1

Thermodynamic Systems

1.1 Overview

Thermodynamics is the science relating heat and work transfers and the associated changes in the properties of the working substance within a predefined working system. A thermodynamic system is one that is concerned with the generation of heat and/or work using a working fluid. In this chapter, thermodynamic system behaviour will be described and the changes in properties will be calculated during the different processes encountered in typical engineering applications.

Learning Outcomes

- To understand the basic units and properties of thermodynamic systems.
- To be able to apply the laws of thermodynamics to closed and open systems.
- To be able to apply the first law of thermodynamics and calculate the changes in properties during a process and a cycle.
- To be able to solve problems related to compression and expansion of steam and gases.

1.2 Thermodynamic System Definitions

A thermodynamic *system* comprises an amount of matter enclosed within a *boundary* separating it from the outside *surroundings*.

There are two types of thermodynamic system:

A closed system has a fixed mass and a flexible boundary.
An open system has a variable mass (or mass flow) and a fixed boundary.

1.3 Thermodynamic Properties

A thermodynamic property of a substance refers to any quantity whose changes are defined only by the end states and by the process. Examples are the pressure, volume

Conventional and Alternative Power Generation: Thermodynamics, Mitigation and Sustainability,
First Edition. Neil Packer and Tarik Al-Shemmeri.

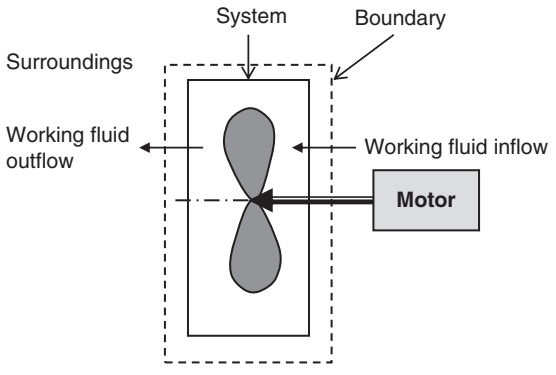


Figure 1.1 Thermodynamic system, boundary and surroundings.

and temperature of the working fluid in the system in Figure 1.1. In addition to these three properties, other thermodynamic properties include enthalpy, entropy and internal energy, which are all important in studying the behaviour of the working fluid in a power plant.

A list of the most common properties and associated terms is given below:

- *Pressure* (P) – The normal force exerted per unit area of the surface within the system. For engineering work, pressures are often measured with respect to atmospheric pressure rather than with respect to absolute vacuum.
  If a pressure gauge is calibrated to read zero at atmospheric pressure then the absolute pressure (P_{abs}) is given by:

$$P_{abs} = P_{atm} + P_{gauge}$$

  In SI units, the derived unit for pressure is the pascal (Pa), where 1 Pa = 1 N/m^2. This is very small for engineering purposes, so usually pressures are quoted in terms of kilopascals (1 kPa = 10^3 Pa), megapascals (1 MPa = 10^6 Pa) or bars (1 bar = 10^5 Pa).
- *Specific volume* (v) – For a system, the specific volume is the space occupied by a unit mass. The units of the specific volume are therefore m^3/kg.
  (Note that the term *specific* and lower-case letters are commonly used to denote thermodynamic property values *per kg of substance*.)
- *Temperature* (T) – Temperature is the degree of hotness or coldness of the system or the working fluid contained in the system. The absolute temperature of a body is defined relative to the temperature of ice at 0 °C.
  In SI units, the Kelvin scale is used, where 0 °C ≡ 273.15 K.
- *Specific internal energy* (u) – The property of a system covering all forms of energy arising from the internal structure of its matter, for example, nuclear, molecular, vibrational etc. The units of specific internal energy are kJ/kg.
- *Specific enthalpy* (h) – An energy property of the system conveniently defined as the sum of the internal energy and flow work, i.e. $h = u + PV$ for a substance.
  The units of specific enthalpy are kJ/kg.
- *Specific entropy* (s) – Entropy refers to the microscopic disorder of the system. It represents the effect of *irreversibilities* due to friction and deviation from the ideal behaviour. Ideal processes are termed *isentropic*.
  The units of specific entropy are kJ/kg K.

- *Phase* – The condition of a substance described by terms such as *solid*, *liquid* or *gas* is known as its phase. Phase change occurs at constant temperature.
  Phase changes are described as follows:
  Condensation: gas (or vapour) to liquid
  Evaporation: liquid to gas (or vapour)
  Melting: solid to liquid
  Freezing: liquid to solid
  Sublimation: solid to gas
  Deposition: gas to solid.
- *Mixed phase* – A multi-phase condition, for example, ice + water, water + vapour etc.
- *Quality of a mixed phase or dryness fraction* (x) – The *dryness fraction* (no units, values 0–1) is defined as the ratio of the mass of pure vapour present to the total mass of a mixed phase. The *quality* of a mixed phase may be defined as the percentage dryness of the mixture.
- *Saturated state* – A saturated liquid is a state in which the dryness fraction is equal to zero. A saturated vapour has a quality of 100% or a dryness fraction of one.
- *Pure substance* – A pure substance is one which is homogeneous and chemically stable. Thus, it can be a single substance that is present in more than one phase, for example, liquid water and water vapour contained in a boiler (in the absence of any air or dissolved gases).
- *Triple point* – A state point in which all solid, liquid and vapour phases coexist in equilibrium.
- *Critical point* – A state point at and above which transitions between liquid and vapour phases are not clear.
- *Superheated vapour* – A gas is described as superheated when its temperature at a given pressure is greater than the phase change (or *saturated*) temperature at that pressure, i.e. the gas has been heated beyond its saturation temperature.
  The *degree of superheat* represents the difference between the actual temperature of a given vapour and the saturation temperature of the vapour at a given pressure.
- *Subcooled liquid* (*or compressed liquid*) – A liquid is described as undercooled or subcooled when its temperature at a given pressure is lower than the saturated temperature at that pressure, i.e. the liquid has been cooled below its saturation temperature.
  The *degree of subcool* represents the difference between the saturation temperature and the actual temperature of the liquid at a given pressure.
- *Specific heat capacity* (C) – An energy storage property dependent on temperature.
  There are two variants:
  Constant volume: C_v
  Constant pressure: C_p
  The ratio of the specific heat capacities, i.e. C_p/C_v, is a parameter that is important in isentropic processes. It represents the expansion/compression process index.

