

CHAPTER 1

INTRODUCTION

The principle of reciprocity is, in its general sense, interpreted as a norm defining the response to the mutual interaction between two entities involved. Within the realm of electromagnetic (EM) theory, the interacting entities are EM field states, and their mutual interaction is prescribed by Lorentz's reciprocity theorem [1, 2]. The theorem has been recognized as a truly universal relation providing a rigorous basis for approaching both direct and inverse problems of wave field physics [3, 4].

Apart from a limited number of EM field problems that can be solved exactly in terms of analytic functions, the vast majority of EM scattering and radiation problems met in practice must be handled approximately by means of analytical approximations or/and numerical techniques. As the EM reciprocity theorem encompasses all “weak” formulations of the EM (differential) field equations, it offers a convenient venue for developing computational schemes [5]. This strategy has been successfully followed in constructing a general finite-element formulation [6] and dedicated time domain (TD) contour-integral strategies for analyzing planar structures [7], for instance.

A sophisticated analytical method for tackling a large class of problems in wave field physics directly in space-time is known as the Cagniard-DeHoop (CdH) method [8]. It has been shown that the CdH method yields the exact solutions to a large class of canonical TD wave field problems in electromagnetics [9], acoustics [10], and elastodynamics [11]. Since the CdH method is also capable of providing useful large-argument asymptotic solutions [12], as well as insightful closed-form solutions based on the (modified) Kirchhoff approximation [13], a natural question arises whether the CdH approach could also be applicable to constructing a reciprocity-based TD integral-equation technique. Introducing such a novel numerical scheme, hereinafter referred to as the Cagniard-DeHoop method of moments (CdH-MoM), is an essential objective of the present book.

An efficient computational approach for analyzing planarly layered structures in the real frequency domain (FD) is well known as the integral equation technique

[14]. Its high computational efficiency stems from the reduction of the solution space to conductive surfaces, which is achieved by including the effect of layering in the pertaining Green's functions constructed traditionally via Sommerfeld's formulation [15, chapter VI]. Although the theory of TD EM field propagation in planarly stratified media is available via the CdH methodology [16], it has never been incorporated into the existing TD integral-equation schemes relying heavily upon the simple form of a Green's function associated with an unbounded, homogeneous, and isotropic medium. Since the CdH-MoM lends itself to the inclusion of mutually parallel layers, it may provide a suitable means to fill the void in computational electromagnetics.

1.1 SYNOPSIS

In the present book, we explore modeling methodologies for analyzing TD EM wave fields associated with fundamental antenna topologies. A common feature of all the methods and solution strategies employed is the use of the EM reciprocity theorem of the time-convolution type as the point of departure.

In chapter 2, the reciprocity theorem is applied to formulate a direct EM scattering problem regarding EM scattering and radiation from a thin-wire antenna. The result is a complex-FD reciprocity relation, the enforcement of the equality in which yields a "weak" solution of the scattering problem. In order to achieve the solution in the (original) TD, we adopt here the strategy behind the CdH method [8]. Namely, the reciprocity relation in the complex-FD is represented in terms of slowness-domain integrals that are subsequently handled analytically with the aid of standard tools of complex analysis. The employed slowness representation is briefly introduced in appendix A, and its illustrative application to the analysis of the electric-current (space-time) distribution along an infinitely-long, gap-excited antenna is given in appendix B. Via the representation, it is shown that a piecewise linear space-time distribution of the induced electric current along a wire antenna can be calculated upon solving a system of equations, whose coefficients are, for thin wire antennas, expressible via elementary functions only. This is demonstrated in appendix C, where the slowness-domain representation of (the elements of) the impedance matrix is transformed back to TD via the CdH method. Chapter 2 is further supplemented with appendix K, where a demo MATLAB[®] implementation of the introduced solution procedure is provided, including both the antenna excitation via a voltage gap source and a uniform EM plane wave.

The analytical handling of the EM reciprocity relation with the aid of the CdH method is also applied in the subsequent chapter 3, where the pulsed EM coupling between parallel thin-wire antennas is analyzed. This chapter is again supplemented with appendix D, where the elements of the pertaining TD mutual-impedance matrix are described in detail. It is shown that, in contrast to the self-impedance matrix elements described in appendix C, the filling of the mutual-impedance matrix calls, in general, for a numerical calculation of double integrals.

In chapter 4, we propose a straightforward methodology for incorporating ohmic losses of the analyzed thin-wire antenna. It is shown that the effect of losses can be included in by modifying the boundary condition on the cylindrical surface of a wire antenna. This way facilitates the incorporation of losses in a separate impedance matrix, which lends itself to modular programming. This chapter is further supplemented with appendix E, where the internal impedance of a solid EM-penetrable wire is derived with the aid of the gap-excited, thin-wire antenna model and the wave-slowness representation from appendix A.

The EM radiation and scattering characteristic of a wire antenna can be effectively tailored via lumped elements connected to its ports. Therefore, in chapter 5, it is shown how a linear lumped element can be incorporated in the thin-wire CdH-MoM formulation. Again, the presence of a lumped element is captured in an isolated impedance matrix, which makes it possible to evaluate its impact without the need for repeating all the calculations over again. This feature may be profitable especially for optimization routines that require an efficient algorithm for evaluating the objective function.

