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Decision-Making in Problems of System Design, Planning, Operation, and Control

Motivation, Objectives, and Basic Notions

The main objective of this chapter is to offer the reader a broad perspective on the fundamentals of decision-making problems, provide their general taxonomy in terms of criteria, objectives, and attributes involved, emphasize the objectivity and relevance of the uncertainty factor, classify the types of uncertainty, discuss ways of considering the uncertainty factor, and highlight the aspects of rationality of decision-making processes. This chapter also highlights the fundamental differences between optimization and decision-making problems, between the concepts of optimal solutions and robust solutions as well as between approaches to their construction. The main objectives and characteristics of group decision-making are discussed. The role of fuzzy sets is stressed in the general framework of decision-making processes. The most important advantages of their application to individual as well as group decision-making are discussed. The chapter also clarifies necessary notations and terminology (for instance, $<X, F>$ models and $<X, R>$ models) used throughout the book.

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1.1 Decision-Making and Its Support

The life of each person is filled with alternatives. From the moment of conscious thought to a venerable age, from morning awakening to nightly sleeping, a person is faced with the need to make certain decisions. This need is associated with the fact that any situation may have two or more mutually exclusive alternatives and it is necessary to choose one among them. The decision-making process, in the majority of cases, consists of the evaluation of alternatives and the choice of the most preferable from them.

Pospelov and Pushkin (1972) indicate that making the "correct" decision means choosing such an alternative from a possible set of alternatives, in which, by considering all the diversified factors and contradictory requirements, an overall value will be optimized. That is, it will be favorable in achieving the goal sought to the maximal possible degree.

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If the diverse alternatives met by a person are considered as a set, then this set usually includes at least three intersecting subsets of alternatives related to personal life, social life, and professional life. As possible examples, we can indicate, for instance, deciding where to study, where to work, how to spend time on a vacation, who to elect, and many others.

At the same time, if we speak about any organization, it faces different goals and achieves them through the use of diverse types of resources (material, energy, financial, human, etc.), and the performance of managerial functions such as organizing, planning, operating, controlling, and so on (Lu et al. 2007). To fulfill these functions, managers need to participate in the continuous decision-making process. Since each decision supposes a reasonable and justified choice realized among different alternatives, the manager can be called a decision-maker (DM). DMs can be managers at various levels, from a technological process manager to a chief executive officer of a large company, and their decision problems can vary in nature. Besides, decisions can be made by individuals or groups (individual decisions are usually made at lower managerial levels and in small organizations and group decisions are usually made at high managerial levels and in large organizations). As possible examples, we can indicate, for instance, deciding what to buy, where to buy, when to begin a production process, whom to employ, and many others. These problems can concern logistics management, customer relationship management, production planning, and so on.

A person makes simple, habitual decisions easily and frequently in an automatic and subconscious way, without too much intensive thinking. However, in many cases, alternatives are related to complex situations that are characterized by a contradiction of requirements and multiple criteria, ambiguity in evaluating situations, errors in the choice of priorities, and so on. All these factors substantially complicate a way in which decisions are being made.

Furthermore, various facets of uncertainty are commonly encountered in a wide range of problems of an optimization character, which are inherently present in the design, planning, operation, and control of complex systems (engineering, economical, ecological, etc.). In particular, diverse manifestations of the uncertainty factor are associated, for instance, with (Ekel 1999; Pedrycz et al. 2011):

- the impossibility or inexpediency of obtaining sufficient amounts of information with the necessary degree of reliability;
- the lack of reliable predictions of the characteristics, properties, and behavior of complex systems that reflect their responses to external and internal actions;
- poorly defined goals and constraints in the design, planning, operation, and control tasks;
- the infeasibility of formalizing a number of factors and criteria and the need to take into account qualitative (semantic) information.

Considering the essence of the manifestations of the uncertainty factor listed here, more concisely, it is possible (Stewart 2005; Durbach and Stewart 2012) to talk about internal uncertainties (related to DM values and judgments) and external uncertainties (defined by environmental conditions lying beyond the control of a DM).

The described situation with the uncertainty involved is to be considered as natural and unavoidable in the context of problems of complex systems. In principle, it is impossible to reduce these problems to exact and well-formulated mathematical problems; to do this, it is necessary, in one way or another, to "discard" the uncertainty and accept some hypothesis (Pedrycz et al. 2011). However, the construction of hypotheses is a prerogative of the substantial analysis; in reality, this is the formalization of informal situations. One of the ways to address the problem is the formation of subjective estimates based on knowledge, experience, and intuition of involved experts, managers, and DMs in general, and the definition of the corresponding preferences.

Thus, DMs are forced to rely on their own subjective ideas of the efficiency of possible alternatives and importance of diverse criteria. Sometimes, this subjective estimation is the only possible basis for combining the heterogeneous physical parameters of a problem to be solved into a unique model, which permits decision alternatives to be evaluated (Larichev 1987). At the same time, there is nothing unusual and unacceptable in the subjectivity itself. For instance, experienced managers perceive, in a broad and well-informed manner, how many personal and subjective considerations they have to bring into the decision-making process. On the other hand, successes and failures of the majority of decisions can be judged by people on the basis of their subjective preferences.

However, the most complicated aspect is associated with the fact that the essence of problems solved by humans in diverse areas has been changed in recent decades (Trachtengerts 1998). New, more complicated and unusual problems have emerged. For many centuries, people made decisions by considering one or two main factors, while ignoring others that were perceived to be marginal to the essence of the problem. They lived in a world where changes in the surroundings were few and new phenomena arose "in turn" but not simultaneously.

Presently, this situation has changed. A considerable number of problems, or probably the majority of them, are multicriteria in nature where it is necessary to take into account many factors. In these problems, a DM has to evaluate a set of influences, interests, and consequences that characterize decision alternatives. For instance, in decision-making dealing with the creation of enterprises, it is necessary to consider not only the expected profits and necessary investments, but also market dynamics, the actions of competitors, and ecological, political, social factors, and so on.

Considering all the aspects listed here, it is necessary to stress that recognition of the factor of subjectivity of a DM in the process of decision-making conflicts with the fundamental methodological principle of operational research: the search for an objectively optimal solution. Recognition of the right of a DM in the subjectivity of decisions is a sign of the appearance of a new paradigm of multicriteria decision-making (Kuhn 1962). However, in problems with multiple criteria, an objective component always exists. Usually, this component includes diverse types of constraints imposed by the environment on possible decisions (availability of resources, temporal constraints, ecological requirements, social situations, etc.).

