CHAPTER 1

Volatility and Options

Volatility is a measure of the uncertainty of the future price of a security. Options are contracts that allow market participants to expose their portfolios to the future volatility of an underlying security. The buyer of an option is said to be *long volatility* because she expects to profit if volatility rises. The converse is true for the seller of an option. She is *short volatility*.

Although options are often also simultaneously used for other purposes, such as to gain exposure to the direction in which the underlying security will move, to the future path of interest rates, or to gain leverage, among other uses, their raison d'etre is that they allow market participants to trade volatility. Indeed, forwards (or futures) already allow market participants to expose their portfolios to direction, to interest rates, and to gain leverage. We explore the other joint uses of options over the course of this book. However, the aim in this chapter is to provide a broad understanding of the concept of volatility and to communicate the central idea that an option is a bet on future volatility.

The exposition in this chapter is deliberately imprecise in order to disseminate the intuition underlying options theory and practical options trading more quickly. We will study these topics in depth and with greater accuracy in later chapters.

1.1 WHAT IS AN OPTION?

Perhaps the quickest and most intuitive way to understand the basics of options is through inspecting their payoff profiles. The payoff profiles of a *vanilla* call option and put option are shown in Figure 1.1. I explain these diagrams in more detail in Chapter 2. For now, the important points to understand are:

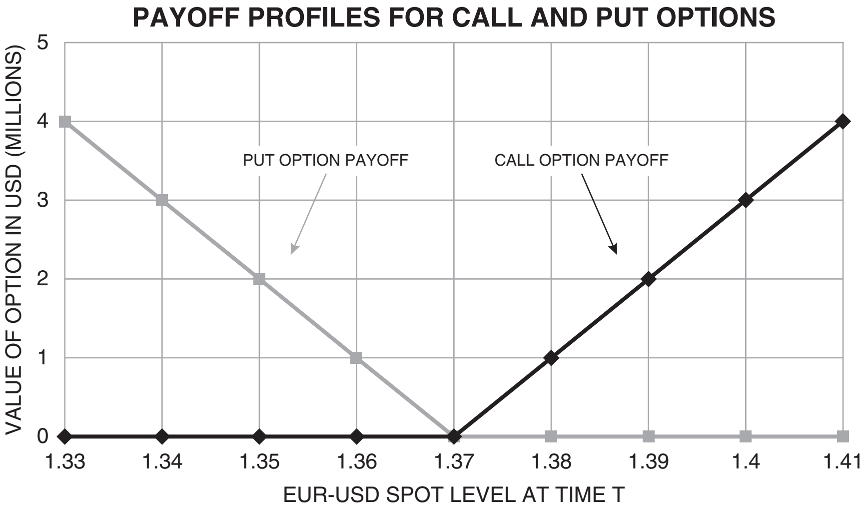

FIGURE 1.1 The black and gray lines show the payoff profiles of a call and put option respectively. In this example, the payoff corresponds to an option with notional of 100 million EUR and a strike of 1.37. The horizontal axis shows the level of EUR-USD at the maturity, or expiry, time of the option. If EUR-USD is above (below) 1.37 on the expiry date of the option, the owner of the call (put) can buy (sell) 100 million EUR-USD at 1.37 if they so choose. In FX markets, the expiry date typically ranges from overnight through to 20 years, although even longer expiries are possible. The vertical axis shows the payoff to the owner of the option in USD. The payoff of the call option is zero if EUR-USD is below 1.37 at the expiry time because the trader would not rationally exercise her option to buy EUR-USD at the higher price of 1.37. However, it is positive if EUR-USD is above 1.37 and rises linearly. The put option behaves similarly, but the payoff is zero if EUR-USD is above 1.37, and positive if EUR-USD is below 1.37.

- The payoffs depend on the value of an underlying security (the EUR–USD exchange rate in the example in Figure 1.1[1]) at the expiry time. We can rephrase this; the payoff is *derived* from the value of EUR-USD. This is why options are members of a class of financial products called *derivatives*.

[1]Readers unfamiliar with quotation conventions in foreign exchange should understand the EUR–USD exchange rate as the number of USD per EUR. For example, EUR-USD at 1.37 means 1.37 USD exchange for 1 EUR. Note that equities are quoted analogously. Microsoft stock trading at 100 USD means Microsoft-USD is 100.

- The call (put) option allows the trader to buy (sell) the underlying security at a prespecified price, called the strike price (EUR-USD at 1.37 in our example) on the expiry date.
- It follows that the payoff of the call option is $\max(S_T - K, 0) \times N$, where S_T is the price of the underlying security at the expiry of the option, K is the strike price, and N is the notional, or number of units of the option purchased. Suppose that the trader has purchased $N = 100$ million EUR of a $K = 1.37$ call option on EUR-USD. If $S_T = 1.39$, then the trader exercises her option and buys 100 million EUR at the strike price $K = 1.37$. She can then sell them in the market at $S_T = 1.39$ to receive $(1.39 - 1.37) \times 100$ million EUR = 2 million USD. However, if $S_T < 1.37$, then she can abandon her option and receive nothing. The formula $\max(S_T - K, 0) \times N$ correctly describes her payoff. Analogously, the put payoff is $\max(K - S_T, 0) \times N$.
- Note that the payoff is in USD in our example on EUR-USD. Had USD-JPY been the underlying currency pair, then the payoff would have been in JPY.
- The call payoff (black) is high when the value of the underlying asset is high. The put payoff (gray) is high when the value of the underlying asset is low. However, losses are floored in that the payoffs of both calls and puts are never negative, regardless of the value of the underlying. Options therefore provide insurance against market moves and their buyer must pay a premium.
- Purchasing a call (put) option is therefore a means to express the view that the underlying currency pair will rise (fall) and the maximum loss associated with this approach is the premium paid to purchase the option. However, we shall see that options and, in particular, portfolios of options, spot and forward positions, provide many ways of expressing these views and option traders do not in general buy calls (puts) to express the pure directional view that the underlying currency pair will rise (fall). Selecting the optimal portfolio requires a clear view on future volatility.
- Option payoffs are nonlinear and convex.

