1 INTRODUCTION TO MECHANICS OF MATERIALS

1.1 WHAT IS MECHANICS OF MATERIALS?

Mechanics is the physical science that deals with the conditions of rest or motion of bodies acted on by forces or by thermal disturbances. The study of bodies at rest is called *statics*, whereas *dynamics* is the study of bodies in motion. You have been introduced to the fundamental principles of statics and dynamics and have applied these principles to *particles* and to *rigid bodies*, which are both simplified idealizations of real physical systems. The principles of statics and dynamics are also fundamental to the *mechanics of solids* and to the *mechanics of fluids*, two major branches of applied mechanics that deal, respectively, with the behavior of materials in solid form and with the behavior of materials that are fluids. This book is an introduction to *mechanics of materials*, a topic that is also known by several other names, including “strength of materials,” “mechanics of solids,” and “mechanics of deformable bodies.” The mechanics of fluids is treated as a separate topic of study in engineering.

This book deals almost exclusively with the behavior of solids under static-loading conditions (the exception is Section 11.9). However, studies of the *dynamics of solids* (e.g., earthquake excitation of buildings and bridges) utilize most of the same concepts that are covered here. This topic is generally referred to as “structural dynamics” or “mechanical vibrations.”[1]

Mechanics of Materials. We can begin to answer the question, “What is mechanics of materials?” by considering Fig. 1.1. First, a **deformable body** is a solid that changes size and/or shape as a result of loads that are applied to it or as a result of temperature changes. The diving board in Fig. 1.1 visibly changes shape due to the weight of the diver standing on it. Changes of size and/or shape are referred to as **deformation.** The deformation may even be so small that it is invisible to the naked eye, but it is still very important. To relate the deformation to the applied loading, it is necessary to understand how *materials* (i.e., solids) behave under loading.

[1]See, for example, *Fundamentals of Structural Dynamics* [Ref. 1-1] in the References section near the back of the book.

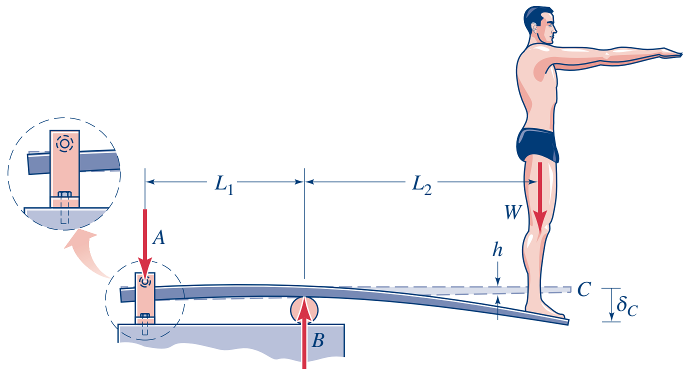


FIGURE 1.1 A diving board as an example of a deformable body.

Whereas it would be possible from rigid-body equilibrium alone, given the weight of the diver and the lengths L_1 and L_2, to determine the diving-board support reactions at A and B in Fig. 1.1, questions of the following type can only be answered by employing the principles and procedures of mechanics of materials:

1. What weight W would cause the given diving board to break, and where would the break occur?
2. For a given diving board and given position of roller B, what is the relationship between the tip deflection at C, δ_C, and the weight, W, of the diver standing on the board at C?
3. Would a tapered diving board be "better" than one of constant thickness? If so, how should the thickness, h, vary from A to C?
4. Would a diving board made of fiberglass be "preferable" to one made of aluminum?

Stress and Strain. All of the preceding questions require consideration of the diving board as a *deformable body;* that is, they require consideration not only of the external forces applied to the diving board by the diver and by the diving-board supports at A and B, but they also involve the localized effects of these forces within the diving board (i.e., the **stress distribution** and the **strain distribution**) and the behavior of the material from which the diving board is constructed (i.e., the **stress-strain behavior** of the material). Stress and strain, the key concepts in the study of mechanics of materials, are formally defined in Chapter 2.

Finite Element Analysis. Throughout this course you will be learning how to analyze the distribution of stress and strain in various one-dimensional deformable bodies, and how they deform, but the principles and procedures you learn here are also the basis for computer programs that are used to analyze very complex deformable bodies. **Finite element analysis** is a very powerful procedure for analyzing complex structures and machines by treating them as assemblages of thousands of small parts, or elements.[2] The photos in Fig. 1.2 illustrate analyses (e.g., stress and strain distributions) that have been obtained by use of various finite element computer programs. In the photos, different colors are used to indicate different levels of stress, etc.

Analysis and Design. All static deformable-body mechanics problems fall into one of two categories—**strength** problems or **stiffness** problems. A structure or machine must be "strong enough"; that is, it must satisfy prescribed strength criteria. It must also be "stiff enough"; that is, its deformation must be within acceptable

[2]See, for example, Refs. [1-2] and [1-3].

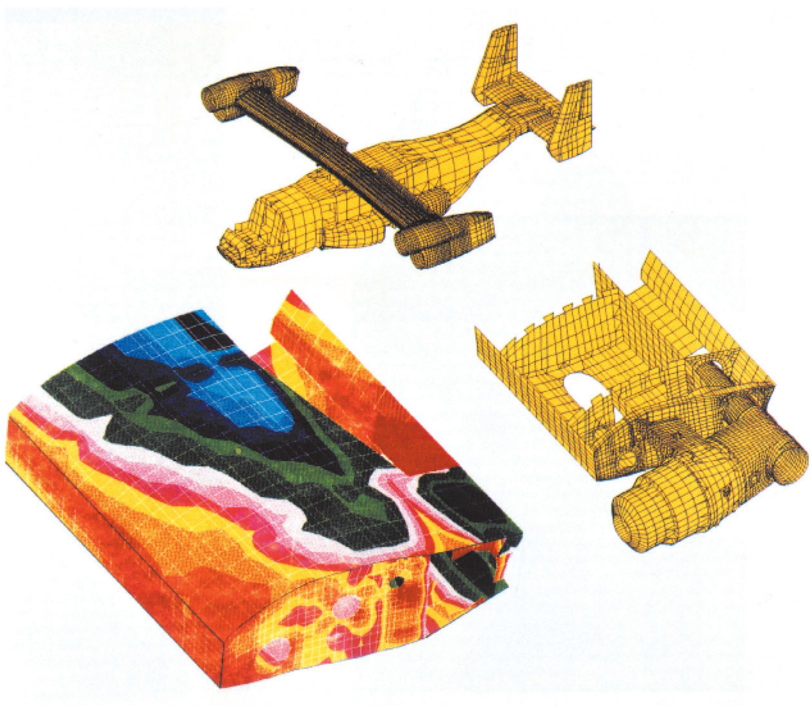


FIGURE 1.2(a) V-22 Osprey tilt-rotor aircraft structure. Shown are the complete aircraft and a wing-tip engine-mount section with finite element grids, and a wing-tip section that illustrates the use of color graphics to interpret strain contours. (*Source:* From *MSC/NASTRAN* finite element analysis by Bell Helicopter Textron. Reprinted with the permission of the American Institute of Aeronautics and Astronautics.)