The numerical procedure introduced in chapter 2 yields the expansion coefficients describing the space-time electric-current distribution along a thin-wire antenna. The thus obtained electric-current distribution may subsequently serve as the input for calculating both the EM radiation and scattering characteristics of the antenna. This is exactly the main objective of chapter 6, where the EM fields radiated from a thin, straight wire segment are expressed in terms of the expansion electric-current coefficients.

In chapter 7, we provide an illustrative numerical example that largely serves for validation of the methodologies described in the previous chapters. Namely, a special form of the (self)-reciprocity antenna relation is first applied to calculate the pulsed EM radiation characteristics from both the antenna self-response, when operating in transmission, and the plane-wave induced response in the relevant receiving situation. Subsequently, the response obtained in the reciprocity-based way is compared with the far-field radiated amplitude computed directly from the electric-current distribution according to chapter 6.

The impact of a wire scatterer on the self-response of a thin-wire transmitting antenna is analyzed in chapter 8. To that end, we make use of a TD reciprocity relation to express the change of the electric-current response of a voltage-gap excited thin-wire antenna using the induced electric-current distribution along the scatterer. The reciprocity-based result is, again, validated directly, via the difference of the electric-current responses calculated in the presence and the absence of the wire scatterer. For the latter, the calculations heavily rely on the methodology introduced in chapter 3.

In chapter 9, we shall analyze the change of EM scattering from a thin-wire antenna due to the change in its localized load. It is first demonstrated that the change of EM scattering characteristics can be expressed using the corresponding EM-radiated field amplitude and the change of the voltage induced across the (varying) antenna lumped load. The result obtained via the reciprocity-based

approach is, again, validated directly by evaluating the relevant difference using the methodology specified in section 6.3.

In chapter 10, the EM reciprocity theorem of the time-convolution type is used to evaluate the impact of a wire scatterer on a thin wire receiving antenna. Namely, it is demonstrated that the change of the equivalent Norton electric current can be, in a similar manner as the current response analyzed in chapter 8, related to the electric-current distribution induced along the scatterer. The direct evaluation of (the difference of) the short-circuit electric-current responses validates the TD reciprocity relation.

The following chapter 11 starts by demonstrating that the gap-excited cylindrical antenna (see appendix B) located above the perfectly conducting ground can approximately be handled via transmission-line theory. Under this approximation, the EM reciprocity theorem of the time-convolution type is applied to derive an EM-field-to-line coupling model interrelating the terminal voltage and current quantities with the weighted distribution of the excitation EM wave fields along the transmission line. It is next shown that the (integral) reciprocity-based coupling model can be understood as a generalization of the classic (differential) EM-field-to-line coupling models. Finally, via the EM reciprocity theorem, again, alternative coupling models are introduced, concerning both the EM plane-wave incidence and a prescribed EM volume-source distribution.

In order to provide the reader with straightforward applications of the EM reciprocity-based coupling model, the EM plane-wave-induced Thévenin-voltage response of transmission lines is analyzed in chapter 12. Namely, we derive closed-form expressions for the induced voltages concerning a finite transmission line above the perfect ground and a narrow trace on a grounded dielectric slab. The validity of approximate expressions for the grounded-slab problem configuration is finally discussed with the aid of a three-dimensional computational EM tool.

Whenever the external EM field that couples to a transmission line cannot be longer approximated by a plane wave, sophisticated analytical techniques can be used to evaluate the induced voltage response. This is exactly demonstrated in chapter 13, where the vertical electric dipole (VED)-induced Thévenin's voltage response of a transmission line is analyzed with the help of the CdH method. It is further shown that the effect of a finite ground conductivity can be readily accounted for via the Cooray-Rubinstein formula, thus providing a computationally efficient, analytical model for lightning-induced voltage calculations. The handling of generic integrals necessary for deriving the corresponding TD closed-form expressions is closely described in appendix F. Moreover, the chapter is supplemented with an illustrative numerical example and demo MATLAB[®] implementations (see appendix L).

In chapter 14, the EM reciprocity theorem is applied to propose a computational technique capable of analyzing planar strip antennas. Following the lines of reasoning similar to those used in chapter 2, it is demonstrated that the electric-current surface density along a narrow perfect electric conductor (PEC) strip follows upon carrying out an updating step-by-step procedure. The elements of the relevant TD

“impeditivity matrix” interrelating the induced electric-current surface density with the excitation voltage pulse are closely specified in appendix G with the aid of the CdH method again. This chapter is further supplemented with appendix M, where the reader is provided with an illustrative MATLAB[®] implementation and appendix H concerning a recursive-convolution method and its numerical implementation. Moreover, it is demonstrated that the proposed CdH-MoM methodology can be readily extended to analyze the performance of a wide-strip antenna supporting a vectorial electric-current surface distribution. Chapter 14 finally concludes with a numerical example demonstrating the validity of the computational procedure via the thin-wire formulation from chapter 2 and the concept of equivalent radius [17].

