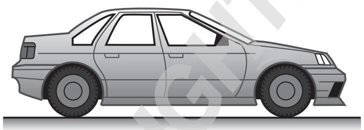
# Straight Lines in Engineering

CHAPTER 1

In this chapter, the applications of straight lines in engineering are introduced. It is assumed that the students are already familiar with this topic from their high school algebra course. This chapter will show, with examples, why this topic is so important for engineers. For example, the velocity of a vehicle while braking, the voltage–current relationship in a resistive circuit, and the relationship between force and displacement in a preloaded spring can all be represented by straight lines. In this chapter, the equations of these lines will be obtained using both the slope-intercept and the point-slope forms.

## 1.1 VEHICLE DURING BRAKING

The velocity of a vehicle during braking is measured at two distinct points in time, as indicated in Fig. 1.1.



| $t$ (s) | $v(t)$ (m/s) |
|---|---|
| 1.5 | 9.75 |
| 2.5 | 5.85 |

**Figure 1.1** A vehicle while braking.



The velocity satisfies the equation

$$v(t) = at + v_o \tag{1.1}$$

where $v_o$ is the initial velocity in m/s and $a$ is the acceleration in m/s$^2$.

(a) Find the equation of the line $v(t)$ and determine both the initial velocity $v_o$ and the acceleration $a$.

(b) Sketch the graph of the line $v(t)$ and clearly label the initial velocity, the acceleration, and the total stopping time on the graph.

The equation of the velocity given by equation (1.1) is in the slope-intercept form $y = mx + b$, where $y = v(t)$, $m = a$, $x = t$, and $b = v_o$. The slope $m$ is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Therefore, the slope $m = a$ can be calculated using the data in Fig. 1.1 as

$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{5.85 - 9.75}{2.5 - 1.5} = -3.9 \text{ m/s}^2.$$

The velocity of the vehicle can now be written in the slope-intercept form as

$$v(t) = -3.9\, t + v_o.$$

The $y$-intercept $b = v_o$ can be determined using either one of the data points. Using the data point $(t, v) = (1.5, 9.75)$ gives

$$9.75 = -3.9\,(1.5) + v_o.$$

Solving for $v_o$ gives

$$v_o = 15.6 \text{ m/s}.$$

The $y$-intercept $b = v_o$ can also be determined using the other data point $(t, v) = (2.5, 5.85)$, yielding

$$5.85 = -3.9\,(2.5) + v_o.$$

Solving for $v_o$ gives

$$v_o = 15.6 \text{ m/s}.$$

The velocity of the vehicle can now be written as

$$v(t) = -3.9\, t + 15.6 \text{ m/s}.$$

The total stopping time (time required to reach $v(t) = 0$) can be found by equating $v(t) = 0$, which gives

$$0 = -3.9\, t + 15.6.$$

Solving for $t$, the stopping time is found to be $t = 4.0$ s.

Figure 1.2 shows the velocity of the vehicle after braking. Note that the stopping time $t = 4.0$ s and the initial velocity $v_o = 15.6$ m/s are the $x$- and $y$-intercepts of the line, respectively. Also, note that the slope of the line $m = -3.90 \text{ m/s}^2$ is the acceleration of the vehicle during braking.

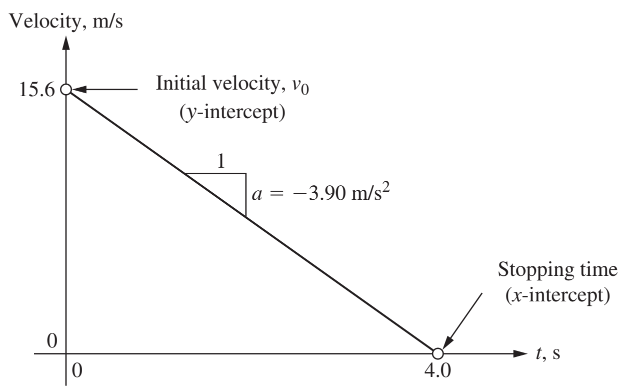
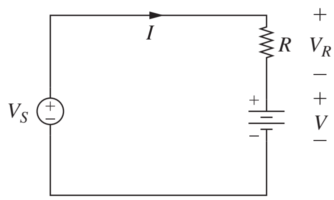



**Figure 1.2** Velocity of the vehicle after braking.

## 1.2 VOLTAGE–CURRENT RELATIONSHIP IN A RESISTIVE CIRCUIT

For the resistive circuit shown in Fig. 1.3, the relationship between the applied voltage $V_s$ and the current $I$ flowing through the circuit can be obtained using **Kirchhoff's voltage law (KVL)** and **Ohm's law**. For a closed-loop in an electric circuit, KVL states that the sum of the voltage rises is equal to the sum of the voltage drops:

$$\text{Kirchhoff's voltage law:} \Rightarrow \quad \sum Voltage\ rise = \sum Voltage\ drop.$$




| $V_s$ (V) | $I$ (A) |
|---|---|
| 10.0 | 0.1 |
| 20.0 | 1.1 |

**Figure 1.3** Voltage and current in a resistive circuit.

Applying KVL to the circuit of Fig. 1.3 gives

$$V_s = V_R + V. \tag{1.2}$$

**Ohm's law** states that the voltage drop across a resistor $V_R$ in volts (V) is equal to the current $I$ in amperes (A) flowing through the resistor multiplied by the resistance $R$ in ohms (Ω):

$$V_R = I\,R. \tag{1.3}$$

Substituting equation (1.3) into equation (1.2) gives a linear relationship between the applied voltage $V_s$ and the current $I$ as

$$V_s = I\,R + V. \tag{1.4}$$

The objective is to find the value of $R$ and $V$ when the current flowing through the circuit is known for two different voltage values given in Fig. 1.3.

The voltage–current relationship given by equation (1.4) is the equation of a straight line in the slope-intercept form $y = mx + b$, where $y = V_s$, $x = I$, $m = R$, and $b = V$. The slope $m$ is given by

$$m = R = \frac{\Delta y}{\Delta x} = \frac{\Delta V_s}{\Delta I}.$$

Using the data in Fig. 1.3, the slope $R$ can be found as

$$R = \frac{20 - 10}{1.1 - 0.1} = 10\ \Omega.$$

Therefore, the source voltage can be written in the slope-intercept form as

$$V_s = 10\,I + b.$$

The $y$-intercept $b = V$ can be determined using either one of the data points. Using the data point $(I, V_s) = (0.1, 10)$ gives

$$10 = 10\,(0.1) + V.$$

Solving for $V$ gives

$$V = 9 \text{ V}.$$

The $y$-intercept $V$ can also be found by finding the equation of the straight line using the point-slope form of the straight line $(y - y_1) = m(x - x_1)$ as

$$V_s - 10 = 10(I - 0.1) \;\Rightarrow\; V_s = 10\,I - 1.0 + 10.$$

Therefore, the voltage–current relationship is given by

$$V_s = 10\,I + 9. \tag{1.5}$$

Comparing equations (1.4) and (1.5), the values of $R$ and $V$ are given by

$$R = 10\ \Omega, \quad V = 9 \text{ V}.$$

Figure 1.4 shows the graph of the source voltage $V_s$ versus the current $I$. Note that the slope of the line $m = 10$ is the resistance $R$ in $\Omega$ and the $y$-intercept $b = 9$ is the voltage $V$ in volts.

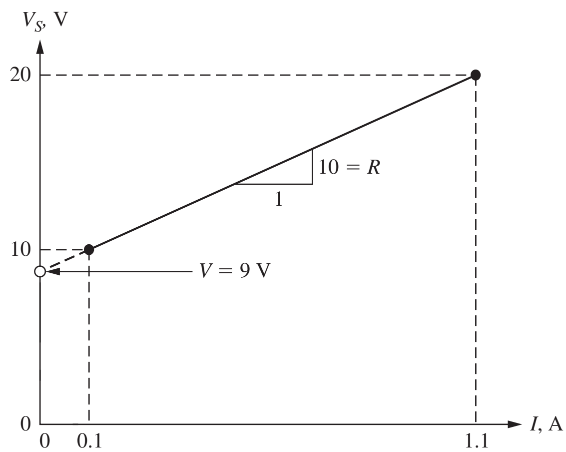



**Figure 1.4** Voltage–current relationship for the data given in Fig. 1.3.

The values of $R$ and $V$ can also be determined by switching the interpretation of $x$ and $y$ (the independent and dependent variables). From the voltage–current relationship $V_s = I\,R + V$, the current $I$ can be written as a function of $V_s$ as

$$I = \frac{1}{R}\,V_s - \frac{V}{R}. \tag{1.6}$$

This is an equation of a straight line $y = m\,x + b$, where $x$ is the applied voltage $V_s$, $y$ is the current $I$, $m = \frac{1}{R}$ is the slope, and $b = -\frac{V}{R}$ is the $y$-intercept. The slope and

$y$-intercept can be found from the data given in Fig. 1.3 using the slope-intercept method as

$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta I}{\Delta V_s}.$$

Using the data in Fig. 1.3, the slope $m$ can be found as

$$m = \frac{1.1 - 0.1}{20 - 10} = 0.1.$$

Therefore, the current $I$ can be written in the slope-intercept form as

$$I = 0.1\ V_s + b.$$

The $y$-intercept $b$ can be determined using either one of the data points. Using the data point $(V_s, I) = (10, 0.1)$ gives

$$0.1 = 0.1\,(10) + b.$$

Solving for $b$ gives

$$b = -0.9.$$

Therefore, the equation of the straight line can be written in the slope-intercept form as

$$I = 0.1\ V_s - 0.9. \tag{1.7}$$

Comparing equations (1.6) and (1.7) gives

$$\frac{1}{R} = 0.1 \quad \Rightarrow \quad R = 10\ \Omega$$

and

$$-\frac{V}{R} = -0.9 \quad \Rightarrow \quad V = 0.9\,(10) = 9\ \text{V}.$$

Figure 1.5 is the graph of the straight line $I = 0.1V_s - 0.9$. Note that the $y$-intercept is $-\frac{V}{R} = -0.9$ A and the slope is $\frac{1}{R} = 0.1$.

