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An Overview of the Art of Decision-making

1.1 Introduction

What motivates one to make a decision? Finding the precise reason behind these motivations might not be as easy as it might seem. Nevertheless, given that these choices are shaping the world around us, it would not be an exaggeration to claim that the answer to the aforementioned question may facilitate understanding the workings of many world phenomena. Just for a moment consider the possibility of knowing the motivations prompting person to make decisions. If that were achieved, predicting humans' behavior from the simple every-day activity to the most sophisticated social, economic, and political contexts would be possible.

Now let us change the scope of the question; *how can one make a good choice*? This time we may be more successful in finding a more proper answer. Let us take a moment to consider the description of the act of decision-making. The Oxford dictionary defines decision-making as "*the process of deciding about something important, especially in a group of people or in an organization*." From a psychological point of view, however, decision-making is regarded as the cognitive process resulting in the selection of a belief or a course of action among several alternative possibilities. Each decision-making process produces a final choice, which may or may not prompt action (Tzeng and Huang 2011). In other words, the decision-making merely refers to the act choosing among a set of solutions, rather than the procedural requirements of executing the selected set of alternatives.

The decision-making process is founded on a four-stage analytical procedure (Vroom and Jago 1974; Bell et al. 1988; Weber and Coskunoglu 1990; Kleindorfer et al. 1993). The first stage of the decision-making process is better known as descriptive analytics or positive analytics, which is concerned with describing observed behaviors of the stakeholders who are involved in the decision-making process, mainly by looking at the their past performance and understanding such behavior either by mining historical data sets, and/or looking for the behavioral and social, psychological, and even neurology reasoning motivations that can best

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describe the course of actions made by the stockholders of the decision-making problem (Tzeng and Huang 2011; Santos and Rosati 2015). Descriptive analysis is outside the scope of this book in spite of its psychological nature being pertinent to decision-making process.

The predictive analytic stage concerns the prediction of what is likely to occur given a set of circumstances, which takes place after discerning the motivational patterns behind decision-making problems through descriptive analytical techniques. The application of predictive analysis is limited to the decision-making under uncertainty and, admittedly, not all decision-making problems require such approach. Nevertheless, if necessary, the historical data sets may be reviewed during this second stage to determine the probability of an event or the likelihood of a situation's occurrence (Bell et al. 1988; Kleindorfer et al. 1993). Exploring this phase of the decision-making process is left to readers given the scope and aims of this book.

The third stage of the decision-making process is the normative analysis. The term "normative" generally refers to relating an item to an evaluative standard through assessing and making judgments about the item's behavior or outcomes (Kahneman and Tversky 1984; Tversky and Kahneman 1986). Normative analysis, subsequently, is concerned with techniques through which the decision-makers would be able to evaluate the feasible alternatives in a mathematical sense (Bell et al. 1988; Kleindorfer et al. 1993). Note that traditional normative analysis is based on the assumption of rationalism through the evolved entities of the decision-making problem, which, loosely speaking, is a term that refers to decision-makers pursuing what was described through the previous stages, as their interests and goals. Naturally, making a decision irrationally is beyond the scope of this book, though methods are introduced throughout this book that would enable decision-makers to cope with different types of criteria, including intangible criteria.

The final stage of the decision-making process is called prescriptive analysis. In this stage, decision-makers go beyond predicting future outcomes to determine which alternatives would be the most advantageous or desirable solutions to the problem at hand (Saad 2001). In other words, prescriptive analytics combine the information gathered through studying the behavioral patterns of the stockholders (descriptive analysis), the likelihood of random events inherent to the decision-making problems (predictive analysis), which would be expressed in mathematical-oriented frameworks (normative analysis), to obtain the best course of actions for the decision-makers. Furthermore, through the realms of prescriptive analysis, decision-makers can explore the possible options on how to take advantage of future opportunities or coping with future risks, and, eventually, evaluate the implication of each feasible decision option based on the nature the decision-making problem at hand (Bell et al. 1988; Kleindorfer et al. 1993; Tzeng and Huang 2011).

Having defined decision-making, we consider what is a good choice or alternative in a decision-making problem. Indeed, the notion of a "good alternative" may differ among decision-makers' viewpoints due to their different personal desires, experiences, and backgrounds. In other words, one's idea of a "good choice" may not necessarily represent every decision-makers' ideal choice. Furthermore, the selection procedure of decision-makers may differ from one another, when facing the same decision-making problem. Nevertheless, the decision-makers' selection procedure is founded on a basic and similar principle, which is that decision-makers would have to choose a set of solutions that would outperform other feasible alternatives based on a set of evaluation criteria defined either explicitly or implicitly by the decision-makers for the specific problem at hand. In fact, this decision paradigm underlies multicriteria decision-making (MCDM) in general. In practice, almost everyone may face an MCDM problem on a daily bases, which most cope with by aggregating the criteria through an intuition-oriented weighting mechanism. Nevertheless implementing a systematic MCDM approach is essential to making informed and logical decision.