In case that a strip antenna is not perfectly conducting, the theory of high-contrast, thin-sheet cross-boundary conditions [18] lends itself to incorporate the effect of strip’s electric conductivity and permittivity. Therefore, this strategy is adopted in chapter 15, where the impact of a finite conductivity and permittivity is accounted for via an additional impeditivity matrix. Elements of the latter are subsequently derived concerning a homogeneous planar strip with conductive or/and dielectric EM properties and with the Drude-type plasmonic behavior. The relevant thin-sheet jump conditions are justified in appendix I by analyzing a simplified two-dimensional problem configuration.

The inclusion of a linear circuit element in the narrow-strip CdH-MoM formulation is addressed in chapter 16. Pursuing the line of reasoning used in chapter 5, it is demonstrated that a lumped element can be incorporated in the computational scheme by modifying the surface boundary condition at the position where the element is connected. In this way, the impact of a lumped element is taken into account in an isolated impeditivity matrix whose elements are closely specified for a lumped resistor, capacitor, and inductor.

In order to demonstrate that the CdH-MoM is capable of analyzing horizontally layered problem configurations, a narrow strip antenna above a PEC ground plane is analyzed in chapter 17. It is shown that the pertaining impeditivity matrix can be understood as an extension of the one from appendix G that accounts for the effect of reflections from the ground plane. Taking into account the conclusions drawn in section 11.1, the proposed computational scheme is finally validated with the aid of transmission-line theory.

1.2 PREREQUISITES

The mathematical description of EM phenomena is effected via Maxwell field quantities that can be viewed as functions of space and time. To register the position in the analyzed problem configuration, we shall employ a Cartesian reference frame with the origin $\mathcal{O}$ and its (standard) base vectors $\{\mathbf{i}_x, \mathbf{i}_y, \mathbf{i}_z\}$. In line with a common typographic convention, vectors will be hence represented by bold-face italic symbols. Consequently, the position vector, defined as the linear combination of

the base vectors, will be represented as $\mathbf{x} = x\mathbf{i}_x + y\mathbf{i}_y + z\mathbf{i}_z$, where $\{x, y, z\}$ are the (Cartesian) coordinates specifying the point of observation with respect to the Cartesian frame. The time coordinate is real-valued and is denoted by t . The partial differentiation with respect to a coordinate is denoted by ∂ that is supplied with the pertaining subscript. For example, the spatial differentiation with respect to x is denoted by ∂_x , while the time differentiation will be denoted by ∂_t . The vectorial spatial differentiation operator is then defined as $\nabla = \partial_x\mathbf{i}_x + \partial_y\mathbf{i}_y + \partial_z\mathbf{i}_z$.

A Cartesian vector, or, equivalently, a Cartesian tensor of rank 1, can be arithmetically represented as a 1-D array. For example, $\mathbf{v}$ may represent a rank-1 Cartesian tensor (vector) whose components are then $\{v_x, v_y, v_z\}$. A natural extension in this respect is hence a Cartesian tensor of rank 2, also frequently referred to as a dyadic, that is representable as a 2-D array [3, section A.4]. Notationally, such quantities will be further denoted by underlined bold-face italic symbols. For instance, $\underline{\boldsymbol{\eta}}$ will then represent a rank-2 Cartesian tensor. The definition of products of tensors is most conveniently expressed through the subscript notation with the summation convention [3, appendix A]. For a definition of selected products between rank-1 and rank-2 Cartesian-tensor quantities, we refer the reader to Ref. [4, section 1].

1.2.1 One-Sided Laplace Transformation

An alternative way to describe a causal EM space-time quantity is offered by the one-sided Laplace transformation (e.g. [19, section 1.2.1], [3, section B.1]). To introduce the concept, we assume that EM sources that generate the EM wave fields are activated at the origin $t = 0$. Consequently, in view of the property of causality, we will analyze the behavior of the EM space-time quantity, say $f(\mathbf{x}, t)$, in $\mathcal{T} = \{t \in \mathbb{R}; t > 0\}$. The one-sided Laplace transformation of the causal quantity is then defined as

$$\hat{f}(\mathbf{x}, s) = \int_{t=0}^{\infty} \exp(-st) f(\mathbf{x}, t) dt \quad (1.1)$$

where $\{s \in \mathbb{C}; \text{Re}(s) \geq s_0\}$ denotes the Laplace-transform parameter (complex frequency). A sufficient condition for the integral to exist is its absolute convergence along the line $\{s \in \mathbb{C}; \text{Re}(s) = s_0\}$ parallel to the imaginary axis. In the analysis presented in the book, we shall limit ourselves to (physical) EM wave quantities that are bounded. In other words, we will apply the one-sided Laplace transformation (1.1) only to EM wave quantities of the zero exponential order, that is, $f(\mathbf{x}, t) = O(1)$ as $t \rightarrow \infty$. Consequently, $\exp(-s_0 t) f(\mathbf{x}, t) = o(1)$ as $t \rightarrow \infty$ for all values $\{s \in \mathbb{C}; \text{Re}(s) \geq s_0 > 0\}$ of the complex s -plane, to the right of the line of convergence (see Figure 1.1). Hence, in the right half $\{s \in \mathbb{C}; \text{Re}(s) > s_0 > 0\}$ of the complex s -plane, the one-sided Laplace transformation of a causal wave quantity $\hat{f}(\mathbf{x}, s)$ does exist and is here, thanks to the analyticity of the transform kernel $\exp(-st)$, an analytic function of s .

