IN THIS CHAPTER

» Lining up the lingo

» Introducing multiple ways to describe mathematical situations

» Looking at applications for probability

» Linking logic logically

» Taking on games with new vigor

Chapter 1

Feeling Fine with Finite Math

What is *finite* mathematics? It seems that there are infinite ways to describe this subject or subjects. When applying the processes from the various topics in finite mathematics, you consider multiple applications and get to solve them in a variety of ways. Finite mathematics has become a gathering spot for many applications in business, social sciences, biological sciences, economics, finance, and so on. This gives the businessman, social scientist, biologist, economist, financial officer, and others many options for dealing with their everyday decisions.

Finite mathematics starts with the basic mathematical processes and draws in all the applications that make the processes interesting, usable, and valuable. And this is just the beginning. In addition to the basic mathematical topics and procedures, you also have all the possibilities for using modern technology to solve a particular problem or organize a situation.

Getting in Line with Linear Statements

Most of the applications in finite mathematics that involve mathematical statements are of the *linear* variety. A linear equation or linear inequality has only first-degree variables. You don't find curves like parabolas or shapes like circles or ellipses in the study of linear algebra.

In Table 1-1, you find some linear statements and their descriptions. A common practice is to have the variables be letters from the end of the alphabet and the constants and coefficients come from the beginning of the alphabet.

TABLE 1-1 Linear Statements

Algebraic Statement	Description
$ax + by = c$	Linear equation in two variables in standard form
$y = mx + b$	Linear equation in two variables in slope-intercept form
$ax + by + cz = d$	Linear equation in three variables in standard form
$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$	Linear equation in *n* variables in standard form
$ax + by < c$	Linear inequality, less than
$ax + by \geq c$	Linear inequality, greater than or equal to
$a < bx + c \leq d$	Compound linear inequality

Note that the power of each variable in a linear statement is equal to 1. The power isn't showing. You don't usually write an equation as $4x^1 + y^1 = 7$; the preferred format is $4x + y = 7$. When there's no exponent showing, you assume that the exponent is 1.

Making the Most with Matrices

What is a matrix? In the movie *The Matrix*, the characters dealt with computers, so you may find a bit of a tie-in there, because matrices provide formats that are conducive to being entered into computer programs and graphing calculators. But matrices are actually very simple structures.

A *matrix* is a rectangular array of numbers or other elements. By rectangular, this means that every row is the same size (making the length uniform) and every column is the same size (making the width uniform). For example, the following matrix A has four rows and two columns, so it's a 4×2 matrix.

$$A = \begin{bmatrix} -1 & 1 \\ 8 & -7 \\ 2 & 5 \\ 3 & 0 \end{bmatrix}$$

The matrix A has eight elements, and the elements are all integers. The elements are inside brackets, and the matrix has a capital letter as its name. In Chapter 5, you find even more details about matrices and the processes that go along with them.

Most graphing calculators have built-in matrix apps so you can enter the elements in the matrix and perform operations on a matrix or multiple matrices. Excel spreadsheets also lend themselves nicely to matrix processes; and the added benefit of using computer spreadsheets is that you can easily view and print them.

You can solve systems of linear equations by the tried-and-true methods from algebra: substitution and elimination. But matrix mathematics also includes methods that you can use to solve systems of linear equations. Matrices also help by changing the format of mathematical statements to make them more usable and understandable. The results are easily read after performing matrix computations. You just have to follow steps provided in Chapter 6.

Staying with the Program

Finite mathematics involves quite a bit of *linear programming,* in one form or another. Basically, this means that the topics covered take applications that involve linear statements and find a solution. Typically, the solution is in the form of finding the maximum or minimum value possible.

For example, say that you're trying to take care of some dietary problems and don't want to spend too much money while doing this. You're trying to *minimize* the cost. You need to add just so much vitamin A, some vitamin D, some iron, and some potassium to your diet. Pill I has certain amounts of each, Pill II has three out of four of those elements, and Pill III has a different three out of four. And, of course, they each cost a different amount of money.

A linear programming process associated with this situation has you write statements that represent the amounts of the vitamins, iron, and potassium and their relative cost. Then you write inequalities expressing that you want at least the minimum of each added to your diet. Finally, you write the statement that you want to minimize — the total cost.

Yes, this may seem very complicated, but all this becomes clear in Chapters 7 and 8. The steps are spelled out and the options for solving the problem presented.