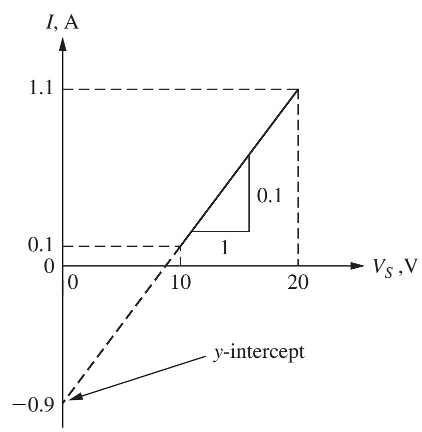


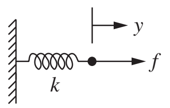

**Figure 1.5** Straight line with $I$ as independent variable for the data given in Fig. 1.3.

## 1.3 FORCE–DISPLACEMENT IN A PRELOADED TENSION SPRING

The force–displacement relationship for a spring with a preload $f_o$ is given by

$$f = k\,y + f_o, \tag{1.8}$$

where $f$ is the force in *Newtons* (N), $y$ is the displacement in *meters* (m), and $k$ is the spring constant in N/m.




| $f$(N) | $y$(m) |
|---|---|
| 1 | 0.1 |
| 5 | 0.9 |

**Figure 1.6** Force–displacement in a preloaded spring.

The objective is to find the spring constant $k$ and the preload $f_o$, if the values of the force and displacement are as given in Fig. 1.6.

**Method 1:** Treating the displacement $y$ as an independent variable, the force–displacement relationship $f = k\,y + f_o$ is the equation of a straight line $y = mx + b$, where the independent variable $x$ is the displacement $y$, the dependent variable $y$ is the force $f$, the slope $m$ is the spring constant $k$, and the $y$-intercept is the preload $f_o$. The slope $m$ can be calculated using the data given in Fig. 1.6 as

$$m = \frac{5 - 1}{0.9 - 0.1} = \frac{4}{0.8} = 5.$$

The equation of the force–displacement equation in the slope-intercept form can therefore be written as

$$f = 5\,y + b.$$

The $y$-intercept $b$ can be found using one of the data points. Using the data point $(y, f) = (0.9, 5)$ gives

$$5 = 5\,(0.9) + b.$$

Solving for $b$ gives

$$b = 0.5 \text{ N}.$$

Therefore, the equation of the straight line can be written in the slope-intercept form as

$$f = 5\,y + 0.5. \tag{1.9}$$

Comparing equations (1.8) and (1.9) gives

$$k = 5 \text{ N/m}, \quad f_o = 0.5 \text{ N}.$$

**Method 2:** Now treating the force $f$ as an independent variable, the force–displacement relationship $f = k\,y + f_o$ can be written as $y = \frac{1}{k} f - \frac{f_o}{k}$. This relationship is the equation of a straight line $y = mx + b$, where the independent variable

$x$ is the force $f$, the dependent variable $y$ is the displacement $y$, the slope $m$ is the reciprocal of the spring constant $\frac{1}{k}$, and the $y$-intercept is the negative preload divided by the spring constant $-\frac{f_o}{k}$. The slope $m$ can be calculated using the data given in Fig. 1.6 as

$$m = \frac{0.9 - 0.1}{5 - 1} = \frac{0.8}{4} = 0.2.$$

The equation of the displacement $y$ as a function of force $f$ can therefore be written in the slope-intercept form as

$$y = 0.2f + b.$$

The $y$-intercept $b$ can be found using one of the data points. Using the data point $(f, y) = (5, 0.9)$ gives

$$0.9 = 0.2\,(5) + b.$$

Solving for $b$ gives

$$b = -0.1.$$

Therefore, the equation of the straight line can be written in the slope-intercept form as

$$y = 0.2f - 0.1. \tag{1.10}$$

Comparing equation (1.10) with the expression $y = \frac{1}{k}f - \frac{f_o}{k}$ gives

$$\frac{1}{k} = 0.2 \quad \Rightarrow \quad k = 5 \text{ N/m}$$

and

$$-\frac{f_o}{k} = -0.1 \quad \Rightarrow \quad f_o = 0.1\,(5) = 0.5 \text{ N}.$$

Therefore, the force–displacement relationship for a preloaded spring given in Fig. 1.6 is given by

$$f = 5\,y + 0.5.$$

## 1.4 FURTHER EXAMPLES OF LINES IN ENGINEERING

**Example 1-1**

The velocity of a vehicle follows the trajectory shown in Fig. 1.7. The vehicle starts at rest (zero velocity) and reaches a maximum velocity of 10 m/s in 2 s. It then cruises at a constant velocity of 10 m/s for 2 s before coming to rest at 6 s. Write the equation of the function $v(t)$ for times between 0 and 2 s, between 2 and 4 s, between 4 and 6 s, and greater than 6 s.

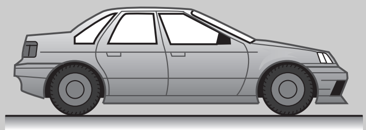
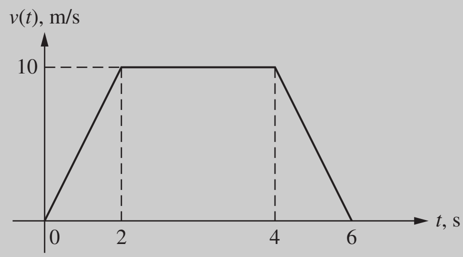




**Figure 1.7** Velocity profile of a vehicle.

**Solution** The velocity profile of the vehicle shown in Fig. 1.7 is a piecewise linear function with three different equations. The first linear function is a straight line passing through the origin starting at time 0 s and ending at time equal to 2 s. The second linear function is a straight line with zero slope (cruise velocity of 10 m/s) starting at 2 s and ending at 4 s. Finally, the third piece of the trajectory is a straight line starting at 4 s and ending at 6 s. The equation of the piecewise linear function can be written as

(a) $0 \le t \le 2$ s:

$$v(t) = mt + b$$

where $b = 0$ and $m = \dfrac{10 - 0}{2 - 0} = 5$. Therefore,

$$v(t) = 5\,t \text{ m/s}.$$

(b) $2 \le t \le 4$ s:

$$v = 10 \text{ m/s}.$$

(c) $4 \le t \le 6$ s:

$$v(t) = m\,t + b,$$

where $m = \dfrac{0 - 10}{6 - 4} = -5$ and the value of $b$ can be calculated using the data point $(t, v(t)) = (6, 0)$ as

$$0 = -5\,(6) + b \quad \Rightarrow \quad b = 0 + 30 = 30.$$

The value of $b$ can also be calculated using the point-slope formula for the straight line

$$v - v_1 = m(t - t_1),$$

where $v_1 = 0$ and $t_1 = 6$. Thus,

$$v - 0 = -5(t - 6).$$

Therefore,

$$v(t) = -5(t - 6).$$

or

$$v(t) = -5t + 30 \text{ m/s}.$$

(d) $t > 6$ s:

$$v(t) = 0 \text{ m/s}.$$

**Example 1-2**

The velocity of a vehicle is given in Fig. 1.8.

(a) Determine the equation of $v(t)$ for
  (i) $0 \leq t \leq 3$ s
  (ii) $3 \leq t \leq 6$ s
  (iii) $6 \leq t \leq 9$ s
  (iv) $t \geq 9$ s

(b) Knowing that the acceleration of the vehicle is the slope of velocity, plot the acceleration of the vehicle.

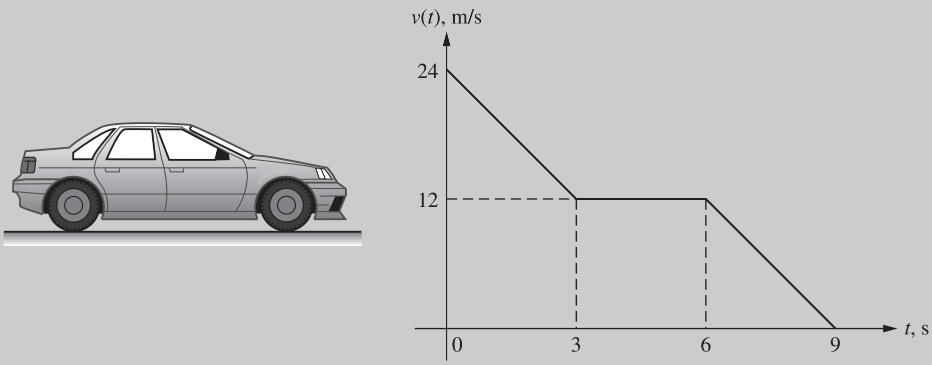



**Figure 1.8** Velocity profile of a vehicle.

**Solution** (a) The velocity of the vehicle for different intervals can be calculated as

(i) $0 \leq t \leq 3$ s:

$$v(t) = m\,t + b,$$

where $m = \dfrac{12 - 24}{3 - 0} = -4 \text{ m/s}^2$ and $b = 24$ m/s. Therefore,

$$v(t) = -4\,t + 24 \text{ m/s}.$$

(ii) $3 \leq t \leq 6$ s:

$$v(t) = 12 \text{ m/s}.$$

(iii) $6 \leq t \leq 9$ s:

$$v(t) = m\,t + b,$$

where $m = \dfrac{0 - 12}{9 - 6} = -4 \text{ m/s}^2$ and $b$ can be calculated in the slope-intercept form using point $(t, v(t)) = (9, 0)$ as

$$0 = -4(9) + b.$$

Therefore, $b = 36$ m/s and

$$v(t) = -4t + 36 \text{ m/s}.$$

(iv) $t > 9$ s:

$$v(t) = 0 \text{ m/s}.$$

(b) Since the acceleration of the vehicle is the slope of the velocity in each interval, the acceleration $a$ in m/s$^2$ is given by

$$a = \begin{cases} -4; & 0 \le t \le 3 \text{ s} \\ 0; & 3 \le t \le 6 \text{ s} \\ -4; & 6 \le t \le 9 \text{ s} \\ 0; & t > 9 \text{ s} \end{cases}$$

The plot of the acceleration is shown in Fig. 1.9.