A large number of investigations of the psychological character demonstrate that DMs, not being provided with additional analytical support, use simplified and sometimes contradictory decision rules (Slovic et al. 1977).

Besides, Lu et al. (2007) share the opinion given here (Trachtengerts 1998) and indicate that decision-making in the activities of organizations is more complicated and difficult because the number of available alternatives is much larger today than ever before. Due to the availability of information technologies and communication systems, especially the Internet and its search engines, it is possible to find more information quickly and therefore more alternatives can be generated. Second, the cost of making errors can be big enough because of the complexity of operations, automation, and the chain reaction that an error can cause in many parts, in both vertical and horizontal levels, of organizations. Third, there are continuous changes in the fluctuating environment and more uncertainties in the impacting elements, including information sources and information itself. It is also very important that the rapid change of the decision environment requires decisions to be made quickly. These reasons mean organizational DMs require increasing technical support to help make high-quality decisions. A high-quality decision related, for example, to bank management, is expected to bring greater profitability, lower costs, shorter distribution times, and increased shareholder value, attracting more new customers or resulting in a certain percentage of customers responding positively to a direct mail campaign.

Decision support consists of assisting a DM in the process of decision-making. For instance, this support may include (Trachtengerts 1998):

- assisting a DM in the analysis of an objective component; that is, in the understanding and evaluation of the existing situation and constraints imposed by the surroundings;
- revealing DM preferences; that is, revealing and ranking priorities, considering the uncertainty in DM estimates, and shaping the corresponding preferences;
- generating possible solutions; that is, shaping a list of available alternatives;
- evaluating possible alternatives, considering DM preferences and constraints imposed by the environment;
- analyzing the consequences of decision-making;
- choosing the best alternative from the DM’s point of view.

Generally, computerized decision support is based on the formalization of methods for obtaining initial and intermediate estimates given by a DM and on the algorithms for a proper decision process. The formalization of methods for generating alternatives, their evaluation, comparison, choice, prioritization, and/or ordering is extremely complicated. One of the main complexities is associated with the fact that a DM, as a rule, is not ready to provide quantitative estimates in the decision process, is not accustomed to the evaluation of proper decisions on the basis of applying formal mathematical methods, and analyzes the consequences of decisions with significant difficulties.

In fact, decision support systems have existed for a long time; for example, one can refer here to councils of war, ministry boards, various meetings, analytical centers, and so on (Trachtengerts 1998). Although they were never called decision support systems, they executed the functions of such systems, at least partially.

The term “decision support system” appeared at the beginning of the 1970s (Eom 1995). There are several definitions of this concept, such as that given in Larichev and Moshkovich (1996): “Decision support systems are man–machine objects, which permit a DM to use data, knowledge, objective and subjective models for the analysis and solution of semi-structured or unstructured problems.”

Taking into account this given definition, it is necessary to indicate that one of the important features of decision-making problems is associated with their structures. In particular, it is possible to distinguish among structured, semi-structured, and unstructured problems of decision-making (Simon 1977; Larichev and Moshkovich 1996; Lu et al. 2007). The last two types of decision-making problems are also called ill-structured.

In *structured* problems (quantitatively formulated problems), essential relationships are established so convincingly that they can be expressed in numbers or symbols that receive, ultimately, numerical estimates. Such problems can be described by existing “traditional” mathematical models. Their analysis becomes possible by applying standard methods.

Unstructured problems (qualitatively expressed problems) include only a description of the most important resources, indicators, and characteristics. Quantitative relationships between them are not known. These problems cannot be described by existing traditional mathematical models and cannot be analyzed by applying standard methods yielding “traditional” solutions.

Finally, *semi-structured* problems (or mixed problems) include quantitative as well as qualitative elements. As these problems are analyzed, qualitative, little-known, poorly explored, uncertain parameters have a tendency to dominate. These problems are positioned between structured and unstructured problems, having both structured and unstructured elements. The solution of these problems involves a combination of both standard solution procedures and active DM participation.

Taking into account this classification, typical problems in operational research can be called *structured*. This class of problem is widely used in the design, planning, operation, and control of engineering systems. For example, it is possible to talk about the design of forms of an aircraft hull, planning of water supply systems, control of power systems, and so on.

The distinctive characteristics of unstructured problems can be outlined as follows (Larichev and Moshkovich 1996).

- uniqueness of choice in the sense that, at any time, the problem is a new one for a DM or it has new properties in comparison to a similar problem solved in the past;
- uncertainty in the evaluation of alternative solutions;
- the qualitative character of the evaluations of problem solutions, most often formulated in verbal form;
- the evaluation of alternatives obtained only on the basis of the subjective preferences of a DM;
- the estimates of criteria obtained only from experts.

Typical unstructured problems are associated, for example, with planning new services, hiring executives, selecting a locale for a new branch, choosing a set of research and development projects, and others.

If we talk about semi-structured problems, their solutions are based on applying traditional analytical models as well as models based on DM preferences. As an example, one can point at the problem related to liquidation of the consequences of extraordinary situations associated with radioactive contamination (Trachtengerts 1998). In forming its solution, analytical models can be applied to define the degree and character of radioactive contamination for given temporal intervals. At the same time, models based on DM preferences can be applied in the choice of measures for liquidation of the consequences of radioactive contamination. It is also possible to qualify many problems associated with economic and political decisions, medical diagnostics, and so on as semi-structured problems.

Returning to the issue of computerized decision support, it should be noted that, due to the large number of components (variables, functions, and parameters) involved in many decisions, this has become a basic requirement to assist DMs in considering and analyzing the implications of various courses of decision-making (Lu et al. 2007). Besides, the impact of computer technologies, particularly Internet- and Intranet-based, on organizational management is increasing rapidly. Here, the main tendency is associated with the fact that computer applications in organizations are moving from transaction processing and monitoring activities to problem analysis and finding solutions (Lu et al. 2007). Internet- or intranet-based online analytical processing and real-time decision support are becoming the cornerstones of modern management, in particular within the elaboration of e-commerce, e-business, and e-government.

There is a trend toward providing managers with information systems that can assist them directly with their most important task, namely making decisions.