Getting Set with Sets

A *set* can be many things, and it can be used in many ways. In mathematics, a set is a grouping or collection of objects. Yes, the objects are usually numbers, but they really can be anything.

When you describe a set in mathematics, you usually name the set with a capital letter, and you list the objects or elements of the set in braces, with the elements separated by commas.

The set of states starting with the letter *i* can be described with $\text{I} = \{\text{Iowa, Idaho, Indiana, Illinois}\}$. This set has four elements. And this isn't the only way to describe the set. You can also say that $\text{I} = \{\text{Idaho, Illinois, Indiana, Iowa}\}$. The order in which you list the elements doesn't matter.

If the set is very large and you don't want to list all the elements, then you can use a rule or an ellipsis. For example, if the set H contains all the positive integers smaller than 100, then you can use one of the following formats:

$$\text{H} = \{\text{positive integers smaller than } 100\}$$

$$\text{H} = \{x \mid 1 \le x < 100\}$$

$$\text{H} = \{1,\ 2,\ 3,\ \ldots,\ 98,\ 99\}$$

Each description of the set H means the same thing — that is, creates the same elements. The positive integers smaller than 100 are 1, 2, 3, 4, . . ., 98, 99. You don't want to list all those numbers, so you can use an alternate form for the set of numbers.

How many elements are there in the set H? You answer that question with the notation $n(\text{H}) = 99$. This says that set H has 99 elements. And, again, they don't have to be listed in order, if you choose to list all the elements.

You can accomplish many operations and other calculations using sets. One of the most popular processes involves Venn diagrams. A Venn diagram usually involves a geometric figure (most often, a circle) that represents a set and its elements, and it shows where the set intersects (shares) with another set or two. Figure 1-1 shows you a Venn diagram illustrating the relationship between sets M and F.

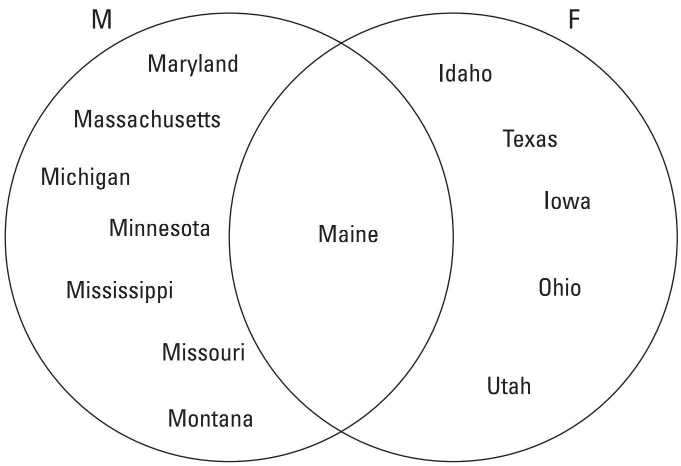

FIGURE 1-1: States starting with M and states with five or fewer letters.

$$\begin{aligned}\text{Set M} &= \{\text{States starting with the letter } M\}\\ &= \{\text{Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi,}\\ &\quad \text{Missouri, Montana}\}\end{aligned}$$

and

$$\begin{aligned}\text{Set F} &= \{\text{States with five or fewer letters in their name}\}\\ &= \{\text{Idaho, Maine, Texas, Iowa, Ohio, Utah}\}.\end{aligned}$$

From both the figure and the set listings, you see that $n(\text{M}) = 8$ and $n(\text{F}) = 6$. The intersection of the two sets is what they share, and that contains one element. The union of the two sets is the combination of the two sets put together. There are $8 + 6 - 1 = 13$ elements in the union, because you don't count Maine twice.

Sets provide a great way of organizing information and making conclusions about how they relate to one another.

Posing the Probability

What is the probability that it will rain tomorrow? What is the probability that you'll land on Park Place in the game Monopoly? Each of these answers or predictions is based on the numbers 0 through 100. If something has 0%

probability, then it isn't supposed to happen, and 100% probability is a sure thing. If you're four spaces away from Park Place, then the probability is about 11% that you'll land on that spot with its hotel!

You write probability amounts as percentages, decimals, or fractions. Each has an equivalence to the other two, and the use of one or another form is usually just a preference or whatever works best in the situation.