1.4 Thermodynamic Processes

A process describes the path by which the state of a system changes and some properties vary from their original values.

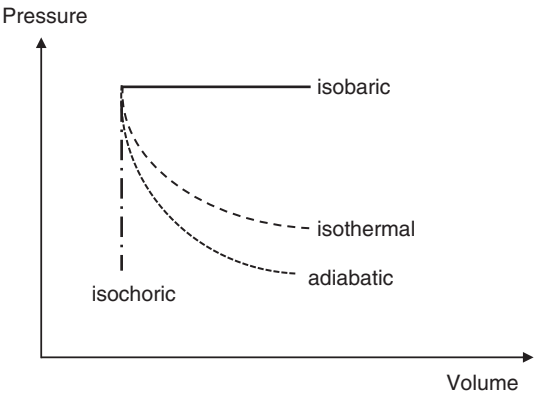


Figure 1.2 Thermodynamic processes.

In thermodynamics, the following types of processes are often encountered:

Adiabatic process: No heat transfers from or to the working fluid take place.
Isothermal process: No change in temperature of the working fluid takes place.
Isobaric process: No change in pressure of the working fluid takes place.
Isochoric process: No change in volume of the working fluid takes place.

These processes are illustrated on a pressure–volume basis in Figure 1.2.
Two other processes are of interest:

Isentropic process: No change in entropy of the fluid.
Isenthalpic process: No change in enthalpy of the fluid.

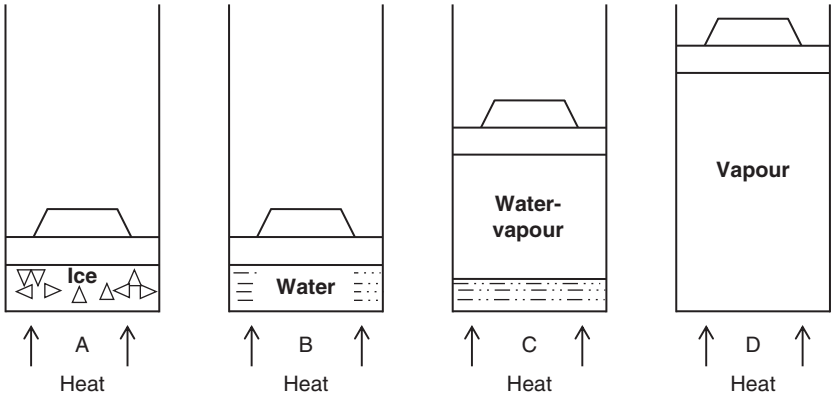
1.5 Formation of Steam and the State Diagrams

Consider the change in volume that occurs due to heating a unit mass of pure substance contained in a closed cylinder at constant pressure; take water (ice) as an example with an initial condition (temperature −10 °C; pressure 100 kN/m^2). The change in specific volume corresponding to equilibrium states is illustrated by the successive piston displacements shown in Figure 1.3a and plotted on the temperature-specific volume diagram of state shown in Figure 1.3b. It can be seen that there is very little increase in the specific volume of ice up to the melting point, B. At the melting point, there is a small but significant contraction to point B'. This is not typical of all pure substances; many substances show a small expansion at the melting point. Further heating in the liquid phase results in a small expansion as the temperature is increased to the vaporization point, C. At this point, the liquid is said to be in the *saturated liquid condition*. Further heating causes the progressive conversion of liquid into vapour, with a large increase in specific volume while the temperature remains constant. The process of conversion from liquid to vapour is completed at point C', which is known as the *saturated vapour condition*. Further heating gives rise to a gradual rise in temperature and specific volume in the vapour (or superheated steam) region, which is typified by point D.