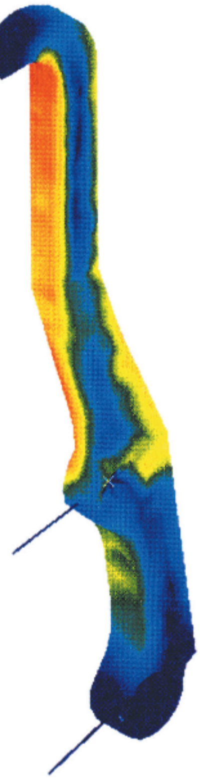


FIGURE 1.2(b) An example of the use of finite element analysis in support of the design of a mechanical latch member from a Space Shuttle payload. (*MSC/NASTRAN* finite element analysis with *SDRC 1-DEAS* postprocessing. *Source:* Courtesy NASA-Johnson Space Center.)

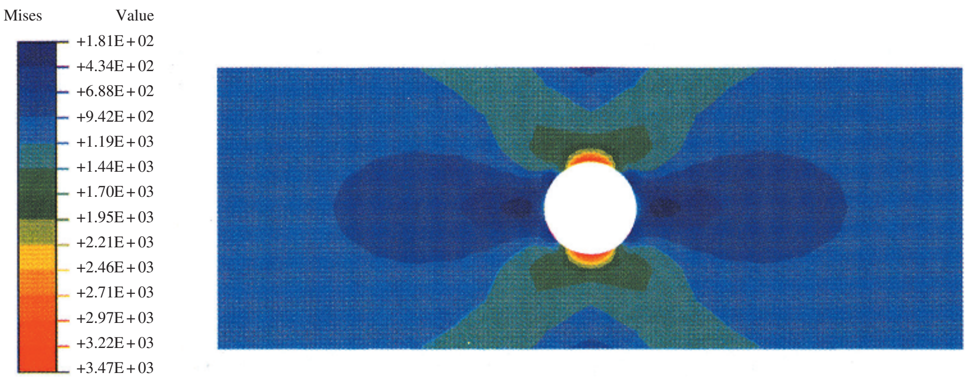


FIGURE 1.2(c) Finite element analysis of the stress concentration due to a hole in an axially loaded flat bar, as described in Section 12.2. The adjacent color bar indicates that the stress depicted is the von Mises stress, which is discussed in Section 12.3. (*Source: ABAQUS* finite element analysis by Courtesy Greg Swadener.)

limits. The first diving-board question is a strength question; the second one addresses stiffness.

The first two diving-board questions above fall under the category of **analysis.** That is, given the system (in this case the diving board) and the loads applied to it, your task is to analyze the behavior of this particular system subjected to this particular loading condition. Questions 3 and 4, on the other hand, are **design** questions. Given certain information about the loading and the performance criteria (e.g., the

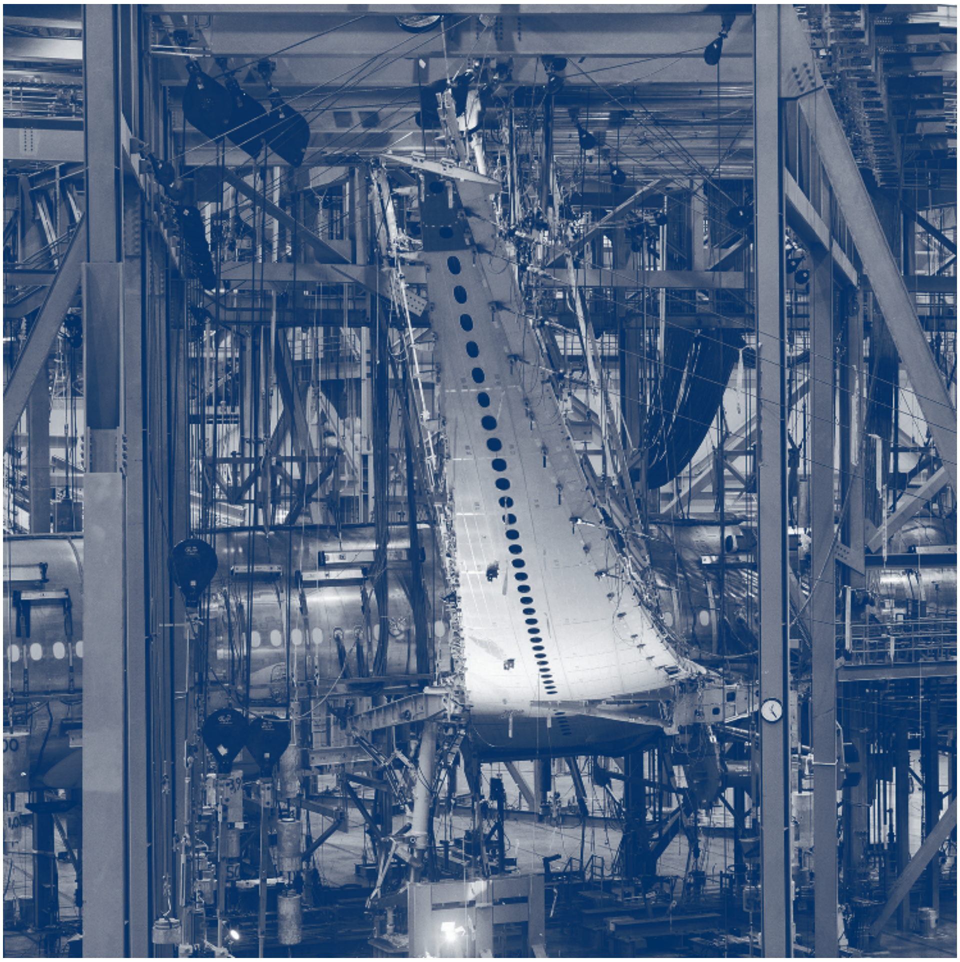


FIGURE 1.3 An airplane wing undergoing an ultimate-load test. (Copyright © Boeing.)

range of diver weight to be accommodated, and what constitutes a better diving board), your task is to select the configuration of the diving board and the material to be used in its construction.