REMEMBER

To change a fraction to a percentage, you first change the fraction to its equivalent decimal form and then that decimal to a percent. For example, the fraction $\frac{5}{8} = 0.625$. Changing the decimal to a percentage, you move the decimal point two places to the right and get 62.5%.

What is the big advantage of using percentages? They're much easier to compare to one another. If you wanted to know which is the greater probability, $\frac{5}{8}$ or $\frac{14}{25}$, you get a better idea by comparing their percentages. The fraction $\frac{5}{8}$ is equal to 62.5%, and the fraction $\frac{14}{25} = 0.56$ or 56%, so $\frac{5}{8}$ represents the greater probability.

What do you do about decimals that don't end? Some don't even repeat! The short answer is to shorten them or round to a certain number of decimal places. If you want the decimal equivalent of $\frac{11}{12}$, you divide 12 into 11 and get 0.9166666 . . . with the digit 6 repeating forever. Choosing to round the percentage to the nearer hundredth, you first change the decimal to a percent, getting 91.6666 . . . % and then round to the nearest hundredth by changing the second 6 to a 7. The fraction $\frac{11}{12}$ is about 91.67%.

REMEMBER

To change a percentage to a fraction, you go backward. Change the percentage to a decimal, and then put the digits of the decimal over a power of ten that has the same number of zeros as decimal places.

The percentage 13.25% becomes 0.1325. Putting 1,325 over 10,000 and reducing the fraction, you have $\frac{1{,}325}{10{,}000} = \frac{53}{400}$.

TIP

Which version do you use? It's whichever version is most helpful and informative in the circumstances. For example, the three circles in Figure 1-2 show you the same circle labelled with fractions, decimals, and percentages. Each is valuable in some format or application. Your choice.

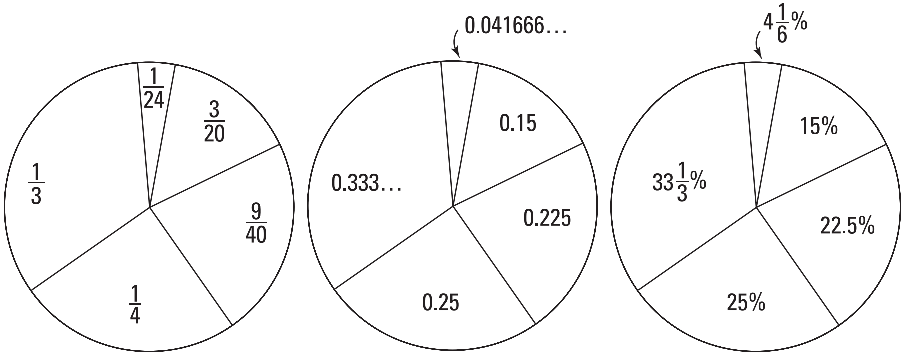

FIGURE 1-2: Comparing fractions, decimals, and percentages.

Figuring in Financial Factors

A big application area in finite mathematics is that involving financial topics. There's interest, dividends, amortized loans, continuous compounding, and more. And each of these topics comes with its own, special formula for performing the computations needed.

In real life, if you end up working with all this financial figuring, you'll have all sorts of apps and programs to do all the hard work. But you still need to understand what you're figuring and whether the result you get makes any sense. You need to know what number or form of the number needs to be input into what value. The financial overview in Chapter 11 will give you much more confidence.

But what if you're not going into the field of finance? You still want to know what's going on in that area. For example, when determining how much money you'll have in your savings account after a certain number of years, you need to know that the initial deposit is entered as a decimal number, the rate of interest is entered as a decimal, the compounding value is in terms of how often each year, and the time is a number of years. So how much will you have after ten years if you deposit $50,000 at an interest rate of 4.75% compounded monthly? Here's the computation:

$$\begin{aligned} A &= 50{,}000\left(1+\frac{0.0475}{12}\right)^{10\times 12} \\ &= 50{,}000\left(1.003958333\right)^{120} \\ &= 80{,}325.36 \end{aligned}$$

You'll have more than $80,000 — or your investment will have earned more than $30,000. You want to do better than that? Then try out some other institutions or investigate into what other processes or investment forms are available.

Finding Statistical Satisfaction

Statistical figures are part of everyone's life. What is the average daily temperature? What does she need on the next test to get an A in the course? Does your IQ score put you in the genius category? What is the median price of a house in that lovely neighborhood?