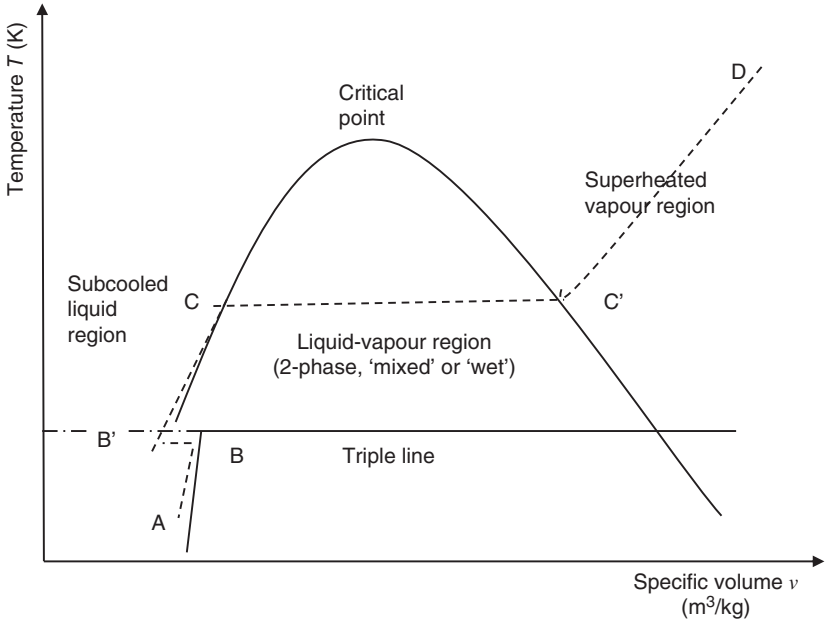
Line ABB'CC'D represents a typical constant pressure line on the temperature-specific volume diagram of state in which the pressure lies above that at the triple point but well below the critical pressure. If the above process was carried out at a higher pressure,

the large change in the specific volume at vaporization would be less pronounced. The abrupt change in specific volume disappears at the critical and higher pressures and a point of inflection occurs in the constant pressure line passing through the critical point.

At pressures above the critical value, there is no marked distinction in transition from the liquid to the vapour phase and it is usual to regard a substance as a gas at temperatures above the critical temperature.



(a)



(b)

Figure 1.3 (a) Formation of vapour (steam); (b) state diagram; (c) two-phase definitions.

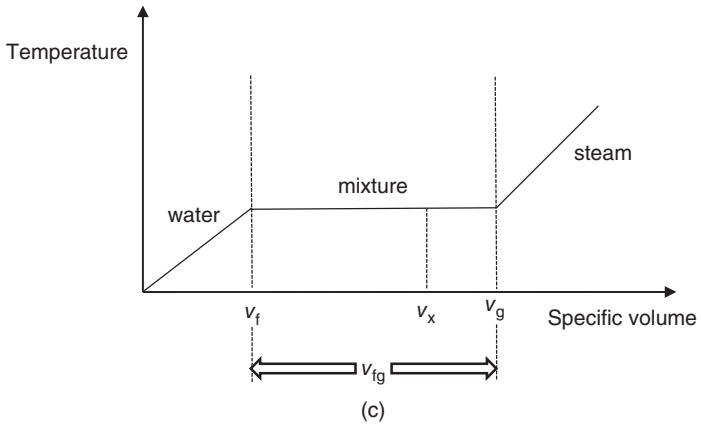


Figure 1.3 (*Continued*)

The term *dryness fraction* (x) is only applicable for the wet region and, as described earlier, is defined as follows:

$$x = \frac{\text{mass of vapour}}{\text{total mass of system (liquid + vapour)}}$$

The total mass of the system = mass of vapour + mass of liquid, hence the system-specific volume along the two-phase line (Figure 1.3c) is:

$$v = (1 - x)v_f + xv_g \tag{1.1}$$

Values of v_f and v_g and other properties for real substances are normally given in tables. Suffix 'f' refers to the liquid; suffix 'g' refers to the dry vapour.

At point C, $x = 0$ and thus $v = v_f$ whereas at point C', $x = 1$ and thus $v = v_g$

Letting suffix 'fg' refer to the change of phase, i.e. $v_{fg} = v_g - v_f$ then

$$v \text{ (at } x) = (1 - x)v_f + xv_g = v_f + x(v_g - v_f) = v_f + xv_{fg} \tag{1.2}$$

Either equation will give the same answer, however the second requires less data manipulation and hence is easier for manual calculation.

Similar expressions can be written for the specific enthalpy, specific internal energy and specific entropy of a substance:

$$h = (1 - x)h_f + xh_g = h_f + x(h_g - h_f) = h_f + xh_{fg}$$

$$u = (1 - x)u_f + xu_g = u_f + x(u_g - u_f) = u_f + xu_{fg}$$

$$s = (1 - x)s_f + xs_g = s_f + x(s_g - s_f) = s_f + xs_{fg}$$

1.5.1 Property Tables and Charts for Vapours

Tables giving data for saturated liquid and saturated vapour conditions, as well as the superheated values of v, u, h and s at a given pressure and over a range of temperatures are readily available. An example is shown in Table 1.1.

Table 1.1 Typical steam table.

T	*P* = 2.00 MPa (212.42 °C)		
	v	*h*	*s*
(°C)	m^3/kg	kJ/kg	kJ/kg K
Sat. liquid	0.00117	908.8	2.4474
Sat. vapour	0.09963	2799.5	6.3409
225	0.10377	2835.8	6.4147
250	0.11144	2902.5	6.5453
300	0.12547	3023.5	6.7664
350	0.13857	3137.0	6.9563
400	0.15120	3247.6	7.1271
500	0.17568	3467.6	7.4317

Note in the example used in Table 1.1 that the saturation temperature, i.e. the liquid–vapour phase change temperature, is supplied in brackets after the pressure value.

Some property tables for water/steam are supplied in Appendix A.

Charts for steam that indicate h or T as ordinate against s (specific entropy) as abscissa are also available. An example is shown in Figure 1.4.