Interpreting Eq. (1.1) as an integral equation to be solved for the unknown function $f(\mathbf{x}, t)$, a question as to its uniqueness arises. Fortunately, the existence of the

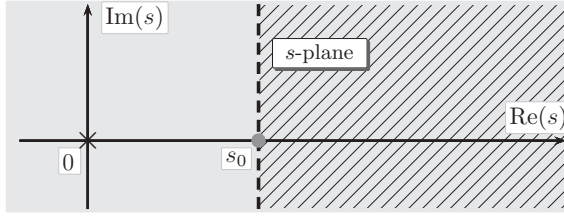


FIGURE 1.1. Complex s -plane with the line of convergence $\text{Re}(s) = s_0$ and the region of regularity $\{s \in \mathbb{C}; \text{Re}(s) > s_0\}$.

one-to-one mapping between a causal quantity $f(\mathbf{x}, t)$ and its Laplace transform $\hat{f}(\mathbf{x}, s)$ is guaranteed by Lerch's uniqueness theorem provided that the equality in Eq. (1.1) is invoked along the Lerch sequence $\mathcal{L} = \{s \in \mathbb{R}; s = s_0 + nh, h > 0, n = 1, 2, \dots\}$ [4, appendix]. The choice of taking the Laplace-transform parameter s real-valued and positive is also employed in the CdH method [8] that heavily relies on the uniqueness theorem due to Matyáš Lerch.

With the uniqueness theorem in mind, we shall frequently represent some TD operators in the s -domain. This applies, in particular, to the operation of (continuous) time convolution. The time convolution of two causal wave quantities, say $f(\mathbf{x}, t)$ and $g(\mathbf{x}, t)$, is defined as

$$[f *_t g](\mathbf{x}, t) = \int_{\tau=0}^t f(\mathbf{x}, \tau) g(\mathbf{x}, t - \tau) d\tau \quad (1.2)$$

for all $t \geq 0$. Applying now (1.1) to Eq. (1.2), we may write

$$\begin{aligned} & \int_{t=0}^{\infty} \exp(-st) \left[\int_{\tau=0}^t f(\mathbf{x}, \tau) g(\mathbf{x}, t - \tau) d\tau \right] dt \\ &= \int_{\tau=0}^{\infty} f(\mathbf{x}, \tau) \left[\int_{t=0}^{\infty} \exp(-st) g(\mathbf{x}, t - \tau) dt \right] d\tau \\ &= \int_{\tau=0}^{\infty} \exp(-s\tau) f(\mathbf{x}, \tau) d\tau \int_{\vartheta=0}^{\infty} \exp(-s\vartheta) g(\mathbf{x}, \vartheta) d\vartheta \\ &= \hat{f}(\mathbf{x}, s) \hat{g}(\mathbf{x}, s) \end{aligned} \quad (1.3)$$

where we have first interchanged the order of the integrations with respect to t and τ , which is permissible thanks to the absolute convergence of the Laplace transforms of both $f(\mathbf{x}, t)$ and $g(\mathbf{x}, t)$, and, secondly, we have used the substitution $\vartheta = t - \tau$. For the Laplace transforms in Eq. (1.3) to make any sense, there clearly must be at least one value of s at which $\hat{f}(\mathbf{x}, s)$ and $\hat{g}(\mathbf{x}, s)$ do converge simultaneously. If $f = O[\exp(\alpha t)]$ as $t \rightarrow \infty$ for some $\alpha \in \mathbb{R}$ and $g = O[\exp(\beta t)]$ as $t \rightarrow \infty$ for some $\beta \in \mathbb{R}$, the region of convergence for Eq. (1.3) extends over the half-plane to the right of the line of convergence $\text{Re}(s) = \max\{\alpha, \beta\}$. Again, if both $f(\mathbf{x}, t)$ and

$g(\mathbf{x}, t)$ are bounded functions whose exponential orders are $\alpha = \beta = 0$, then their region of regularity extends over the right half of the complex s -plane. The property that the Laplace transform of the time convolution of two causal wave quantities is equal to the product of their Laplace transforms (see Eq. (1.3)) will be frequently (and tacitly) used throughout the book. For further useful properties of the Laplace transform, we refer the reader to more complete accounts on this subject (e.g. [3, appendix B.1], [20]).

1.2.2 Lorentz's Reciprocity Theorem

The point of departure for analyzing the space-time EM problems posed in the present book is the TD reciprocity theorem of the time-convolution type (see [3, section 28.2] [4, section 1.4.1]) that is in literature widely known as Lorentz's reciprocity theorem (e.g. [21, section 8.6]).