1.2 OPTIONS ARE BETS ON VOLATILITY

If the trader purchases both the call option and the put option, then her payoff profile is given by the *V* shape shown in Figure 1.2. This common strategy is called a *straddle*.

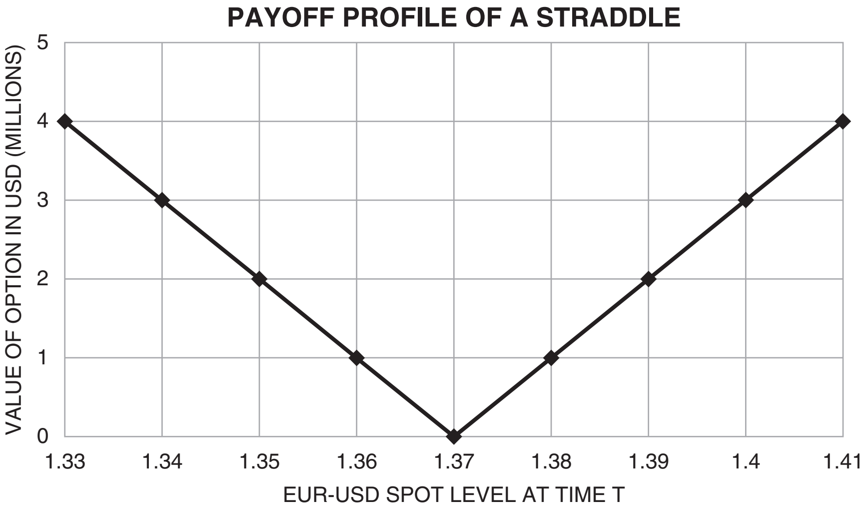

FIGURE 1.2 The figure shows the payoff of a straddle option strategy applied to EUR-USD. This is the purchase of a call option and a put option both with the same strike and notional. I use a strike $K = 1.37$, and notional $N = 100$ million EUR each. The horizontal axis shows the level of EUR-USD at the expiry time of the option. The vertical axis shows the payoff to the owner of the option in USD.

Unlike with individual calls and puts, it is clear that with a *V*-shaped symmetrical payoff profile the trader is not, initially at least, concerned as to whether the underlying security rises or falls because her payoff is the same if EUR-USD moves from 1.37 up to 1.39 or if it moves down to 1.35; in our example in Figure 1.2 it is 2 million USD in both cases. Her main concern is that EUR-USD moves away from 1.37. The further it moves, the higher her payoff. If EUR-USD were to stay at 1.37, then her payoff is zero and she loses the premium she paid to purchase the options. More so than any other option portfolio, the straddle makes clear that options are really bets on volatility. If EUR-USD volatility is high, then her chances of EUR-USD moving away from 1.37 and thereby earning a higher payoff are higher. For this reason, the buyer of the straddle is said to be *long volatility* and the seller is said to be *short volatility*.

One can show that individual call options and put options are also bets on volatility. Instead of the straddle, suppose that EUR-USD is trading at 1.37 and that the trader purchases a $K = 1.37$ EUR call option with $N = 200$ million EUR. The payoff of this position is shown in the left part of Figure 1.3. Simultaneously, the trader sells 100 million EUR-USD spot

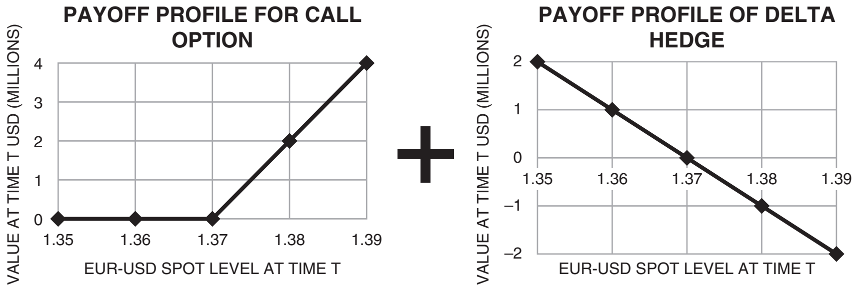

FIGURE 1.3 The left diagram shows the payoff of a call option with strike 1.37 and notional of 200 million EUR. The right diagram shows the payoff of selling 100 million EUR at 1.37. The payoff of the portfolio containing these two position is shown in Figure 1.4.

at 1.37. The payoff of this transaction is shown in the right-hand diagram in Figure 1.3. The selling of 100 million EUR-USD is called a *delta hedge*.[2] The payoff of the portfolio formed by purchasing the call option and executing the delta hedge is equivalent to that of a straddle (see Figure 1.4), with the notional of the call and the put each equaling 100 million EUR. Accordingly, a delta hedged call option is a straddle, which we have already established is a bet on volatility! An analogous result holds for the portfolio formed by purchasing the put option on EUR-USD and then delta hedging by purchasing EUR-USD spot.