1.6 Ideal Gas Behaviour in Closed and Open Systems and Processes

An ideal gas is one which, if pure, will behave according to the ideal gas equation, also known as the equation of state. The equation of state expresses the relationship between pressure (P) and volume (V) as a function of the mass of gas (m), its specific gas constant (R) and the temperature (T) as follows:

$$PV = mRT \tag{1.3}$$

The value of the gas constant depends on the gas molar mass, thus:

$$R = R_o/M \tag{1.4}$$

where R_o is the universal gas constant = 8.314 kJ/kmol K and M is the gas molar mass (kg/kmol).

For gas going through a process between conditions 1 and 2, the equation of state becomes:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \tag{1.5}$$

An *adiabatic reversible* process is characterized by the following equation: $PV^n = c$

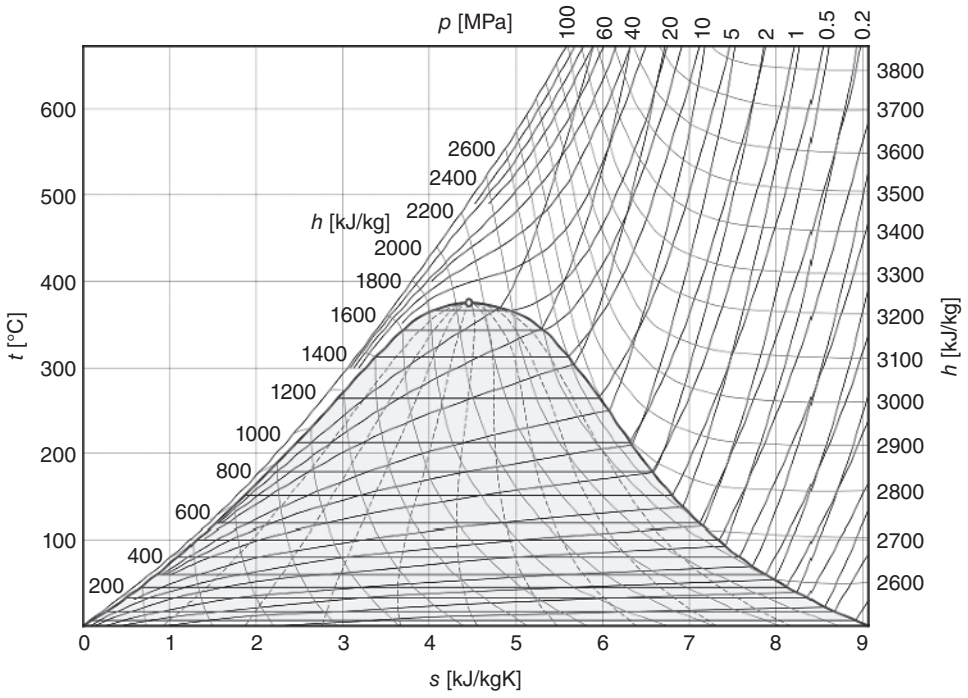


Figure 1.4 Temperature–entropy chart for water/steam. Reproduced with permission from Moran, Shapiro, Boettner and Bailey (2012) Principles of Engineering Thermodynamics, 7th edition, John Wiley & Sons.

Therefore, between states 1 and 2, the following relationship can be deduced:

$$P_1 V_1^n = P_2 V_2^n \tag{1.6}$$

where n is the index describing the path of the process.

Combining Equations (1.5) and (1.6), the process can be described in terms of pressure and temperature thus:

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{n-1/n} \tag{1.7}$$

Consider a working fluid undergoing a compression process. Equation (1.7) is used to determine the ideal temperature of the gas at the end of the compression process in terms of the initial temperature and the pressure ratio.

Since entropy is defined as a property that remains constant during an *adiabatic reversible* process, it follows that a temperature–entropy diagram would indicate such a process by a straight line perpendicular to the entropy axis if the process was purely isentropic (Figure 1.5). The friction in an irreversible process will cause the temperature of the gas to be higher than it would have been in a frictionless (reversible) process. Thus, the entropy increases during an *irreversible* process.

The isentropic process (1–2) is found from Equation (1.7) and the relation between the ideal temperature difference and that in the real process (1–2a) forms the isentropic

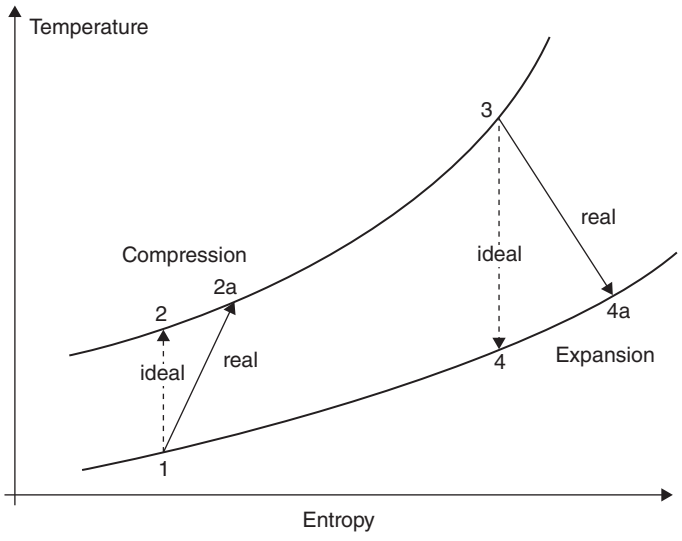


Figure 1.5 Concept of isentropic efficiency.

efficiency for a compressor:

$$\eta_{ic} = \frac{T_2 - T_1}{T_{2a} - T_1} \tag{1.8}$$

Thus, an irreversible compression process requires more work input than an ideal process due to the unwanted temperature rise.