In technical terms, MCDM is a procedure by which the decision-maker explicitly evaluates a set of alternatives with regard to multiple, usually conflicting, criteria. Decision makers apply MCDMs to restructure and redefine the decision-making problem to make an informed decision. Although developing and implementing MCDM methods are not novel ideas, there have been undeniable advances in this field since the blooming era of computational intelligence (CI) during the early 1960 and 1970s, especially in the form of mathematically oriented methods that recapture and redefine MCDM. MCDM has been an active area of research that has played a crucial role in an array of disciplines, ranging from politics and business to the environment and energy (Zolghadr-Asli et al. 2018a).

Hwang and Yoon (1981) proposed clustering MCDM problems based on the nature of solutions that are available for the problem in hand into two main categories, namely, multiobjective decision-making (MODM), and multiattribute decision-making (MADM). Essentially, the aforementioned classification is based on whether the solutions are explicitly or implicitly defined (Mendoza and Martins 2006; Tzeng and Huang 2011; Velasquez and Hester 2013).

MODM problems describe a situation in which decision-makers are searching for a set of solutions that would satisfy the constraints imposed on the given problem and obtain results that constitute an optimal set of solutions based on the decision-makers objectives (Hwang and Yoon 1981). In essence, MODM is suitable for tackling design and planning problems, in which the decision-makers aim to achieve states objectives or goals by considering the various interactions within the given constraints. The decision space of MODM problems can be described as a multidimensional Cartesian space, with each (conflicting) objective acting as an axis, defined by a set of constraints that separate the feasible and infeasible solutions. MODM can solve problems with continuous or discrete decision spaces.

MODM solution methods are usually associated with mathematical programming methods (Tzeng and Huang 2011).

In general, MODM involve trade-off and scale problems (Tzeng and Huang 2011; Zolghadr-Asli et al. 2018a). MODM involves more than one objective, therefore, the optimal solutions to a MODM problem must be posed in terms of Pareto fronts or production possibility frontier (after the Italian economist Vilfredo Pareto 1848–1923), which are sets of points representing combinations of the values of the objective functions with the best tradeoffs among objectives that are achievable for the problem being solved. In classic MODM techniques, an optimal solution is commonly obtained with mathematical programming. This means multiple objectives are merged into a single-objective problem through a weighting of the various objectives. The process of obtaining a proper weighting scheme for the objectives is a trade-off problem. If such trade-off information is unavailable, Pareto solutions must be derived. Pareto solutions to MODM problems are expressed as a set of nondominated solutions. A nondominated solution has the property that it is not possible to improve the solution's utility or degree of preference without degrading at least one objective (Zolghadr-Asli et al. 2017, 2018a). The MODM's scaling problem, on the other hand, is a computational challenge surrounding most real-world, practical, decision-making problems, whereby the stakeholders must consider several conflicting objectives. As the number of objectives increases the decision makers face *the curse of dimensionality*, whereby the computational costs of solving a MODM problem become burdensome in the extreme, and sometimes computationally unassailable (Bozorg-Haddad et al. 2017; Zolghadr-Asli et al. 2017, 2018a). In an attempt to surmount this challenge, meta-heuristic algorithms have arisen to search within the decision-space and identify potential solutions to a MODM problem (Bozorg-Haddad et al. 2017, Zolghadr-Asli et al. 2018b,c,d).

MADM problems describe a situation in which the decision-makers evaluate a finite number of predefined alternatives. The alternatives are known at the beginning of the solution process. The decision-makers attempt to systematically assess each alternative via a discrete preference rating mechanism. The rating mechanism used by decision-makers to evaluate and compare the performance of each of the alternatives under consideration is defined either explicitly or implicitly (Hwang and Yoon 1981). Table 1.1 compares the main characteristic of MCDM approaches, namely, MODM and MADM (Malczewski 1999; Mendoza and Martins 2006; Tzeng and Huang 2011; Velasquez and Hester 2013).

Nowadays, decision-makers have numerous methods and techniques at their disposal to deal with MCDM problems, ranging from simple, easy-to-use approaches to complex techniques. Given the important role of MADM, it is vital for decision-makers to know the merits and drawbacks of MADM methods. This book introduces some of the fundamental MADM methods that have been

Table 1.1 Comparison of MODM and MADM approaches.