We have shown that calls and puts are bets on volatility when the strike price is equal to the spot level (1.37 in our example). Such options are called *at-the-money spot*, or ATMS, options. In Chapter 9, I show that this argument is (locally) true for all options.

The discussion here goes some way toward convincing the reader that, despite the common usage of call and put options to bet on the direction in which the underlying security will move, the ability to delta hedge means that options are fundamentally not bets on direction, but are bets on volatility.

The example in this section also provides the reader with an early and intuitive introduction to the more general concept of delta hedging. Delta hedging means trading an amount of the underlying currency pair such that the options trader becomes indifferent to the direction in which spot moves.

[2]More precisely, the trader should sell 100 million of the EUR-USD forward contract to be *forward hedged*. More on forward hedging in later chapters.

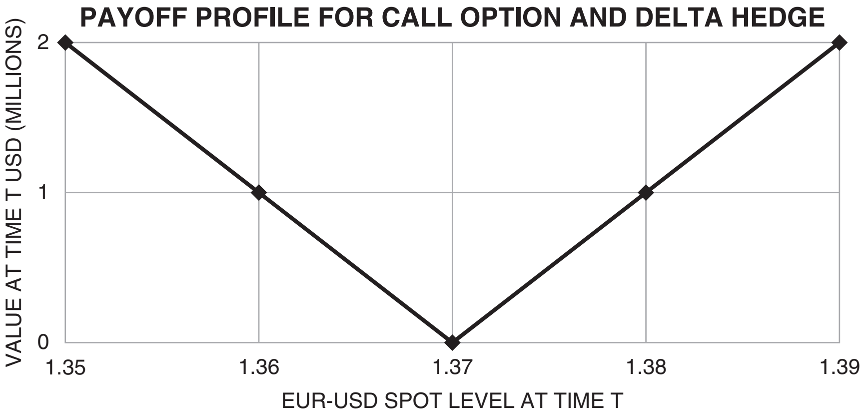

FIGURE 1.4 The figure shows the payoff formed from the purchase of a EUR-USD call option with strike 1.37 and notional of 200 million EUR, and selling 100 million EUR-USD at 1.37. The result is a straddle. Even though the trader purchased a call option, she is now indifferent to the direction in which EUR-USD moves, and is only concerned that the volatility is high so that the probability that EUR-USD moves away from 1.37 is high and she is able to collect a larger payoff.

1.3 OPTION PREMIUMS AND BREAKEVENS

Earlier, we established that the owner of the straddle requires the spot rate to move in order to profit (EUR-USD must move away from 1.37 in our example). An important question is, *how much* do we require spot to move so that the owner of the straddle is able to earn back the premium that she spent to purchase it? The points in spot space beyond which the payoff of the straddle is greater than the initial premium paid are called the *breakevens*. Let us investigate this idea further. The first step is to understand how option premiums are quoted.

1.3.1 Understanding Option Premiums

There are three main conventions in which option premiums are quoted. Again, let us take the example of a EUR-USD option.

% EUR If the trader wishes to purchase the option using cash held in EUR, she may ask for a price in % EUR. Suppose that she is quoted a price of 0.75% EUR to purchase $N = 200$ million EUR of a $K = 1.38$ EUR-USD option. Then, quite simply, she must pay $0.75\% \times 200$ million EUR = 1.5 million EUR to purchase the option.

% USD If the trader wishes to pay the premium in USD, then she can ask for a price in % USD. Suppose EUR-USD spot is trading at 1.37. She will be quoted 0.745% USD to purchase the $K = 1.38$ option. The reason is that the USD notional of the option is 276 million USD. This is calculated based on the definition of a EUR-USD call option. By definition the trader may purchase 200 million EUR at 1.38 if she chooses. This is equivalent to selling 200 million EUR × 1.38 = 276 million USD. Her cost is therefore 0.745% × 276 million USD = 2.055 million USD, which is equivalent to 1.5 million EUR at the current spot rate of 1.37. Note that the price in % EUR and % USD are only equal if the strike equals the current spot level, and they are different otherwise.

USD Pips The third is USD pips. A *pip* or *price interest point* is loosely defined as the smallest price move that a given exchange rate makes based on market convention. So, for example, if EUR-USD moves from 1.3700 to 1.3710, we say that it has moved by 10 pips because market convention is to quote EUR-USD to four decimal places. However, if USDJPY moves from 115.50 to 115.60, then this is still 10 pips because market convention is to quote USD-JPY to two decimal places.

The USD pip price is defined by N × USD pip price = Cash price in USD, where N is the notional of the option in EUR. In the context of the example above, the USD pip price of the option is 102.75 USD pips. The reason is that 200 million EUR × 102.75 USD pips = 2.055 million USD.

A final important and related point to note is that option prices are most often quoted in terms of *Black-Scholes implied volatility*, which I denote by $\sigma_{implied}$. We shall study the meaning of $\sigma_{implied}$ in more detail later in the chapter and over the course of this book. At this stage, it is important for the reader to understand simply that it is the number that one must plug into a function (the Black-Scholes-Merton [BSM] function) to retrieve a premium price in one of the three conventions described previously.