When considering a working fluid undergoing an expansion, a reversible adiabatic or isentropic process (3–4) would, again, be illustrated by a vertical line when plotted on a T–s basis.

A renumbered Equation (1.7) is used to determine the ideal temperature of the gas at the end of the expansion process:

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{n-1/n} \tag{1.9}$$

However, a real expansion (3–4a) would again suffer from irreversibilities, with its final condition having a higher temperature and an increased entropy relative to the isentropic case (see Figure 1.5). In this case, the isentropic efficiency for the expansion is given by:

$$\eta_{it} = \frac{T_3 - T_{4a}}{T_3 - T_4} \tag{1.10}$$

1.7 First Law of Thermodynamics

The first law of thermodynamics is based on the conservation of energy within a system and, between any two state points, can be written as follows:

$$\text{Initial system energy} + \text{Energy in} = \text{Final system energy} + \text{Energy out} \tag{1.11}$$

1.7.1 First Law of Thermodynamics Applied to Open Systems

The first law applied to an open thermodynamic system, i.e. one where mass may cross a thermodynamic boundary, is termed the Steady Flow Energy Equation (SFEE) and is expressed as:

$$\dot{Q} - \dot{W} = (\Delta H + \Delta KE + \Delta PE) = \dot{m}\left[(h_2 - h_1) + \frac{\left(\bar{V}_2^2 - \bar{V}_1^2\right)}{2000} + g\frac{(z_2 - z_1)}{1000}\right] \tag{1.12}$$

Here, the LHS represents the heat flow and work transfers. On the RHS, the terms are, respectively: change in enthalpy, kinetic energy change and potential energy change between the start and end states (process).

Note that for an ideal gas, the change in enthalpy can be written as:

$$\Delta h = C_p(T_2 - T_1) \tag{1.13}$$

where C_p is the specific heat capacity at constant pressure (kJ/kg K) for the working fluid.

1.7.2 First Law of Thermodynamics Applied to Closed Systems

The first law applied to a closed thermodynamic system, i.e. one where mass does not cross a thermodynamic boundary, is termed the Non-Flow Energy Equation (NFEE) and is expressed as:

$$Q - W = \Delta U \tag{1.14}$$

where Q and W represent the heat and work during a process and the RHS represents the change in internal energy.

In general, the work done during a non-flow process is represented by the following expression:

$$W = \int P\mathrm{d}V \tag{1.15}$$

Note that for ideal gas, the change in internal energy can be written as:

$$\Delta U = mC_v(T_2 - T_1) \tag{1.16}$$

where C_v is the specific heat capacity at constant volume (kJ/kg K) for the gas.

Some important points to note:

- For both SFEE and NFEE there is a sign convention for heat and work. It is common to regard heat leaving a thermodynamic system as heat *loss* (−ve) and heat entering the system as positive. The opposite is used with work; hence, work input to the system is negative whereas work exported is positive.
- In both forms, it is common to regard electrical energy and power as work or work transfer.
- The term *adiabatic* is used to denote zero heat or heat transfer, i.e. Q or $\dot{Q} = 0$.

1.8 Worked Examples

All property values for the following questions can be found in Appendix A.

Worked Example 1.1 – Use of steam tables
Determine the enthalpy of steam at 2 MPa using steam tables for the following conditions:

(a) Dryness fraction $x = 0$.
(b) Dryness fraction $x = 1.0$.
(c) Dryness fraction $x = 0.5$.
(d) At $T = 300\,^{\circ}\text{C}$.

Solution
Given: $P = 2$ MPa.
Find x, T(d).

(a) h (at $x = 0$) = 908.8 kJ/kg
(b) h (at $x = 1$) = 2799.5 kJ/kg
(c) h (at $x = 0.5$) using the two-phase formula

$$h = h_f + x(h_g - h_f)$$
$$= 908.8 + 0.5 \times (2799.5 - 908.8) = 1854.15 \text{ kJ/kg}$$

(d) h (at 300 °C) = 3023.5 kJ/kg

Worked Example 1.2 – Interpolation from steam tables
Determine the enthalpy of steam at 10 MPa and 340 °C.

Solution
Given: $P = 10$ MPa, $T = 340\,^{\circ}\text{C}$.
Find h.

This example illustrates the way in which steam data are estimated when they are not found in the table at the exact conditions.

Let us define the following:

x_1 – initial value (given)
x_2 – final value (given)
y_1 – initial value (given)
y_2 – final value (given)
x_3 – required datum (given)
y_3 – value/property to be found, which can be determined as follows:

$$y_3 = y_1 + (x_3 - x_1) \times \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

Define the parameters for linear interpolation:
$x_1 = 325$; $x_2 = 350$; $y_1 = 2809.1$; $y_2 = 2923.4$; $x_3 = 340$.

Hence,

$$y_3 = y_1 + (x_3 - x_1) \times \frac{(y_2 - y_1)}{(x_2 - x_1)} = 2809.1 + (340 - 325) \times \frac{(2923.4 - 2809.1)}{(350 - 325)}$$
$$= 2877.7 \text{ kJ/kg}.$$

Worked Example 1.3 – Thermodynamic property changes
Steam at 2 MN/m^2 in a saturated dry vapour condition is heated at constant pressure until its temperature reaches a steady value of 400 °C. Determine, for a unit mass (i.e., 1 kg) of the steam:

(a) The change in its volume.
(b) The change in its entropy.
(c) The heat received by the steam.