To introduce the relation, let us assume two states of EM fields, say (A) and (B), that are on a spatial domain $\mathcal{D}$ governed by Maxwell's EM field equations in the s -domain [3, section 24.4]

$$-\nabla \times \hat{\mathbf{H}}^{A,B} + \hat{\underline{\eta}}^{A,B} \cdot \hat{\mathbf{E}}^{A,B} = -\hat{\mathbf{J}}^{A,B} \quad (1.4)$$

$$\nabla \times \hat{\mathbf{E}}^{A,B} + \hat{\underline{\zeta}}^{A,B} \cdot \hat{\mathbf{H}}^{A,B} = -\hat{\mathbf{K}}^{A,B} \quad (1.5)$$

for all $\mathbf{x} \in \mathcal{D}$, in which

- $\hat{\mathbf{E}}^{A,B}$ = electric field strength in V/m;
- $\hat{\mathbf{H}}^{A,B}$ = magnetic field strength in A/m;
- $\hat{\mathbf{J}}^{A,B}$ = electric current volume density in A/m²;
- $\hat{\mathbf{K}}^{A,B}$ = magnetic current volume density in V/m²;
- $\hat{\underline{\eta}}^{A,B}$ = transverse admittance (per unit length) of the medium in S/m;
- $\hat{\underline{\zeta}}^{A,B}$ = longitudinal impedance (per unit length) of the medium in Ω/m .

The EM field Eqs. (1.4) and (1.5) are further supplemented with the constitutive relations

$$\hat{\underline{\eta}}^{A,B}(\mathbf{x}, s) = \hat{\underline{\sigma}}^{A,B}(\mathbf{x}, s) + s\hat{\underline{\epsilon}}^{A,B}(\mathbf{x}, s) \quad (1.6)$$

$$\hat{\underline{\zeta}}^{A,B}(\mathbf{x}, s) = s\hat{\underline{\mu}}^{A,B}(\mathbf{x}, s) \quad (1.7)$$

for all $\mathbf{x} \in \mathcal{D}$, in which

- $\hat{\underline{\sigma}}^{A,B}$ = electric conductivity in S/m;
- $\hat{\underline{\epsilon}}^{A,B}$ = electric permittivity in F/m;
- $\hat{\underline{\mu}}^{A,B}$ = permeability in H/m.

The point of departure is the following local interaction quantity [3, section 28.4]

$$\nabla \cdot \left[\hat{\mathbf{E}}^{\text{A}}(\mathbf{x}, s) \times \hat{\mathbf{H}}^{\text{B}}(\mathbf{x}, s) - \hat{\mathbf{E}}^{\text{B}}(\mathbf{x}, s) \times \hat{\mathbf{H}}^{\text{A}}(\mathbf{x}, s) \right] \quad (1.8)$$

applying throughout $\mathcal{D}$ that can be with the aid of Eqs. (1.4)–(1.5) written as

$$\begin{aligned} & \nabla \cdot \left[\hat{\mathbf{E}}^{\text{A}}(\mathbf{x}, s) \times \hat{\mathbf{H}}^{\text{B}}(\mathbf{x}, s) - \hat{\mathbf{E}}^{\text{B}}(\mathbf{x}, s) \times \hat{\mathbf{H}}^{\text{A}}(\mathbf{x}, s) \right] \\ &= \hat{\mathbf{H}}^{\text{A}}(\mathbf{x}, s) \cdot \left\{ \hat{\underline{\zeta}}^{\text{B}}(\mathbf{x}, s) - [\hat{\underline{\zeta}}^{\text{A}}(\mathbf{x}, s)]^{\mathcal{T}} \right\} \cdot \hat{\mathbf{H}}^{\text{B}}(\mathbf{x}, s) \\ &\quad - \hat{\mathbf{E}}^{\text{A}}(\mathbf{x}, s) \cdot \left\{ \hat{\underline{\eta}}^{\text{B}}(\mathbf{x}, s) - [\hat{\underline{\eta}}^{\text{A}}(\mathbf{x}, s)]^{\mathcal{T}} \right\} \cdot \hat{\mathbf{E}}^{\text{B}}(\mathbf{x}, s) \\ &\quad + \hat{\mathbf{J}}^{\text{A}}(\mathbf{x}, s) \cdot \hat{\mathbf{E}}^{\text{B}}(\mathbf{x}, s) - \hat{\mathbf{K}}^{\text{A}}(\mathbf{x}, s) \cdot \hat{\mathbf{H}}^{\text{B}}(\mathbf{x}, s) \\ &\quad - \hat{\mathbf{J}}^{\text{B}}(\mathbf{x}, s) \cdot \hat{\mathbf{E}}^{\text{A}}(\mathbf{x}, s) + \hat{\mathbf{K}}^{\text{B}}(\mathbf{x}, s) \cdot \hat{\mathbf{H}}^{\text{A}}(\mathbf{x}, s) \end{aligned} \quad (1.9)$$