1.3.2 Relation Between Premium and Breakeven

One may ask, why have the quoting conventions described earlier developed as they have? There are at least two reasons.

First, and very plainly, quoting in terms of % of notional or pips is convenient because it takes the option notional out of the discussion, whereas a cash price is tied to a given notional.

Second, and more important in the context of practical options trading, is that thinking in terms of % or pips allows the option trader to perform instant *rule-of-thumb* analysis on the option that she is considering buying or selling. Consider, for example, the 1.38 strike call option priced at 102.75 USD pips earlier. The trader knows that her breakeven is therefore 1.390275.

Next, consider the 1.37 strike straddle. Suppose that the spot rate is 1.37, the cost of the 1.37 EUR-USD call option is 0.5% USD, and the cost of the 1.37 put option is also 0.5% USD. The trader purchases both options. She buys notional N EUR of the call and N EUR of the put. The total cost in USD is therefore 1% of $N \times 1.37$. The trader therefore requires spot to move away from 1.37 by 1% in order to break even because if spot moves higher to 1.3837, she makes 1% on notional $1.37N$ and similarly if spot moves lower to 1.3563. Figure 1.5 assumes N = 100 million EUR and makes this point clear.

Equivalently, the price of the straddle in USD pips is 68.5 for the call and 68.5 for the put. Therefore the total is 137 USD pips, to give breakevens of 1.3563 and 1.3837.

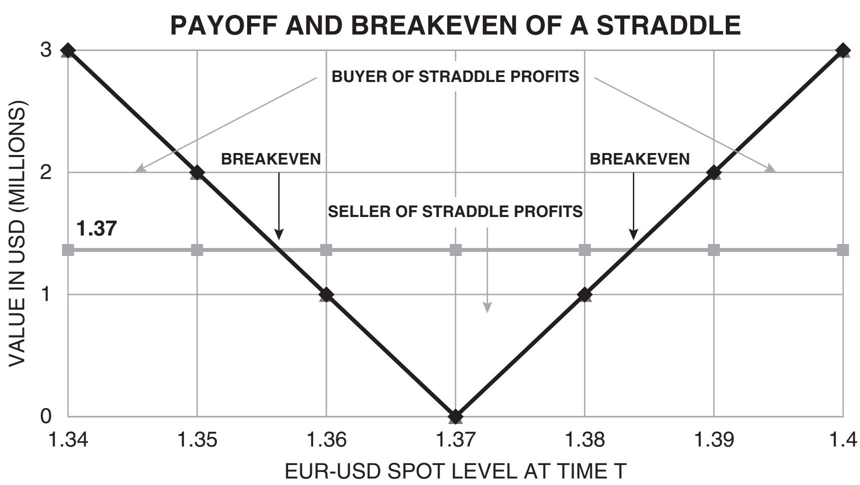

FIGURE 1.5 The trader has purchased 100 million EUR each of the call and the put. This is equivalent to 137 million USD each of the call and the put. The cost is 1.37 million USD, or 1%. The trader's breakeven is therefore 1%. We see from the diagram that if spot is below 1.3563 or above 1.3837 at expiry, the payoff is greater than the premium paid and the buyer of the straddle profits. However, if spot is between 1.3563 and 1.3837 at expiry, then the seller profits. This analysis assumes that the trader takes no other action toward hedging her straddle position.

In short, the advantage of prices being quoted using one of the conventional methods described earlier is that they tell you how far spot must move in percentage or pip terms for the option position to break even. The concept of a breakeven provides a starting point for option traders to think about whether options are over- or under-priced.

At this stage, it is sufficient for the reader to think of the breakeven as follows. If the breakeven is small (large) compared with the amount that the trader expects the spot market to move, then she should consider buying (selling) the straddle. However, in later chapters, we see that this idea is too simplistic. The ability of the options trader to delta hedge means that spot may exceed the breakeven and yet the seller of the option profits.

1.4 STRIKE CONVENTIONS

The term *at-the-money* is commonly used in three contexts in FX options. They are at-the-money spot (ATMS), at-the-money forward (ATMF) and at-the-money (ATM). I discuss each of these below.

ATMS I discussed ATMS earlier. To recapitulate, this is when the strike K is equal to the rate at which the underlying currency's spot market is trading, which I denote by S_t. That is, $K = S_t$ for an ATMS option.

ATMF An ATMF option is when the strike K is equal to the rate at which the underlying currency pair's forward is trading. I assume zero interest rates for both currencies in the first part of this book. Readers already familiar with the basics of forwards will note that this means that the spot and the forward are equal to each other. Therefore, unless otherwise specified, there is no distinction between ATMF and ATMS options until Chapter 9.

ATM Finally, there are at-the-money (ATM) options. Readers already working with options will commonly hear the term *ATM* because this is the most commonly traded and liquid contract in the OTC market. We have not yet built the background knowledge to understand the ATM contract. To do so requires an understanding of implied volatility, the topic of the next section. I therefore explain the ATM contract later in the chapter, but at this stage the reader should think of an ATM contract in a similar way to the ATMS contract.

1.5 WHAT IS VOLATILITY?

Until now we have understood volatility only conceptually as the uncertainty associated with the future price of the spot rate. High volatility implies that the spot rate may change by a large amount and vice versa for low volatility. Here, I make the concept of volatility more precise.