Solution
Given: $P_1 = 2$ MPa and dry (saturated vapour)
$P_2 = 2$ MPa, 400 °C.
Find Δv, Δs, Δh.
At the beginning of the process (all values refer to the saturated vapour condition):

$v_1 = 0.09963$ m^3/kg
$h_1 = 2799.5$ kJ/kg
$s_1 = 6.3409$ kJ/kg K

At the end state (400 °C, superheated):

$v_2 = 0.15120$ m^3/kg
$h_2 = 3247.6$ kJ/kg
$s_2 = 7.1271$ kJ/kg K

Therefore, the changes during the process are calculated as the difference in values:

(a) $\Delta v = v_2 - v_1 = 0.05157$ m^3/kg (increase)
(b) $\Delta s = s_2 - s_1 = 0.7862$ kJ/kg K (increase)
(c) $\Delta h = h_2 - h_1 = 448.1$ kJ/kg (increase) – this is the heat received.

Worked Example 1.4 – SFEE (steam boiler)
A boiler receives feed water as saturated liquid at 20 bar and delivers steam at 20 bar and 500 °C. If the furnace of this boiler is oil-fired (calorific value of oil = 42 000 kJ/kg), determine the quantity of oil required to process 45 000 kg/h of steam, given a combustion efficiency of 90%.

Solution
Given: $P_1 = P_2 = 20$ bar $= 2$ MPa, $x_1 = 0$, $T_2 = 500$ °C, $CV_{\text{oil}} = 4200$ kJ/kg, $\dot{m}_s = 45\,000$ kg/hr, $\eta_{\text{comb}} = 0.9$.
Find $\dot{m}_{\text{oil}}$.

This is a constant pressure process, for which the enthalpy values at the initial and final conditions are extracted from a table: $h_1 = h_f = 908.8$ kJ/kg (saturated liquid at 20 bar).

$h_2 = 3467.6$ kJ/kg (superheated steam at 20 bar, 500 °C).

The SFEE for the boiler (no work transfer, and ignoring changes in ΔKE and ΔPE) becomes:

$$\dot{Q} = \dot{m}_s(h_2 - h_1)$$
$$= (45\,000/3600) \times (3467.6 - 908.8) = 32 \text{ MW}$$

The heat generated by burning oil in the furnace is:

Mass of oil burned × Calorific value × Efficiency of combustion

Hence, the mass flow rate of oil:

$$\dot{m}_{oil} = \frac{\dot{Q}}{CV_{oil} \times \eta_{comb}} = \frac{32 \times 10^6}{42 \times 10^6 \times 0.9} = 0.846 \text{ kg/s.}$$

Worked Example 1.5 – NFEE (gas compression)

1 kg of air at 1 bar, 27 °C undergoes adiabatic compression to 7 bar. Determine the heat transfer (kJ) during the process.

For air, take $R = 287$ J/kg K, $C_v = 714$ J/kg K and $n = 1.4$.

Solution

Given: $m = 1$ kg, $P_1 = 1$ bar, $P_2 = 7$ bar, $R = 287$ J/kg K, $C_v = 714$ J/kg K, $n = 1.4$.
Find Q.

The final temperature is:

$$T_2 = T_1\left[{}^{P_2}/_{P_1}\right]^{\frac{n-1}{n}} = 300 \times \left[{}^{7}/_{1}\right]^{\frac{(1.4-1)}{1.4}} = 523 \text{ K}$$

The work done during the process is:

$$W = \int P \mathrm{d}V$$

Since $PV^n = C$, $P = CV^{-n}$, i.e.

$$W = C\int V^{-n}\mathrm{d}V = \frac{P_2V_2^n(V_2^{-n+1})}{-n+1} - \frac{P_1V_1^n(V_1^{-n+1})}{-n+1} = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$W = \frac{1 \times 287 \times (300 - 523)}{1.4 - 1} = -160\,000 \text{ J}$$

The internal energy change is:

$$\Delta U = m.C_v(T_2 - T_1) = 1 \times 714(523 - 300) = +159287 \text{ J}$$

According to the first law:

$$Q - W = \Delta U$$

Hence, $Q = \Delta U + W = +159287 - 160000 = -780$ J

The low value is not surprising, as the process is adiabatic.

Worked Example 1.6 – SFEE (steam turbine)

A steam turbine receives 120 kg/minute of steam at 1 MPa, 350 °C and a velocity of 20 m/s. The exit condition from the turbine is 100 kPa, $x = 1$ and a velocity of 10 m/s. Neglecting changes in potential energy and assuming the process to be adiabatic, estimate the power output (kW) from the turbine.

Solution

Given: $P_1 = 1$ MPa, $P_2 = 100$ kPa $= 0.1$ MPa, $T_1 = 350\,^{\circ}$C, $\bar{V}_1 = 20$ m/s, $\bar{V}_2 = 10$ m/s, $\dot{m}_s = \frac{120 \text{ kg/min}}{60 \text{ s/min}} = 2$ kg/s.

Assumptions: $\dot{Q} = 0$ (adiabatic), $g(z_2 - z_1) = 0$.

Find $\dot{W}$.

From steam tables:

$h_1 = 3157.7$ kJ/kg

$h_2 = 2675.5$ kJ/kg

Using the SFEE:

$$\dot{Q} - \dot{W} = \dot{m}_s \left[(h_2 - h_1) + \frac{\bar{V}_2^2 - \bar{V}_1^2}{2 \times 1000} + g(z_2 - z_1) \right]$$

$$0 - \dot{W} = \frac{120}{60} \left[(2675.5 - 3157.7) + \frac{10^2 - 20^2}{2 \times 1000} + 0 \right]$$

Hence, $\dot{W} = +964.7$ kW.