The *design process* usually involves an iterative procedure whereby a *design* (a specific configuration made of specific materials) is proposed, the response of the designed system to given loads is analyzed, and the response is compared with the response of other designs and with the stated design criteria. The "best" of several candidate designs is then selected. Finally, there may be a requirement that a prototype based on the selected design be manufactured and tested to verify that the system meets all the design requirements in an acceptable manner. Figure 1.3 shows an airplane wing undergoing an ultimate-load test, that is, a test to determine the maximum wing loading that can be applied without causing the wing to break. The designers' expectations were exceeded when, during the test, the wings were pulled to approximately 24 ft above their normal position. (Of course, in normal service the wing will only undergo deflections that are much smaller than those experienced in an ultimate-load test like this.)

Applications of Mechanics of Materials. The applications of deformable-body mechanics are practically endless and can be found in every engineering discipline. The impressive Brooklyn Bridge, shown in Fig. 1.4, is truly an engineering marvel. It was designed by Johann Roebling in 1867–1869 and built under the supervision of his son Washington Roebling in 1870–1883 at a cost of $25 million. Its span of 1595 ft is suspended from towers that are 271 ft tall.

FIGURE 1.4 The Brooklyn Bridge, New York (Andreas Feininger/Time & Life Pictures/Getty Images, Inc.)

Since the 1960s, finite element analysis (FEA) computer programs (e.g., NASTRAN, ANSYS, ABAQUS, SAP 2000) have been employed extensively for carrying out the numerical computations required in applying deformable-body mechanics principles to the design of mechanical systems and structures. Very detailed finite element models were used to create the color plots shown in Fig. 1.2. In designing modern airplanes, automobiles, and other mechanical and structural systems, Computer-Aided-Design (CAD) plays an essential role in defining the geometry of components, creating mathematical models of these components, and then performing the deformable-body analysis of these components.

Not only are the principles and procedures of deformable-body mechanics used to analyze and design large objects like bridges and airplanes (e.g., the tilt-rotor vehicle in Fig. 1.2a), but they also find application to very small objects as well. For example, FEA has been used to determine the stress distribution in a small computer chip at one stage in the heating-cooling cycle that occurs when the computer is turned on and off. Figure 1.2c illustrates how FEA can be used to depict the effect of **stress concentration**, as discussed in Section 12.2.

Although many of the tasks involved in the design and analysis of the systems illustrated in Fig. 1.2 require a knowledge of the mechanics of deformable bodies that is beyond the scope of this introductory textbook, the principles and procedures introduced in this book form a foundation on which more advanced topics build, and on which the design of complex applications, like those illustrated in Fig. 1.2, depends.

1.2 THE FUNDAMENTAL EQUATIONS OF DEFORMABLE-BODY MECHANICS

Throughout this textbook, the three *fundamental types of equations* that are used in solving strength and stiffness problems of deformable-body mechanics will be stressed repeatedly. They are:

1. The **equilibrium** conditions must be satisfied.
2. The **geometry of deformation** must be described.
3. The **material behavior** (i.e., the force-temperature-deformation relationships of the materials) must be characterized.

Here these fundamental equations are applied to fairly simple deformable bodies, but the same three basic types of equations apply, in more advanced mathematical form in many cases, to all studies of deformable solids.

Equilibrium. We have already noted that the principles of statics, that is, the *equations of equilibrium*, are fundamental to the study of deformable-body mechanics. Section 1.4 gives a brief review of static equilibrium and introduces the equilibrium concepts that are particularly important in the study of mechanics of solids. It also stresses the importance of drawing complete, accurate free-body diagrams. An entire chapter, Chapter 5, is devoted to the topic of equilibrium of beams.

Geometry of Deformation. There are several ways in which the *geometry of deformation* enters the solution of deformable-body mechanics problems, including:

1. Definitions of extensional strain and shear strain (Chapter 2).
2. Simplifications and idealizations (e.g., "rigid" member, "fixed" support, plane sections remain plane, displacements are small).
3. Connectivity of members, or geometric compatibility.
4. Boundary conditions and other constraints.

Several of these may be illustrated by a comparison of Fig. 1.1 with Fig. 1.5. In Fig. 1.1, the diving board itself was considered to be deformable, but the supports at A and B were assumed to be rigid. Therefore, the *idealized model* in Fig. 1.1 is a *deformable beam* with *rigid constraints* at A and B. By contrast, the beam BD in Fig. 1.5 is assumed to be "rigid" under the loading and support conditions shown. Although BD does actually deform, that is, change shape, its deformation is assumed to be small in comparison to the rotation, θ, that it undergoes if the rod AB stretches significantly when load W is applied to the beam at D. Hence, the *idealized model* depicted in Fig. 1.5 is a *rigid beam, BD*, connected by a frictionless pin at end B to a *deformable rod AB*. As rod AB stretches, beam BD rotates through a small angle about a fixed, frictionless pin at C.

Material Behavior. The third principal ingredient in deformable-body mechanics is *material behavior*. Unlike equilibrium and geometry of deformation, which are purely analytical in nature, the constitutive behavior of materials, that is, the *force-temperature-deformation relationships* that describe the material behaviors, can only be established by conducting experiments. These are discussed in Chapter 2.

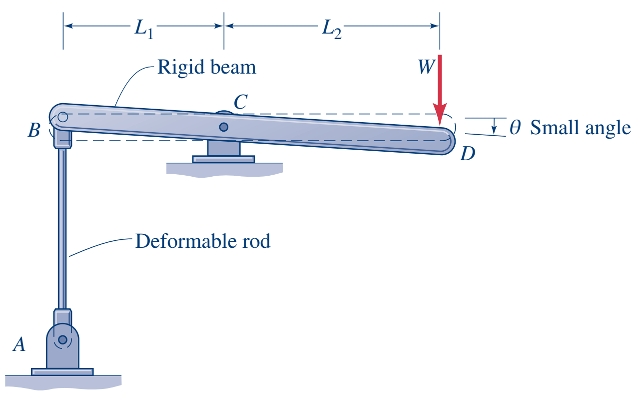


FIGURE 1.5 A system that illustrates several deformation assumptions.