Let us overlay the payoff profile of the straddle with a probability density function (PDF) of EUR-USD.[3] This is shown in Figure 1.6. The standard deviation of the PDF, denoted by σ, is closely related to the probability that EUR-USD moves away from 1.37. Clearly, this probability is higher if σ is larger. Options traders usually annualize σ and then refer to it as *volatility* (more on how to annualize later).

There are two types of σ that are most commonly used for options trading, *implied volatility*, $\sigma_{implied}$, and *realized volatility*, $\sigma_{realized}$. Let us discuss each of these in turn.

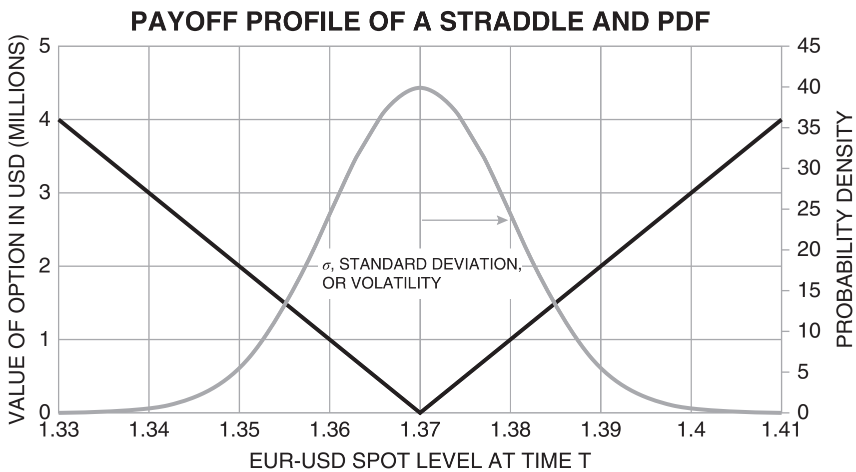

FIGURE 1.6 The figure shows the payoff of a straddle option strategy and the PDF of EUR-USD overlayed. The larger σ, the more chance that EUR-USD moves far from 1.37 and the higher the payoff of the straddle.

[3] Readers unfamiliar with the concept of a PDF may consult Section A.1 of the appendix.

1.5.1 Implied Volatility, $\sigma_{implied}$

Suppose an FX option is trading in the market at certain cash price. $\sigma_{implied}$ is the number that one must plug into the Black-Scholes-Merton (BSM) option pricing function, along with several other parameters, in order to match this price. The reader may be confused as to why $\sigma_{implied}$ is a useful concept. After all, one can create an arbitrary function $f(x)$ of a parameter x that is completely different to the BSM function and then back out x such that $f(x)$ matches an option price! x would not represent any economic meaning. The reason that $\sigma_{implied}$ remains interesting is because BSM derived their formula using a plausible description for the PDF of the exchange rate. If the BSM PDF had turned out to describe observed exchange rate movements, then $\sigma_{implied}$ would be a useful quantity with economic meaning; it would be the market's forecast of the volatility that will occur in the future.

Although it does not capture several important features of real markets, fortunately the BSM approach does capture many of them. Therefore, options traders nevertheless think of $\sigma_{implied}$ as an approximation of the market's forecast of the volatility that will occur in the future and as the width or standard deviation of the PDF of spot that is inferred from the prices of options.[4] I continue for the rest of this chapter on this basis. It should be intuitive that the larger $\sigma_{implied}$, the higher the price of the straddle.

The value of $\sigma_{implied}$ changes many times in a typical trading day, but for the purpose of this discussion, let us suppose that $\sigma_{implied} = 10\%$ and the option expires in 1 year. Economically, this means that to a good approximation, over the year, option prices imply that there is a 68% probability that the return of spot is between −10% and +10%. That is, $\sigma_{implied}$ is the *one standard deviation* spot return. The (optional) feature box explains this calculation in more detail. Appendix A.1.3 provides a refresher on standard results relating to normal distributions.

[4]Readers familiar with the concept of risk premiums will note that $\sigma_{implied}$ contains a so-called *volatility risk premium* and it is therefore not the standard deviation of the objective PDF but the risk-neutral, or risk-adjusted PDF. I assume risk-neutral investors at this stage in order to build intuition and refer readers to Cochrane (2005) for a detailed treatment of risk premia. Readers familiar with *smile* will also note that the standard deviation must be calculated via volatility swap prices, not $\sigma_{implied}$. Again, I sacrifice accuracy for intuition.

THE BLACK-SCHOLES-MERTON APPROACH

In their seminal work, Black & Scholes (1973) and Merton (1973) priced options by assuming that the log spot return, rather than the absolute return, of the underlying security is normal (see Appendix A.1.3 for more on log-normal returns). I describe the BSM model in detail in Chapter 9 but, in short, their assumption is that,

$$r_t^T \equiv \ln \frac{S_T}{S_t} \sim \mathcal{N}\left(-\frac{1}{2}\sigma^2\tau, \sigma^2\tau\right).$$

Here, r_t^T is the log return between the present time t and T, the remaining time until expiry is $\tau = T - t$, and $\mathcal{N}(\mu, \sigma^2)$ denotes a normal or Gaussian distribution with mean μ and variance σ^2. For simplicity I have assumed zero interest rates at this stage.