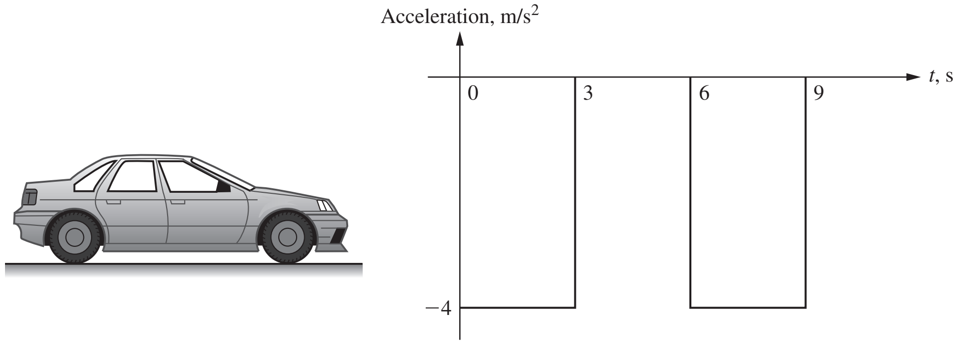



**Figure 1.9** Acceleration profile of the vehicle in Fig. 1.8.

**Example 1-3**

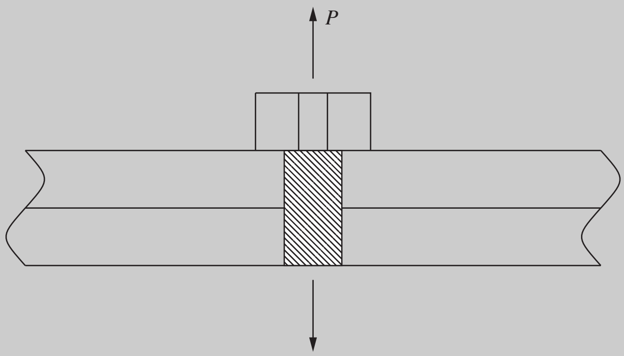
In a bolted connection shown in Fig.1.10, the force in the bolt $F_b$ is related to the external load $P$ as

$$F_b = C\,P + F_i,$$

where $C$ is the joint constant and $F_i$ is the preload in the bolt.

(a) Determine the joint constant $C$ and the preload $F_i$ given the data in Fig. 1.10.

(b) Plot the bolt force $F_b$ as a function of the external load $P$, and label $C$ and $F_i$ on the graph.




| $P$ (lb) | $F_b$ (lb) |
|---|---|
| 400 | 500 |
| 800 | 600 |

**Figure 1.10** External force applied to a bolted connection.

**Solution** (a) The force–load relationship $F_b = CP + F_i$ is the equation of a straight line, $y = mx + b$. The slope $m$ is the joint constant $C$, which can be calculated as

$$C = \frac{\Delta F_b}{\Delta P} = \frac{600 - 500}{800 - 400} = \frac{100 \text{ lb}}{400 \text{ lb}} = 0.25.$$

Therefore,

$$F_b = 0.25\,P + F_i. \tag{1.11}$$

Now, the $y$-intercept $F_i$ can be calculated by substituting one of the data points into equation (1.11). Substituting the second data point $(P, F_b) = (800, 600)$ gives

$$600 = 0.25 \times 800 + F_i.$$

Solving for $F_i$ yields

$$F_i = 600 - 200 = 400 \text{ lb}.$$

Therefore, $F_b = 0.25\,P + 400$ is the equation of the straight line, where $C = 0.25$ and $F_i = 400$ lb. Note that the joint constant $C$ is dimensionless.

(b) The plot of the force $F_b$ in the bolt as a function of the external load $P$ is shown in Fig. 1.11.

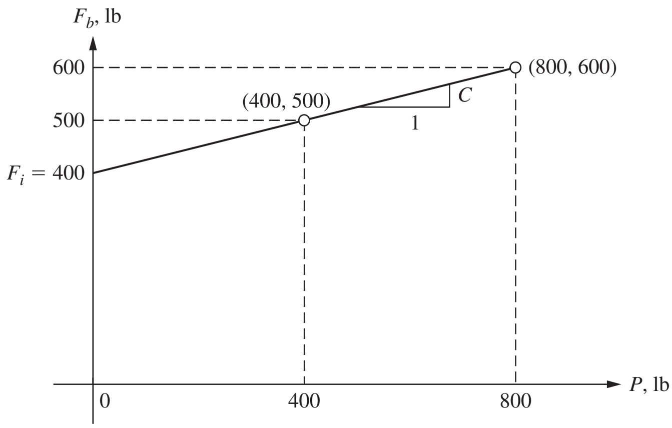
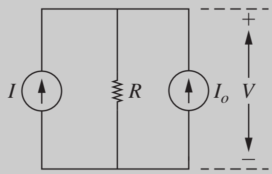



**Figure 1.11** Plot of the bolt force $F_b$ as a function of the external load $P$.

**Example 1-4**

For the electric circuit shown in Fig. 1.12, the relationship between the voltage $V$ and the applied current $I$ is given by $V = (I + I_o)R$. Find the values of $R$ and $I_o$ if the voltage across the resistor $V$ is known for the two different values of the current $I$ as shown in Fig. 1.12.




| $I$ (A) | $V$ (V) |
|---|---|
| 0.1 | 1.2 |
| 0.2 | 2.2 |

**Figure 1.12** Circuit for Example 1-4.

**Solution** The voltage–current relationship $V = R\,I + R\,I_o$ is the equation of a straight line $y = mx + b$, where the slope $m = R$ can be found from the data given in Fig. 1.12 as

$$R = \frac{\Delta V}{\Delta I} = \frac{2.2 - 1.2}{0.2 - 0.1} = \frac{1}{0.1}\,\frac{\text{V}}{\text{A}} = 10\ \Omega.$$

Therefore,

$$V = 10\,I + 10\,I_o. \tag{1.12}$$

The $y$-intercept $b = 10\,I_o$ can be found by substituting the second data point (0.2, 2.2) in equation (1.12) as

$$2.2 = 100 \times 0.2 + 10\,I_o.$$

Solving for $I_o$ gives

$$10\,I_o = 2.2 - 2 = 0.2,$$

which gives

$$I_o = 0.02\ \text{A}.$$

Therefore, $V = 10\,I + 0.2$, $R = 10\ \Omega$ and $I_o = 0.02\ \text{A}$.

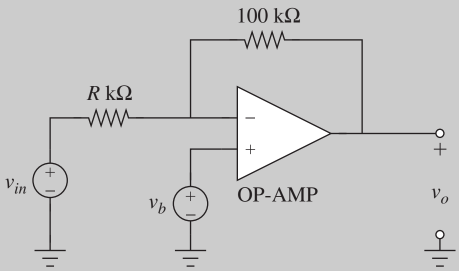
**Example 1-5**

The output voltage $v_o$ of the operational amplifier (OP-AMP) circuit shown in Fig. 1.13 satisfies the relationship $v_o = \left(-\frac{100}{R}\right) v_{in} + \left(1 + \frac{100}{R}\right) v_b$, where $R$ in k$\Omega$ is the unknown resistance and $v_b$ is the unknown voltage. Fig. 1.13 gives the values of the output voltage for two different values of the input voltage.

(a) Determine the value of $R$ and $v_b$.

(b) Plot the output voltage $v_o$ as a function of the input voltage $v_{in}$. On the plot, clearly indicate the value of the output voltage when the input voltage is zero

($y$-intercept) and the value of the input voltage when the output voltage is zero ($x$-intercept).




| $v_{in}$, V | $v_o$, V |
|---|---|
| 5 | 5 |
| 10 | − 5 |

**Figure 1.13** An OP-AMP circuit as a summing amplifier.

**Solution** (a) The input–output relationship $v_o = \left(-\frac{100}{R}\right) v_{in} + \left(1 + \frac{100}{R}\right) v_b$ is the equation of a straight line, $y = mx + b$, where the slope $m = -\frac{100}{R}$ can be found from the data given in Fig. 1.13 as

$$-\frac{100}{R} = \frac{\Delta v_o}{\Delta v_{in}} = \frac{-5-5}{10-5} = \frac{-10}{5} = -2.$$

Solving for $R$ gives $R = 50\ \Omega$. Therefore,

$$\begin{aligned} v_o &= \left(-\frac{100}{50}\right) v_{in} + \left(1 + \frac{100}{50}\right) v_b \\ &= -2\, v_{in} + 3\, v_b. \end{aligned} \tag{1.13}$$

The $y$-intercept $b = 3\, v_b$ can be found by substituting the first data point $(v_{in}, v_o) = (5, 5)$ in equation (1.13) as

$$5 = -2 \times 5 + 3\ v_b.$$

Solving for $v_b$ yields

$$3\ v_b = 5 + 10 = 15,$$

which gives $v_b = 5$ V. Therefore, $v_o = -2\ v_{in} + 15$, $R = 50\ \Omega$, and $v_b = 5$ V. The $x$-intercept can be found by substituting $v_o = 0$ in the equation $v_o = -2\ v_{in} + 15$ and finding the value of $v_{in}$ as

$$0 = -2\ v_{in} + 15,$$

which gives $v_{in} = 7.5$ V. Therefore, the $x$-intercept occurs at $v_{in} = 7.5$ V.