for all $\mathbf{x} \in \mathcal{D}$ and $\mathcal{T}$ is the transpose operator. The global form of the interaction quantity is derived upon integrating the local interaction over the union of sub domains constituting $\mathcal{D}$ in each of which we assume that the terms of Eq. (1.9) are continuous functions with respect to $\mathbf{x}$. Upon applying Gauss' divergence theorem and adding the contributions of the integrations, we arrive at

$$\begin{aligned} & \int_{\mathbf{x} \in \partial \mathcal{D}} \left(\hat{\mathbf{E}}^{\text{A}} \times \hat{\mathbf{H}}^{\text{B}} - \hat{\mathbf{E}}^{\text{B}} \times \hat{\mathbf{H}}^{\text{A}} \right) \cdot \boldsymbol{\nu} \, dA \\ &= \int_{\mathbf{x} \in \mathcal{D}} \left\{ \hat{\mathbf{H}}^{\text{A}} \cdot \left[\hat{\underline{\zeta}}^{\text{B}} - (\hat{\underline{\zeta}}^{\text{A}})^{\mathcal{T}} \right] \cdot \hat{\mathbf{H}}^{\text{B}} \right. \\ &\quad \left. - \hat{\mathbf{E}}^{\text{A}} \cdot \left[\hat{\underline{\eta}}^{\text{B}} - (\hat{\underline{\eta}}^{\text{A}})^{\mathcal{T}} \right] \cdot \hat{\mathbf{E}}^{\text{B}} \right\} dV \\ &\quad + \int_{\mathbf{x} \in \mathcal{D}} \left(\hat{\mathbf{J}}^{\text{A}} \cdot \hat{\mathbf{E}}^{\text{B}} - \hat{\mathbf{K}}^{\text{A}} \cdot \hat{\mathbf{H}}^{\text{B}} \right. \\ &\quad \left. - \hat{\mathbf{J}}^{\text{B}} \cdot \hat{\mathbf{E}}^{\text{A}} + \hat{\mathbf{K}}^{\text{B}} \cdot \hat{\mathbf{H}}^{\text{A}} \right) dV \end{aligned} \quad (1.10)$$

where we have invoked the continuity of $\boldsymbol{\nu} \times \hat{\mathbf{E}}^{\text{A,B}}$ and $\boldsymbol{\nu} \times \hat{\mathbf{H}}^{\text{A,B}}$ across the common interfaces of the subdomains. The resulting relation (1.10) will be further referred to as the EM reciprocity theorem of the time convolution type or Lorentz's theorem in short. Its left-hand side consists of contributions from the outer boundary of domain $\mathcal{D}$ that is denoted by $\partial \mathcal{D}$. The first integral on the right-hand side then represents the contrasts in the EM properties of the media in states (A) and (B). For the sake of conciseness, this term will be referred to as the interaction of the field and material states. Apparently, the field-material

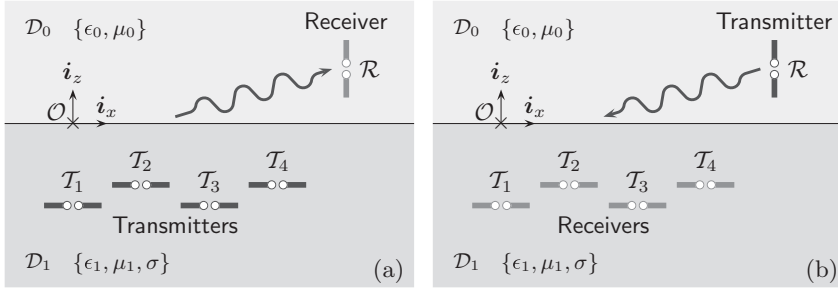


FIGURE 1.2. An example of the use of EM reciprocity to replace (a) the actual problem with an (b) equivalent one that is easier to analyze.

interaction vanishes whenever $\hat{\underline{\zeta}}^B = (\hat{\underline{\zeta}}^A)^T$ and $\hat{\underline{\eta}}^B = (\hat{\underline{\eta}}^A)^T$ for all $\mathbf{x} \in \mathcal{D}$, that is, whenever the media in the both states are each other's adjoint. Finally, the second integral on the right-hand side of Eq. (1.10) is interpreted as the interaction of the field and source states.

Having defined the interacting EM field states and the domain to which the reciprocity theorem applies, Eq. (1.10) can be established. Depending on the choice of EM field states, the resulting relation can be interpreted as a mere relation, an integral representation, an integral equation, or, eventually, a complete solution. Actually, a wide range of venues offered by reciprocity relations is exactly the reason why they are among the most intriguing relations in wave field physics. Since the use of EM reciprocity is largely a matter of ingenuity, it is hardly possible to give a comprehensive application manual. One may, however, provide some typical choices of (A) and (B) states covering a broad spectrum of applications. In computational electromagnetics, for instance, state (A) is typically associated with the (actual) scattered EM wave fields via induced (unknown) current densities, while state (B) is taken to be the computational (or testing) state representing the manner in which the EM field quantities in state (A) are calculated [5]. This strategy is also followed in the present book to formulate a CdH-method-based TD integral equation technique. In antenna theory, states (A) and (B) typically represent receiving and transmitting modes of an antenna system [4]. Similarly, in the context of electromagnetic compatibility (EMC), the reciprocity theorem may serve to link susceptibility and emission properties of the analyzed system at hand. A result from this category is the EM-field-to-line coupling model introduced in chapter 11.