For readers unfamiliar with log returns the important point is to note that they are much like standard returns when returns are small. For example, if $S_t = 1.00$ and $S_T = 1.01$, then the return $S_T/S_t - 1 = 1\%$, and $\ln(S_T/S_t) \approx 1\%$. When returns become large, the log return and standard return diverge Among others, one of the advantages of the log-normal approach is that it ensures that the underlying price of the currency can never become negative.

Applying standard results relating to normal distributions, we see that

$$\text{Prob}\left(-\sigma\sqrt{\tau} - \frac{1}{2}\sigma^2\tau < \ln\frac{S_T}{S_t} < \sigma\sqrt{\tau} - \frac{1}{2}\sigma^2\tau\right) = 0.68 \qquad (1.1)$$

Finally, substituting $\sigma = \sigma_{implied} = 10\%$ and $\tau = 1$ year we find that there is a 68% probability that $\ln\frac{S_T}{S_t}$ is less than 9.5% and greater than −10.5%. Therefore, there is a 68% probability that the actual spot return, $\frac{S_T}{S_t} - 1$, over 1 year is between −10% and 10%, because $\exp(-0.105) - 1 = -10\%$ and $\exp(0.095) - 1 = 10\%$.

So far we have considered a 1-year time horizon. Suppose instead that we are interested in a 3-month option. In this case, apply $\tau = \frac{1}{4}$ and $\sigma_{implied} = 10\%$ into the equation above to find that options have priced a 68% probability that spot will be within ±5% of its present value.

An important point to note is that $\sigma_{implied}$ is an annualized number. The one standard deviation return priced into options depends on $\sigma_{implied}$ appropriately scaled by the tenor of the option under consideration.

Another important point to note is that there is typically a different $\sigma_{implied}$ for options of different tenors. So, for example, it is feasible for $\sigma_{implied}$ to be 8% for a 3-month option, and to be 10% for a 1-year option. The variation in $\sigma_{implied}$ as a function of tenor is known as *term structure*. I discuss term structure in detail in Chapter 6. There I change to the more appropriate notation of $\sigma_{implied}(T)$ to denote the $\sigma_{implied}$ belonging to a particular expiry date T.

Therefore, as a heuristic, if the trader thinks that the probability that spot will move by more than ±10% over the next year is greater than 32%, she should consider buying the option and going long volatility. If she has the opposite view, then she should consider selling the option.

ATM Options If interest rates are zero, then an ATM option has strike

$$K = S_t e^{\frac{1}{2}\sigma^2_{implied}\tau}, \tag{1.2}$$

where $\tau \equiv T - t$ is the time remaining until expiry of the option. The main point to note at this stage is that for typical FX market parameters, $K \approx S_t$. For example, for a 1-year expiry option and $\sigma_{implied} = 10\%$, $K = 1.005 \times S_t$. The difference is smaller for shorter-dated expiries. Therefore, an ATM option looks much like an ATMS option.

This equation comes from the BSM formula, which I discuss in detail in Chapter 10. However, for curious readers, it is the strike K such that the delta of a straddle of strike K is zero, according to the BSM function. It is therefore also known as the delta-neutral straddle (DNS) strike.

Implied Volatility and Breakevens An options trader may prefer to think about $\sigma_{implied}$ in terms of the breakeven that it implies for an ATM straddle, rather than thinking in terms of PDFs. In this subsection, I introduce a simple, approximate rule of thumb to allow option traders to convert between $\sigma_{implied}$ and straddle breakevens.

The breakeven points can be approximately calculated using the following formula,

$$\text{Breakeven(s)} = \text{Spot} \times \left(1 \pm \frac{4.2 \times \sigma_{implied} \times \sqrt{n}}{100}\right), \qquad (1.3)$$

where n is the number of calendar days until the option expires. For example, suppose that $\sigma_{implied} = 10\%$ and the option expires tomorrow, $n = 1$. Then the breakeven is $\pm 0.42\%$ of the current spot rate. Similarly, for a 1-year option, $n = 365$, priced with $\sigma_{implied} = 10\%$, the breakeven is $\pm 8.02\%$ of the current spot rate. Recall from the example in Section 1.3 that the breakeven is also the price of the straddle struck at the current spot rate. The previous formula can therefore be used to calculate option prices.

In practical options trading, three of the most informative (and liquid) options contracts are the overnight, 1-month, and 1-year expiries. To facilitate fast analysis and decision making, it may be worthwhile committing to memory that the breakevens for each of these contracts based on Equation (1.3) are approximately $\pm 4.2\sigma_{implied}$, $\pm 23\sigma_{implied}$, and $\pm 80\sigma_{implied}$ basis points from the current spot level, respectively.

The trader can apply the same rule, but replace the 4.2 with 2.1 if she wishes to consider the breakeven of holding the call option (or put option) alone, rather than the straddle. For example, if $\sigma_{implied}$ for the ATM call (put) option that expires in one day is 10%, the trader requires spot to appreciate (depreciate) by approximately 0.21% in order to break even.

The above rule provides a quick and easy method to convert between $\sigma_{implied}$ and breakevens. However, there are at least two important caveats to note. First, I have ignored the effects of interest rates and *forward carry*. I return to this in Chapter 10. Second, I do not wish to suggest that if the trader's view is that spot will move by more than the breakeven, then she should purchase the option. We shall see in this chapter and in later chapters that because of the possibility of delta hedging, it is the difference between the $\sigma_{implied}$ and $\sigma_{realized}$ that determines the value associated with holding the straddle position and not simply the size of the movement in spot.