Worked Example 1.7 – Isentropic gas expansion

Air is expanded *isentropically* in a nozzle from 13.8 bar and 150 °C to a pressure of 6.9 bar. The process occurs under steady-flow, steady-state conditions. Calculate the exit velocity from the nozzle given that the nozzle is installed in a horizontal plane and that the inlet velocity is 10 m/s. Take the mass flow rate as 1 kg/s, specific heat capacity at constant pressure as 1.005 kJ/kg K and $n = 1.4$.

Solution

Given: $P_1 = 13.8$ bar, $P_2 = 6.9$ bar, $T_1 = 150\,^{\circ}$C, $\bar{V}_1 = 10$ m/s , $\dot{m}_a = 1$ kg/s, $C_p = 1.005$ kJ/kg K, $n = 1.4$.

Assumptions: $\dot{Q} = 0$ (adiabatic), $g(z_2 - z_1) = 0$, $\dot{W} = 0$ (no moving parts).

Find $\bar{V}_2$.

$$T_2 = T_1 \left(\frac{P_2}{P_1} \right)^{n-1/n} = (150 + 273) \times \left(\frac{6.9}{13.8} \right)^{0.4/1.4} = 347 \text{ K}$$

The situation is an open system for which the SFEE applies:

$$\dot{Q} - \dot{W} = \dot{m}_a \left[(h_2 - h_1) + \frac{\left(\bar{V}_2^2 - \bar{V}_1^2 \right)}{2000} + g \frac{(z_2 - z_1)}{1000} \right]$$

$$= \dot{m}_a \left[C_p (T_2 - T_1) + \left(\frac{\bar{V}_2^2 - \bar{V}_1^2}{2000} \right) + g \left(\frac{z_2 - z_1}{1000} \right) \right]$$

Hence, the SFEE reduces to:

$$0 = \dot{m}_a \left[C_p(T_2 - T_1) + \left(\frac{\bar{V}_2^2 - \bar{V}_1^2}{2000} \right) \right]$$

$$0 = 1 \times \left[1.005(347 - 423) + \frac{1}{2000} \left(\bar{V}_2^2 - 10^2 \right) \right]$$

$$\bar{V}_2 = 390 \text{ m/s}.$$

Worked Example 1.8 – SFEE (gas compression)

A compressor takes in air at 1 bar and 20 °C and discharges into a line. The average air velocity in the line at a point close to the discharge is 7 m/s and the discharge pressure is 3.5 bar. Assuming that the compression occurs adiabatically, calculate the work input (kW) to the compressor. Assume that the air inlet velocity is very small.

For air, take $C_p = 1005$ J/kg K, and $n = 1.4$.

Solution

Given: $P_1 = 1$ bar, $T_1 = 20\,^{\circ}\text{C} = 293$ K, $P_2 = 3.5$ bar, $\bar{V}_2 = 7$ m/s , $\dot{m}_a = 1$ kg/s, $C_p = 1005$ J/kg K, $n = 1.4$.

Assumptions: $\dot{Q} = 0$ (adiabatic), $g(z_2 - z_1) = 0$ (horizontally mounted), $\bar{V}_1 = 0$.

Find $\dot{W}$

For adiabatic compression, the final temperature is calculated as:

$$T_2 = T_1 \left\{ \frac{P_2}{P_1} \right\}^{(n-1)/n} = 293 \times \left(\frac{3.5}{1} \right)^{0.4/1.4} = 419.1 \text{ K}$$

Using the SFEE:

$$\dot{Q} - \dot{W} = \dot{m}_a \left[(h_2 - h_1) + \left(\frac{\bar{V}_2^2 - \bar{V}_1^2}{2000} \right) + g \left(\frac{z_2 - z_1}{1000} \right) \right]$$

$$-\dot{W} = \dot{m}_a \left[C_p(T_2 - T_1) + \left(\frac{\bar{V}_2^2 - \bar{V}_1^2}{2000} \right) \right]$$

Hence,

$$\dot{W} = -1 \times \left[1005 \times (419.1 - 293) + \left(\frac{7^2 - 0}{2000} \right) \right]$$

$$= -[126\,728 + 0.0245]$$

$$= -126.7 \text{ kW}$$

Worked Example 1.9 – NFEE (work)

An ideal gas occupies a volume of 0.5 m^3 at a temperature of 340 K and a given pressure. The gas undergoes a constant pressure process until the temperature decreases to 290 K. Determine:

(a) The final volume.
(b) The work done during the process if the pressure is 120 kPa.

Solution

Given: $V_1 = 0.5\ \text{m}^3,\ T_1 = 340\ \text{K},\ P = 120 \times 10^3\ \text{Pa},\ T_2 = 290\ \text{K}.$
Find V_2, W.

(a) For an isobaric process, P = constant so the ideal gas equation for the process is written as:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Therefore:

$$V_2 = V_1 \times \frac{T_2}{T_1} = 0.5 \times \frac{290}{340} = 0.426\ \text{m}^3$$

(b) $W = \int P\text{d}V$

For a constant pressure process:

$$W = P\int \text{d}V = P(V_2 - V_1)$$

$$= 120 \times 10^3(0.426 - 0.5)$$

$$= -8823\ \text{J} = -8.823\ \text{kJ}.$$

The negative sign indicates that work is imported from an external source to the system.

