CHAPTER 1

Motivating Students

What Is It?

Motivation—*why* people do what they do—affects every aspect of schooling. Without motivation, student learning becomes difficult, if not impossible (Artzt, Armour-Thomas, & Curcio, 2008, p. 48). Motivated students tend to have better performance, higher self-esteem, and improved psychological well-being (Fong, Patall, Vasquez, & Stautberg, 2019, p. 123; Gottfried, Marcoulides, Gottfried, & Oliver, 2013, p. 83; Liu & Hou, 2017, p. 49; Reeve, Deci, & Ryan, 2004, p. 22). Conversely, unmotivated students can become disengaged from academics and, in the worst cases, drop out of school (National Research Council, 2004, p. 24).

According to *self-determination theory*, a theory of motivation developed by researchers Edward L. Deci and Richard M. Ryan, motivation can be *intrinsic* (doing something because it is inherently satisfying) or *extrinsic* (doing something because it leads to some other result) (Ryan & Deci, 2000, p. 55). Many times, motivation is difficult to characterize as purely intrinsic or extrinsic. A student may be drawn by an extrinsic reward but may eventually internalize the values and adapt a more intrinsic motivation (Usher & Kober, 2012b, p. 3).

In addition, motivation is not a fixed quantity (Ryan & Deci, 2000, p. 54). Factors like schools, parents, communities, teachers, and life experiences can positively or negatively affect motivation (Usher & Kober, 2012a, p. 7). Students' motivation can vary from class to class—a student who is highly motivated in one class may be completely disengaged in another (National Research Council, 2004, p. 33).

As a result, educators often need to foster both intrinsic and extrinsic motivation. Students who are intrinsically interested in a topic are more likely to seek challenging tasks, think more creatively, and learn at a conceptual level (National Research Council, 2004, p. 38). However, since many academic tasks may not be inherently interesting, teachers also need to learn how to promote different methods of extrinsic motivation (Ryan & Deci, 2000, p. 55).

To sustain motivation, educators often seek ways to encourage students to internalize values. When students do so, they become more persistent and have a more positive sense of themselves (Ryan & Deci, 2000, pp. 60–61).

Why We Like It

In our experience, keeping motivational strategies in mind can enhance student engagement, academic achievement, and confidence to do math. Boosting their confidence is particularly important since many of our students experience math anxiety (we discuss it more in the Introduction), which can hinder their academic growth.

Supporting Research

Many studies on motivation focus on ways to build inclusive communities that promote learning for *all* students (Kumar, Zusho, & Bondie, 2018, p. 78). Proponents of self-determination theory argue that people are motivated to complete a task if doing so fulfills basic psychological needs, such as autonomy, relatedness, and competence (Ryan & Deci, 2000, p. 64).

However, some researchers have begun to challenge the idea of a universal theory of motivation, arguing that most of the existing work ignores the experiences and members of historically marginalized groups, such as people of color (Usher, 2018, p. 132). These researchers seek a more culturally responsive framework in which motivation is viewed not just as an individual characteristic but as the product of the social and historical context that shapes students' emotions and beliefs (King & McInerney, 2016, p. 2).

Other studies have focused on the effect of emotions on student motivation (Hannula, 2019, p. 310). Students who feel more anxious about math often have decreased motivation and do more poorly in school (Gunderson, Park, Maloney, Beilock, & Levine, 2017, pp. 34–35; Mo, 2019, p. 2; Passolunghi, Cargnelutti, & Pellizzoni, 2018, p. 282). Discouragement from parents, inappropriate or overly difficult work, and lack of support from teachers can further erode students' *self-efficacy*—the realistic expectation that making a good effort will lead to success (Usher, 2009, p. 308). In other words, social-emotional learning is tied to motivation.

Research indicates that as students move through the K–12 school system, their attitudes toward math become less positive (Batchelor, Torbeyns, & Verschaffel, 2019, p. 204; Gottfried et al., 2013, p. 70). As a result, keeping middle and high school students motivated in math class can be particularly challenging.

Despite the different approaches and areas of emphasis in the literature, researchers agree on several ways to improve student motivation:

- Teachers should have meaningful and challenging instruction (Kumar et al., 2018, p. 90).
- Teachers should empathize with students and accept them unconditionally (Wormeli, 2014).
- Administrators should ensure that school personnel are culturally diverse (Usher, 2018, p. 140).