It will be of great help to you in solving problems in the mechanics of deformable bodies if you will always keep in mind these three distinct ingredients: **equilibrium, geometry of deformation,** and **material behavior.**

1.3 PROBLEM-SOLVING PROCEDURES

A consistent, systematic procedure is required for solving most problems in engineering practice, and this certainly applies to solving problems involving the mechanics of deformable bodies. The five steps in such a problem-solving procedure are:

1. Select the **system** of interest. This may be based on an existing physical system, or it may be defined by a set of design drawings and specifications.
2. Make simplifying assumptions that reduce the real system to an **idealized model,** or idealization of the system. For example, Figs. 1.1 and 1.5 illustrate two different idealized models of diving boards. The idealized model should provide the basis for a mathematical model that is practical to solve and provides predictions within the accuracy required for the given problem. It is often necessary to choose simplifying assumptions that balance the ease of solution with the accuracy of predictions from that solution.
3. Apply the principles of deformable-body mechanics to the idealized model to create a **mathematical model** of the system, and solve the resulting equations to predict the *response* of the system to the applied disturbances (applied forces and/or temperature changes). Interpret the results that you obtain, and seek to understand the behavior (stress and deformation) that the system exhibits in response to the applied disturbances.
4. Perform a *test* to compare the predicted responses to the behavior of the actual system. (A full-scale prototype or a scale model may have to be constructed if the physical system does not already exist.) This is often the only method to determine whether model predictions meet the accuracy required.
5. If the response predicted in Step 3 does not agree with the response of the tested system, repeat Steps 1–4, making changes as necessary until agreement is achieved.

In the future, as a practicing engineer you will find Steps 1, 2, and 4 to be very important and very challenging. However, the main purpose of this textbook is to introduce you to the fundamental concepts of mechanics of solids and to enable you to solve deformable-body mechanics problems. Therefore, attention here will be devoted primarily to carrying out Step 3, in which a mathematical model is formulated and its behavior analyzed.

So, how do you apply the principles of deformable-body mechanics to create mathematical equations, and how do you solve these equations to obtain the response of the system? A glance at the Example Problems in this textbook will indicate that the following four steps are clearly identified:

1. State the Problem
2. Plan the Solution
3. Carry Out the Solution
4. Review the Solution

1. State the Problem — This step involves:

- listing the given data,
- drawing any figures needed to describe the problem data, and
- listing the results that are to be obtained.

2. Plan the Solution — While you probably have not seen this step treated in a formal manner in previous textbooks, your success in solving problems quickly and accurately depends on how carefully you plan your solution strategy in advance. You should think about the given data and the results desired, identify the basic principles involved and recall the applicable equations, and plan the steps that will be needed to carry out the solution.

3. Carry Out the Solution — As noted in Section 1.2, your solution will involve three basic ingredients: equilibrium, geometry of deformation, and material behavior. Example problems and homework problems in this textbook appear in one of two forms—problems where numerical values are employed directly in the solution, and ones where algebraic symbols are used to represent the quantities involved and where the final answer is essentially a formula. One advantage of the symbolic form of solution is that a check of dimensional homogeneity may be easily made at each step of the solution. A second advantage is that the symbols serve to focus attention on the physical quantities involved. For example, the effect of a force designated by the symbol P can be traced through the various steps of the solution. Finally, since the end result is an equation in symbolic form, different numerical values can be substituted into the equation if desired.

The importance of checking for *dimensional homogeneity* at each step of a solution cannot be overemphasized! The principal quantities involved in static solid mechanics problems are force (dimension F) and length (dimension L). If, at some step in the solution of a problem, you check an equation for dimensional homogeneity and obtain the result that

$$F \cdot L = F/L^2$$

you should realize that some error has been made. It is a waste of time and effort to proceed further without rectifying the error and establishing a dimensionally homogeneous equation! Other checks for accuracy should also be made frequently as the solution progresses.

Appendix B provides a discussion of the *units* used in solving deformable-body mechanics problems, and Appendix A discusses the number of *significant digits* required.

4. Review the Solution — This step, like the Plan the Solution step, may be one that you have not previously encountered in a formal manner. However, it is important for you to get into the habit of always checking your results by asking yourself the following types of questions:

- Is the answer dimensionally correct?
- Do the quantities involved appear in the final solution in a reasonable manner?
- Is the sign of the final answer reasonable, and is the numerical magnitude reasonable?
- Is the final result consistent with the assumptions that were made in order to achieve the solution? (For example, an assumption that "the slope is small" is violated if the final slope turns out to be 45°.)

You may be tempted to think that Review the Solution means for you to compare your answer with one in the back of the book. However, in the "real world" there are no "answer books." Hence, you should begin now, if you have not already done so, to make a habit of testing by any means possible the reasonableness, dimensional homogeneity, and accuracy of your own answers. But, it is not enough just to solve problems and get correct answers. You should also learn to *interpret your answers*, so that you begin to develop that indispensable quality called *engineering judgment*.

Although the Carry Out the Solution step will undoubtedly occupy a major portion of your problem-solving time as you work on problems from this textbook, you will probably find that, as a practicing engineer, most of your time is involved in the other three steps, since a computer may quickly and obediently carry out the actual numerical solutions of your engineering problems. It then becomes your task to set up the problem correctly (State the Problem and Plan the Solution) and to evaluate carefully the results of the computer solution (Review the Solution).

Finally, you should heed the advice of the authors of a popular statics text[3]:

> It is also important that all work be neat and orderly. Careless solutions that cannot be easily read by others are of little or no value. The discipline involved in adherence to good form will in itself be an invaluable aid to the development of the abilities for formulation and analysis. Many problems that at first may seem difficult and complicated become clear and straightforward when begun with a logical and disciplined method of attack.

Communication is a vital part of any engineering project, and only work that is neat and orderly can serve to communicate technical information, like the solution of a deformable-body mechanics problem.

Finally, engineers who design machines and structures (e.g., commercial airplanes or high-rise office buildings) using the principles of deformable-body mechanics assume not only legal responsibility, but also moral and ethical responsibility that their designs will not fail in service in ways that could lead to serious loss of life or property.[4]

1.4 REVIEW OF STATIC EQUILIBRIUM; EQUILIBRIUM OF DEFORMABLE BODIES

In this textbook we consider deformable bodies at rest, that is, bodies whose acceleration and velocity are both zero. In your previous study of statics, you learned the equations of equilibrium and you learned how to apply these equations to particles and to rigid bodies through the use of free-body diagrams. In this section we will review the fundamental equations and problem-solving procedures of statics and will begin to indicate how they apply to the study of deformable bodies.

[3] See Ref. [1-4], p. 14.

[4] The National Transportation Safety Board "cited a design flaw as the likely cause of the collapse" of the I-35W Mississippi River Bridge in Minneapolis, Minnesota, on August 1, 2007. Thirteen people were killed and 145 people were injured in the collapse of this bridge. See, for example, wikipedia.com.

Equations of Equilibrium. Recall that the *necessary conditions* for equilibrium of a body (rigid or deformable) are[5]:

$$\sum \mathbf{F} = \mathbf{0}, \quad \left(\sum \mathbf{M}\right)_O = \mathbf{0} \tag{1.1}$$

That is, if a body is in equilibrium,

- the sum of the external forces acting on the body is zero, and
- the sum of the moments, about any arbitrary point O, of all the external forces acting on the body is zero.