A detailed description of the advantages generated by applying computerized decision support systems for individual as well group decision-making is given in Lu et al. (2007). At the same time, the authors of Lu et al. (2007) indicate that the important issue is that, with computerized decision support technologies, many complex decision-making problems can now be handled effectively. However, these technologies can be better used in analyzing structured problems rather than semi-structured and unstructured problems. In an unstructured problem, only part of the problem can be supported by advanced tools such as intelligent decision support systems. For semi-structured problems, the computerized decision support technologies can improve the quality of information on which the decision is based by providing not just a single, unique solution, but a range of alternative solutions from the decision uncertainty regions. Their occurrence and their essence will be discussed in the next section.

1.2 Problems of Optimization and Decision-Making

Is there any difference between the notions of “optimization” and “decision-making?” Are these notions synonymous or not? Partial answers to these questions have been given in the previous section. However, deeper and more detailed considerations are required.

A traditional optimization problem is associated with the search for an extremum (minimum or maximum, in accordance with the essence of the problem) of a certain objective function, which reflects interests of a DM, when observing diverse types of constraints (related to allowable resources, physical laws, standards, industrial norms, etc.). Formally, it is possible to represent an optimization problem as follows:

$$F(x) \to \underset{x \in L}{\text{extr}} \tag{1.1}$$

where L is a set of feasible solutions in $\mathbf{R}^n$ defined by the considered constraints.

To solve the problem Eq. (1.1), it is necessary to find x^0 such that

$$x^0 = \arg \underset{x \in L}{\text{extr}}\, F(x) \tag{1.2}$$

If numerical details to Eq. (1.1) have been provided and we can obtain a unique solution without any guidance or assistance of a DM, then Eq. (1.1) forms an optimization problem.

Generally, an optimization problem may be complicated from the mathematical point of view, and a large amount of time might be required to generate a solution. Can human participation in the search for a solution be useful? Without any doubt, such participation could be useful, because, for instance, the

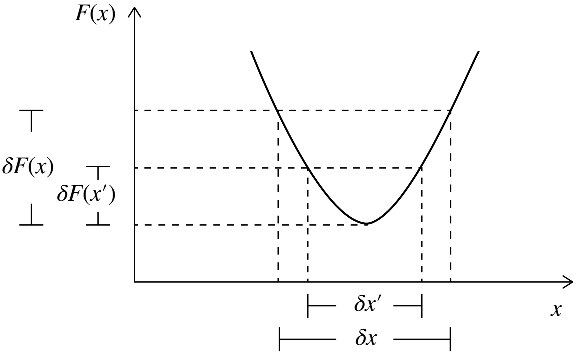

Figure 1.1 Decision uncertainty region and its reduction through the reduction of the uncertainty of information.

indication or change of initial points for a search or the introduction of heuristics can reduce the time necessary to obtain an optimal solution. However, this cannot change the solution and, in principle, a unique solution to the problem can be obtained without human participation.

At the same time, the presence of any type of uncertainty can require human participation in order to generate a unique solution to the problem.

For instance, the uncertainty of information gives rise to some decision uncertainty regions. As an example, Figure 1.1 (Pedrycz et al. 2011) demonstrates that the uncertainty of information $\delta F(x)$ in the estimation of an objective function $F(x)$ leads to a situation where formally the solutions coming from a region δx cannot be distinguished, thus generating a decision uncertainty region. Taking this into consideration, the formal formulation Eq. (1.1) can be transformed to the following form:

$$F(x,\theta) \rightarrow \underset{x \in L(\theta)}{\text{extr}} \tag{1.3}$$

where θ is a vector of uncertain parameters, whose existence changes the essence of Eq. (1.1). In particular, we can say that the solution Eq. (1.2) is an optimal one for a concrete realization of θ (a concrete hypothesis); however, for some other realization (another hypothesis), it is no longer optimal.

What are the ways to reduce this uncertainty region? The first way is to elicit information (let us not forget that any information has some cost), for example, by acquiring additional measurements or examining experts to reduce the level of uncertainty. As shown in Figure 1.1, the reduction of the uncertainty $\delta F(x)$ to $\delta F(x')$ permits one to obtain a reduced decision uncertainty region with $\delta x' < \delta x$.

However, if there is no possibility of reducing the uncertainty of information, it is possible to resort to another approach. It is associated with introducing

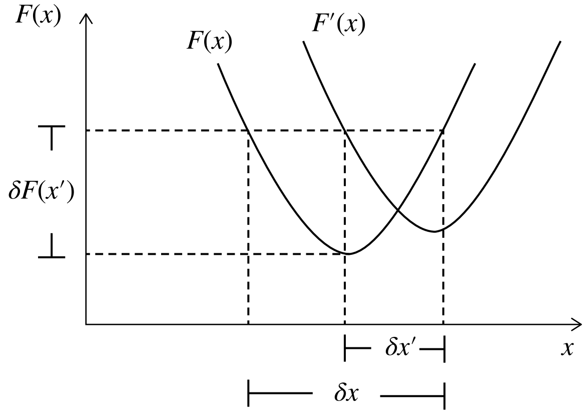

Figure 1.2 Decision uncertainty region and its reduction through the introduction of additional criteria.

additional criteria to try to reduce the decision uncertainty regions. As demonstrated in Figure 1.2, the introduction of the objective function $F'(x)$ allows one to reduce the decision uncertainty region as well, providing $\delta x' < \delta x$.

On the other hand, the existence of more than one objective function may be considered as uncertainty as well (Pedrycz et al. 2011): uncertainty of goals because this situation may be characterized as "we do not know what we want." Although the nature of this type of uncertainty is not the same as the uncertainty of available information, it also leads to the generation of decision uncertainty regions.

In particular, let us consider the simple problem of minimizing two objective functions $F_1(x) = F_1(x_1, x_2)$ and $F_2(x) = F_2(x_1, x_2)$, considering a set of feasible solutions L. We can transform L from the decision space to a certain region L_F of the space of objective functions $F_1(x)$ and $F_2(x)$ (or, simply, the objective space). In Figure 1.3, it is possible to observe that point a corresponds to the best solution $(\min_{x\in L} F_1(x))$ from the point of view of the first objective function. On the other hand, point b corresponds to the best solution $(\min_{x\in L} F_2(x))$ considered from the viewpoint of the second objective function.