In the Application section of this chapter, we discuss some strategies for improving motivation for all students.

Common Core Connections

Many of the motivational techniques that we describe in this chapter are related to Common Core standards. For example:

- Showing the usefulness of a topic relates to standards such as modeling with mathematics (Mathematical Practice [MP].4), interpreting division of fractions by fractions (6-NS.A.1), using a linear equation to model bivariate data (8-SP.A.3), and interpreting equations as viable in a modeling context (A-CED.A.3) (National Governors Association & Council of Chief State School Officers [NGA & CCSSO], 2010, pp. 7, 42, 56, 65).
- Finding a pattern connects to such standards as describing patterns in bivariate data (8-SP.A.1), writing expressions in equivalent forms (A-SSE.B.3), analyzing functions using different representations (F-IF.C.7), and verifying the properties of dilations (G-SRT.A.1) (NGA & CCSSO, 2010, pp. 56, 64, 69, 77).
- Promoting student autonomy relates to making sense of problems and persevering in solving them (MP.1) and using appropriate tools strategically (MP.5) (NGA & CCSSO, 2010, p. 10). Problem solving and decision making are important elements of self-determination (Heroux, Peters, & Randel, 2014, p. 200).
- Using technology is encouraged in such standards as drawing geometric shapes (7.G.A.2), interpreting scientific notation (8.EE.A.4), finding the solution to the equation $f(x) = g(x)$ (A-REI.D.11), showing key features of the graphs of functions (F-IF.C.7), and describing transformations of functions (F-BF.B.3) (NGA & CCSSO, 2010, pp. 50, 54, 66, 69, 70).

Application

Researchers seeking to merge self-determination theory with culturally responsive teaching (which we describe in more detail in Chapter 2: Culturally Responsive Teaching) have identified five characteristics of effective motivation:

1. **Culture:** Students' learning is affected by the culture that surrounds them. Understanding it can shed light on how inequitable aspects of the dominant culture can negatively affect student learning (Usher, 2018, p. 139).
2. **Meaningfulness:** Students are most likely to see learning as valuable when teachers connect lessons to their lives in meaningful ways (Kumar et al., 2018, p. 83).
3. **Competence:** Competence includes both *cultural competence* (understanding the cultural identities of oneself and others) and *academic competence* (the belief that one can complete a task) (Kumar et al., 2018, p. 83; Usher & Kober, 2012b, p. 2).
4. **Autonomy:** Teaching students how to set goals and make decisions can foster students' sense of *autonomy* (the extent to which they believe they can control their goals and actions) and lead to both individual growth and societal change (Kumar et al., 2018, p. 87).
5. **Relatedness:** When educators develop authentic relationships with students (by learning about their home lives, culture, and values) and use that knowledge to communicate effectively with them, they are more likely to succeed academically (Bonner, 2014, p. 397).

Here are some strategies that apply these characteristics in supporting student motivation.

NURTURING STUDENT CONFIDENCE

As we said in the Supporting Research section in this chapter, building students' self-efficacy and autonomy can help alleviate their math anxiety and improve their motivation, which can in turn improve their academic performance.

One way that we nurture students' self-confidence is to use language that supports their choice whenever possible. Saying, "I recommend that you rephrase this definition in your own words," can often be more effective than simply commanding students to write it down. Explaining why completing a task is necessary can help students understand how they can benefit from doing so (Reeve & Halusic, 2009, p. 150).

Having private conversations with students about areas of concern can also help them feel more in control of their learning ("I've noticed that your work has slipped. Is everything OK? What can we do to improve it?"). Acknowledging and addressing their concerns can often turn complaints into more positive discussions ("Yes, I agree that this wording sounds confusing, but that's what they use on the state tests! Let's make it clearer.") (Reeve & Halusic, 2009, p. 151).

In fact, building meaningful relationships with students—learning about their interests and activities, demonstrating authentic care, and genuinely trying to connect with students of all backgrounds—is the cornerstone of culturally responsive teaching (Heroux et al., 2014, p. 198; Kumar et al., 2018, p. 89). We believe that doing so can help not just students of color but *all* students.

While the techniques we describe above are often successful, they have limitations since many students face constraints that limit their autonomy (Kumar et al., 2018, p. 87). In order for students to become more self-determined, they need more opportunities to set goals for themselves and regulate their learning (Heroux et al., 2014, p. 200). Some students may not complete online assignments because they lack Internet access at home, so simply telling them to "try harder" without figuring out what causes their behavior would accomplish little. Instead, they may need more time or an offline method to complete assignments. We discuss some helpful strategies in Chapter 11: Building a Productive Classroom Environment, Chapter 14: Differentiating Instruction, and Chapter 17: Cooperative Learning.