These equations are usually expressed in component form with the components referred to a set of rectangular Cartesian axes x, y, z. Then, the resulting scalar equations are:

$$\begin{aligned} \sum F_x &= 0, & \left(\sum M_x\right)_O &= 0 \\ \sum F_y &= 0, & \left(\sum M_y\right)_O &= 0 \\ \sum F_z &= 0, & \left(\sum M_z\right)_O &= 0 \end{aligned} \tag{1.2}$$

When the number of *independent* equilibrium equations available is equal to the number of unknowns, the problem is said to be *statically determinate*. When there are more unknowns than available independent equations of equilibrium, the problem is said to be *statically indeterminate*. For example, if a body has more external supports or constraints than are required to maintain the body in a stable equilibrium state, the body is statically indeterminate. Supports that could be removed without destroying the equilibrium of the body are called *redundant supports*, and their reactions are called, simply, *redundants*.

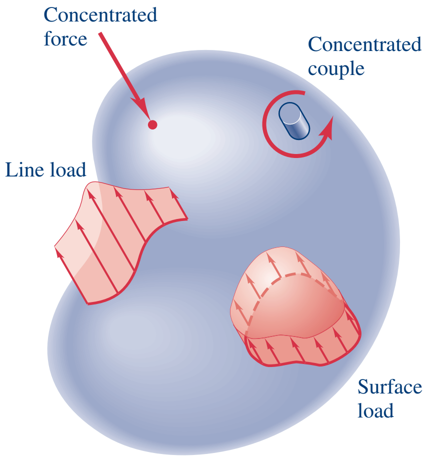
In order to apply equilibrium equations to a body, it is <u>always</u> wise to draw a *free-body diagram* (FBD) of the body. However, before reviewing the procedure for drawing a free-body diagram, let us consider the types of external loads that may act on a body and several ways in which the body may be supported or connected to other bodies.

External Loads. The **external loads** acting on a deformable body are the known force and moments that are applied to the body. They may be classified in four categories, or types. These types, together with their appropriate dimensions, are:

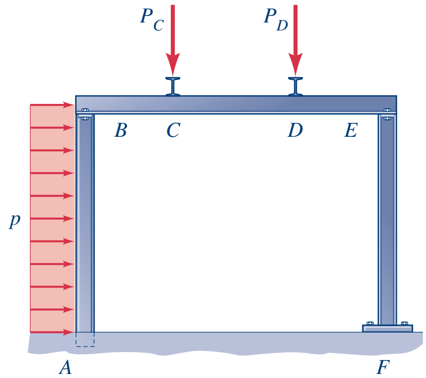
- *Concentrated loads*, including *point forces* (F) and *couples* ($F \cdot L$),
- *Line loads* (F/L),
- *Surface loads* (F/L^2), and
- *Body forces* (F/L^3).

The first three types of external loads are illustrated on the generic deformable body in Fig. 1.6*a*. Body forces are produced by action-at-a-distance. Like the force of

[5]For a rigid body, Eqs. 1.1 also constitute *sufficient conditions* for equilibrium of the body. That is, if Eqs. 1.1 are satisfied, then the rigid body is in equilibrium. However, the *necessary and sufficient condition* for equilibrium of a deformable body is that the sets of external forces that act on the body and on every possible subsystem isolated out of the original body all be sets of forces that satisfy Eqs. 1.1. (See, for example, [Ref. 1-5], p. 16.)



(*a*) A generic deformable body.



(*b*) A portal frame.

FIGURE 1.6 External loads acting on deformable bodies.

gravity (weight), they are typically proportional to volume, and they act on particles throughout the body.

Although, in reality, all external loads that act on the surface of a deformable body must act on a finite area of that surface, line loads and concentrated loads are considered to act along a "line" or to act at a single "point," respectively, as indicated in Fig. 1.6*a*. Concentrated loads and line loads are, therefore, idealizations. Nevertheless, they permit accurate analysis of the behavior of the deformable body, except in the immediate vicinity of the loads.[6] In Fig. 1.6*b* a cross-beam at *C* (shown in end view as an "I" shape) exerts a downward concentrated force P_C on the horizontal frame member *BE*, and a horizontal line load of uniform intensity *p* acts on the vertical frame member *AB*.

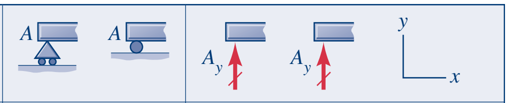
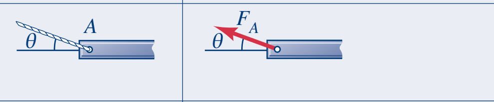
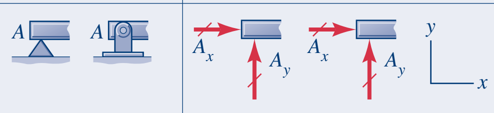
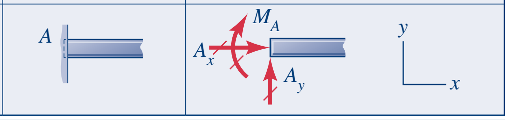
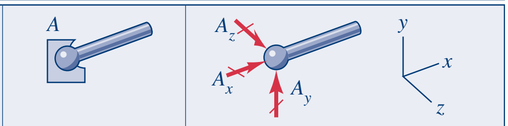
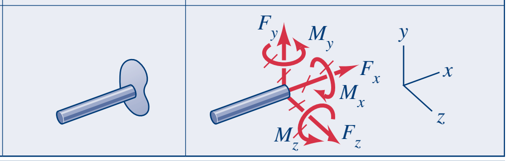
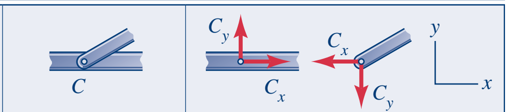
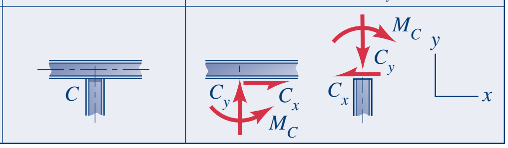
Concentrated forces have the units of force [e.g., newtons (N) or pounds (lb)], and line loads have the units of force per unit length (e.g., N/m or lb/ft). Other external loads are expressed in units appropriate to their dimensionality.