(b) The plot of the output voltage of the OP-AMP as a function of the input voltage if $v_b = 5$ V is shown in Fig. 1.14.

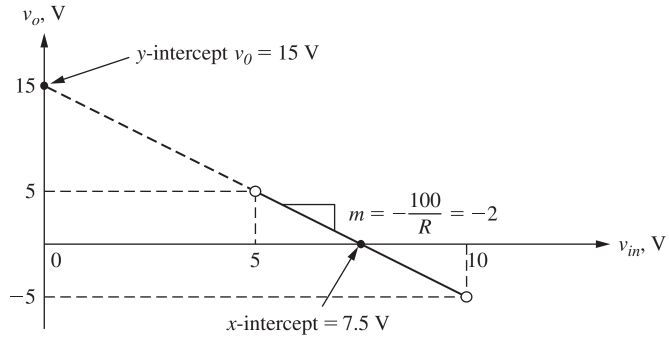



**Figure 1.14** An OP-AMP circuit as a summing amplifier.

**Example 1-6**

An actuator used in a prosthetic arm (Fig. 1.15) can produce a different amount of force by changing the voltage of the power supply. The force and voltage satisfy the linear relation $F = kV$, where $V$ is the voltage applied and $F$ is the force produced by the prosthetic arm. The maximum force the arm can produce is $F = 44.5$ N when supplied with $V = 12$ V.

(a) Find the force produced by the actuator when supplied with $V = 7.3$ V.

(b) What voltage is needed to achieve a force of $F = 6.0$ N?

(c) Using the results of parts (a) and (b), sketch the graph of $F$ as a function of voltage $V$. Use the appropriate scales and clearly label the slope and the results of parts (a) and (b) on your graph.

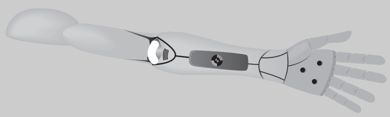


**Figure 1.15** Prosthetic arm.

**Solution** (a) The input–output relationship $F = k\,V$ is the equation of a straight line $y = m\,x$, where the slope $m = k$ can be found from the given data as

$$k = \frac{44.5}{12} = 3.71 \;\; \text{N/V}.$$

Therefore, the equation of the straight line representing the actuator force $F$ as a function of applied voltage $V$ is given by

$$F = 3.71\;V. \qquad (1.14)$$

Thus, the force produced by the actuator when supplied with 7.3 V is found by substituting $V = 7.3$ in equation (1.14) as

$$\begin{aligned} F &= 3.71 \times 7.3 \\ &= 27.08 \text{ N.} \end{aligned}$$

(b) The voltage needed to achieve a force of 6.0 N can be found by substituting $F = 6.0$ N in equation (1.14) as

$$6.0 = 3.71\,V$$
$$V = \frac{6.0}{3.71}$$
$$= 1.62 \text{ V}. \tag{1.15}$$

(c) The plot of force $F$ as a function of voltage $V$ can now be drawn as shown in Fig. 1.16.

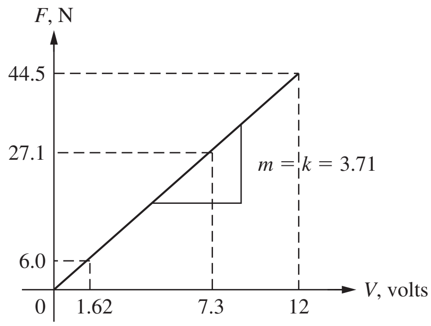
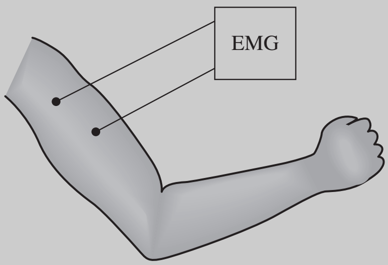



**Figure 1.16** Plot of the actuator force versus the applied voltage.

**Example 1-7**

The electrical activity of muscles can be monitored with an electromyogram (EMG). The following root mean square (RMS) value of the amplitude measurements of the EMG signal were taken when a woman was using her hand grip muscles to ensure a lid was tight on a jar.




| $A$ (V) | $F$ (N) |
|---|---|
| 0.0005 | 110 |
| 0.00125 | 275 |

**Figure 1.17** Amplitude measurements of the EMG signal.

The RMS amplitude of the EMG signal satisfies the linear equation

$$A = mF + b, \tag{1.16}$$

where $A$ is the RMS value of the EMG amplitude in V, $F$ is the applied muscle force in N, and $m$ is the slope.

(a) Determine the values of $m$ and $b$.

(b) Plot the RMS amplitude $A$ as a function of the applied muscle force $F$.

(c) Using the equation of the line from part (a), find the RMS value of the amplitude for a muscle force of 200 N.

**Solution** (a) The input–output relationship $A = mF + b$ is the equation of a straight line $y = mx + b$, where the slope $m$ can be found from the EMG data given in the table (Fig. 1.17) as

$$m = \frac{\Delta A}{\Delta F} = \frac{0.00125 - 0.0005}{275 - 110} = \frac{0.00075}{165} = 4.55 \times 10^{-6}\ \frac{\text{V}}{\text{N}}.$$

The $y$-intercept $b$ can be found by substituting the first data point $(F, A) = (110, 0.0005)$ in equation (1.16) as

$$0.0005 = 4.55 \times 10^{-6}(110) + b.$$

Solving for $b$ yields

$$b = 5 \times 10^{-7} \approx 0.$$

Therefore, the equation of the straight line representing the RMS amplitude as a function of applied force is given by

$$A = 4.55 \times 10^{-6}\ F. \tag{1.17}$$

(b) The plot of the RMS amplitude as a function of the applied muscle force is shown in Fig. 1.18.

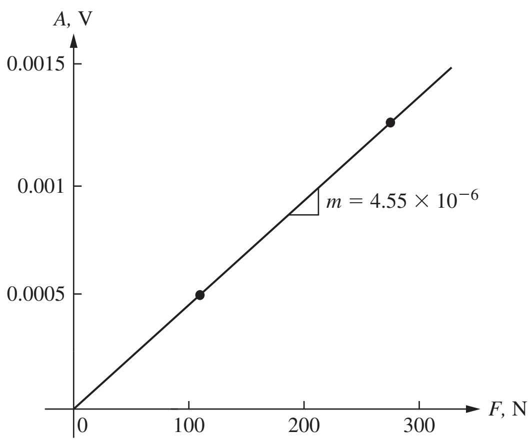


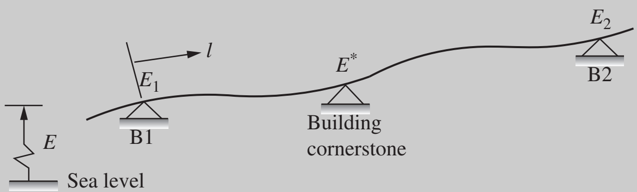

**Figure 1.18** Plot of the RMS amplitude versus the applied muscle force.

(c) The RMS value of the amplitude for a muscle force of 200 N can be found by substituting $F = 200$ N in equation (1.17) as

$$A = 4.55 \times 10^{-6} \times (200) = 0.91 \times 10^{-3} \text{ V}.$$

**Example 1-8**

A civil engineer needs to establish the elevation of the cornerstone for a building located between two benchmarks, B1 and B2, of known elevations as shown in Fig. 1.19.




| | $l_1$ (m) | $E$ (m) |
|---|---|---|
| B1 | 0 | 428.4 |
| B2 | 1001.2 | 476.8 |

**Figure 1.19** Elevations along a uniform grade.

The elevation $E$ along the grade satisfies the linear relationship

$$E = m\,l + E_1, \tag{1.18}$$

where $E_1$ is the elevation of B1, $l$ is the distance from B1 along the grade, and $m$ is the average slope of the grade.

(a) Find the equation of the line $E$ and determine the slope $m$ of the grade.

(b) Using the equation of the line from part (a), find the elevation of the cornerstone $E^*$ if it is located at a distance $l = 565$ m from B1.

(c) Sketch the graph of $E$ as a function of $l$ and clearly indicate both the slope $m$ and elevation $E_1$ of B1.

**Solution** (a) The equation of elevation given by equation (1.18) is a straight line in the slope-intercept form $y = mx + b$, where the slope $m$ can be found from the elevation data given in Fig. 1.19 as

$$m = \frac{\Delta E}{\Delta l} = \frac{476.8 - 428.4}{1001.2 - 0} = \frac{48.4}{1001.2} = 0.0483.$$

The $y$-intercept $E_1$ can be found by substituting the first data point $(l, E) = (0, 428.4)$ in equation (1.18) as

$$428.4 = 0.0483 \times (0) + E_1.$$

Solving for $E_1$ yields

$$E_1 = 428.4 \text{ m}.$$

Therefore, the equation of the straight line representing the elevation as a function of distance $l$ is given by

$$E = 0.0483\,l + 428.4 \text{ m.} \tag{1.19}$$

(b) The elevation $E^*$ of the cornerstone can be found by substituting $l = 565$ m in equation (1.19) as

$$E^* = 0.0483 \times (565) + 428.4 = 455.7 \text{ m.}$$

(c) The plot of the elevation as a function of the length is shown in Fig. 1.20.