MOTIVATING THROUGH MATH

Many times, we use the math we teach as a motivational tool. These techniques work best when teachers use them to introduce the concept and elicit the lesson's goals (Posamentier, Smith, & Stepelman, 2010, p. 71).

When implementing these strategies, we also try to be sensitive to their emotional needs. We try to give them meaningful, scaffolded work that presents a moderate challenge. Giving students a reasonable chance to succeed can strengthen their self-efficacy and ease their math anxiety (Margolis, 2014).

Connecting to the Real World

Many times, we try to connect what students are learning to the real world. Practical applications should be brief and accessible enough to advance the lesson, not detract from it (Posamentier et al., 2010, p. 66). For example, students can analyze college loan payments when discussing compound interest or determine which mobile phone plan is most economical when learning about piecewise functions. Real-world applications often provide a strong motivation for mathematical learning (Walkington, Sherman, & Howell, 2014, p. 277).

Personalizing problems based on student interests or cultures can make mathematical tasks more relevant and improve their persistence and learning. Personalizing problems can connect what students already know to abstract mathematical concepts (a phenomenon known as *grounding*) and help them determine if their work is reasonable (Walkington et al., 2014, p. 275). Teachers can get information from students by asking them how they use numbers in their lives or what they are interested in. We talk about specific strategies for learning more about students in Chapter 11: Building a Productive Classroom Environment.

Here are some examples of problems that connect to students' lives and experiences:

- Showing how one unusually low score can negatively affect an average can lead to a meaningful discussion of the harmful effects of outliers and the value of using other statistical measures like the median.
- Students can use pattern blocks or other models of regular polygons to explore why honeycombs are shaped like hexagons. This can introduce a lesson on rotational symmetry, perimeter, and area. Students can relate this topic to chemistry by examining why hexagons appear in molecular structures.
- Precalculus students can discover the need for a polar coordinate system by examining the maps of ancient cities like Paris, Moscow, or Beijing. We first ask students to give directions in a familiar city or town with a rectangular street grid, such as Manhattan. Students are then asked to give directions in a city in which streets radiate outwards from a center. This lesson can lead to a discussion of how math is used in other fields like city planning and navigation.

Students can even create their word problems (we discuss this more in Chapter 4: Promoting Mathematical Communication).

Finding Patterns

Many students learn how to recognize patterns as early as kindergarten. This skill helps them make connections with more complicated ideas in middle and high school (Markworth, 2016, p. 23). Pattern recognition boosts students' feelings of competence since they usually need little prior knowledge (Smith, Hillen, & Catania, 2007, p. 39). Research indicates that the ability to process complex patterns is one of the brain's most important features since it underlies many other cognitive functions, including thought, imagination, invention, and reasoning (Mattson, 2014, p. 13). Pattern recognition is used to make many real-life decisions, including finding the fastest route home, predicting what people are thinking based on their body language, and estimating how much money to spend on a construction project (Barkman, 2018; Miemis, 2010). In our experience, pattern recognition can be incorporated into almost any lesson, often as an introductory exercise (which we discuss in Chapter 7: How to Plan Lessons).

Here are some examples in which we find patterns in lessons:

- Figure 1.1: Pattern Blocks shows an example using triangular numbers that can introduce sequences for middle school or Algebra I students.
- Figure 1.2: Rotational Symmetry has an activity that middle school or Geometry students can use to discover the formula for the minimum angle of rotation that maps a regular polygon back onto itself.
- Figure 1.3: Exponential Growth contains an activity in which students explore exponential growth by looking at the number of layers created when papers are folded in half.

Identifying a Void in Knowledge

Students often want to complete their understanding of a topic, so making them aware of a gap in that knowledge may motivate them to learn more (Posamentier et al., 2010, p. 62). We like this strategy because it allows us to make connections to knowledge that is familiar to students. This helps us promote retrieval practice, which we also discuss in Chapter 4: Promoting Mathematical Communication and Chapter 7: How to Plan Lessons.