Criteria for comparison	MODM	MADM
Criteria defined	Objectives	Attributes
Objective defined	Explicitly	Implicitly
Attributes defined	Implicitly	Explicitly
Constraints defined	Explicitly	Implicitly
Alternatives defined	Implicitly	Explicitly
Number of alternatives	Infinite	Finite
Decision-makers' control	Significant	Limited
Decision modeling paradigm	Process-oriented	Outcome-oriented
Relevant to	Design/operation	Evaluation/choice

proven to be effective and practical solution-searching tools. Section 1.2 describes the foundations of MADM methods and their classification.

1.2 Classification of MADM Methods

There are numerous ways through which one can classify the MADM methods. A sound classification relies on the core principles and assumptions of MADM to categorize these methods. Familiarity with MADM methods is paramount to choose adequately among them to solve a decision problem at hand. Accordingly, based on the decision-makers' prioritizing system, the interactions among attributes, the mathematical nature of attributes' values, and the number of decision-makers, numerous classifications have been proposed for MADM methods.

1.2.1 Preference Evaluation Mechanism

Every MADM method requires a preference evaluation mechanism for the purpose of reflecting the stockholders' preferences in the decision-making process. These mechanisms act as a measure that enables decision-makers evaluating alternatives according to their attributes. The mechanism can be defined either explicitly (where the preference values are computable through a set of predefined boundaries or mathematical functions), or they can be defined implicitly so that the decision-makers' experiences, expertise, perception, and instincts are reflected in the alternatives' preference evaluation.

A classification based on the notion of preference evaluation mechanism divides MADM methods into multiattribute utility theory (MAUT) and outranking methods (Belton and Stewart 2002; Mendoza and Martins 2006). On the basis of Bernoulli's utility theory MAUT methods obtain the decision-makers' preferences, which can usually be represented as a hierarchical structure by using an appropriate utility function. By evaluating the utility function the alternative with the highest utility value can be identified as the solution to the MADM problem at hand. In spite of their reliance of solid axiomatic background of MAUT methods, they are criticized by their unrealistic assumption of preferential independencies (Tzeng and Huang 2011). Preferential independence describes situations in which the preferred outcome of one criterion over another is not influenced by the remaining criteria. However, it so happens that the criteria are usually interactive in real-world MADM problems. Alternatively, instead of building complex utility functions, outranking methods compare the preference relations among alternatives to determine on the best alternatives. The outranking methods were introduced to overcome the empirical difficulties experienced with the utility function in handling practical problems. Yet they lack axiomatic foundation, such as is the case with classical aggregate problems and structural problems (Tzeng and Huang 2011).

The previous classification categorizes classical MADM methods, yet it may face difficulties categorizing some of the modern MADM methods whose features do not fit either one of the previously cited categories. Belton and Stewart (2002) proposed a more sophisticated classification system for MADM methods that addresses the latter classification difficulties. The alternate classification categorizes the MADM methods within three classes, namely, value measurement, goal aspirations or reference level, and outranking methods. This classification is a reviewed next.

Value-measurement MADM methods implement numerical scales to represent the degree to which a feasible alternative may be preferable to another. The scores obtained for each alternative are developed initially for each individual evaluation criterion and are then synthesized to rate the overall performance of the alternatives. The scores assigned to each of the feasible alternatives reflect a preference order. These preferences must be consistent with a set of axioms, which are as follows (Belton and Stewart 2002; Mendoza and Martins 2006):

(I) Constant discipline and roles must be imposed by the decision-maker in the construction procedure of preference measurement scales;
(II) Provide a framework through which the decision-makers are able to systematically analyze the obtained preference values and gain a deeper understanding of the process that led to the final results; and
(III) Promoting explicit statements, rather than implicit judgments regarding the trade-offs between evaluation criteria.

Desirable or satisfactory levels of achievement must be defined by the decision-makers for each evaluation criterion. Through these reference level methods, those alternatives that are closest to achieving the goals or aspirations are identified. These types of MADM methods are recommended for those cases in which decision-makers may not be able to express trade-offs or identify importance weights of the evaluation criteria. Nevertheless, the most desirable outcome can be portrayed through arbitrary aspirations or goals for each criterion. As far as these branches of MADM is concerned, an alternative that represents the most similarities with the arbitrary defined ideal solution can best reflect the stakeholders' interests in the process of decision-making. Through the framework represented by this branch of MADM methods, the feasible alternatives, which are available courses of actions, are systematically eliminated until achieving a solution that best fits the stakeholders' ideal outcome for the MADM problem at hand (Belton and Stewart 2002; Mendoza and Martins 2006).