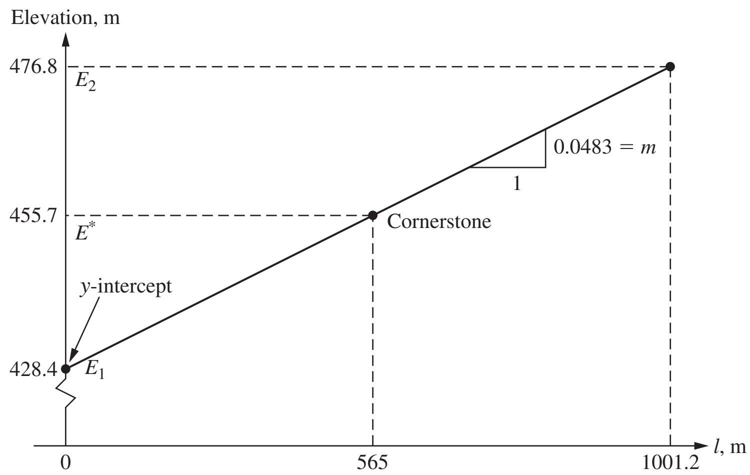
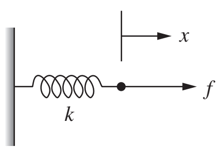



**Figure 1.20** Elevation along a uniform grade.

## PROBLEMS

**1-1.** A constant force $F = 2.5$ N is applied to a spring, and the displacement $x$ is measured as 0.05 m. If the spring force and displacement satisfy the linear relation $F = kx$, find the stiffness $k$ of the spring.



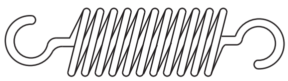

| $F$ (N) | $x$ (m) |
|---|---|
| 2.5 | 0.05 |

**Figure P1.1** Displacement of a spring in problem P1-1.

**1-2.** The spring force $F$ and displacement $x$ for a close-wound tension spring are measured as shown in Fig. P1.2. The spring force $F$ and displacement $x$ satisfy the linear equation $F = kx + F_i$, where $k$ is the spring constant and $F_i$ is the preload induced during manufacturing of the spring.

(a) Using the given data in Fig. P1.2, find the equation of the line for the spring force $F$ as a function of the displacement $x$, and determine the values of the spring constant $k$ and preload $F_i$.

(b) Sketch the graph of $F$ as a function of $x$. Use appropriate axis scales and clearly label the preload $F_i$, the spring constant $k$, and both given data points on your graph.



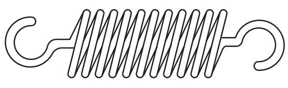
| $F$ (N) | $x$ (cm) |
|---|---|
| 34.5 | 1.5 |
| 57.0 | 3.0 |

**Figure P1.2** Close-wound tension spring for problem P1-2.

**1-3.** The spring force $F$ and displacement $y$ for a close-wound tension spring are measured as shown in Fig. P1.3. The spring force $F$ and displacement $y$ satisfy the linear equation $y = \frac{1}{k}F - \frac{F_i}{k}$, where $k$ is the spring constant and $F_i$ is the preload induced during manufacturing of the spring.

(a) Determine the spring constant $k$ and the pre-load $F_i$ using the given data in Fig. P1.3.

(b) Sketch the graph of the line $y(F)$ and clearly indicate both the spring constant $k$ and preload $F_i$ on the graph.



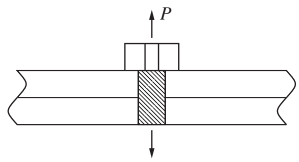
| $F$ (lbf) | $y$ (in.) |
|---|---|
| 100 | 1.0 |
| 75 | 0.5 |

**Figure P1.3** Close-wound tension spring.

**1-4.** In a bolted connection shown in Fig. P1.4, the force in the bolt $F_b$ is given in terms of the external load $P$ as $F_b = C\,P + F_i$.

(a) Given the data in Fig. P1.4, determine the joint constant $C$ and the preload $F_i$.

(b) Plot the bolt force $F_b$ as a function of the load $P$ and label $C$ and $F_i$ on the graph.




| $P$ (lb) | $F_b$ (lb) |
|---|---|
| 100 | 200 |
| 600 | 400 |

**Figure P1.4** Bolted connection.

**1-5.** The number of college courses that Janice has completed toward her engineering degree after her first and third year of college is summarized in the table shown in Fig. P1.5. Assume that the total number of college courses that Janice has completed satisfies the linear relationship $C_{total}(t) = m\,t + C_{HS}$, where $m$ is the number of college courses completed per year and $C_{HS}$ is the number of college courses she completed while still in high school.

(a) Using the data given in Fig. P1.5, find the equation of the line representing the total number of college courses $C_{total}$, and determine both the number of college courses per year $m$ and the number of courses she completed in high school $C_{HS}$.

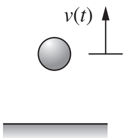
(b) Sketch the graph of the line $C_{total}(t)$, and clearly indicate both the number of college courses per year $m$ and the number of courses she completed while still in high school $C_{HS}$ on the graph.

(c) If it takes 40 total courses for Janice to complete her engineering degree and she keeps taking courses at the same annual rate, how long will it take for Janice to finish her degree?

| $t$ (years) | $C_{total}(t)$ |
|---|---|
| 1 | 10 |
| 3 | 26 |

**Figure P1.5** Number of courses completed.

**1-6.** The velocity $v(t)$ of a ball thrown upward satisfies the equation $v(t) = v_o + at$, where $v_o$ is the initial velocity of the ball in ft/s and $a$ is the acceleration in ft/s$^2$.




| $v(t)$ (ft/s) | $t$ (s) |
|---|---|
| 67.8 | 1.0 |
| 3.4 | 3.0 |

**Figure P1.6** A ball thrown upward with an initial velocity $v_o$.

(a) Given the data in Fig. P1.6, find the equation of the line representing the velocity $v(t)$ of the ball, and determine both the initial velocity $v_o$ and the acceleration $a$.

(b) Sketch the graph of the line $v(t)$, and clearly indicate both the initial velocity and the acceleration on your graph. Also determine the time at which the velocity is zero.

**1-7.** The alternating strength $S_a$ and mean strength $S_m$ for the modified Goodman fatigue criterion are measured for the two load cases as shown in the table given below. The alternating strength $S_a$ and mean strength $S_m$ satisfy the linear equation $S_a = S_e - \frac{S_e}{S_{ut}} S_m$, where $S_e$ is the endurance limit and $S_{ut}$ is the ultimate tensile strength of the material being used.

(a) Determine the endurance limit $S_e$ and the ultimate tensile strength $S_{ut}$ using the given data. Clearly write the equation of the line $S_a(S_m)$.

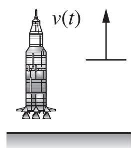
(b) Sketch the graph of the alternating strength versus mean strength and clearly indicate $S_e$ and $S_{ut}$ on the graph.

| $S_m$ (ksi) | $S_a$(ksi) |
|---|---|
| 60 | 10 |
| 20 | 30 |

**1-8.** A model rocket is fired in the vertical plane. The velocity $v(t)$ is measured as shown in Fig. P1.8. The velocity satisfies the equation $v(t) = v_o + a\,t$, where $v_o$ is the initial velocity of the rocket in m/s and $a$ is the acceleration in m/s$^2$.

(a) Given the data in Fig. P1.8, find the equation of the line representing the velocity $v(t)$ of the rocket, and determine both the initial velocity $v_o$ and the acceleration $a$.

(b) Sketch the graph of the line $v(t)$ for $0 \leq t \leq 8$ s, and clearly indicate both the initial velocity and the acceleration on your graph. Also determine the time at which the velocity is zero (i.e., the time required to reach the maximum height).




| $v(t)$ (m/s) | $t$ (s) |
|---|---|
| 34.3 | 0.5 |
| 19.6 | 2.0 |

**Figure P1.8** A model rocket fired in the vertical plane.

**1-9.** The electrical resistivity $\rho_t$ (Ω-nm) at two different temperatures $T$ (°C) of a copper nickel alloy is measured as shown in Fig. P1.9. The resistivity $\rho_t$ and temperature $T$ satisfy the linear equation $\rho_t = \rho_o + \alpha T$, where $\alpha$ and $\rho_o$ are material constants measured in Ω-nm/°C and Ω-nm, respectively.

(a) Given the data in Fig. P1.9, find the equation of the line representing the resistivity $\rho_t$ as a function of temperature $T$, and determine the values of the material constants $\alpha$ and $\rho_o$. Note that "Ω-nm" is pronounced "ohm-nanometers" and you may leave the units as given.

(b) Sketch the graph of the line $\rho_t$ as a function of $T$, and clearly indicate $\alpha$ and $\rho_o$ on your graph.

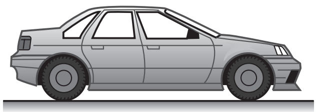
| $T$ (°C) | $\rho_t$ (Ω-$nm$) |
|---|---|
| 50.0 | 45 |
| −100 | 35 |

**Figure P1.9** Electrical resistance of copper nickel alloy.

**1-10.** The velocity of a vehicle is measured at two distinct points in time as shown in Fig. P1.10. The velocity satisfies the relationship $v(t) = v_o + at$, where $v_o$ is the initial velocity in m/s and $a$ is the acceleration in m/s$^2$.

(a) Find the equation of the line $v(t)$, and determine both the initial velocity $v_o$ and the acceleration $a$.

(b) Sketch the graph of the line $v(t)$, and clearly label the initial velocity, the acceleration, and the total stopping time on the graph.



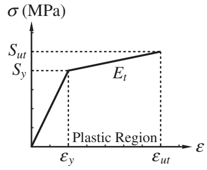
| $v(t)$ (m/s) | $t$ (s) |
|---|---|
| 30 | 1.0 |
| 10 | 2.0 |

**Figure P1.10** Velocity of a vehicle during braking in problem P1-10.

**1-11.** The behavior of a material that exhibits bilinear kinematic hardening can be approximated with a linear relationship in the plastic region, as shown in Fig. P1.11. The stress, $\sigma$ (MPa) and strain, $\varepsilon$ (unitless) satisfy the linear equation $\sigma(\varepsilon) = E_t\,\varepsilon + \sigma_o$, where $E_t$ is the strain hardening or tangent modulus and $\varepsilon_o$ is the nominal stress, both measured in MPa.