Here is an example from Algebra I involving solving systems of linear equations algebraically:

1. Solve for x and y:

$$2x + 5y = 34$$
$$-2x - 4y = -28$$

2. Solve for x and y:

$$2x + y = 7$$
$$3x - y = 3$$

3. Solve for x and y:

$$2x + 5y = 34$$
$$x + 2y = 14$$

In the first two systems, students should be able to add the equations to eliminate one variable and solve for both. However, adding the equations in the third system will not eliminate the variable. Students will then realize that their prior knowledge (in this case, solving by adding equations) will not work for the third system. They could then be motivated to learn what they can do to solve the third system—in this case, multiplying the second equation by -2 so that they can eliminate a variable by adding the equations.

- Figure 1.4: Identify a Void shows an example from Geometry. The first two examples can be solved using the Pythagorean theorem since two side lengths from right triangles are given and students must find the third side. However, the third example can't be solved using the Pythagorean theorem since only one side length is given. This prepares students for seeing the value of trigonometry, which deals with angle measures and the ratios of side lengths in right triangles.

REWARDS

The use of *rewards*—extrinsic incentives like prizes or points—to motivate students is controversial. Many researchers agree that rewards can work when individuals are not initially motivated. However, the effect of incentives on individuals who are already motivated is less clear (Hidi, 2015, p. 87). Some studies conclude that rewards generally undermine intrinsic motivation (Kohn, 1994; Deci, Ryan, & Koestner, 2001, p. 50). Others find that the detrimental effect of rewards on motivation is overstated and that rewards may sometimes have a positive effect (Cameron, Banko, & Pierce, 2001, p. 21; Eisenberger & Shanock, 2003, p. 128).

We believe that rewards, when used with the other strategies that we describe in this chapter, can help keep students motivated to learn. In reality, rewards are necessary for most tasks. After all, almost everyone works for *some* kind of compensation, such as money, awards, or high grades.

When possible, we make rewards meaningful and immediate. Incentives work best when students see an immediate benefit—offering extra computer time now is usually more effective than offering an end-of-year party. Minimizing the gap between the effort and the reward reduces *delay discounting*, in which people assign less value to future rewards (Cheng, 2016).

We use a variety of incentives. Rewards like extra credit, stars, or "good work" tickets are cheap, easy, and accessible to all students (we mention several apps that can keep track of points in the Technology Connections section of this chapter). Students could even be involved in distributing stars or "good work" tickets (Lewis, 2017). To encourage autonomy, teachers can allow students to convert such tickets into more tangible rewards, like extra credit (Chapter 10: How to Develop an Effective Grading Policy describes how we incorporate extra credit into our grades). Teachers may also consider awarding classroom privileges, such as extra computer time (we discuss rules and procedures more in Chapter 11: Building a Productive Classroom Environment).

We sometimes give candy or some other inexpensive prize, such as pens or erasers. However, we find that regularly offering these prizes can be problematic. They obviously require time and money, and they may put students in awkward situations. Some students may be allergic to candy but may not want to tell us publicly.

In our opinion, teachers should *not* use incentives like "no-homework" passes. We feel that such rewards send the message that homework is an unpleasant burden instead of a necessary part of learning (we talk more about the importance of homework in Chapter 8: How to Plan Homework).

Finally, we try to remember that many times, the best reward for a student can be as simple as a smile, high five, or word of encouragement (Wong & Wong, 2004, p. 162). We find that such emotional supports are often more effective or lasting than any incentives.

MOTIVATING THROUGH POPULAR CULTURE

We often use music, art, literature, movies, and other elements from popular culture to motivate students. When we find a pop culture reference to a specific topic that we teach, we can give students a short excerpt at the beginning of a lesson and ask students to discuss it. These excerpts can also be used as a summary at the end of a lesson, in which case students can answer the question that is posed. Teachers can customize pop culture references by finding sources that appeal to students' cultures and interests.

Here are some of our favorite examples of math in pop culture:

- The opening song "Seasons of Love" from the musical *Rent* has the lyrics "Five hundred twenty-five thousand six hundred minutes. How do you measure a year in the life?" Students can use proportional reasoning to calculate the number of hours, minutes, or seconds in a year.
- The satirical book *Flatland* by Edwin Abbott refers to a square living in two-dimensional space that is visited by a sphere. Geometry teachers can use this book or show an excerpt from the movie when talking about cross-sections of three-dimensional objects.
- In the movie *Stand and Deliver*, teacher Jaime Escalante explains to his primarily Hispanic class that the Mayans discovered the concept of zero long before the Greeks and Romans. Teachers can use this example to show that *all* students can do math because mathematicians come from all cultures.
- In the movie *Mean Girls*, Cady Heron and her classmates solve algebra word problems in a math competition and ultimately win the tie-breaker when she determines that $\lim_{x \to 0} \frac{\ln(1-x)-\sin(x)}{1-\cos^2(x)}$ does not exist. Algebra or calculus teachers can play these excerpts when discussing systems of equations or limits.

