wall" between the solid surface and the vapour serves as a resistance to heat transfer. The velocity and temperature profiles of the condensate are also given in Figure 1.18. Due to no-slip condition, the velocity of the condensate at the wall is zero and it reaches a maximum at the liquid–vapour interface. At the interface, the temperature of the condensate is saturation temperature and it decreases gradually to surface temperature at the wall. In forced convection, involving a single phase, heat transfer in condensation also depends on whether the condensate flow is laminar or turbulent. Again, the Reynolds number provides the criterion for the flow regime.

1.9.2   Drop-wise Condensation

It is characterized by countless droplets of varying diameters on the condensing surface instead of a continuous liquid film. This is one of the most effective mechanisms of heat transfer, and extremely large heat transfer coefficients can be achieved with this mechanism and is shown in Figure 1.19.
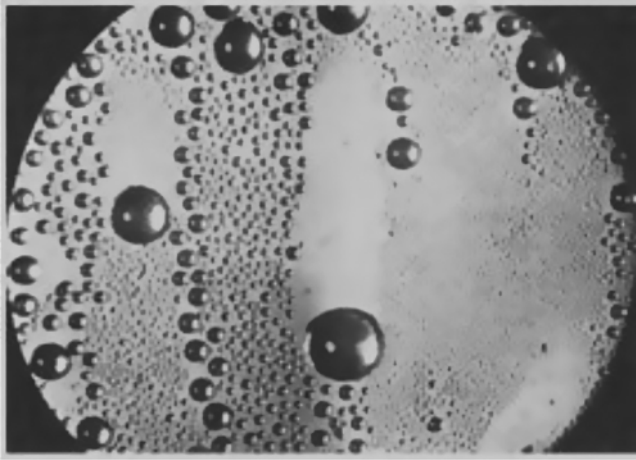
Figure 1.19 Drop-wise condensation. (Collier and Thome [19])

In this mechanism, the small droplets that form at the nucleation sites on the surface grow as a result of continued condensation, coalesce into large droplets, and slide down when they reach a certain size. As a result, with drop-wise condensation, heat transfer coefficients can be achieved that are 10 times larger than those associated with film condensation. Therefore, drop-wise condensation is the preferred mode of condensation in heat transfer applications and is achieved by adding a promoting chemical into the vapour or coating the surface with a polymer such as Teflon or a noble metal such as gold, silver, rhodium, palladium or platinum. The drop-wise condensation achieved does not last long and converts to film condensation after some time, so it is common practice to be conservative and assume film condensation in the design of heat transfer equipment (Collier and Thome [19]).

For details of the condensation data, refer to Chapter 5 for condensation experiments on different types of compact heat exchanger fins and Chapter 6 for worked example for estimation of condensation heat transfer coefficient and pressure drop estimation.

Nomenclature

A	cross-sectional area or heat transfer area, m^2
A_w	wall area, m^2
b	length, m
$C_{f,x}$	local skin friction coefficient, dimensionless
C_p	specific heat at constant pressure, J/kg.K
D	spacing, diameter, m
D_h	hydraulic diameter, m
$\Delta x, \Delta y, \Delta z$	differential distance, m

∂t	time
E	heat flux, W/m^2
f	friction factor, dimensionless
G	emission from block body, irradiation
G_z	Graetz number, dimensionless
H	height, m
h	heat transfer coefficient, $W/m^2 \cdot K$
	specific bulk enthalpy, J/kg
k	thermal conductivity, W/m.K
L	flow/ heat transfer length, m
$\dot{m}$	mass flow rate, kg/s
m', n'	direction cosines
Nu	Nusselt number, dimensionless
Nu_x	local Nusselt number, dimensionless
P	pressure, Pa
ΔP	pressure difference, Pa
Pr	Prandtl number, dimensionless
Pr_t	turbulent Prandtl number, dimensionless
p	perimeter, m
Q	Heat flow (W)
q'' or q	heat flux, W/m^2
r	radial position, m
r_h	hydraulic radius, m
r_0	tube radius, m
Ra	Rayleigh number, dimensionless
Re_D	Reynolds number based on D, dimensionless
Re_{Dh}	Reynolds numbers based on D_h, dimensionless
Re_L	Reynolds number based on L, dimensionless
S_t	Stanton number, dimensionless
Sh_x	Sherwood number, dimensionless
Sc	Schmidt number
S_{opt}	optimum distance, m
t	plate thickness, m
T	temperature, K
T_{in}	inlet temperature, K