Lastly, outranking MADM methods evaluate alternatives' relative performances against one another using a comparison-oriented framework. Outranking MADM methods, the first evaluate feasible alternatives by the decision-makers in terms of evaluation criteria to establish their merits. This is followed by an aggregation stage whereby the gathered information is used as evidence to obtain an alternative that outrank others and emerges as the optimal solution. The aggregation stage establishes the relations between the alternatives in terms of preference, indifference, and incomparability. Consequently, a complete ranking of alternatives is produced.

1.2.2 Attributes' Interactions

In any MADM, the decision-maker is dealing with the presence of a number of evaluation criteria. In essence, each MADM method offers a different approach to aggregate each criterion's value to obtain an optimal solution. Based on that notion MADM methods are divided into two main categories, namely, compensatory and noncompensatory (Jeffreys 2004). In compensatory techniques, the poor performance of an alternative in some criteria can be compensated by high performance in some other criteria. Therefore, the aggregated performance of an alternative might not reveal its weakness area. In contrast, in noncompensatory techniques, the significant poor performance of an alternative in some criteria cannot be compensated with high performance in other criteria. The aggregated performance reflect this fact. In other words, each criterion can independently play a crucial rule in the aggregated performance of an alternative (Banihabib et al. 2017).

1.2.3 The Mathematical Nature of Attributes' Values

From a mathematical point of view, variables, and in this case evaluating criteria, can have different nature, such as, deterministic vs. nondeterministic, and fuzzy vs. crisp. MADM methods can be divided into the following categories.

1.2.3.1 Deterministic Vs. Nondeterministic

Deterministic MADM methods involve decision-makers who are certain about the occurrence of the set of outcomes in a decision-making problem. On the other hand, nondeterministic problems involve the occurrence of outcomes with stochastic components of a random-based nature (Pearl 1996; Tzeng and Huang 2011). In such case, the likelihood of an outcome would play a direct role in selecting the most suitable alternative (Coombs and Pruitt 1960). Nondeterministic methods are beyond the scope of this book.

1.2.3.2 Fuzzy Vs. Crisp

Crip MADM modeling expresses the decision-makers' preferences with numeric values. However, there are cases in which the subjective uncertainties that are surrounding decision-makers prevent the stockholders to express their preferences with a crisp number (Tzeng and Huang 2011). In such situations, decision-makers may rely on a fuzzy set that can best describe the stockholders' preferences. Fuzzy sets offer the benefit of implying linguistic evaluation, which in turn, would ease the evaluation process of the decision-makers (Bellman and Zadeh 1970).

It is vital for decision-makers to distinguish the fuzzy-uncertainty logic from the probability-uncertainty logic, and to use them in the proper context. In cases where the certainty of outcomes is in question, the probability-uncertainty logic is the recommended tool. In such situations, the decision-makers' decision-tree is founded on at least one uncertain event. Consequently, the probability of each outcome would play a role in determining the most suitable alternative. On the other hand, when the decision-makers are not certain on how to express the preference of an alternative, the fuzzy logic becomes the favored option. Fuzzy evaluation enables decision-makers to describe an alternative's preference through a fuzzy set employing membership functions. In essence, while the probability-uncertainty logic deals with the probability of outcomes in a decision-tree, the fuzzy logic offers the possibility of preference evaluation by the decision-makers. Exploring the realms of nondeterministic evaluation and fuzzy description of performances lays beyond the scope of this book.

1.2.4 Number of Involved Decision-makers

MADM methods can be classified as single or group decision-making methods depending on the number of decision-makers involved (Black 1948). In the case of single decision-maker methods, the opinion of that single individual forms the preference evaluation mechanism of the decision-making process. On the other hand, group decision-making enables a number of experts and stakeholders to contribute and influence the decision-making process (Kiesler and Sproull 1992). Group decision-making methods are founded on the basis of single decision-making methods; yet, they require an additional strategy through which, each decision-maker's opinion is aggregated and integrated with others' viewpoints to form the final result. Exploring such strategies falls outside the scope of this book.