- In the Edgar Allan Poe short story "The Pit and the Pendulum," the protagonist is strapped to a frame while a swinging pendulum slowly descends. Algebra students can calculate the time required for a pendulum to descend and determine if the story is realistic. Poe might not be considered "pop culture" by all—especially by students. However, there are appropriately "gruesome" scenes from B-grade movies available online that many would find engaging.
- If fashion dolls were real-life people, would their body measurements be reasonable? Geometry students can use measure dolls and scale the measurements using proportional reasoning.
- Students can examine artwork from various cultures, such as Islamic tiles or Native American quilts. Teachers can ask questions related to pattern recognition, fractions, symmetry, and geometric transformations. Students can even design their own quilts as a project (Paznokas, 2003, p. 255).

One challenge that we find when using pop culture references is that they often depict math in a negative way. The movie *A Beautiful Mind*, which tells the story of Princeton professor John Nash's struggles with schizophrenia, can reinforce stereotypes of mathematicians as being socially awkward or mentally ill. We suggest either incorporating difficult issues into student discussions or avoiding controversial examples entirely if you feel that you can't address them adequately in class. When used appropriately, pop culture references can make math more interesting and fun.

MOTIVATING ENGLISH LANGUAGE LEARNERS AND STUDENTS WITH LEARNING DIFFERENCES

Some motivational strategies, like the ones we discuss in this section that promote self-confidence in math, can work particularly well with both English Language Learners (ELLs) and with students who have learning differences. However, the challenges facing each group can be very different. Most ELLs are just as intellectually capable as most English-proficient students, but face language and cultural challenges. Many students with learning differences may be more English-proficient, but have specific needs related to the way they process new information and concepts. We believe that *always* keeping these distinctions in mind is critical to helping both groups succeed.

English Language Learners

We try to create an environment in which ELLs feel comfortable speaking and writing in another language while learning content. We allow students to ask or answer questions in their own language. In these cases, we ask a different student to translate or use a tool like Google Translate. We encourage ELLs to use glossaries or dictionaries to help them with unfamiliar words. We discuss other strategies to help students build their language skills in Chapter 3: Teaching Math as a Language and Chapter 4: Promoting Mathematical Communication.

In addition, we point out to the class that understanding other languages and cultures is important in today's diverse world. Sometimes we even ask ELLs to teach *us* words and phrases in their native languages. Making an honest effort to learn other languages can not only build a rapport with students (which we discuss more in Chapter 11: Building a Productive Classroom Environment) but can also help remind us of the challenges that they face when learning English.

Students with Learning Differences

We see students with learning differences as *capable* instead of disabled by identifying their strengths as well as ways that we can make them more successful. For example, we may give them specific tasks to perform in class. An autistic student in Larisa's class became the person that students asked for help. His ability to explain problems clearly to his classmates not only made him a valuable member of the class but also helped him improve his social skills.

To help students with learning differences visualize difficult mathematical concepts, we often use *manipulatives* (concrete objects like pattern blocks and algebra tiles that students can use to represent abstract mathematical concepts) and technology. These strategies can often benefit all students, not just those with learning differences. Research indicates that manipulatives and technology have a modest positive effect on student motivation (Bouck & Park, 2018, p. 97; Jensen, 2005; Preston et al., 2015, p. 180). Simply using these strategies does not guarantee that student achievement will improve. They should be supported by high-quality instruction with enough guidance for students (Carbonneau, Marley, & Selig, 2013, p. 396).

Although manipulatives are often associated with elementary school math, we find them especially useful for introducing topics or making connections with prior knowledge. Many of our students find manipulatives less intimidating than other strategies, which can reduce their math anxiety and makes them more receptive to learning. For example, operations with polynomials can be difficult to visualize, so we introduce the topic with algebra tiles, which many students use in elementary school. Algebra tiles help students transition to more abstract tasks, such as multiplying polynomials or completing the square. Chapter 5: Making Mathematical Connections contains other examples of how manipulatives can be used.