Is point c a solution to the problem? Yes, it is. Is it possible to improve this solution? Yes, we can do it by passing to point d. Can we improve this solution? Yes, this is possible by passing to point e. Can we improve this solution? This is possible by passing to point f. Is it improving? No, we cannot advance here. It is possible to pass to point g but this step does not make the resulting solution any better: we can improve it from the point of view of $F_1(x)$, but deteriorate its quality from the point of view of $F_2(x)$. In a similar way, by passing to point h, we can improve the solution from the point of view of but deteriorate it from the point of view of $F_1(x)$.

Thus, formally, the solution to the problem presented in the objective space is a boundary Ω_L^P of L_F located between points a and b. The set $\Omega^P \subseteq L$

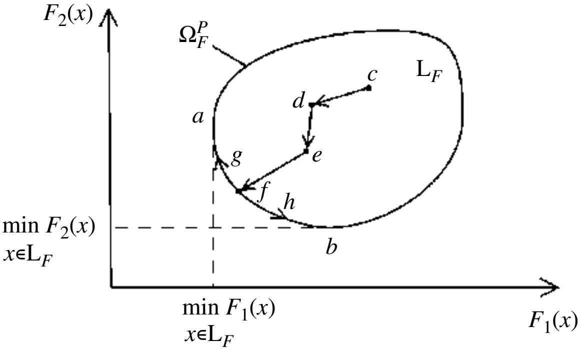

Figure 1.3 The concept of Pareto-optimal solutions.

corresponding to Ω_F^P is the problem solution, which is called a Pareto-optimal solution set. This concept of optimality was proposed by Edgeworth (1881) and was further generalized by Pareto (1886). Although we say that Ω^P is the problem solution, from a formal point of view, this is not a solution that can be implemented. In reality, it is the decision uncertainty region. The choice of a particular Pareto-optimal solution is based on the DM's involvement.

The more difficult situations are associated with problems where uncertainty of information exists as well as the uncertainty of goals. Some features of their analysis are discussed in the next section.

The problems of an optimization character, which include the uncertainty of information and/or the uncertainty of goals and demand human participation in their solution, are inherent problems in decision-making. Taking this into consideration, it is necessary to make some additional observations.

One of the most important criteria (Larichev 1984) for classifying decision-making problems is the existence or lack of an objective model established for the corresponding problem. Note here that it is not uncommon to encounter situations where it is impossible to talk about the existence of objective functions in decision-making problems. The models that can be used for analyzing these problems reflect "a point of view" and, in a more general sense, the "world outlook" of a DM. In these cases, an obvious question is how to choose actions that correspond, in the best way, to the preferences of a DM (Keeney and Raifa 1976; Pedrycz et al. 2011) and are based on his/her knowledge, experience, and intuition. Taking this into account, semi-structured and unstructured problems, classified in the previous section, are subjects of decision-making.

In conclusion, the following general tendency is visible. If we intend to solve an optimization problem, we generally look for the best solution. If we talk about a decision-making problem, the methodology used to solve it is quite distinct: we do not look for the best solution, but apply information arriving from different sources and try to eliminate some alternatives, which are dominated by other alternatives, in order to reduce the decision uncertainty regions.

1.3 Uncertainty Factor and Its Consideration

The incorporation of the uncertainty factor in constructing mathematical models, related to problems of an optimization character, serves as a means for increasing their adequacy and, as a result, the credibility and factual efficiency of decisions based on their analysis.

Durbach and Stewart (2012) classify the following five formats of uncertainty:

- probabilities;
- decision weight;
- explicit risk measures;
- fuzzy numbers;
- scenarios.

However, it is possible to distinguish two general approaches to taking into account the uncertainty of initial information: probabilistic and possibilistic. The first one is based on the preferential use of statistical data. Sometimes, this approach can be useful. However, generally, it is difficult or impossible to speak about the future, applying only information of the past. Taking this into account, we try to utilize the possibilistic approach, which consists of constructing and analyzing diverse representative combinations of initial data, states of nature, or scenarios, on the basis of applying information of the past as well as guesses, assumptions, and so on, based on knowledge, experience, and intuition of the involved experts.

The presence of several representative combinations of initial data, states of nature, or scenarios makes the process of decision-making under conditions of uncertainty completely different from traditional ways of solving problems by traditional methods of operational research. The majority of approaches, methods, and techniques of operational research is based on the conception of the existence of an "optimal solution." Considering this, most solution search strategies of operational research are directed at obtaining this "optimal solution." Under conditions of uncertainty, the notion of the "optimal solution" does not work: the "optimal solution" obtained for one scenario is not the "optimal solution" for another scenario. Taking this into account, it is necessary to talk about generating so-called "robust solutions": solutions that are intended to satisfy all possible scenarios in a maximal degree. However, if attempts at constructing these solutions for monocriteria problems are well reflected in the literature (for instance, Luce and Raiffa 1957; Raiffa 1968; Belyaev 1977; Webster 2003), the questions of their generation for multicriteria problems are not covered in the literature. Considering this, one of the main objectives of this work is to fill this gap, in particular, on the basis on combining two branches of mathematics of uncertainty such as the elements of game theory and fuzzy set theory.

Taking this into account, it becomes necessary to indicate that the general methodology of analyzing diverse decision-making problems that will pass through the book is different from the methodology of operational research: as was emphasized in the previous section, we do not search for a unique problem solution, but search for alternatives dominated by others ones, reducing, step-by-step, the number of non-dominated solution alternatives and, therefore, reducing the regions of solution uncertainty.

1.4 Multicriteria Decision-Making: Multiobjective and Multiattribute Problems

The uncertainty of goals in decision-making is an important manifestation of uncertainty that relates to a multicriteria character of many problems encountered in the project, planning, operation, and control of complex systems of different nature. Some professionals in the field of decision-making and systems analysis (for example, Lyapunov 1972) affirm that, from a general point of view, this type of uncertainty is the most difficult to overcome and handle because “we simply do not know what we want.” In reality, this type of uncertainty cannot be effectively captured only on the basis of applying formal models and methods, as sometimes the unique information sources are the individuals who make decisions.