(a) Using the data from the tensile test given in the table, find the equation of the line for the measured stress $\sigma$ as a function of the strain $\varepsilon$, and determine the values of the tangent modulus $E_t$ and nominal stress $\sigma_o$.

(b) Sketch the graph of the line $\sigma$ as a function of $\varepsilon$, and clearly indicate both the tangent modulus $E_t$ and the nominal stress $\sigma_o$ on your graph.




| Stress $\sigma$ (MPa) | Strain $\varepsilon$ |
|---|---|
| 370 | 0.01 |
| 440 | 0.15 |

**Figure P1.11** Stress–strain relation in the plastic region.

**1-12.** The velocity $v(t)$ of a vehicle during braking is given in Fig. P1.12. Determine the equation for $v(t)$ for

(a) $0 \le t \le 2$ s

(b) $2 \le t \le 4$ s

(c) $4 \le t \le 6$ s

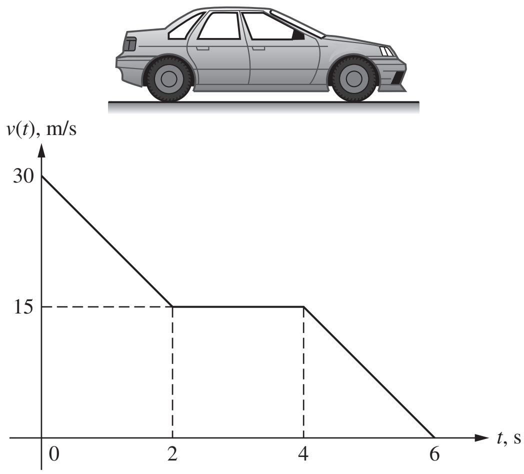



**Figure P1.12** Velocity of a vehicle during braking in problem P1-12.

**1-13.** A linear trajectory is planned for a robot to pick up a part in a manufacturing process. The velocity of the trajectory of one of the joints is shown in Fig. P1.13. Determine the equation of $v(t)$ for

(a) $0 \le t \le 1$ s

(b) $1 \le t \le 3$ s

(c) $3 \le t \le 4$ s

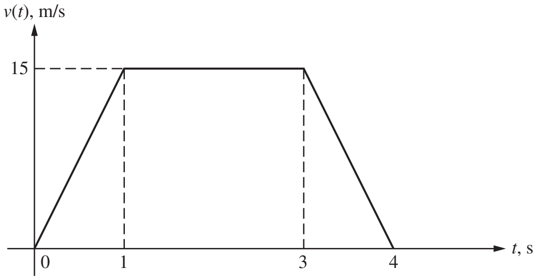



**Figure P1.13** Velocity of a robot trajectory.

**1-14.** The acceleration of the linear trajectory of problem P1-13 is shown in Fig. P1.14. Determine the equation of $a(t)$ for

(a) $0 \le t \le 1$ s
(b) $1 \le t \le 3$ s
(c) $3 \le t \le 4$ s

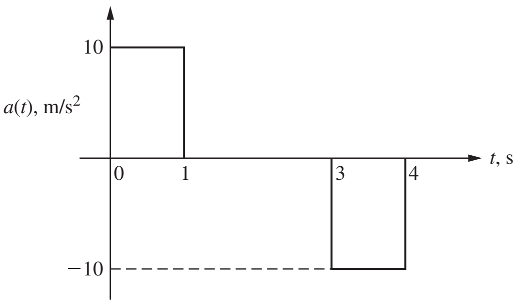
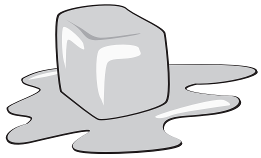



**Figure P1.14** Acceleration of the robot trajectory.

**1-15.** The relationship between the Celsius (°C) and Fahrenheit (°F) temperature scales is a linear equation with a slope and $y$-intercept. To obtain a formula for conversion, the freezing and boiling points of water for each scale are used, as shown in the table in Fig. P1.15. Using the data in the table, the resulting conversion from $T_F$ to $T_C$ satisfies the linear equation $T_C = k\,T_F + T_O$, where $k$ is the slope and $T_O$ is the temperature offset in (°C).



| $T_C$ (°C) | $T_F$ (°F) |
|---|---|
| 100 | 212 |
| 0 | 32 |

**Figure P1.15** Temperature relationship between the Celsius (°C) and Fahrenheit (°F).

(a) Determine the slope $k$ and $y$-intercept $T_O$ and write the equation of the line for $T_C$ as a function of $T_F$.
(b) Sketch the equation of the line for $T_C$ and clearly indicate $k$ and $T_O$ on the graph.

**1-16.** The temperature distribution in a well-insulated axial rod varies linearly with respect to distance when the temperature at both ends is held constant as shown in Fig. P1.16. The temperature satisfies the equation of a line $T(x) = C_1\,x + C_2$, where $C_1$ and $C_2$ are constants of integration with units of °C/m and °C, respectively.

(a) Find the equation of the line $T(x)$, and determine both constants $C_1$ and $C_2$.
(b) Sketch the graph of the line $T(x)$ for $0 \le x \le 0.5$ m, and clearly label $C_1$ and $C_2$ on your graph. Also, clearly indicate the temperature at the center of the rod ($x = 0.25$ m).


| $T(x)$ (°C) | $x$ (m) |
|---|---|
| 0 | 0.0 |
| 20 | 0.5 |

**Figure P1.16** Temperature distribution in a well-insulated axial rod in problem P1-16.

**1-17.** The voltage–current relationship for the circuit shown in Fig. P1.17 is given by Ohm's law as $V = I\,R$, where $V$ is the applied voltage in volts, $I$ is the current in amps, and $R$ is the resistance of the resistor in ohms.

(a) Sketch the graph of $I$ as a function of $V$ if the resistance is 8 Ω.

(b) Find the current $I$ if the applied voltage is 12 V.

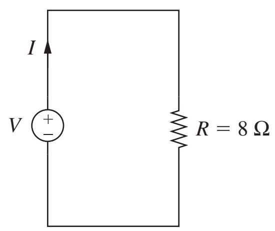
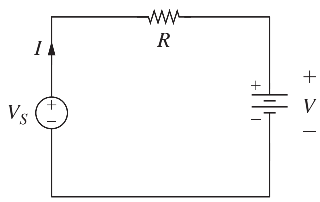



**Figure P1.17** Resistive circuit for problem P1-17.

**1-18.** A voltage source $V_s$ is used to apply two different voltages (12 V and 18 V) to the single-loop circuit shown in Fig. P1.18. The values of the measured current are shown in Fig. P1.18. The voltage and current satisfy the linear relation $V_s = IR + V$, where $R$ is the resistance in ohms, $I$ is the current in amps, and $V_s$ is the voltage in volts.

(a) Using the data given in Fig. P1.18, find the equation of the line for $V_s$ as a function of $I$, and determine the values of $R$ and $V$.

(b) Sketch the graph of $V_s$ as a function of $I$ and clearly indicate the resistance $R$ and voltage $V$ on the graph.



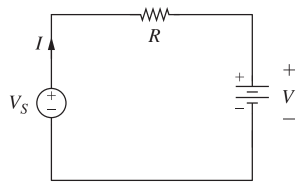

| $V_s$ (V) | $I$ (A) |
|---|---|
| 12.0 | 0.75 |
| 18.0 | 1.5 |

**Figure P1.18** Single-loop circuit for problem P1-18.

**1-19.** The voltage source $V_s$ and current $I$ for a single-loop circuit are measured as shown in Fig. P1.19. The voltage source $V_S$ and current $I$ satisfy the linear equation $I = \frac{1}{R}V_S - \frac{V}{R}$, where $R$ is the resistance in ohms and $V$ is an unknown voltage in volts.

(a) Using the data given in Fig. P1.19, find the equation of the line for the current $I$ as a function of voltage $V_S$, and determine the values of the resistance $R$ and unknown voltage $V$.

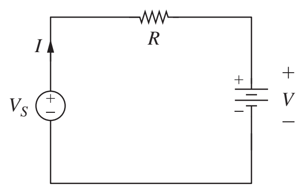
(b) Sketch the graph of $I$ as a function of $V_S$ and clearly indicate the resistance $R$ on the graph.




| $V_S$ (V) | $I$ (A) |
|---|---|
| 12 | 0.05 |
| 24 | 0.20 |

**Figure P1.19** Single-loop circuit for problem P1-19.

**1-20.** Repeat problem P1-18 for the data shown in Fig. P1.20.



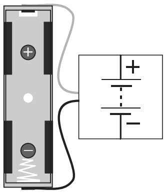

| $V_s$ (V) | $I$ (A) |
|---|---|
| 24 | 1.5 |
| 32 | 2.5 |

**Figure P1.20** Single-loop circuit for problem P1-20.

**1-21.** A fuel cell's thermodynamic voltage change $\Delta V$ measured in millivolts is linearly dependent on the operating temperature $T$ measured in °C. This relationship follows the equation $\Delta V = \Delta V_o + \frac{S}{nF}T$, where $S$ is the entropy of the system, $\Delta V_o$ is the initial voltage difference, and $nF$ is a constant.

(a) If the quantity $nF = 193$ (J/mV), determine both the initial voltage difference $\Delta V_o$ and the entropy $S$ and write the equation of the line $\Delta V(T)$.

(b) Sketch the graph of $\Delta V$ as a function of $T$, and clearly indicate the slope and $y$-intercept on your graph.