Statistics provide a way of explaining situations, but you have to understand what is being presented and understand the possible misunderstandings or misuses when statistics are used.

One of the basic measures studied in statistics is the *average*. The average can be the mean, the median, or the mode. And the mean can be arithmetic or geometric. In Figure 1-3, you see a graph representing the salaries, in thousands of dollars, of the employees at a certain firm. Just looking at the figure, you can determine one of the measures for average: the mode.

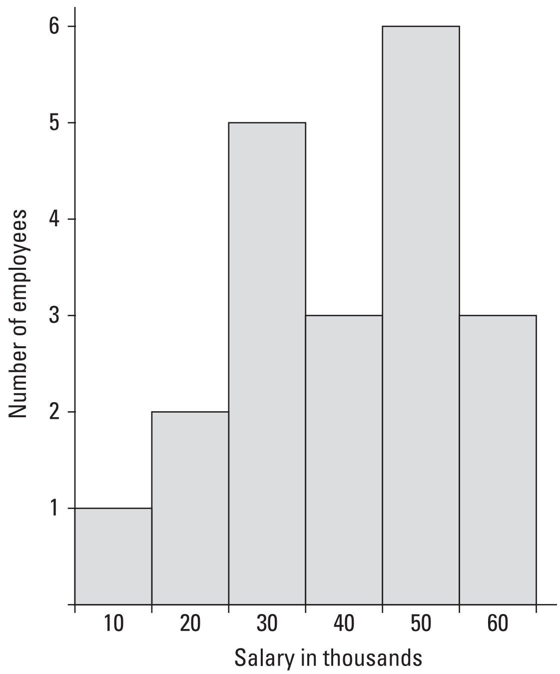

FIGURE 1-3: The salaries at XYZ Manufacturing.

The *mode* is the most frequently occurring score. In this case, the mode is $50,000. So the owner of the company can say that the average salary is $50,000. Is this a good representation?

You can also quickly find the *median* from this graph. The median is the middle score, when you line up all the numbers in order. Looking at the graph, how many people or salaries are represented here? You see that one person is earning $10,000, two people are earning $20,000, and so on. Add them all up, and you'll find 20 salaries listed. The middle is really between the 10th and the 11th numbers. So adding up the numbers associated with the salaries, you have $1+2+5+3$, and you can stop there. The three people represented in the $40,000 column are the 9th, 10th, and 11th in an ordered list. The middle is between the 10th and 11th, which are both $40,000, so the salary $40,000 is the median. Is this a better representation than the mode of $50,000?

There's one more average to check — the one you're probably most familiar with when talking average scores — and that's the *arithmetic mean*. The arithmetic mean is what you get when you add up all the scores or salaries and divide by how many there are. Adding up the 20 salaries and dividing by 20, you get

$$\frac{1(10{,}000)+2(20{,}000)+5(30{,}000)+3(40{,}000)+6(50{,}000)+3(60{,}000)}{1+2+5+3+6+3}$$
$$=\frac{800{,}000}{20}=40{,}000$$

The mean average is $40,000. This is the same as the median, so it looks like this salary is the better representation of what the employees earn, on average. But someone reporting that the average is $50,000 wouldn't be lying — they just may be misrepresenting for one reason or another. If you know what is going on, you can make a better judgment based on the statistics given.

There's a lot more to investigate in terms of the statistics of a situation, and you get much more information in Chapter 12, to help satisfy your statistical cravings.

Considering the Logical Side of Mathematics

You hear someone make the following argument:

> All cats have four legs.
>
> All cats are mammals.
>
> Therefore, all mammals have four legs.

You can probably do some convincing reasoning, with examples, to show why this argument is false, but what is basically wrong here? Are the assumptions wrong? Is the structure of the argument wrong? What structures work?

Aristotle is usually credited with being the first person to use — or at least record his use — of a formal logic system. Many others followed him, tweaking the subject and format and applying it to the sciences and other areas of endeavor.

Mathematics has long been a part of logic, coming from both directions. Principles of logic have been applied and incorporated into mathematical systems, and, going the other way, some mathematical findings have been utilized in further developments in logic.

In Chapter 13, you find the basics of logic, truth tables, and some applications of logic. And then perhaps, you can weigh in on Mr. Spock's quote: "You may find that *having* is not so pleasing a thing as *wanting*. This is not logical, but it is often true."