Multicriteria decision-making is related to making decisions in the presence of multiple and conflicting criteria. Multicriteria decision-making problems may range from our daily life decision problems, such as a purchase of a car, to those affecting entire nations, as in the judicious use of money for the preservation of national security (Lu et al. 2007).

However, even with the existing diversity, all multicriteria decision-making problems share the following common characteristics (Hwang and Yoon 1981):

- multiple criteria: each problem has multiple criteria, which can be objectives or attributes;
- conflicting criteria: multiple criteria conflict with each other;
- incommensurable units: criteria may have different units of measurement;
- design/selection: solutions to multicriteria decision-making problems are either to design the best alternative(s) or to select the best one among previously specified finite alternatives.

Taking this into account, two types of criterion should be distinguished: objectives and attributes. In such a manner, multicriteria decision-making problems can be classified into two wide classes (Hwang and Masud 1979; Hwang and Yoon 1981):

- multiobjective decision-making;
- multiattribute decision-making.

Although the main difference between these two classes, as the authors of Hwang and Masud (1979) and Hwang and Yoon (1981) indicate, is that the first concentrates on continuous decision spaces and the second focuses on problems with discrete decision spaces, the first class may include problems with integer, Boolean, and discrete variables, for instance Villareal and Karwan (1982) and Ramesh et al. (1986).

To proceed with the next steps, some basic concepts and terminology are given next based on Hwang and Masud (1979); Hwang and Yoon (1981); and Lu et al. (2007).

Criteria are the standard of judgment or rules to test acceptability. In the multicriteria decision-making literature, they indicate objectives and/or attributes.

Objectives are the reflection of the desire of DMs and indicate the direction in which DMs want to concentrate. Multiobjective decision-making problems, as a result, involve the design of alternatives that optimize or at least satisfy the objectives of DMs.

Goals are entities desired by DMs and expressed in terms of a specific state in space and time. Thus, while objectives give the desired direction, goals give a desired (or target) level to achieve.

Attributes are the characteristics, qualities, or performance parameters of alternatives. Multiattribute decision-making problems involve the selection of the “best” alternative from a pool of preselected alternatives described in terms of their attributes.

Generally, multiobjective decision-making is known as the continuous type of multicriteria decision-making and its main characteristics are that the DM needs to achieve multiple objectives while these objectives are non-commensurable and conflict with each other. A multiobjective decision-making model includes a vector of decision variables, objective functions that describe the objectives, and constraints. The DM attempts to maximize or minimize the objective functions.

At the same time, multiattribute decision-making is related to making preference decision (that is, comparison, choice, prioritization, and/or ordering) over the available alternatives that are characterized by multiple, usually conflicting, attributes. The main peculiarity of multiattribute decision-making problems is that there are usually a limited number of predetermined alternatives that are associated with a level of achieving the attributes. Based on the attributes, the final decision is to be made.

Finally, it is necessary to discuss in detail the concept of alternatives. How to generate alternatives is an important moment of the process of multiobjective and multiattribute decision-making model construction (Lu et al. 2007). In almost all multiobjective decision-making models, the alternatives are generated automatically by the models. In the case of multiattribute decision-making models, however, it is necessary to generate alternatives manually. Sometime, the essence of the problem defines the number of alternatives. However, in general, how and when to stop generating alternatives becomes a very important issue. Generating alternatives significantly depends on the availability and cost of information, and also requires reliance on expertise in the problem area. Alternatives can be generated with the use of heuristics as well, and they could come from either individuals or groups.

The issues related to the necessity of setting up and solving multicriteria problems as well as the classification of decision-making situations, which need the application of the multicriteria approach, have been discussed in many works (for instance, Larichev 1984; Gomes et al. 2002). However, it is possible to identify two major types of situation that require the application of a multicriteria approach (Ekel 2002; Pedrycz et al. 2011):

- Problems whose solution consequences cannot be estimated with a single criterion: these problems are associated with the analysis of models including economic as well as physical indices (when alternatives cannot be reduced to comparable form) and also by the need to consider indices whose cost estimation is hampered or impossible (for example, many power engineering problems are considered on the basis of technological, economical, ecological, and social nature criteria: Berredo et al. 2011; Ekel et al. 2016).
- Problems that may be solved on the basis of a single criterion (or several criteria). However, if the uncertainty of information does not permit one to derive unique solutions, it is possible to reduce these problems to multicriteria decision-making by applying additional criteria, including those of a qualitative character (for example, "flexibility of development," "complexity of maintenance," "attractiveness of investments," and so on, whose utilization is based on the knowledge, experience, and intuition of involved experts). This can serve as a convincing means to contract the corresponding decision uncertainty region. It could be regarded as an intuitively appealing approach exercised in the practice of decision-making.

In accordance with the major types of situations outlined before, two classes of models, so-called $<X, F>$ models and $<X, R>$ models (Ekel 2001, 2002; Pedrycz et al. 2011) can be constructed. Both of these classes of models are comprehensively discussed in detail in the book. The $<X, F>$ models correspond to multiobjective decision-making problems, which include a vector of objective functions F. At the same time, the $<X, R>$ models correspond to the multiattribute decision-making problems and include a vector of fuzzy preference relations R (Orlovsky 1981; Fodor and Roubens 1994), which play the role of attributes. In the book, the construction and analysis of both types of models are illustrated by considering diverse practical problems.

The $<X, R>$ models are also used in the statement and solution of problems of group decision-making, which are briefly discussed in the next section.

Finally, two fundamental questions such as "What to do?" and "How to do it?" arise in the design, planning, operation, and control of complex systems of diverse nature (Pedrycz et al. 2011). Taking this into account, it is necessary to indicate the following.

To decide on the first fundamental question, if we speak, for instance, about diverse types of planning (strategic, new business, innovation, expansion, maintenance, operational, etc.) activities, usually, it is necessary to evaluate, compare, choose, prioritize, and/or order solutions or alternatives (strategic actions, new business projects, innovation projects, expansion alternatives, maintenance actions, operational strategies, etc.) that permit one to achieve the stated planning objectives. The answers to the first fundamental question are based on processing of information related to different perspectives, such as "investment attractiveness," "implementation cost," "political impact," "development flexibility," and so on and can be elaborated by constructing and analyzing $<X, R>$ models (Pedrycz et al. 2011; Ekel et al. 2016).