We use technology like calculators or online animations to illustrate complicated mathematical ideas. Students who have difficulty factoring polynomials can work backwards by using the calculator to find the zeros. We discuss other benefits, pitfalls, and examples of using technology in the classroom in Chapter 19: Using Technology.

Students can develop their sense of competence and autonomy by receiving work that is properly scaffolded to match their current level of readiness (Usher & Kober, 2012a, p. 4). We do this for all students, and keep it especially in mind for ELLs and students with learning differences. We try to give students work that "meets them where they are" and gradually increases the level of difficulty. For example, a Geometry lesson on applying properties of parallelograms can start with identifying highlighted parts of parallelograms and move to algebraic problems before culminating in formal proofs.

Chapter 14: Differentiating Instruction and Chapter 15: Differentiating for Students with Unique Needs contain other useful strategies.

Student Handouts and Examples

Figure 1.1: Pattern Blocks

Figure 1.2: Rotational Symmetry

Figure 1.3: Exponential Growth

What Could Go Wrong

When *we* try to motivate students, many things can go wrong.

USING FEAR TO MOTIVATE

Research indicates that threatening students generally has a negative effect on their motivation and academic growth (Von der Embse, Schultz, & Draughn, 2015, p. 630). Intimidating students often decreases their intrinsic motivation and causes a fear of failure (Putwain & Remedios, 2014, p. 512). That doesn't mean, however, that we can't communicate frustration or disappointment to students. If we have a good relationship with students, saying something like, "I'm feeling disappointed by what is happening now" can be an effective and appropriate motivational strategy.

In our experience, expressing motivation by explaining how the task will lead to a desired goal ("Doing homework will help you know what types of questions will be on the test") generally works better than browbeating students, especially by linking academic outcomes to unrelated consequences ("If you don't do well on this test, you won't be allowed to go to the prom").

STEREOTYPE THREAT

In a misguided attempt to boost students' confidence, teachers may connect their performance to a prejudice—for example, by saying that girls often struggle with math or praising a Chinese student by saying that Asian students typically excel. This phenomenon, called *stereotype threat*, is a fear of confirming a stereotype about one's gender, ethnicity, or other self-identified group (Steele, 1997, p. 617). People who experience this idea worry so much about being identified with the stereotype that their academic performance suffers (Graham & Morales-Chicas, 2015, p. 25; Laldin, 2016).

To minimize the effect of stereotype threat, we try not to imply that their academic performance is in any way related to their identity. Praising students for being "naturally smart" inadvertently implies that their intelligence is somehow fixed to their identity group. Instead, we try to promote a growth mindset (which we discuss more in Chapter 4: Promoting Mathematical Communication).

In addition, we try to foster a sense of "belonging" among all students in our class. Research indicates that a sense of belonging in school correlates with achievement, self-efficacy, and motivation (Chiu, Chow, McBride, & Mol, 2015, pp. 189–190; Lam, Chen, Zhang, & Liang, 2015, p. 402; Osterman, 2000, p. 327; Reynolds, Lee, Turner, Bromhead, & Subasic, 2017, p. 89). Exposing students to cultural concepts from different ethnic groups can promote self-understanding (Gray, Hope, & Matthews, 2018, p. 101). We discuss this more in Chapter 2: Culturally Responsive Teaching, Chapter 4: Promoting Mathematical Communication, and Chapter 7: How to Plan Lessons. Making *all* students feel welcome in our class and encouraging students from different identity groups to interact in our classrooms (through activities like group work or peer teaching) can also strengthen the ethnic climate in our classroom. A strong ethnic climate can minimize the negative effects of perceiving few classmates of one's own identity group (Graham & Morales-Chicas, 2015, p. 23).

"WHY DO WE NEED TO KNOW THIS?"

During a lesson, students occasionally ask us, "When are we going to use this in life?" or "Why are we learning this?" Since students usually want to know how our lesson is used in their lives, we try to make those connections whenever possible. When discussing compound interest, we talk about student loans. When discussing measures of central tendency, we talk about grades.

However, we can't make real-world connections in every lesson. Many topics—such as formal geometric proofs or rules of exponents—have no immediate connection outside of a math class. These lessons are important as necessary tools for larger mathematical concepts. Trying to find an application in every lesson can backfire, especially if the connections are strained. Constantly connecting math to the real world can actually de-motivate struggling students who may feel even *more* pressure to do well (Hulleman, Godes, Hendricks, & Harackiewicz, 2010, p. 891). Some of our most successful lessons involve straightforward ideas like finding a pattern. Many times, the simplest solution is the most effective one!