Unlocking the Chains

The study of Markov chains has helped in many applications in the real world. When making a prediction about a coming event, using a Markov chain, you consider only the present state, not the history of events or any other outside influences. Not all situations are appropriate for the use of these chains, but they still have been important enough to continue to study.

Consider a situation where a diet enthusiast has decided to limit her lunches to either broccoli, carrots, or kale. Each lunch consists of that vegetable, only, and nothing else. Figure 1-4 shows her choices after eating one of those vegetables and the percentage of the time she makes that choice.

If the dieter eats broccoli on one day, then 40% of the time she'll have broccoli the next day, 40% of the time she'll have carrots, and 20% of the time she'll have kale. If she has carrots one day, then the next day her two choices are only carrots again (70% of the time) or broccoli the other 30% of the time.

The diagram gives you lots of information about her eating habits, and a picture is often very helpful when trying to figure out patterns and make predictions, but there's another format that's even more useful for the predictions part.

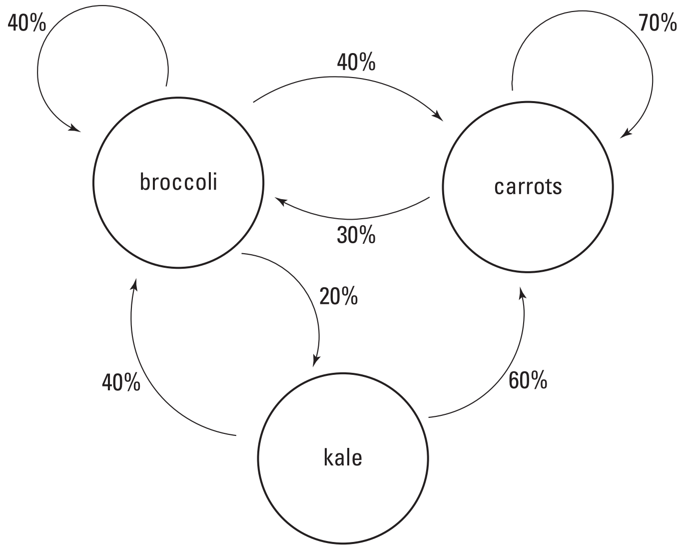

FIGURE 1-4: What is she having for lunch tomorrow?

You can put the information from the diagram into a rectangular format — yes, a matrix. To read the matrix that corresponds to Figure 1-4, you read from the left side, representing the current state or what was eaten today, and then down from the top, representing the next day's choice.

	broccoli	carrots	kale
broccoli	40%	40%	20%
carrots	30%	70%	0
kale	40%	60%	0

Reading from the matrix, if the dieter eats kale one day, there's a 60% chance she'll eat carrots the next day and a 0% chance she'll repeat the kale. You see that each row adds up to 100% — covering all the possibilities for the next day's choice. Also, what you find in the long run is that when the dieter uses this particular pattern of choices, she ends up eating broccoli 34% of the time, carrots 59% of the time, and kale 7% of the time. This is useful information when planning on future purchases. How were these percentages determined? You find all you need to create the same figures and matrices and the resulting patterns in Chapter 14.

Getting into Gaming

When you hear or read the words *game theory*, you may dismiss them as being something to do with gambling or with video games or with some of those fun apps on your tablet or phone. You wouldn't be completely incorrect, but there's so much more to game theory than just fun and games.

Game theory is applied to adversarial situations, which can be wars, competing for business, gaining votes in an election, making money, and much more.

When studying game theory, you see many of the mathematical structures and processes that are also used for other topics — matrices and sketches and solving equations are all incorporated into the study of game theory.

You can use some game theory when deciding how to invest that $100,000 you inherited from your great-aunt Lucy. You go to an investment firm and are given some figures on what may happen if you invested all the money into either a money market, some bonds, or some growth stocks for about five years. Of course, the gain (or loss) will depend on whether the economy is stable or inflationary. Here's what you're shown:

	Stable	Inflationary
Money Market	+$2,500	+$3,000
Bonds	+$17,000	+$10,000
Growth Stocks	+$50,000	–$20,000

So what's the game here? How would you play it? Safe or risky? In Chapter 16, you find different strategies and net results. This still doesn't guarantee success, but it gives you important information.