The second fundamental question is associated with the allocation of various types of resources (financial, human, logistics, etc.) or their shortages between solutions or alternatives (strategic actions, business projects, innovation projects, expansion alternatives, maintenance actions, operational strategies, etc.) to achieve the stated objectives in the maximal degree. The rational allocation of resources and the answers to the second fundamental question can be obtained by constructing and analyzing $<X, F>$ models (Pedrycz et al. 2011; Ekel et al. 2016).

1.5 Group Decision-Making: Basic Notions

Although in this book we do not consider in detail the issues of group decision-making, many of its results are very helpful for group decision-making (first of all, the results of Chapter 5). Taking this into account, we discuss the basic notions of group decision-making next.

Group decision-making is defined as a decision situation in which there is more than one DM.

The group members have their own attitudes and motivations, recognize the existence of a common problem, and attempt to reach a collective decision (Lu et al. 2007). The necessity of applying procedures of group decision-making is associated with the following considerations.

There are many real-world situations, for instance, at the high managerial levels of organizations, when the decision problems involve wide knowledge areas that are beyond a single individual (this is particularly true when the decision environment becomes more complex and multifaceted). As a consequence, it is usually necessary to attract more than one professional to the decision process. This is particularly valid in environments with a diverse workforce, where decisions require multiple perspectives and different areas of expertise of the individuals involved in the decision process. It is possible to indicate the following advantages of group decision-making over individual decision-making (Tan et al. 1995):

- Group decision-making allows more intellectual resources to be gathered to support the decision. The resources available to the group include the individual competencies, intuition, and knowledge.
- With the participation of multiple experts, it becomes possible to distribute among them the labor related to acquiring and processing the vast amount of information pertaining to the decision.
- If the group members exhibit divergent interests, the final decision tends to be more representative of the needs of the organization.

In addition, it is necessary to indicate an important factor of the possibility to share the responsibility for the decision between members of the group.

Lu et al. (2007) highlight the following important characteristics of group decision-making:

- the group performs a decision-making task;
- group decision-making may cover the whole process of transfer from generating ideas for solving a problem to implementing solutions;
- group members may be located at the same place or at different places;
- group members may work at the same or different times;
- group members may work for the same or different departments or organizations;
- the group can be at any managerial level;
- there may be conflicting opinions in the group decision process among group members;
- the decision might have to be accomplished in a short time;
- group members might not have complete information for the decision;
- some required data, information, or knowledge for a decision may be located in many sources and some may be external to the organization.

Considering these characteristics, it should be noted that quite often, the group members may be at different locations and may be working at different times. Thus, they need to communicate, collaborate on, and access a diverse set of information sources, which can be met by the development of the internet and its derivates (intranets and extranets). The questions of constructing and utilizing web-based group decision support systems are discussed, for instance, in Lu et al. (2007) and Kokshenev et al. (2014).

With respect to the common goals and interests of the experts in the group, it is possible to distinguish two environments; namely, cooperative and non-cooperative work (Lu et al. 2007). In cooperative decision-making, all the experts are supposed to work together to achieve a decision for which they will share the responsibility. In non-cooperative decision-making, the experts play the role of antagonists or disputants over some common interest for which they are to negotiate. Taking this into account, it should be made clear that this book addresses problems of group decision-making in the cooperative environment.

As in cooperative work, the experts share responsibility for the decision (and, as indicated before, they also may participate in the implementation of the selected solution), it is important to guarantee that each member is satisfied with the selected solution (Pedrycz et al. 2011). Obviously, the commitment of the group to the implementation of the outcomes depends on the level of consensus achieved by the group. Therefore, a group decision constructed by means of domination and enforced concessions should be considered inferior to an individual decision, because it will probably face more difficulties in its implementation. Therefore, achieving a veritable consensus on the solution is an important task for the group. However, it should be noted that achieving unimprovable concordance among the experts is very difficult or impossible. Although, ideally, the condition for terminating decision-making process under group settings should be the achievement of a unanimous solution, in reality, because that unanimous solution hardly ever exists, it is sufficient to meet the alternative that is the most satisfactory for the group as a whole (Pedrycz et al. 2011). Otherwise, the decision will probably take longer than is admissible or affordable.

Among the reasons for the occurrence of discordance among the group members, it is possible to indicate the following (Pedrycz et al. 2011):

- Although members of the group are to share the primary goal, which obviously is to generate the solution that provides the maximum benefit for the organization, their secondary goals may be just partially shared. For instance, when experts are representatives of different departments, it is natural that they would have specific interests associated with the priorities and needs of the corresponding departments.
- Each group member usually has a distinct perception of the problem and own intuition that may be difficult to formalize and communicate to the other group members.
- Generally, no single expert knows the entire domain of the decision problem. Each expert usually has access to different sources and different profiles of information. In particular, certain group members may have privileged access to secure information.

In general, the influence of these factors can be reduced by realizing discussions among the experts, trying to pool all relevant information related to the decision. In fact, by pooling the undistributed information, it is possible to raise chances to achieve decisions better than each group member could obtain without help. However, the availability of rich intellectual resources is not sufficient to provide high-quality decisions, as some group members may fail to wisely consider, evaluate, and integrate the profiles of information and perspectives held by the other group members (Bonner et al. 2002; van Ginkel and van Knippenberg 2009). The existing literature identifies some factors that can adversely affect the decision process, leading to low-quality decisions. In particular, the authors of Pedrycz et al. (2011) distinguish the following factors:

- The pressure for early consensus that is due to the need to obtain a solution rapidly.
- The pressure of concordant majorities on the other experts, which is reflected by the group's tendency to prematurely converge on a single solution, once a majority supports a position (even if such a solution is not high-quality).
- The problem of critical pooling of non-distributed information, which can be described as follows: the information supporting the best alternative is not shared among all experts, whereas all group members have information supporting the inferior alternatives. In this case, the group may prematurely achieve a consensus on a bad solution that is apparently good, as the information shared among most experts has more chance of being recalled. The authors Stasser and Vaughan (2000) indicate that one way to reduce this specific problem is to stimulate the group members to focus on information related to their proper areas of expertise during the discussion.

Taking this into account, it is necessary to stress the importance of the moderator (or facilitator) in the discussion among the group members. As indicated in Wong and Aiken (2003), the participation of a moderator (human or automated) in the decision process always generates better outcomes. The moderator is to act as an arbiter responsible for controlling the information flows across the group. Thus, the moderator does not participate directly in the decision, but is supposed to enhance the ability of the group to make decisions (Griffith et al. 1998).