In our experience, students often ask us, "Why do we need to know this?" when they get frustrated or bored with our lesson. Since students in these situations are really saying that they don't understand what we're trying to teach, we start by first acknowledging students' concerns ("I can see that you're getting frustrated with this"). We then try to determine the source of those concerns, using some of the questioning and discussion techniques we discuss in Chapter 4: Promoting Mathematical Communication. If necessary, we come up with a clearer or simpler explanation.

MISREADING STUDENTS

If we underestimate our students' potential, we often wind up watering down our instruction by ignoring connections between topics or reducing math to calculator shortcuts and mindless drills. Unfortunately, this can erode student effort, especially for students who face multiple challenges (Mo, 2019, p. 5; Usher & Kober, 2012a, p. 4). In our experience, students can often tell when we have less confidence in their ability to learn and react by being less interested in the lesson, which leads us to slow down even more.

Conversely, giving them tasks that are too difficult can heighten their anxiety. Students may develop perfectionist tendencies, get discouraged when they make a mistake, and even burn out (Mo, 2019, p. 5). Not giving students appropriate support—for example, by teaching without paying attention to their current level of understanding—can make lessons inaccessible to struggling students and discourage them from learning (National Research Council, 2004, p. 47).

We recommend focusing instruction on identifying what students *currently* know, what they *need* to know, and scaffolding instruction with appropriate techniques (such as the ones we discuss throughout this book) so they can learn independently.

In addition, we try to address any emotional concerns that can adversely affect student motivation. Getting to know students better helps us give them emotional comfort when they seem stressed. We also remind them frequently that learning to deal with failure is an important life lesson and share examples from our own lives when we encountered difficulty. We discuss ways to improve the teacher–student relationship in Chapter 11: Building a Productive Classroom Environment.

Sometimes, we mistakenly assume that students have complete control of their environment. Criticizing students for not buying an expensive graphing calculator or not doing homework assumes that they *choose* not to finish a task and ignores economic hardships or personal situations that serve as obstacles. Often, students who we might think are “lazy” have too many responsibilities at home or worry that they won’t be valued if they try and fail. Letting students know that we won’t humiliate them, empathizing with them, and providing adequate academic and emotional support for tasks can help students move past their fears (Wormeli, 2014).

LIMITATIONS TO MOTIVATION

Occasionally, despite our best efforts, we just can’t seem to motivate some students. Getting help from a guidance counselor, psychologist, or some other school professional can be helpful since they often have experience and training to deal with challenging situations. In these cases, we find that putting too much academic pressure on students can often backfire. This can decrease their motivation to stay in school and may make them withdraw even deeper.

Even if we can’t motivate them to learn all of the math that we’d like, we can help develop important life skills, such as working with others and coping with adversity. Giving them a friendly greeting and showing genuine concern can build trust and eventually encourage them to open up when ready.

Technology Connections

More information about self-determination theory, including a more detailed explanation and related research, can be found at the Center for Self-Determination Theory’s website (http://selfdeterminationtheory.org).

Some sources of real-world math problems include Mathalicious (www.mathalicious.com), which offers real-world middle and high school lessons; Math in Daily Life (www.learner.org); and Real World Math (http://realworldmath.org), which offers free math activities for Google Earth.

Teachers can use virtual manipulatives as motivations for lessons. Didax (http://www.didax.com/virtual-manipulatives-for-math) has a wide variety of virtual manipulatives, including two-color counters, pattern blocks, spinners, and algebra tiles. The National Library of Virtual Manipulatives (http://nlvm.usu.edu) has a wide range of online tools for pre-K to grade 12 math.

Printable algebra tiles can be downloaded from sites like The Math Lab (http://themathlab.com/toolbox/algebra%20stuff/algebra%20tiles.htm). Proof Blocks (www.proofblocks.com) are two-dimensional blocks (with a mathematical fact on each block) that can be physically linked together to form logical chains of reasoning.

In addition, Desmos (www.desmos.com) and Geogebra (www.geogebra.org) have many activities that can be used to motivate students.