T_{out}	outlet temperature, K
T_m	mean temperature, K
ΔT_{avg}	average temperature difference, K
ΔT_{lm}	log mean temperature difference, K
u	longitudinal velocity, m/s
u_*	friction velocity, m/s
U	mean velocity, m/s
V	volume, m^3
υ	transversal velocity, m/s
w	width, m
x,y,z	positions, m
X	flow entrance length, m
X_T	thermal entrance length, m
y	transversal position, m
y_{VSL}	viscous sub-layer thickness, m

Greek Symbols

α	thermal diffusivity, m^2/s
ε	emissivity
λ	wave length
σ	Stefan–Boltzmann constant
θ_{*m}	bulk temperature, dimensionless
Γ	boundary
μ	viscosity, kg/s-m
ν	kinematic visocity, m^2/s
ρ	density, kg/m^3
τ_{app}	apparent sheer stress, Pa

Subscripts

a	atmosphere
r,θ,z	coordinates
lm	log mean
w	wall

References

1 Incropera F.P., Dewitt D.P., Bergman T.L., Lavine A.S., Seetharamu K.N. and Setharam T.R., *Fundamentals of Heat and Mass Transfer*, Wiley publications, New Delhi, 2014.

2 Bejan A. *Heat Transfer*, John Wiley & Sons, New York, 1993.

3 Holman, *Heat Transfer*, McGraw-Hill, New York, 1989.

4 Incropera F.P.,and Dewitt D.P. *Fundamentals of Heat and Mass Transfer*, John Wiley & Sons, New York, 1990.

5 Sukhatme S.P. *A Text Book on Heat Transfer*, Third Edition, Orient Longman Publishers, 1992.

6 Ozisik M.N. *Boundary Value Problems of Heat Conduction*, International Text Book Company, Scranton, P.A., 1968.

7 Carslaw H.S. and Jaeger J.C. *Conduction of Heat in Solids*, Second Edition, Oxford University Press, Fairlawn, N.J., 1959.

8 Roland W. Lewis, Perumal Nithiarasu and Kankanhally N. Seetharamu, Fundamentals of the Finite Element Method for Heat and Fluid Flow, John Wiley & Sons Ltd., 2004.

9 Jaluria Y., *Natural Convection Heat and Mass Transfer*, Pergamon Press, 1986.

10 Massarotti N., Nithiarasu P. and Zienkiewicz O.C. Characteristic – based – split (CBS) algorithm for incompressible flow problems with heat transfer, *International Journal of Numerical Methods for Heat and Fluid Flow*, 8, 969–990, 1998.

11 Minkowycz W.J., Sparrow E.M., Schneider G.E. and Pletcher R.H. *Handbook of Numerical Heat Transfer*, John Wiley & Sons, 1988.

12 Aung W. and Worku G. B., Theory of fully developed, combined convection including flow reversal, *Trans. of ASME J. Heat Transfer*,108, 485–488, 1986.

13 Nithiarasu P., Seetharamu K.N. and Sundararajan T. Finite element analysis of transient natural convection in an odd-shaped enclosure, *International Journal of Numerical Methods for Heat and Fluid Flow*, 8, 199–216, 1998.

14 Blasius H.Z., Math. Phys., 56, 1, English translation in National Advisory Committee for Aeronautics Technical Memo no. 1256, 1908.

15 Chilton T.H. and A.P. Colburn, *Ind. Eng. Chem.*, 26, 1183, 1934.

16 Rohsenow W.M., James P. Hartnett and Young I. Cho, *Handbook of Heat Transfer*, McGraw-Hill Education, U.S., 1998.

17 Siegel R. and Howell J.R. *Thermal Radiation Heat Transfer*, Third Edition, Hemisphere Publishing Corporation, 1992.

18 Kandlikar S.G. and A. Mariani, in S.G. Kandlikar, M. Shoji and V.K. Dhir, Eds., *Handbook of phase change: Boiling and Condensation*, Chp 15, Taylor & Francis, New York, 1999.

19 Collier J.G. and J.R. Thome, *Convective Boiling and Condensation*, 3rd Ed., Oxford University Press, New York, 1996.

20 Butterworth, D., in D. Butterworth and G.F. Hewitt, Eds., *Two-phase Flow and Heat Transfer*, Oxford University press, London, pp. 426–462, 1977.