In real-world applications (Pedrycz et al. 2011), sometimes it is impossible to promote the consensus and thereby the exchange of information among the experts, for instance, due to logistic, timing, monetary, or other constraints. In this case, the invited professionals may give their opinions individually and then the group decision is dictatorially built with the use of aggregation rules, despite the existence of significant discordances among the experts. The authors of (Pedrycz et al. 2011) indicate the following approaches for dealing with this situation:

- the use of a majority rule, according to which the group decision is constructed in concordance with the opinion of the majority in the group (Lu et al. 2007);
- the use of a rule determined by a member of the group with authority to make the ultimate decision for the group (Lu et al. 2007);

- the search for a collective opinion that minimizes the major discordance in the group, in such a way that no expert is extremely dissatisfied with the group outcomes (Parreiras et al. 2010).

1.6 Fuzzy Sets in Problems of Decision-Making

As elaborated in Section 1.1, various types of uncertainty are commonly arising in a wide range of problems of an optimization character, which are inherently encountered in the design, planning, operation, and control of complex systems. Besides, as indicated in Section 1.3, taking these types of uncertainty into account when constructing mathematical models serves as a vehicle for increasing the adequacy of the models and, as a result, the credibility and factual efficiency of decisions based on their analysis.

Taking this into account, it is necessary to indicate two fundamental points, related to applying fuzzy set theory to solve the problems of the design, planning, operation, and control of complex systems:

- The theory of fuzzy sets can serve as a basis for considering diverse types of uncertainty, as well as diverse combinations of diverse types of uncertainty.
- Traditional models used in the design, planning, operation, and control of complex systems (for example, power systems, transportation systems, etc.) exhibit a high level of complexity. However, the professionals who utilize these models not are always satisfied with the results obtained on the basis of their utilization. At the same time, experienced planners and operators can plan or operate complex systems without the use of these models, applying the proper knowledge, experience, and intuition. However, their capacity to perceive and to process large volumes of information is too limited. So, for instance, operators are weak elements in the control loops. Considering this, it is necessary to create systems where the technology of thinking of operators may work, but which have no these limitations. The fuzzy sets (Dubois and Prade 1980; Zimmermann 1996; Pedrycz and Pedrycz and Gomide 1998) and/or their modifications (for instance, Pavlak 1982; Atanassov 1986) may serve as an universal language for transferring the knowledge, experience, and intuition to the computer that does not have these limitations.

The starting point in the formation of mathematical models is the requirement of a strict correspondence of these models to the level of uncertainty of information used for their building. Observing just this correspondence, we can talk about the adequacy of the presentation of the object, system, or process and the possibility of obtaining a real effect as a result of solving the corresponding problems of an optimization character. Any simplification of reality or its idealization, undertaken with the goal of using rigorous mathematical models, distorts the nature of many problems and reduces the practical value of results obtained on the basis of analyzing these models. Considering this, researchers, for instance Belyaev and Krumm (1983) and Rommelfanger (2004), for a number of reasons, have doubts about the validity or, at least, the expediency of taking into account the uncertainty factor within the framework of traditional approaches (first of all, approaches based on probability theory; for instance, Dantzig 1955; Grassman 1981; Wagner 1982). In particular, the authors of Belyaev and Krumm (1983) indicate that, similar to the solution of problems on the basis of deterministic methods, when we assume exact knowledge of the information, which usually does not correspond to reality, the application of probabilistic methods also supposes exact knowledge of the distribution laws and their parameters, which does not always correspond to the real possibilities of obtaining the entire spectrum of the probabilistic description.

In general, the approaches indicated previously do not ensure an adequate or sufficiently rational consideration of the uncertainty factor along with an entire spectrum of its manifestations.

Giving up the traditional approaches to the construction of mathematical models (Ekel and Popov 1985; Popov and Ekel 1987) and the application of the concepts of fuzziness to the studied objects and systems and processes associated with them, the application of fuzzy set theory (Dubois and Prade 1980; Zimmermann 1996; Pedrycz and Gomide 1998), established by Zadeh (1965), may play and plays a significant positive role in overcoming the existing difficulties. The use of this theory opens a convincing avenue of giving up "excessive" precision, which is inherent in the traditional modeling approaches, while preserving reasonable rigor. The principle of incompatibility coined by Zadeh (1973) offers an interesting view of the tradeoffs between precision and relevance of the models: "As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics."

Furthermore, operating in a fuzzy parameter space allows one not only to be oriented toward the contextual or intuitive aspect of the qualitative analysis as a fully substantial aspect, but, by means of fuzzy set theory, to use this aspect as a sufficiently reliable source for obtaining quantative information (Popov and Ekel 1987). Finally, fuzzy sets allow one to reflect in an adequate way the essence of the decision-making process. In particular, since the "human factor" has a noticeable effect and occupies a very visible position in making decisions in many real-world problems, we can capitalize on the way in which fuzzy sets help quantify the linguistic facets of available data and preferences (Dubois and Prade 1980; Zimmermann 1996; Pedrycz and Gomide 1998).

Besides, we also have to bear in mind that the aspiration for attaining the maximum effectiveness in decision-making in the presence of uncertainty requires, first of all, that a significant effort be directed toward finding ways to remove or, at least, partially overcome the uncertainty factor (Popov and Ekel 1987). In particular, this can be attained by aggregating information coming from diverse sources of both a formal and informal nature. This aggregation allows one (Ekel and Popov 1985) to supplement the characteristics of the uncertain initial information by justified assumptions about the differentiated confidence (reliability) of its various values that could be reflected by choosing appropriate membership functions (Dubois and Prade 1980; Zimmermann 1996; Pedrycz and Gomide 1998).

However, taking this into account, it is necessary to indicate that the issues related to the relationships between probability theory and fuzzy set theory, as well as an interpretation of membership functions, have been the subject of intensive discussions of methodological and philosophical character over the years. Considering this, it should be emphasized (Pedrycz et al. 2011) that the decision-making approaches based on fuzzy set theory do not compete with probabilistic methods, but these two approaches can complement each other. In particular, we can observe the appearance of some hybrid approaches in which fuzzy sets and probability are used synergistically.