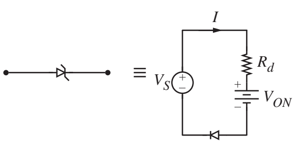
| $T$(°C) | $\Delta V$ (mV) |
|---|---|
| 25 | 49.0 |
| 80 | −28.0 |

**Figure P1.21** Thermodynamics of a fuel cell.

**1-22.** A linear model of a diode is shown in Fig. P1.22, where $R_d$ is the forward resistance of the diode and $V_{ON}$ is the voltage that turns the diode ON. To determine the resistance $R_d$ and voltage $V_{ON}$, two voltage values are applied to the diode and the corresponding currents are measured. The applied voltage $V_S$ and the measured current $I$ are given in Fig. P1.22. The applied voltage and the measured current satisfy the linear equation $V_s = I\,R_d + V_{ON}$.

(a) Find the equation of the line for $V_s$ as a function of $I$ and determine the resistance $R_d$ and the voltage $V_{ON}$.

(b) Sketch the graph of $V_S$ as a function of $I$, and clearly indicate the resistance $R_d$ and the voltage $V_{ON}$ on the graph.



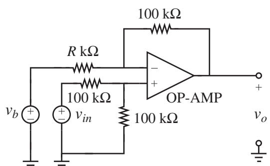

| $V_s$ (V) | $I$ (A) |
|---|---|
| 2.0 | 0.035 |
| 6 | 0.135 |

**Figure P1.22** Linear model of a diode for problem P1-22.

**1-23.** The output voltage, $v_o$, of the OP-AMP circuit shown in Fig. P1.23 satisfies the relationship $v_o = \left(1 + \frac{100}{R}\right)\left(\frac{v_{in}}{2}\right) - \left(\frac{100}{R}\right) v_b$, where $R$ is the unknown resistance in kΩ and $v_b$ is the unknown voltage in volts. Fig. P1.23 gives the values of the output voltage for two different values of the input voltage.

(a) Determine the equation of the line for $v_o$ as a function of $v_{in}$ and find the values of $R$ and $v_b$.

(b) Plot the output voltage $v_o$ as a function of the input voltage $v_{in}$. On the plot, clearly indicate the value of the output voltage when the input voltage is zero ($y$-intercept) and the value of the input voltage when the output voltage is zero ($x$-intercept).



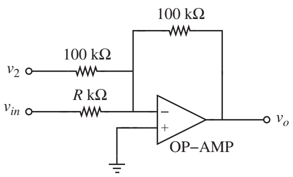

| $v_{in}$ (V) | $v_o$ (V) |
|---|---|
| 5 | 7 |
| 10 | 22 |

**Figure P1.23** An OP-AMP circuit as a summing amplifier for problem P1.23.

**1-24.** The output voltage, $v_o$, of the OP-AMP circuit shown in Fig. P1.24 satisfies the relationship $v_o = -\left(v_2 + \frac{100}{R} v_{in}\right)$, where $R$ is the unknown resistance in kΩ, $v_{in}$ is the input voltage, and $v_2$ is the unknown voltage. Fig. P1.24 gives the values of the output voltage for two different values of the input voltage $v_{in}$.

(a) Find the equation of the line for $v_o$ as a function of $v_{in}$ and determine the values of $R$ and $v_2$.

(b) Plot the output voltage $v_o$ as a function of the input voltage $v_{in}$. Clearly indicate the value of the output voltage when the input voltage is zero ($y$-intercept) and the value of the input voltage when the output voltage is zero ($x$-intercept).




| $v_{in}$ (V) | $v_o$ (V) |
|---|---|
| 5 | −4.5 |
| 10 | −7.0 |

**Figure P1.24** An OP-AMP circuit for problem P1-24.

**1-25.** A manually operated controlled descent device utilizes a cam assembly to apply friction on a rope to control the speed of descent. The user must actively apply pressure to the handle to allow the rope to slide. No pressure at all locks the device and stops the descent. The interference $I$, measured in inches and controlled by the grip pressure, is linearly related to the descent velocity $v$ as $v = kI + v_f$, where $k$ is a constant of proportionality between how fast an object falls and the engagement of the device and $v_f$ is the velocity of rapid descent.

(a) Using the given data points given in Fig. P1.25 , determine both the rapid descent velocity $v_f$ and the constant $k$ and explicitly write the equation of the line $v(I)$.

(b) Sketch the graph of $v$ as a function of $I$, and clearly indicate the slope and $y$-intercept on your graph.

(c) Determine how much interference is needed to completely stop descent and label this point on your graph in part (b).

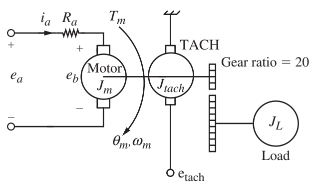
| $v$ (ft/s) | $I$ (in.) |
|---|---|
| 2.5 | 3/16 |
| 7.5 | 1/16 |

**Figure P1.25** Manually operated controlled descent device for problem P1-25.

**1-26.** A DC motor is driving an inertial load $J_L$ shown in Fig. P1.26. To maintain a constant speed, two different values of the voltage $e_a$ are applied to the motor. The voltage $e_a$ and the current $i_a$ flowing through the armature winding of the motor satisfy the relationship $e_a = i_a R_a + e_b$, where $R_a$ is the resistance of the armature winding in ohms and $e_b$ is the back-emf in volts. Figure P1.26 gives the values of the current for two different values of the input voltage applied to the armature of the DC motor.

(a) Find the equation of the line for $e_a$ as a function of $i_a$ and determine the values of $R_a$ and $e_b$.

(b) Plot the applied voltage $e_a$ as a function of the current $i_a$. Clearly indicate the value of the back-emf $e_b$ and the winding resistance $R_a$.




| $i_a$ (A) | $e_a$ (V) |
|---|---|
| 1.0 | 5.0 |
| 2.25 | 10.0 |

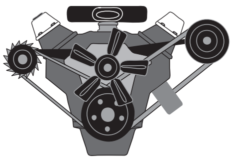
**Figure P1.26** Voltage–current data of a DC motor in problem P1-26.

**1-27.** The accelerator of a vehicle controls the rpm (rev/min) of an engine by adjusting how much air enters into the intake through a throttle cable. The engine rpm behaves linearly as a function of the position of the accelerator controlled by the driver as $N(\phi) = N_o + S\,\phi$, where $N(\phi)$ is the engine rpm, $\phi$ is the position of the accelerator measured in degrees, and $S$ is the sensitivity of the accelerator.

(a) Using the data points given in Fig. P1.27, determine both the sensitivity $S$ and idling rpm $N_o$, and explicitly write the equation of the line $N(\phi)$.

(b) Sketch the graph of $N$ as a function of $\phi$, and clearly indicate the slope and $y$-intercept on your graph.

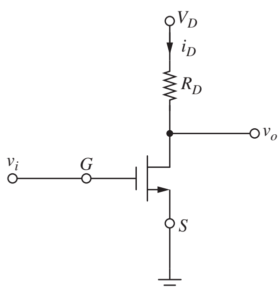
(c) If the maximum rpm of the engine is $N_{max} = 2000$ rpm, determine the maximum position of the accelerator, $\phi_{max}$.



| $\phi$ (degrees) | $N$ (rpm) |
|---|---|
| 10 | 900 |
| 30 | 1300 |

**Figure P1.27** Vehicle engine rpm data for problem P1-27.

**1-28.** In the active region, the output voltage $v_o$ of the n-channel enhancement-type MOSFET (NMOS) circuit shown in Fig. P1.28 satisfies the relationship $v_o = V_D - R_D\, i_D$, where $R_D$ is the unknown drain resistance and $V_D$ is the unknown drain voltage. Figure P1.28 gives the values of the output voltage for two different values of the drain current. Plot the output voltage $v_o$ as a function of the input drain current $i_D$. On the plot, clearly indicate the values of $R_D$ and $V_D$.




| $v_o$ (V) | $i_D$ (mA) |
|---|---|
| 0 | 10 |
| 5 | 5 |

**Figure P1.28** NMOS for problem P1-28.

**1-29.** An actuator used in a prosthetic arm can produce different amounts of force by changing the voltage of the power supply. The force and voltage satisfy the linear relation $F = kV$, where $V$ is the voltage applied and $F$ is the force produced by the prosthetic arm. The maximum force the arm can produce is 50.0 N when supplied with 10 V.

(a) Find the force produced by the actuator when supplied with 6.0 V.

(b) What voltage is needed to achieve a force of 5.0 N?

(c) Using the results of parts (a) and (b), sketch the graph of $F$ as a function of voltage $V$. Use the appropriate scales and clearly label the slope and the results of parts (a) and (b).

**1-30.** The following two measurements of maximum heart rate $R$ (in beats per minute, bpm) were recorded in an exercise physiology laboratory.

| $R$ (bpm) | $A$ (years) |
|---|---|
| 183 | 30 |
| 169.5 | 45 |

**Figure P1.30** Maximum heart rate recorded in an exercise physiology laboratory.

The maximum heart rate $R$ and age $A$ satisfy the linear equation

$$R = mA + B,$$

where $R$ is the heart rate in beats per minute and $A$ is the age in years.

(a) Using the data provided in Fig. P1.30, find the equation of the line for $R$.

(b) Sketch $R$ as a function of $A$.

(c) Using the relationship developed in part (a), find the maximum heart rate of a 60-year-old person.

**1-31.** The electrical activity of muscles can be monitored with an electromyogram (EMG). The RMS amplitude measurements of the EMG signal when a person is using the hand grip muscle to tighten the lid on a jar is given in the table below:

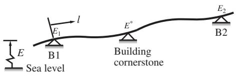
| $A$ (V) | $F$ (N) |
|---|---|
| 0.5 E–3 | 100 |
| 1.25 E–3 | 250 |

The RMS amplitude of the EMG signal satisfies the linear equation $A = mF + B$, where $A$ is the RMS amplitude in volts, $F$ is the applied muscle force in N, and $m$ is the slope of the line.

(a) Using the data provided in the table, find the equation of the line for $A(F)$.

(b) Sketch $A$ as a function of $F$.
(c) Using the relationship developed in part (a), find the RMS amplitude for a muscle force of 200 N.