At this stage I ask the reader to take Equation (1.3) as given. It may not look like it to readers with prior exposure to options theory, but it is a special case of the BSM function applied to the ATM straddle with interest rates set to zero. The many terms in the BSM function have been subsumed into the number 4.2. I derive this equation using a simple normal approximation in Chapter 3 and then formally in Chapter 10.

1.5.2 Probabilities and Breakevens

In addition to the breakeven formula in Equation (1.3), another useful quantity to note to aid decision making is the probability that the payoff of an ATM straddle exceeds its breakeven. This turns out to be approximately 42%. That is, the shaded area in Figure 1.7 is approximately 42%. This quantity turns out to be close to independent of both $\sigma_{implied}$ and expiry.

I derive this result in the context of the normal distribution in Chapter 6 and in the context of the BSM model in Chapter 9. If the trader's view is that the probability that the spot return exceeds the breakeven is greater than 42%, then she may consider buying the straddle. Else, she may consider selling it.

1.5.3 Implied Volatility and Realized Volatility

In addition to directly considering the probability that spot moves by more than $\pm\sigma_{implied}$ or the probability that spot moves beyond the breakeven point, there are many additional metrics that option traders can use to form

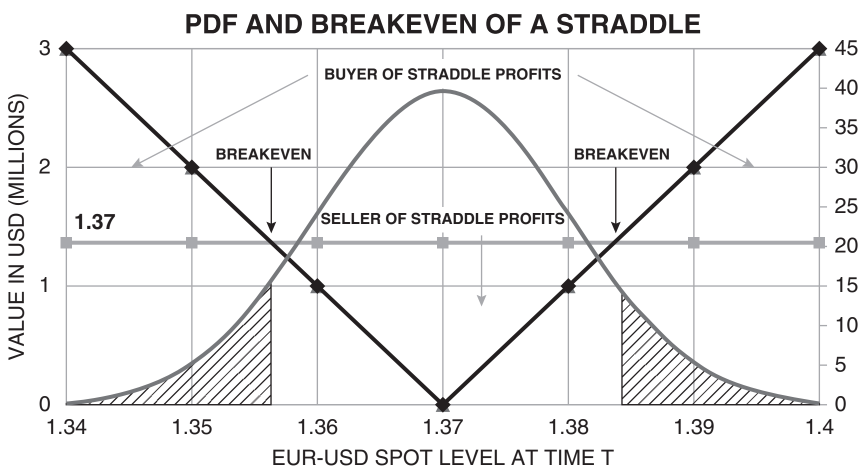

FIGURE 1.7 The figure shows the payoff of the straddle (black), the breakeven points (gray), and the PDF of the EUR-USD spot rate (dark gray). The shaded area is 42%, meaning that the probability that the option payoff exceeds its breakeven is 42%.

a view on $\sigma_{implied}$. One may apply a volatility forecasting model or make a subjective judgment based on an assessment of the present macroeconomic backdrop combined with intuition gained from past experience, among other techniques. However, almost every option trader will study $\sigma_{realized}$ and use it to form at least part of their analysis. This is the topic of the next subsection.

The key point to take away from this subsection is that $\sigma_{implied}$ is a forward-looking metric. It is, to a good approximation, the option market's guess at the width of the future spot PDF and it therefore determines the price that the buyer of the option must pay.

1.5.4 Realized Volatility, $\sigma_{realized}$

Next, there is *realized volatility*, which I denote by $\sigma_{realized}$. Unlike $\sigma_{implied}$, $\sigma_{realized}$ is a backward-looking metric. Crudely put, it is a measure of how much spot has been moving over a particular time period in the past; it measures the standard deviation of the PDF of spot that has been realized. One can see this from a common formula that is used to calculate $\sigma_{realized}$,

$$\sigma^2_{realized} = \frac{\alpha}{k}\sum_{i=1}^{k}(r_i^{i+1})^2 = \frac{\alpha}{k}\sum_{i=1}^{k}\left(\ln\frac{S_{i+1}}{S_i}\right)^2. \tag{1.4}$$

Here, $r_i^{i+1} \equiv \ln\frac{S_{i+1}}{S_i}$ is the log return of spot over a discrete period of time. In practice, this ranges from 5 minutes through to 1 day. There is no market standard sampling frequency and the *best* frequency to use remains an open topic for debate.[5] k is the number of discrete returns in the sample and α is a normalization factor. α is chosen to make sure that $\sigma_{realized}$ is directly comparable to $\sigma_{implied}$.

Recall that $\sigma_{implied}$ is an annualized quantity. Therefore, for example, if one were to use hourly log returns, then $\alpha = 8760$, the number of hours in a year. If instead one were to use daily log returns, then $\alpha = 365$.

Since $\sigma_{realized}$ is related to the average of squared returns, it does not matter if the spot rate has moved higher or lower. All that matters is the size of the returns.

[5]For interested readers, Zhang, Mykland, and Ait-Sahalia (2005) provide discussion and suggestions relating to sampling frequencies and methods.

The feature boxes that follow provide some simple realized volatility example calculations and rules of thumb. Here, I focus on the practical issues of how this number is used, and why it is important.