Recent years saw intensive investigations, which took advantage of applying fuzzy set theory directly or in combination with other branches of mathematics of uncertainty to deal with diverse manifestations of the uncertainty factor. Its use in problems of an optimization character offers advantages both of the fundamental nature (the possibility of validly obtaining more effective, less "cautious" solutions, as well as the possibility of considering simultaneously different manifestations of the uncertainty factor) and of a computational character (Ekel 2002; Pedrycz et al. 2011).

Besides, it is possible to indicate two principal ways for solving problems under conditions of uncertainty. In the use of the first way, one obtains (at least, theoretically) an exact solution for fixed values of the uncertain parameters, and then estimates its stability for variations of such parameters (for example, by performing multi-variant computations). The second way presupposes the tracking of the effect of the uncertainty at all stages along the path toward the final decision. This approach can be implemented on the basis of the theory of fuzzy sets. It is more complicated than the first one, but is also more fruitful and highly promising (Ekel and Popov 1985).

As mentioned before, in many real-world problems we have to take into account the criteria, constraints, indices, and so on of a qualitative character. Thus, it should be emphasized that this type of information was taken into account in the past. However, it was used only after obtaining solutions on the basis of the use of formal models, with the disruption of the solutions obtained on their basis (to consider information of a qualitative character) without any sufficient justification (Pedrycz et al. 2011). Since this approach significantly reduces the value of the obtained solutions, it remains necessary to develop ways of introducing this type of information directly into the decision-making processes. This book demonstrates that fuzzy sets can be considered as a sound way of proceeding along this path.

Returning to the considerations of Section 1.1, it is necessary to highlight that one of the most important criteria for classifying decision-making problems (Larichev 1987) is the existence or lack of an objective model for the problem. Considering this, it should be noted that it is not uncommon to encounter situations, as mentioned in Section 1.1, where it is impossible to talk about the existence of objective functions in decision-making problems. Thus, the models corresponding to these problems are to reflect the "world outlook" of a DM. In this case, an obvious question is how to choose actions that correspond, in the best way, to the preferences of the individual (Keeney and Raifa 1976). Considering that the manner of human thinking, including the perception of preferences, is vague and subjective, fuzzy set theory can play an important role in individual and group preference modeling (Fedrizzi and Kacprzyk 1990; Fodor and Roubens 1994).

The application of fuzzy sets to preference modeling and analysis of the corresponding decision-making problems provides a flexible environment that permits one to deal with the inherent fuzziness of perception and, in this manner, to incorporate more human consistency into preference models. Besides, a stimulus for using fuzzy set theory stems, as indicated before, from one of its most important facets that concerns the linguistic aspect commonly applied to different decision-making problems and different preference structures (Herrera and Viedma 2000; Xu 2005). In particular, it is possible to distinguish among several directions in decision-making by applying the linguistic aspect of fuzzy set theory, such as multicriteria decision-making (Buckley 1995; Rasmy et al. 2002), group decision-making (Yager 1993b; Herrera et al. 1995), diverse consensus schemes (Herrera et al. 1995; Bordogna et al. 1997), decision-making on the basis of information granularity (Borisov et al. 1989; Herrera et al. 2000), and others. In principle, all these directions are associated with analyzing the $<X, R>$ models mentioned before. Taking into account the rationality of analyzing the $<X, R>$ models on the basis of fuzzy sets as well, it is possible to assert that their use in the statement and solution of decision-making problems, as indicated previously, provides answers to the fundamental questions "What to do?" and "How to do it?" arising in the project, planning, operation, and control of complex systems of a diverse nature.

Finally, it is important to note that some specific advances are created by the application of fuzzy set theory in solving decision-making problems:

- Any expert involved in the process of decision-making or any considered criterion may demand different forms (or formats) of preference representation (for instance, direct ordering of alternatives, levels of utility functions, multiplicative preference, fuzzy estimates, etc., see Zhang et al. 2004, 2007). Diverse preference formats can be reduced to fuzzy preference relations, using so-called transformation functions (for example, Chiclana et al. 1998; Herrera-Viedma et al. 2002) to prepare homogeneous information for decision-making processes.

- Any expert can give no information on some preferences, due to the lack of knowledge or because of unwillingness to respond (for example, because of political considerations), creating information "holes." Applying the interpolative nature of fuzzy set theory, it is possible to fill these "holes" (for instance, Herrera-Viedma et al. 2007; Chen et al. 2014).
- It is not uncommon that information related to the preferences is inconsistent (for instance, this information does not have the necessary level of transitivity). The theory of fuzzy sets can offer efficient means to identify and to correct these situations (for example, Ma et al. 2006; Xu et al. 2013).
- The use of aggregation operators, including those offered by the theory of fuzzy sets, offered by the theory of fuzzy sets (for instance, Yager 1993a; Yager and Kacprzyk 1997), permits one to reflect all requirements of DMs; for example, related to the level of mutual compensation among criteria in the decision-making processes.
- Diverse indicators created with the use of fuzzy sets can serve for constructing flexible consensuses to help a human or computational moderator (Lu et al. 2006: Parreiras et al. 2012).

1.7 Conclusions

In this chapter, we have discussed the fundamental questions of the appearance and essence of problems of an optimization character related to the design, planning, operation, and control of complex systems of diverse nature. The fundamental differences between optimization and decision-making problems as well as between the notions of optimal solutions and robust solutions have been emphasized. The relevance and omnipresence of the uncertainty factor and its influence on the character of the analyzed decision-making models have been considered. The structured, semi-structured, and unstructured problems of decision-making have been classified with a focus on unstructured problems. The attention has been drawn to the necessity of constructing robust solutions in the multicriteria analysis under conditions of uncertainty. The models of multicriteria decision-making have been characterized and classified with the split into two main categories of so-called $<X, F>$ models (as multiobjective models) and $<X, R>$ models (as multiattribute models), which are the subject of comprehensive considerations in this book. The essence, main concepts, and characteristics of group decision-making have been discussed. Finally, the role of fuzzy set theory in decision-making processes has been discussed, including consideration of its advantages. First of all, we have stressed the fundamental benefit stemming from the use of fuzzy sets that is the possibility of obtaining more effective, less "cautious" solutions to the decision-making problems as well as the abilities of the incorporation, including mutual incorporation, of different manifestations of the uncertainty factor.

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