The reciprocity theorem is frequently exploited to replace the actual (tough) task by an equivalent one that is smoothly amenable to an analytical analysis, to measurements or computer simulations. A typical problem from this category is the EM coupling between (a set of) insulated transmitting antennas and a receiving (victim) antenna above the conductive layer (see Figure 1.2), which is of interest to designing wireless inter chip or submarine communication systems [22–24]. In accordance with linear time-invariant system theory, the interaction between such

antennas can be characterized in terms of transfer-impedance matrices that are, in virtue of EM reciprocity, symmetrical [4, chapter 7]. Accordingly, instead of carrying out multiple analyses to evaluate the EM field transfers from each buried antenna to the receiver, it may be more convenient to analyze the equivalent problem (see Figure 1.2b) in a single simulation. Furthermore, the equivalent problem configuration, where the antenna above the interface acts as a transmitter, is suitable for its approximate analysis. Indeed, if the medium in $\mathcal{D}_1$ described by its (scalar, real-valued, and positive) permittivity ϵ_1 , conductivity σ , and permeability μ_0 is sufficiently (electromagnetically) dense with respect to the one in the upper half-space $\mathcal{D}_0$, then the EM field penetrated in $\mathcal{D}_1$ varies dominantly in the normal direction with respect to the interface, thus resembling a plane wave. Consequently, the relevant electric-field strength (i.e. polarized along the insulated antennas' axes) as observed at a horizontal offset $r > 0$ and at a depth $z = -\zeta < 0$ can approximately be expressed via the electric-field distribution at the level of the interface, that is

$$E_x(r, -\zeta, t) \simeq E_x(r, 0, t) *_t \Psi(\zeta, t) \quad (1.11)$$

where (cf. [3, eq. (26.5–29)])

$$\begin{aligned} \Psi(\zeta, t) &= \delta(t - \zeta/c_1) \exp(-\alpha t/2) \\ &+ \frac{\alpha \zeta/2c_1}{(t^2 - \zeta^2/c_1^2)^{1/2}} I_1[\alpha(t^2 - \zeta^2/c_1^2)^{1/2}/2] \exp(-\alpha t/2) H(t - \zeta/c_1) \end{aligned} \quad (1.12)$$

with $I_1(t)$ being the modified Bessel function of the first kind and order one, $H(t)$ denotes the Heaviside unit-step function, $\alpha = \sigma/\epsilon_1$ and $c_1 = (\mu_0\epsilon_1)^{-1/2}$. Furthermore, under the assumption given previously, the electric-field distribution at $z = 0$ can be related to the corresponding magnetic-field strength via the surface-impedance (Leontovich) boundary condition

$$E_x(r, 0, t) \simeq -Z(t) *_t H_y(r, 0, t) \quad (1.13)$$

where $Z(t) = \zeta_1 \partial_t [I_0(\alpha t/2) H(t)]$ with $\zeta_1 = (\mu_0/\epsilon_1)^{1/2}$ is the TD surface impedance described via $I_0(t) \triangleq I_0(t) \exp(-t)$, which is defined as the (scaled) modified Bessel function of the first kind and order zero (see [25, figure 9.8]). Upon combining Eq. (1.11) with the surface-impedance boundary condition (1.13), the horizontal component of the electric-field strength in the lossy medium can be related to the magnetic-field distribution over the planar interface, that is

$$E_x(r, -\zeta, t) \simeq -Z(t) *_t H_y(r, 0, t) *_t \Psi(\zeta, t) \quad (1.14)$$

Under certain conditions that are met for typical lightning-induced calculations [26, 27], for instance, the resulting approximate expression (1.14) can be further simplified by replacing the actual magnetic-field distribution at $z = 0$ with the one