Support Reactions and Member Connections. The external loads that are applied to a member must generally be transmitted to adjacent members that are connected to the given member, or carried directly to some form of support.[7] For example, the vertical loads P_C and P_D that act on the horizontal beam *BE* in Fig. 1.6*b* are eventually "transmitted" to the ground at supports *A* and *F*. Where there is a *support*, as at points *A* and *F* in Fig. 1.6*b*, the displacement (i.e., the change of position) is specified to be zero, but the force is unknown. Therefore, forces (including couples) at supports are called **reactions**, since they react to the loads that are applied elsewhere. We say that the support enforces a *constraint*, that is, the support constrains (i.e., makes) the displacement to be zero.

Table 1.1 gives the symbols that are used to represent idealized supports and member connections. Also shown are the force components and the couples that correspond to these. In this textbook, all *reactions* are indicated by an arrow (straight for forces, curved for couples) that has a single slash through its shank, as illustrated in Table 1.1. For the most part, in this text we will consider loading and supports that lie in a single plane, that is, coplanar loading and support. Occasionally, however, we will consider a three-dimensional situation.

[6] See the discussion of St. Venant's Principle in Section 2.10.

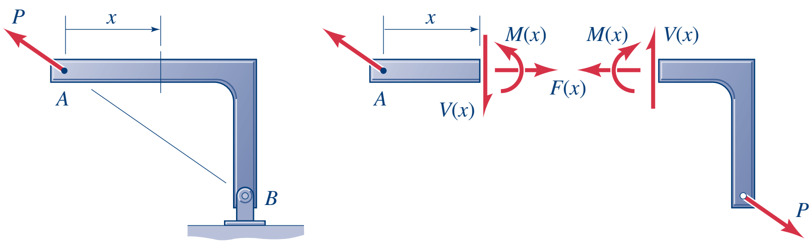
[7] The exception is a self-equilibrated system, like an airplane, whose (upward) distributed lift force equals its (downward) weight.

Table 1.1 Support Reactions and Member Connections		
Description	Symbol	Required Forces/Couples
REACTIONS – 2-D		
1. Roller support		
2. Cable or rod		
3. Pin support		
4. Cantilever support (fixed end)		
REACTIONS – 3-D		
5. Ball joint		
6. Cantilever support (fixed end)		
CONNECTIONS – 2-D		
7. Pinned connection		
8. Rigid connection (e.g., welded, bolted)		

Internal Resultants. In the study of mechanics of deformable bodies, we must consider not only external forces and couples, that is, the applied loads and reactions, but we also must consider **internal resultants,** that is, forces and couples that are internal to the original body. For example, to analyze the L-shaped two-force linkage in Fig. 1.7*a*,[8] it is necessary to imagine a *cutting plane*, like the one indicated in Fig. 1.7*a*, and to show the (unknown) internal resultants acting on this plane, as has been done in Fig. 1.7*b*. This procedure may be called the **method of sections.**

The engineering theories that are developed in this textbook apply to deformable bodies for which one dimension is significantly greater than the other two dimensions;

[8]You may recall from statics that, if a body in equilibrium is subjected only to concentrated forces acting at two points in the body, the forces must be equal and opposite and must be directed along the line joining the points of application of the forces, as illustrated in Fig. 1.7*a*. Such a body is referred to as a *two-force member*.



(*a*) A two-force member. (*b*) Internal resultants: *F*, *V*, and *M*.

FIGURE 1.7 An illustration of internal resultants.

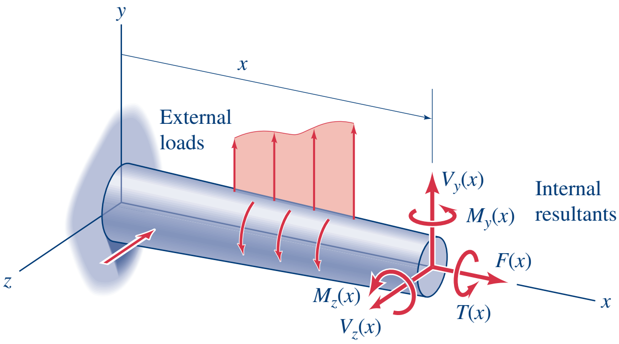


FIGURE 1.8 The six internal resultants on an arbitrary cross section of a slender member of length $L > X$.

that is, we will consider long, thin members.[9] The six internal resultants that result from general loading of such a member are indicated on the sketch in Fig. 1.8, where the x axis is taken to lie along the longitudinal direction of the member, and a cutting plane normal to the x axis, called a **cross section,** is passed through the member at coordinate x $(x < L)$.

On an arbitrary cutting plane through a body subject to general three-dimensional loading there will be three components of the *resultant force* and three components of the *resultant moment*. When the body is slender, as in Fig. 1.8, these resultants are given special names. The force normal to the cross section, labeled $F(x)$, is called the *normal force*, or **axial force.** The two components of the resultant force that are tangent to the cutting plane, $V_y(x)$ and $V_z(x)$, are called transverse shear forces, or just **shear forces.** The component of moment about the axis of the member is called the **torque,** or *twisting moment*, and is labeled $T(x)$. Finally, the other two components of moment, $M_y(x)$ and $M_z(x)$, are called **bending moments.** Much of the remainder of this book is devoted to the determination of how these six resultants are distributed over the cross section.

Free-Body Diagram (FBD). Let us now review the steps that are involved in drawing a complete **free-body diagram.** They may be summarized as follows:

- Determine the extent of the body to be included in the FBD. Completely *isolate* this body from its supports and from any other bodies attached to it. When internal resultants are to be determined, pass a sectioning plane through the member at the desired location. Sketch the outline of the resulting *free body*.
- Indicate on the sketch all of the *applied loads*, that is, all known external forces and couples, acting on the body. These include distributed and concentrated

[9]The only exception is the thin-wall pressure vessels discussed in Section 9.2.

loads applied to the body and also, when it is not negligible, the distributed weight of the body itself. The location, magnitude, and direction of each applied load should be clearly indicated on the sketch.

- Where the body is supported or is connected to other bodies, or where it has been sectioned, show the *unknown forces and couples* that are exerted on this body by the adjacent bodies. Assign a symbol to each such force (or force component or couple) and, where the direction of an unknown force or couple is known, use this information. Often there is a *sign convention* that establishes the proper sense to be assumed as positive. This is particularly true for the internal resultants. However, in some cases the sense of an unknown can be assumed arbitrarily.
- Label significant points and include significant *dimensions*. Also, if reference axes are needed, show these on the sketch.
- Finally, keep the FBD as simple as possible so that it conveys the essential equilibrium information quickly and clearly.

Except in Chapter 10, where we will examine *stability of equilibrium* and where it will be necessary to draw a free-body diagram of the deformed system, we will assume that deformations are small enough that the **free-body diagram can be drawn showing the body in its undeformed configuration,** even though the forces acting on it are those associated with the deformed configuration.