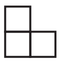
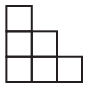
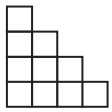
Apps like ClassDojo (www.classdojo.com) and LiveSchool (www.whyliveschool.com) allow teachers to track student behavior with a point system. However, if you choose to use one of these tools, we suggest reading articles that share reservations, including "On Using and Not Using ClassDojo: Ideological Differences?" by Larry Cuban (http://larrycuban.wordpress.com/2014/03/15/on-using-and-not-using-classdojo-ideological-differences) and "Classes of Donkey" by David Truss (http://pairadimes.davidtruss.com/classes-of-donkeys).

Harvard math professor Oliver Knill has a list of math-related movie clips on his website (http://www.math.harvard.edu/~knill/mathmovies/index.html) .

The book *Living Proof: Stories of Resilience Along the Mathematical Journey* (Heinrich, Lawrence, Pons, & Taylor, 2019, http://www.maa.org/press/ebooks/living-proof-stories-of-resilience-along-the-mathematical-journey-2), which is available for free online, is a collection of biographical stories from real-life mathematicians about their struggles learning math and how they overcame them.

Figures

1. Complete the table below.

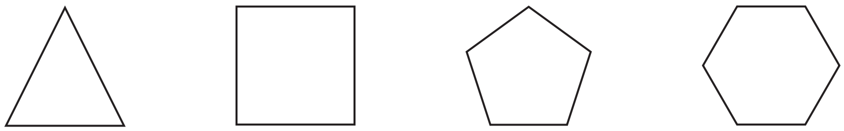
n	**1**	**2**	**3**	**4**	**5**	**6**
Group						
Number of blocks per group						

2. Describe the pattern from the table above in words.

3. Use your pattern to determine the number of blocks in the 20^{th} group. Try to do it without drawing 20 groups of blocks.

Figure 1.1 Pattern Blocks

1. Draw the lines of symmetry that exist for each regular polygon below.



2. Besides line symmetry, regular polygons have another kind of symmetry called rotational symmetry.

a. For each regular polygon above, fill out the chart below to determine the minimum angle of rotation required to map the figure back onto itself.

Number of sides (*n*)	3	4	5	6
Measure of angle of rotation				

b. Based on the table above, fill in the blank in the following sentence: In a regular polygon of *n* sides, the measure of the smallest angle of rotation that maps the figure back onto itself is ___________ .

Figure 1.2 Rotational Symmetry

1. In this problem, you will examine folding a paper in half.

Number of folds (*x*)	**Number of layers (*y*)**
0	
1	
2	
3	
4	
5	

a. Fill out the accompanying table to determine the number of layers of paper that are being made with each fold.

b. Write an equation that relates the number of folds (x) to the number of layers (y).

c. Graph the equation you wrote in part *b* using a calculator or other technology. Sketch this graph below.

2. In this problem, you will examine folding three papers in half.

Number of folds (*x*)	**Number of layers (*y*)**
0	
1	
2	
3	
4	
5	

Figure 1.3 Exponential Growth

a. Fill out the accompanying table to determine the number of layers of paper that are being made with each fold.

b. Write a equation that relates the number of folds (x) to the number of layers (y).

c. Graph the equation you wrote in part *b* using a calculator or other technology. Sketch this graph below.

3. Based on your work above, write a formula for each of the following situations. Let x = the total number of folds and let y = the number of layers of paper.

a. folding 5 pieces of paper in half

b. folding 2 pieces of paper in half

c. folding 4 pieces of paper in half

4. How are these equations different from the equations that you have worked with before?

Figure 1.3 (Continued)

Find the value of *x*.

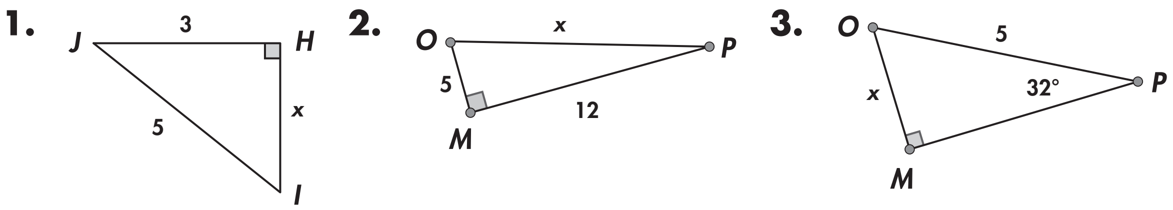


Figure 1.4 Identify a Void