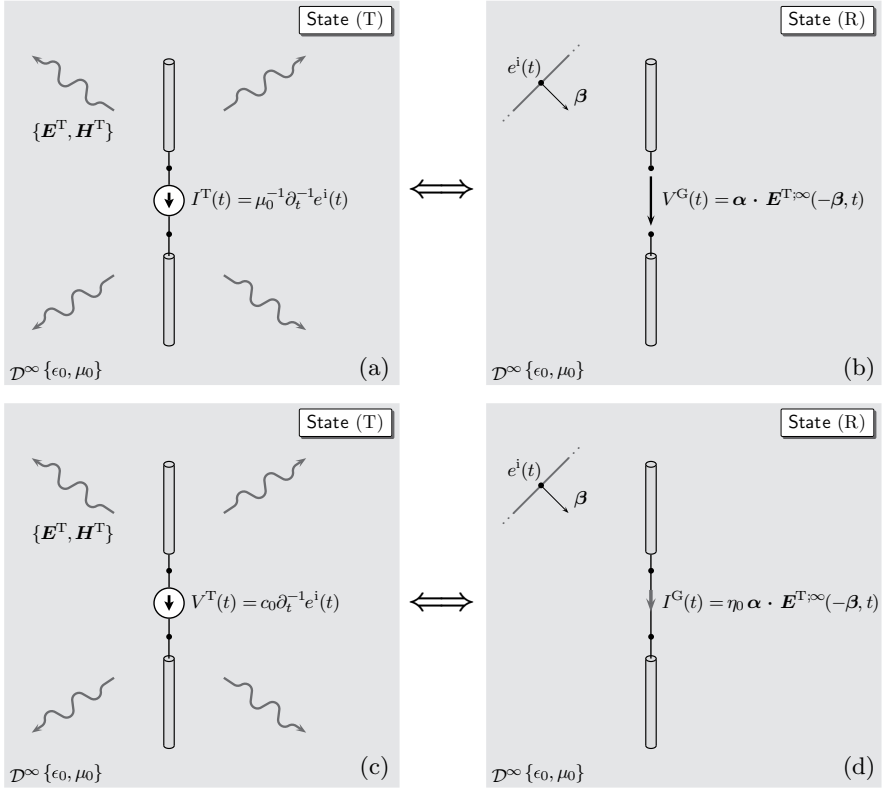


FIGURE 1.3. (a) Electric-current-excited transmitting antenna and (b) the corresponding receiving antenna whose open-circuit voltage response is related to the radiation characteristics; (c) voltage-excited transmitting antenna and (d) the corresponding receiving antenna whose short-circuit electric-current response is related to the radiation characteristics.

pertaining to the PEC surface. As the latter field distribution can be for many fundamental EM sources expressed in closed form, a compact approximate expression for $E_x(r, -\zeta, t)$ follows.

Analyzing mutually reciprocal scenarios is not only useful for simplifying the problem solution itself but may also be beneficial for the validation of purely computational tools. Typical transmitting (T) and receiving (R) scenarios regarding a wire antenna that can be analyzed for validation purposes are shown in Figure 1.3. In them, the transmitting antenna radiating EM wave fields $\{E^T, H^T\}$, whose far-field amplitudes are denoted by $\{E^{T;\infty}, H^{T;\infty}\}$ (see [3, section 26.12]), are related to port responses induced by a uniform EM plane wave defined by its polarization vector α , by a unit vector in the direction of propagation β and its pulse shape $e^i(t)$. Consequently, by virtue of EM reciprocity, the open-circuit (Thévenin) plane-wave

TABLE 1.1. Application of the Reciprocity Theorem

Domain $\mathcal{D}$		
Time-Convolution	State (A)	State (B)
Source	$\{\hat{\mathbf{J}}^A, \hat{\mathbf{K}}^A\}$	$\{\hat{\mathbf{J}}^B, \hat{\mathbf{K}}^B\}$
Field	$\{\hat{\mathbf{E}}^A, \hat{\mathbf{H}}^A\}$	$\{\hat{\mathbf{E}}^B, \hat{\mathbf{H}}^B\}$
Material	$\{\hat{\underline{\eta}}^A, \hat{\underline{\zeta}}^A\}$	$\{\hat{\underline{\eta}}^B, \hat{\underline{\zeta}}^B\}$

induced voltage response pertaining to the receiving state can be directly related to the co-polarized far-field amplitude observed in the backward direction

$$V^G(t) = \boldsymbol{\alpha} \cdot \mathbf{E}^{T;\infty}(-\boldsymbol{\beta}, t) \quad (1.15)$$

provided that the exciting electric-current pulse $I^T(t)$ is proportional to the (time-integrated) plane-wave signature according to

$$I^T(t) = \mu_0^{-1} \partial_t^{-1} e^i(t) \quad (1.16)$$

where ∂_t^{-1} denotes the time-integration operator [28, eq. (20)]. The corresponding scenarios are depicted in Figure 1.3a and b. Alternatively, if the port of the receiving wire antenna is short-circuited, the induced (Norton) electric-current response can be found from

$$I^G(t) = \eta_0 \boldsymbol{\alpha} \cdot \mathbf{E}^{T;\infty}(-\boldsymbol{\beta}, t) \quad (1.17)$$

with $\eta_0 = (\epsilon_0/\mu_0)^{1/2}$ provided that the excitation voltage pulse $V^T(t)$ is related to the plane-wave pulse shape via (see Figure 1.3c and d)

$$V^T(t) = c_0 \partial_t^{-1} e^i(t) \quad (1.18)$$

Yet another and more general example interrelating the transmitting (T) and receiving (R) states of a wire antenna is numerically analyzed in chapter 7. In conclusion, the EM reciprocity theorem may also be viewed as a powerful tool for validation of EM solvers. This (reciprocity-based) strategy has been applied in chapters 7, 8, 9, and 10 to check the consistency of the proposed solution methodologies.

To ease the use of EM reciprocity, we will adopt the tabular representation of the EM reciprocity theorem (see [4, 19]). In this fashion, the source, field, and material states pertaining to the interacting EM field states are clearly summarized in a table. For instance, the table corresponding to the generic form of the EM reciprocity theorem of the time-convolution type (see Eq. (1.10)) is given here as Table 1.1.