Applying Realized Volatility Perhaps the best way to understand how an option trader may use the calculated value of $\sigma_{realized}$ in practice is via an example. Suppose that the trader is considering purchasing a 1-year expiry option. She may begin by considering the 365 daily log returns over the past year and inserting them into Equation (1.4). Note that the returns over the weekend may be zero and so there are likely to be closer to 260 non-zero returns. Suppose that the outcome of the calculation is $\sigma_{realized} = 11\%$. This tells her that her best estimate of the standard deviation of the PDF of spot returns over the period of time under study is 11%. Suppose next that $\sigma_{implied}$ corresponding to the 1-year option is 10%. The trader must now make a judgment. If she believes that the next year will look somewhat like the previous year, or that the forthcoming macroeconomic conditions appear even more volatile, then she should purchase the option. The expected profit from doing so is approximately 1% multiplied by the so-called *Vega* of the option (see Chapter 5). Alternatively, if the trader believes that the volatility will diminish over the year to below 10%, then she may consider selling the option. Making this judgment is arguably the greatest challenge in successful options trading.

OPTIONS TRADERS' RULES OF THUMB

- For an option with n days until expiry, the market implies that there is a 68% probability that spot is within $\pm\sigma_{implied}\sqrt{\frac{n}{365}}\%$ in n days.
- Therefore, for a 1-year option, the market implies that there is a 68% probability that spot is within $\pm\sigma_{implied}\%$ of its present price in 1 year.

This can be understood via Equation (1.1). Often in FX markets, $\sigma_{implied}$ is a number of the order of 10% and the most liquidly traded options have τ of the order of 1 year (or less). Therefore, again setting

(*Continued*)

$\sigma = \sigma_{implied}$, $\sigma\sqrt{\tau} \gg \frac{1}{2}\sigma^2\tau$. Also, to first order, $\ln\frac{S_T}{S_t} = \frac{S_T}{S_t} - 1$. Approximately at least, (1.1) reduces to

$$\text{Prob}\left(-\sigma_{implied}\sqrt{\tau} < \frac{S_T}{S_t} - 1 < \sigma_{implied}\sqrt{\tau}\right) = 0.68.$$

We see that, for a given $\sigma_{implied}$, the standard deviation of the spot return scales with the square root of time. We will see that this idea pops up frequently in options theory, particularly when we study the concept of option *Vega* in later chapters.

Our next set of rules of thumb relate to option breakevens.

- For an ATM straddle with n days until expiry, the amount that spot must move in basis points for the position to break even is $4.2\sqrt{n}\sigma_{implied}$. This equates to $4.2\sigma_{implied}$ for an option that expires tomorrow, $23\sigma_{implied}$ for an option that expires in 30 days, and $80.25\sigma_{implied}$ for an option that expires in 1 year.
- Using this formula, traders can easily convert $\sigma_{implied}$ into breakevens.

There are several other important considerations that the trader should make relating to *smile* or *surface*, to the standard error of the estimate $\sigma_{realized}$, and to the volatility of interest rates (or forwards) that we shall return to in later chapters. However, the difference between $\sigma_{realized}$ over the life of the option trade and $\sigma_{implied}$ that determined the initial price of the option provides a measure of the profits that could have been earned from the option position.

REALIZED VOLATILITY

Suppose that the (log) return of spot is ±1% each weekday. At weekends it is stationary. Since we are taking daily data, α is set to 365. Let's assume that we have approximately 6 months' worth of data ($N = 182$ data points). Of these, 130 will be ±1% and 52 will be zeros, because they occur on weekends. Substituting these numbers into Equation (1.4) we see that

$$\sigma_{realized} = \sqrt{\frac{365}{182} * 130 * 1\%^2} \approx 16\%.$$

The rule of thumb that option traders should recall is then simply that a daily 1% move in spot corresponds to an annualized volatility of 16%. Similarly, if spot (log) returns 0.5% per day, then its annualized volatility is 8% and so on. So, for example, if the trader's view is that spot will return 0.5% every business day for, say, the next 3 months, and $\sigma_{implied}$ for the 3-month option is 7%, then the trader should purchase the option because she thinks that the option is worth 8%.

1.6 TRADER'S SUMMARY

- Options are bets on volatility. The straddle owner does not care which way spot moves because her payoff is symmetric. She simply requires spot to move a long way, in order to receive a higher payoff.
- ATMS call options and put options can be converted into straddles by adding a delta hedge. This shows that calls and puts are bets on volatility.
- Implied volatility, $\sigma_{implied}$, is a forward-looking metric. It is approximately the standard deviation of the PDF of spot that is priced into options. It is approximately the amount of realized volatility that is predicted by options traders to take place in the future.
- Realized volatility, $\sigma_{realized}$, determines the amount of volatility that actually takes place over a given period of time. Loosely put, if $\sigma_{realized}$ is greater than $\sigma_{implied}$ over the life of an option trade, then the owner of the option has had the opportunity to profit. It is also a commonly used backward-looking metric to determine whether the trader should make the decision to buy or sell an option.
- The price of an option as a percentage of the notional or in pips tells us how much the owner of the option requires spot to move to break even (assuming no other trading).
- It is straightforward to convert between $\sigma_{implied}$ and breakeven points using the approximate (special case of the) BSM formula in Equation (1.3).

- Option prices imply that the probability that spot exceeds the breakeven point is 42%. If the trader's view is that the true probability is higher (lower), she may consider buying (selling) the option.

This chapter has provided a whirlwind tour of the basics of options trading. The following chapters gradually make these ideas more precise and introduce the reader to new concepts, such as options *Greeks*, while retaining a model-free setting.