Worked Example 1.10 – SFEE (mass flow)

Steam at a pressure of 6 MPa and a temperature of 500 °C enters an adiabatic turbine with a velocity of 100 m/s and expands to a pressure of 100 kPa. The steam leaves the turbine as just dry saturated vapour, with a velocity of 10 m/s. If the turbine is required to develop 1 MW, determine the mass flow rate of steam (kg/s) through the turbine.

Solution

Given: $P_1 = 6\ \text{MPa},\ T_1 = 500\ ^\circ\text{C}, P_2 = 100\ \text{kPa} = 0.1\ \text{MPa},\ \bar{V}_1 = 100\ \text{m/s}$, $\bar{V}_2 = 10\ \text{m/s},\ \dot{W} = 1\ \text{MW} = 1000\ \text{kW}.$
Assumptions: $\dot{Q} = 0$ (adiabatic), $g(z_2 - z_1) = 0.$
Find $\dot{m}$

From steam tables:

$h_1 = 3422.2\ \text{kJ/kg}$
$h_2 = 2675.5\ \text{kJ/kg}$

$$\dot{Q} - \dot{W} = \dot{m}\left[(h_2 - h_1) + \frac{1}{2}\left(\bar{V}_2^2 - \bar{V}_1^2\right) + g(z_2 - z_1)\right]$$

Neglecting the changes in potential energies, $\dot{Q} = 0$ for an adiabatic process:

$$-\dot{W} = \dot{m}_\text{s}\left[(h_2 - h_1) + \frac{1}{2000}\left(\bar{V}_2^2 - \bar{V}_1^2\right)\right]$$

$$-1000 = \dot{m}_\text{s}\left[(2675.5 - 3422.2) + \frac{10^2 - 100^2}{2 \times 1000}\right]$$

$$\dot{m}_\text{s} = 1.33\ \text{kg/s}.$$

1.9 Tutorial Problems

Thermodynamic properties of water and steam are available in Appendix A.

1.1 Determine the specific enthalpy of steam (kJ/kg) at 20 MPa using steam tables for the following conditions:
(a) Dryness fraction $x = 0$.
(b) Dryness fraction $x = 1.0$.
(c) Dryness fraction $x = 0.5$.

[Answers: 1826.3, 2409.7, 2118 kJ/kg]

1.2 Steam at 4 MPa, 400 °C expands at constant entropy until its pressure is 0.1 MPa. Determine, using steam tables, the dryness fraction of the steam at the end of the expansion process and the energy liberated per kilogram of steam under 100% isentropic conditions.

[Answers: 0.902, 758 kJ/kg]

1.3 Repeat Problem 1.2 for when the expansion process is 80% isentropic.

[Answers: 0.969, 606 kJ/kg]

1.4 An ideal gas occupies a volume of 0.5 m^3 at 600 K and 100 kPa. The gas undergoes a constant pressure process until its temperature reaches 300 K. Determine:
(a) The final volume (m^3).
(b) The work done during the process (kJ).

[Answers: 0.25 m^3, −25 kJ]

1.5 *Self-ignition* would occur in an engine using a certain type of gasoline if the temperature due to compression reached 350 °C. Assume inlet conditions of 27 °C and 1 bar. Calculate the highest ratio of compression that may be used to avoid pre-ignition (base your maximum temperature to 1 degree below that of self-ignition) if the law of compression is: $PV^{1.3} = C$.

[Answer: 11.36]

1.6 A gas turbine operates between 9 bar and 1 bar. Helium is the working fluid, expanding adiabatically from an initial temperature of 1000 K and a mass flow of 0.1 kg/s. Determine the exit temperature (K) and the work done (kW) if changes in kinetic energy and potential energy across the turbine are negligible. For Helium, $R = 2.08$ kJ/kg K and $C_p = 5.19$ kJ/kg K.

[Answers: 414 K, 304 kW]

1.7 A turbine manufacturer claims that his turbine will have an output of about 9 MW under the following conditions:
Steam flow 10 kg/s at 6 MPa and 500 °C inlet condition.
100% isentropic expansion to a final pressure of 100 kPa.
The heat transfer from the casing to the surroundings represents 1% of the overall change of enthalpy of the steam.

The exit is 5 m above entry.
The initial velocity of the steam is 100 m/s whereas the exit velocity is 10 m/s.
Do you agree with the manufacturer or not? Support your answer by calculations.

[Answer: yes]

1.8 A compressor takes in air at 1 bar and 20 °C and discharges at an average air velocity of 10 m/s and a pressure of 5 bar. Assuming that the compression occurs isentropically, calculate the specific work input to the compressor (kJ/kg).
Assume that the air inlet velocity is very small. Take, for air, $C_p = 1.005$ kJ/kg K and $n = 1.4$.

[Answer: −172 kJ/kg]

1.9 A steam turbine receives 120 kg/minute of steam at 1 MPa, 350 °C and a velocity of 20 m/s. The exit condition from the turbine is 10 kPa, $x = 1$ and a velocity of 10 m/s. Neglect changes in potential energy and assume the process to be adiabatic. Estimate the power output from the turbine.

[Answer: +1135 kW]

1.10 Find the change in value of specific enthalpy (kJ/kg) for water (steam) between an initial state where $P = 100$ kPa, saturated liquid, and a final state where $P = 100$ kPa and $T = 350$ °C.

[Answer: 2758.85 kJ/kg]