The following two example problems will serve as a review of the way that free-body diagrams are chosen, and will illustrate how equilibrium equations are used to determine internal resultants. Example Problem 1.1 reviews the *method of joints* and the *method of sections* for solving truss equilibrium problems. Example 1.2 applies the *method of sections* to solve a frame-type problem.

EXAMPLE 1.1

A sign weighing 600 N hangs from a pin-jointed planar truss, as shown in Fig. 1. (a) Use the *method of joints* to determine the internal axial forces F_1 and F_2 in members CD and DE, respectively. Neglect the weight of the truss members. (b) Use the *method of sections* to determine the axial force F_5 in member FG.

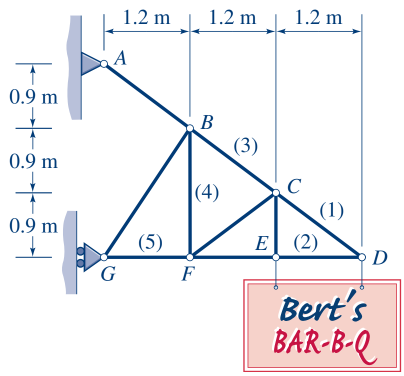


Fig. 1

Plan the Solution This problem asks for the internal axial force in each of three members, and specifies the types of free-body diagrams to use in obtaining the appropriate equilibrium equations. In your Statics course you were introduced to the two methods indicated, so this will give you a good opportunity to review those methods.

The weight of the sign will tend to pull the truss away from the wall at joint A, forcing the wall to push to the right at joint G. Therefore, members along the top of the truss will be in tension, while the axial forces in members along the bottom of the truss will be compressive.

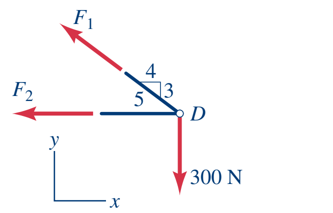


Fig. 2 Free-body diagram of joint D.

Solution (a) *Using the method of joints, determine F_1 and F_2.* We draw a free-body diagram of the joint at D (Fig. 2), taking F_1 and F_2 to be positive in tension. Also, we select reference axes x and y as shown. Equilibrium must be satisfied in both the x and y directions.

$$\overset{+}{\rightarrow}\sum F_x = 0: \qquad -F_2 - (4/5)F_1 = 0$$

$$+\uparrow \sum F_y = 0: \qquad (3/5)F_1 - 300\text{ N} = 0$$

$$F_1 = (5/3)(300\text{ N}) = 500\text{ N}$$

$$F_2 = -(4/5)F_1 = -(4/5)(500\text{ N}) = -400\text{ N}$$

$$F_1 = 500\text{ N (T)}, \quad F_2 = 400\text{ N (C)} \qquad \textbf{Ans. (a)}$$

(b) *Using the method of sections, determine F_5, the force in truss member FG.* We make an imaginary sectioning "cut" as shown in Fig. 3, and draw a free-body diagram of the portion of the truss to the right of this cut, taking the axial forces in the three cut members to be positive in tension. With this free-body diagram, there will be three unknown member forces, but by taking moments about joint B, only one equation of equilibrium will be needed to obtain the required unknown force F_5.

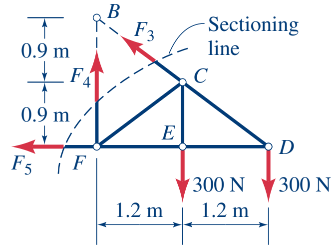


Fig. 3 Free-body diagram of section with cut through member FG.

$$+\circlearrowright\left(\sum M\right)_B = 0: \quad (300\text{ N})(1.2\text{ m}) + (300\text{ N})(2.4\text{ m}) + F_5(1.8\text{ m}) = 0$$

$$F_5 = -600\text{ N}, \text{ or } F_5 = 600\text{ N(C)} \qquad \textbf{Ans. (b)}$$

Review the Solution The force F_1 has turned out to be in tension, and forces F_2 and F_5 in compression, as we expected. Also, the magnitudes are reasonable, so our solution appears to be correct.[10]

Based on our experience in solving equilibrium problems, we could have assumed at the outset that the unknown force in member FG acts to the right on joint F (i.e., that member FG is in compression). Had we done so (by reversing the sense of the arrow representing force F_5), we would have gotten an answer of $F_5 = 600$ N, without the minus sign. Instead, we chose to show all unknown axial forces on the free-body diagram assuming tension to be positive. As a consequence, the answer for F_5 turned out to be $F_5 = -600$ N. That is, the minus sign indicates that the force F_5 is a compressive force rather than a tensile force.

As problems get more complex (e.g., several interconnected bodies) it will become impossible to mentally solve all of the resulting equilibrium equations to

[10]Note that the units (N) and (m) are stated when numerical values are used. It is good practice to show the proper force units (F) and length units (L) in the solution of numerical problems. Also, note that the answers are marked, and that tension (T) and compression (C) are identified in the answers.

the extent that the "correct" sense of every force can be established at the outset when the free-body diagram is drawn. **The procedure of assuming internal axial forces to be positive in tension makes it both easy to draw the free-body diagram and easy to interpret the meaning of the answers (positive forces are tension; negative forces are compression).** Hence, this sign convention will be followed throughout this textbook.

EXAMPLE 1.2

An electrical worker stands in the bucket that hangs from a pin at end D of the boom of the cherry picker in Fig. 1. The worker and bucket together weigh a total of 300 lb. Between A and C the boom weighs 1.1 lb/in., and between C and D it weighs 0.8 lb/in. Assume that AC and CD are uniform beams.

Determine the normal force, the transverse shear force, and the bending moment that act at cross section E, midway between A and B.

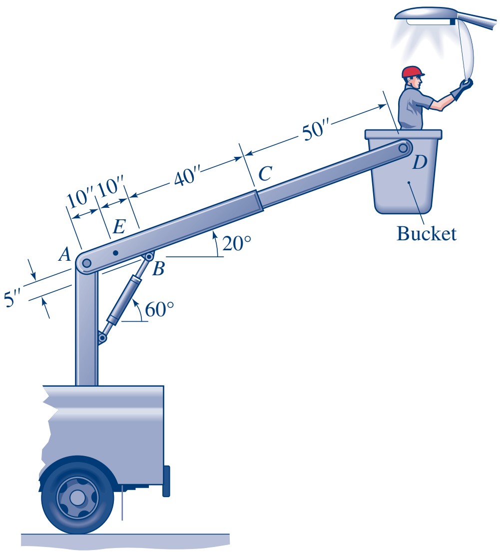


Fig. 1 (*a*) "Bucket truck."