**1-32.** A civil engineer needs to establish the elevation of the cornerstone for a building located between two benchmarks, B1 and B2 of known elevations, as shown in Fig. P1.32.




| | $l_1$ (m) | $E$ (m) |
|---|---|---|
| B1 | 0 | 500 |
| B2 | 500 | 600 |

**Figure P1.32** Elevations along a uniform grade for problem P1-32.

The elevation $E$ along the grade satisfies the linear relationship

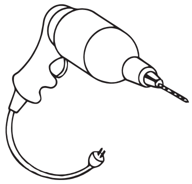
$$E = m\,l + E_1, \qquad (1.20)$$

where $E_1$ is the elevation of B1, $l$ is the distance from B1 along the grade, and $m$ is the rate of change of $E$ with respect to $l$.

(a) Find the equation of the line $E$ and determine the slope $m$ of the linear relationship.
(b) Using the equation of the line from part (a), find the elevation of the cornerstone $E^*$ if it is located at a distance $l = 300$ m from B1.
(c) Sketch the graph of $E$ as a function of $l$ and clearly indicate both the slope $m$ and elevation $E_1$ of B1.

**1-33.** During machining of some polycrystalline metals, the shear strength $\tau_s$ increases linearly with the normal stress $\sigma_s$ applied to the shear plane, as tabulated in Fig. P1.33. The shear strength $\tau_s$ and applied normal stress $\sigma_s$ satisfy the linear equation $\tau_s = \tau_{so} + k\,\sigma_s$, where $k$ is a material property and $\tau_{so}$ is the shear strength of the uncut material.

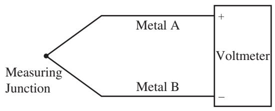
(a) Using the data given in Fig. P1.33, determine the material property $k$ and the uncut material shear strength $\tau_{so}$ using the given data for Aluminum 6061. Clearly write the equation of the line $\tau_s(\sigma_s)$.
(b) Sketch the graph of the line $\tau_s(\sigma_s)$, and clearly indicate $k$ and $\tau_{so}$ on the graph.



| $\tau_s$ (MPa) | $\sigma_s$ (MPa) |
|---|---|
| 277 | 280 |
| 282 | 300 |

**Figure P1.33** Normal and shearing stress during machining.

**1-34.** The voltage across a thermocouple is calibrated using the boiling point of water (373 K) and the freezing point of silver (1235 K), as shown in Fig. P1.34.



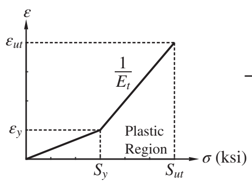

| $T$ (K) | V (mV) |
|---|---|
| 373 | 10.0 |
| 1235 | 70.0 |

**Figure P1.34** Thermocouple to measure temperature in Kelvin.

The junction temperature $T$ and the voltage across the thermocouple $V$ satisfy the linear equation $T = \frac{1}{\alpha}\,V + T_R$, where $\alpha$ is the thermocouple sensitivity in mV/K and $T_R$ is the reference temperature in K.

(a) Using the calibration data given in Fig. P1.34, find the equation of the line for the measured temperature $T$ as a function of the voltage $V$ and determine the value of the sensitivity $\alpha$ and the reference temperature $T_R$.
(b) Sketch the graph of $T$ as a function of $V$ and clearly indicate both the reference temperature $T_R$ and the sensitivity $\alpha$ on the graph.

**1-35.** The behavior of a material that exhibits bilinear kinematic hardening can be approximated with a linear relationship in the plastic region. For a force-controlled experiment, the data in the plastic region is given in the table below Fig. P1.35



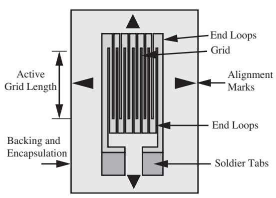

| Stress, $\sigma$ (ksi) | Strain |
|---|---|
| 25.5 | 0.2 |
| 18.23 | 0.1 |

**Figure P1.35** Stress–strain relationship of a material.

The stress $\sigma$ (ksi) and strain, $\varepsilon$ (dimensionless) satisfy the linear equation $\varepsilon(\sigma) = \frac{1}{E_t}\sigma - \frac{\sigma_o}{E_t}$, where $E_t$ is the strain hardening or tangent modulus and $\sigma_o$ is the nominal stress, both measured in ksi.

(a) Determine the slope and $y$-intercept of $\varepsilon(\sigma)$ and write the equation of the line for the measured strain $\varepsilon$ as a function of the stress $\sigma$.
(b) Determine the values of the tangent modulus $E_t$ and nominal stress $\sigma_o$.
(c) Sketch the graph of the line $\varepsilon(\sigma)$, and clearly indicate $E_t$ and $\sigma_o$ on the graph.

**1-36.** Strain is a measure of the deformation of an object. It can be measured using a foil strain gauge shown in Fig. P1.36.




| $R$ ($\Omega$) | $\varepsilon$ (m/m) |
|---|---|
| 100 | 0 |
| 102 | 0.01 |

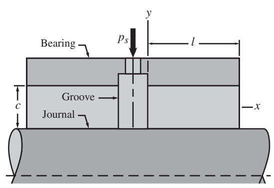
**Figure P1.36** Foil strain gauge to measure strain in problem P1-36.

The strain being measured ($\varepsilon$) and resistance of the sensor satisfy the linear equation $R = R_o + R_o S_\varepsilon \varepsilon$, where $R_o$ is the initial resistance (measured in ohms, $\Omega$) of the sensor with no strain, and $S_\varepsilon$ is the gauge factor (a multiplier with NO units).

(a) Using the given data, find the equation of the line for the sensor's resistance $R$ as a function of the strain $\varepsilon$, and determine the values of the gauge factor $S_\varepsilon$ and initial resistance $R_o$.
(b) Sketch the graph of $R$ as a function of $\varepsilon$, and clearly indicate $R_o$ on your graph.

**1-37.** In a pressure-fed journal bearing, forced cooling is provided by a pressurized lubricant flowing along the axial direction of the shaft (the $x$-direction). The lubricant pressure $p(x)$ satisfies the linear equation $p(x) = -\frac{p_s}{l}x + p_s$, where $p_s$ is the supply pressure and $l$ is the length of the bearing.

(a) Using the data given in the table, find the equation of the line for the lubricant pressure $p(x)$ and determine the values of the supply pressure $p_s$ and the bearing length $l$.
(b) Sketch the graph of the lubricant pressure $p(x)$, and clearly indicate both the supply pressure $p_s$ and the bearing length $l$ on your graph.




| $p(x)$ (psi) | $x$ (in) |
|---|---|
| 30 | 1.0 |
| 10 | 3.0 |

**Figure P1.37** Pressure-fed journal bearing.

**1-38.** To determine the concentration of a purified protein sample, a graduate student used spectrophotometry to measure the absorbance given in the table below:

| $c$ ($\mu$g/ml) | $a$ |
|---|---|
| 3.50 | 0.342 |
| 8.00 | 0.578 |

**Figure P1.38** Concentration of a purified protein sample.

The concentration–absorbance relationship for this protein satisfies a linear equation $a = m\,c + a_i$, where $c$ is the concentration of a purified protein, $a$ is the absorbance of the sample, $m$ is the rate of change of absorbance $a$ with respect to concentration $c$, and $a_i$ is the $y$-intercept.

(a) Find the equation of the line that describes the concentration–absorbance relationship for this protein and determine the slope $m$ of the linear relationship.

(b) Using the equation of the line from part (a), find the concentration of the sample if this sample had an absorbance of 0.486.

(c) If the sample is diluted to a concentration of 0.00419 $\mu$g/ml, what would you expect the absorbance to be? Would this value be accurate?

(d) Sketch the graph of absorbance $a$ as a function of concentration $c$ and clearly indicate both the slope $m$ and the $y$-intercept.

**1-39.** A thermostat control with dial marking from 0 to 100 is used to regulate the temperature of an oil bath. To calibrate the thermostat, the data for the temperature $T$ (°F) versus the dial setting $R$ was obtained as shown in the table below:

| $T$ (°F) | $R$ |
|---|---|
| 110.0 | 20.0 |
| 40.0 | 40.0 |

**Figure P1.39** Calibration of a thermostat.

The relationship between the temperature $T$ in Fahrenheit and the dial setting $R$ satisfies the linear equation $T(°\text{F}) = a\,R + b$.

(a) Using the given data, find the equation of the line relating the temperature to the dial setting.

(b) Sketch the graph of $T$(°F) as a function of $R$, and clearly indicate $a$ and $b$ on your graph.

(c) Calculate the thermostat setting needed to obtain a temperature of 320°F.

**1-40.** A chemistry student is performing an experiment to determine the temperature-volume behavior of a gas mixture at constant pressure and quantity. Due to technical difficulties, the student could only obtain values at two temperatures as shown in the table below:

| $T$ (°C) | $V$ (L) |
|---|---|
| 50 | 1.08 |
| 98 | 1.24 |

**Figure P1.40** Temperature–volume behavior of a gas mixture at constant pressure.

The student knows that the gas volume linearly depends on temperature, that is, $V(T) = m\,T + K$, where $V$ is the volume in L, $T$ is the temperature in °C, $K$ is the $y$-intercept in L, and $m$ is the slope of the line in L/°C.

(a) Find the equation of the line that describes the temperature–volume relationship of the gas mixture and determine the slope $m$ of the linear relationship.

(b) Using the equation of the line from part (a), find the temperature of the gas mixture if the volume is 1.15 L.

(c) Using the equation of the line from part (a), find the volume of the gas if the temperature is 70°C.

(d) Sketch the graph of the volume–temperature relationship for the gas mixture from −300 °C to 100 °C and clearly indicate both the slope $m$ and the $y$-intercept. What is the significance of the temperature when $V = 0$ L?