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A Brief History of Reference Frame Theory

In the late 1920s, R. H. Park, a young MIT graduate working for GE, wrote a paper on a new method of analyzing a synchronous machine [1]. He formulated a change of variables that, in effect, replaced the variables (voltages, currents, and flux linkages) associated with the stator windings of a synchronous machine with variables associated with fictitious, sinusoidally distributed windings rotating at the electrical angular velocity of the rotor. This change of variables, which eliminated the position-varying inductances from the voltage equations, is often described as transforming or referring the stator variables to the rotor reference frame. Although he did not refer to this as "reference frame theory," it was its beginning.

This new approach to machine analysis found limited use until the advent of the computer. Perhaps a discussion written by C. H. Thomas in the late 1950s was the first to set forth a method of using Parks equations to establish a stable computer simulation of a synchronous machine which is still being used today [2]. This opened the door to the use of a change of variables to analyze problems involving electric machines since this same method of simulation is used today for the simulation of all synchronous and induction-type machines.

There have been numerous changes of variables that have been set forth after Park's work. In the late 1930s, C. H. Stanley [3] employed a change of variables in the analysis of induction machines. He showed that the rotor-position-dependent inductances in the voltage equations of an induction machine, which are due to electric circuits in relative motion, could be eliminated by replacing the rotor variables with substitute variables associated with sinusoidally distributed stationary windings. This is often described as transforming or referring the rotor variables to a frame of reference fixed in the stator or the stationary reference frame. About the same time, E. Clarke [4] set forth an algebraic transformation for three-phase stationary circuits to facilitate their steady-state and transient analyses of three-phase ac power systems. She referred to these substitute variables as alpha, beta, and zero components.

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In [5], G. Kron introduced a change of variables that eliminated the rotor-position-dependent inductances of a symmetrical induction machine by transforming both the stator and the rotor variables to a reference frame rotating in synchronism with the fundamental electrical angular velocity of the stator variables. This reference frame is commonly referred to as the synchronously rotating reference frame.

D. S. Brereton et al. [6] employed a change of variables that also eliminated the rotor-position-varying inductances of a symmetrical induction machine. This was accomplished by transforming the stator variables to a reference frame rotating at the electrical angular velocity of the rotor.

Park, Stanley, Kron, and Brereton et al. developed changes of variables each of which appeared to be unique. Consequently, each transformation was derived and treated separately in the literature until it was noted in 1965 [7] that all known real transformations used in machine analysis were contained in one transformation. The Arbitrary Reference Frame was introduced in [7] as a general reference frame that contained all known transformations simply by assigning the speed of the reference frame. For example, when ω, the speed of the q and d axes, is set equal to zero, we have Stanley's and Clarke's transformations; with $\omega = \omega_r$, we have Park's and Brereton's transformations; and when $\omega = \omega_e$, we have Kron's. Although this was an interesting observation, the connection to Tesla's rotating magnetic field was not made. Although it should have been, since moving from one reference frame to another changes only the frequency that we observe Tesla's rotating magnetic field.

In a recent paper [8], the connection between Tesla's rotating magnetic field and the arbitrary reference frame was set forth. It was shown that the transformation to the arbitrary reference frame was contained in Tesla's expression for the rotating magnetic field. Moreover, once the symmetrical stator and rotor are transformed to the arbitrary reference frame, we have the q and d voltage equations for all machines. The only thing that must be transformed are the flux-linkage equations for the machine being considered [9].

Up until the writing of [10], the transformations were given without any explanation as to the basis of the transformation. It was accepted without question. Although it was possible to obtain the transformation by referring the *abc* axes to a *qd*-axis, there was not an analytical basis for the transformation. This plagued machine analysts for nearly a hundred years.

During the writing of [10] it was found that the equation for Tesla's rotating magnetic field contained the basis we had all been trying to find since Park's work. This forms the machine analysis in [10] and was explained in [9]. This approach to machine analysis is the subject of the next two chapters. In Chapter 2, we refer Tesla's rotating magnetic field to a rotating axis. In Chapter 3, we establish the connection between Tesla's rotating magnetic field and reference frame theory.

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