(*b*) Telescopic aerial device. (Used with permission of Altec Industries.)

Plan the Solution Since there will be three unknowns on the cross section at E and, in addition, an unknown force at the pin B, we cannot solve for all four unknowns using a single free-body diagram. Hence, we will have to determine the pin force first using a separate free-body diagram; then we can determine the two components of the internal force at E and the moment at E.

Solution *Pin Reaction at B:* First, we use the free-body diagram in Fig. 2 to determine the pin reaction at B.

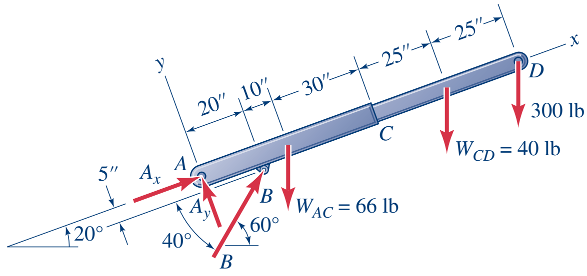


Fig. 2 A free-body diagram of boom AD.

$$W_{AC} = (1.1 \text{ lb/in.})(60 \text{ in.}) = 66 \text{ lb}$$

$$W_{CD} = (0.8 \text{ lb/in.})(50 \text{ in.}) = 40 \text{ lb}$$

$$+\circlearrowleft \left(\sum M\right)_A = 0:$$

$$-(66 \text{ lb})(30 \text{ in.})(\cos 20°) - (40 \text{ lb})(85 \text{ in.})(\cos 20°)$$
$$-(300 \text{ lb})(110 \text{ in.})(\cos 20°) + (B \cos 50°)(20 \text{ in.})$$
$$+ (B \cos 40°)(5 \text{ in.}) = 0$$
$$B = 2161.4 \text{ lb}$$

Internal Resultants: Next, we pass a section through the beam at E and determine the forces and moment on this cross section (Fig. 3). (Let point E be on the centerline of the beam AD.) On the section at E we show the unknown normal force F_E, the unknown transverse shear force V_E, and the unknown bending moment M_E.

$$W_{EC} = (1.1 \text{ lb/in.})(50 \text{ in.}) = 55 \text{ lb}$$

$$+\nearrow \sum F_x = 0:$$

$$-F_E + (2161.4 \text{ lb})(\sin 50°) - (55 \text{ lb} + 40 \text{ lb} + 300 \text{ lb})(\sin 20°) = 0$$
$$F_E = 1520.6 \text{ lb}$$

$$+\nwarrow \sum F_y = 0:$$

$$V_E + (2161.4 \text{ lb})(\cos 50°) - (55 \text{ lb} + 40 \text{ lb} + 300 \text{ lb})(\cos 20°) = 0$$
$$V_E = -1018.2 \text{ lb}$$

(i.e., 1018.2 lb acting opposite to the direction shown on the free-body diagram).

$$+\circlearrowleft \left(\sum M\right)_E = 0:$$

$$-M_E - (55 \text{ lb})(\cos 20°)(25 \text{ in.}) - (40 \text{ lb})(\cos 20°)(75 \text{ in.})$$
$$-(300 \text{ lb})(\cos 20°)(100 \text{ in.}) + (2161.4 \text{ lb})(\cos 50°)(10 \text{ in.})$$
$$+(2161.4 \text{ lb})(\sin 50°)(5 \text{ in.}) = 0$$
$$M_E = -10{,}129.9 \text{ lb} \cdot \text{in.}$$

The answers, rounded to the proper number of significant digits are

$$F_E = 1520 \text{ lb}, \quad V_E = -1020 \text{ lb}, \quad M_E = -10{,}100 \text{ lb} \cdot \text{in.} \qquad \textbf{Ans.}$$

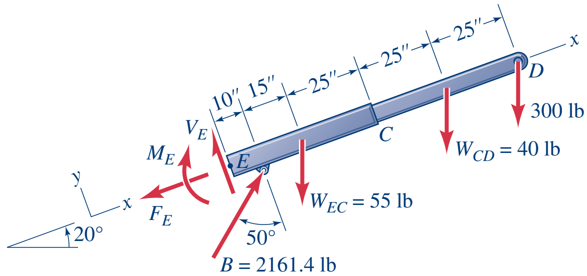


Fig. 3 A free-body diagram showing the resultants at section *E*.

Review the Solution Because of the long moment arm of the 300-lb load compared with the moment arm of the force at *B*, it is reasonable for the magnitude of *B* to be much larger than the magnitude of the total load. The magnitude and sense of F_E, V_E, and M_E also seem to be reasonable in view of the magnitude of *B* and the magnitude and location of the other loads.

CHAPTER 1 REVIEW — INTRODUCTION TO MECHANICS OF MATERIALS

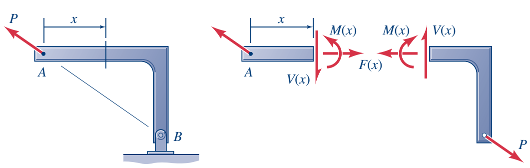
<table>
<tr><th>Section</th><th colspan="2"></th><th>Suggested Review Problems</th></tr>
<tr><td>1.1</td><td>Section 1.1 gives you definitions of the following: mechanics of materials, deformable body, stress and strain, and analysis and design.

It also introduces you to the finite element method, a powerful computational tool for analyzing complex structural and mechanical systems.</td><td>You should familiarize yourself with this material</td><td></td></tr>
<tr><td>1.2</td><td>Section 1.2 introduces the three fundamental type of equation that you will be studying throughout this course. They are:
1. equilibrium equations;
2. geometry of deformation equations; and
3. material behavior (i.e., force-temperature-deformation) equations.</td><td>You should always remember this Big 3 of mechanics of materials!</td><td></td></tr>
<tr><td>1.3</td><td>Section 1.3 discusses a four-step problem-solving procedure for you to use. The four steps are:
1. State the Problem;
2. Plan the Solution;
3. Carry Out the Solution; and
4. Review the Solution.</td><td>You should use this <u>four-step</u> problem-solving procedure.</td><td></td></tr>
<tr><td>1.4</td><td>Section 1.4 reviews important topics that were covered in your Statics course.

It emphasizes the importance of carefully drawn free-body diagrams.

Section 1.4 also introduces the topic of internal resultants—axial force, transverse shear force, and bending moment.

Table 1.1 describes symbols for external reactions.</td><td>The statics review problems involve:
• pin-jointed planar trusses,
• beams, and
• planar frames.



(a) A two-force member. (b) Internal resultants: F, V, and M.

Internal resultants (Fig. 1.7)</td><td>1.4-5
1.4-11
1.4-17</td></tr>
</table>