1.3 Brief Chronicle of MADM Methods

The historical origins of MADM can be traced back to series of correspondence letter between Nicolas Bernoulli (1687–1759) and Pierre Rémond de Montmort (1678–1719), while discussing a mathematical brain teaser, known as the *St. Petersburg paradox* (Tzeng and Huang 2011). In brief, the St. Petersburg paradox can be portrayed as follows (Bernstein 1996):

> "This is a game of chance for a single player who tosses a fair coin at each stage of the game. The player keeps tossing the coin until it turns tails. If the first flip is tails the player wins \$2; if the first tails is on the second flip the player wins \$4; if the first tails is on the third flip the player wins \$8, etc. Concretely if first tails is on the nth flip the player wins \$2^{n}." The question here is: *how much would a prospective gambler be willing to pay to play this game?*

To grasp the magnitude of the described conundrum, consider for a moment, the answer of classical mathematics to the described question. The expected value of the prize resulting from playing this game is (Bernoulli 1738):

$$EV = \frac{1}{2} \times \text{US\$2} + \frac{1}{4} \times \text{US\$4} + \frac{1}{8} \times \text{US\$8} + \ldots = \sum_{n=1}^{\infty} \frac{1}{2^n} \times \text{US\$}2^n = \infty \tag{1.1}$$

in which EV = the expected value turns out to be infinity. Accordingly, a player would be willing to pay any price to participate in the described game. However, this result defies human behavior since no one would be willing to pay a limitless amount of cash to engage in this game (Rieger and Wang 2006). The answer to the St. Petersburg paradox, which revolutionized the way in which decision-making problems were analyzed, did not surface itself until Daniel Bernoulli (1700–1782) published his influential research on utility theory in 1738. The concrete discussions describing the solution of the St. Petersburg paradox in detail are skipped here; yet, it is noteworthy that the remarkable solution that enabled *Daniel Bernoulli* to solve the aforementioned paradox relied on the fact that humans make decisions based not on the expected value, but rather, on the utility value. Specifically, assume that a prospective player has a wealth of w dollars, that the charge for entering the game equals c dollars, and that the player's utility function is $U(w) = \ln(w)$. It can be shown that under these circumstances, the expected incremental (or marginal) utility of playing this game $[E\Delta(U)]$ is finite:

$$E\Delta(U) = \sum_{n=1}^{\infty} \frac{1}{2^k} \left[\ln\left(w + 2^k - c\right) - \ln(w)\right] < \infty \tag{1.2}$$

Therefore, a prospective player whose wealth equals US$$10^6$ should be willing to pay up to US$20.88 to play the game; or US$10.95 if the wealth is US$$10^3$, and so on and so forth, because the amounts the player would be willing to pay maximize his expected incremental utility. The implication of the utility value is that humans choose the alternative with the highest expected utility value when confronting the MADM problems. A chronologic overview of the most fundamental and influential MADM methods, which would be discussed within this book, is presented in Table 1.2.

1.4 Conclusion

Almost everyone, on a daily bases, faces decision-making problems. It would not be exaggerated to state that these decisions constitute the nature of mankind and of the society that humans form. When it comes to real-world decision-making problems, the decision-makers often find judgment a challenging task. This is so because of the notion that the interest of the stakeholders can be only represented through the evaluation of a set of conflictive criteria. Whenever the decision-makers face a set of feasible, discrete, alternatives, the problem at hand involves MADM. Numerous methods have been presented by been reported to ensure a sound and reliable decision-making process. MADM is one of the main branches of operational research; it is an active field of study with multiple overlaps with many scientific disciplines, and has numerous practical applications. This chapter reviewed the principles of MADM. Furthermore, the best well-known MADMs were herein classified and reviewed.

Table 1.2 A chronologic overview of the most influential MADM methods.

MADM Methods	Utility function	Bernoulli (1738)
	Weighted sum method (WSM)	Churchman and Ackoff (1954)
	ELECTERE I	Benayoun et al. (1966)
	ELECTERE II	Roy and Bertier (1971)
	Analytic hierarchy process (AHP)	Saaty (1977)
	ELECTERE III	Roy (1978)
	TOPSIS	Hwang and Yoon (1981)
	ELECTERE IV	Roy and Hugonnard (1982)
	PROMETHEE I	Brans (1982)
	PROMETHEE II	Vincke and Brans (1985)
	PROMETHEE III	Brans et al. (1986)
	PROMETHEE IV	Mladineo et al. (1987)
	Grey relational analysis	Deng (1989)
	Analytic network process (ANP)	Saaty (1996)
	VIKOR	Opricovic (1998)
	Superiority and inferiority ranking (SIR)	Xu (2001)
	PAPRIKA	Hansen and Ombler (2008)
	Best-worst method (BWM)	Rezaei et al. (2015)
Weighting Methods	Entropy method	Shannon (1948)
	Delphi method	Dalkey and Helmer (1963)
	Eigenvector method	Saaty (1977)
	Weighted least square method	Chu et al. (1979)
	Multiple objective programming model	Choo and Wedley (1985)
	Principal element analysis	Fan (1996)
	Modified Delphi method	Custer et al. (1999)

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