1

Fundamental Units and Constants in Metrology

> *"When you can measure what you are speaking about and express it in numbers, you know something about it."*
>
> —*Lord Kelvin (1883). Source: Public Domain.*

1.1 Introduction

Metrology is the science of measurement with various applications. It is derived from the Greek words *metro* – measurement and *Logy* – science. The BIMP (Bureau of Weights and Measures in France) defines metrology as "the science of measurement embracing both experimental and theoretical determinations at any level of uncertainty in any field of science and technology."

Five pivots define the functions of metrology:

1) To establish the units of measurements;
2) To replicate these units as standards;
3) To guarantee the measurement uniformity;
4) To develop measurement methods;
5) To investigate the accuracy of methods-related errors.

Based on this, the objectives of metrology are:

1) Selection of proper measuring instrument;
2) Proper measuring standards;
3) Minimizing inspection cost;
4) Defining process capabilities;
5) Standardization;
6) Maintaining accuracy and precision during inspection or as component of an instrument over time of use [1].

Therefore, two types of metrology exist:

1) Deterministic, or industrial, metrology.
2) Legal, or scientific, metrology.

Measurement is the process of revealing a single or multiple values to the characteristics of an object or property by conducting experiments to determine the value of this particular property. These properties may be physical, mechanical, or chemical, such as length, weight, force, strain, volume, angle, and molls.

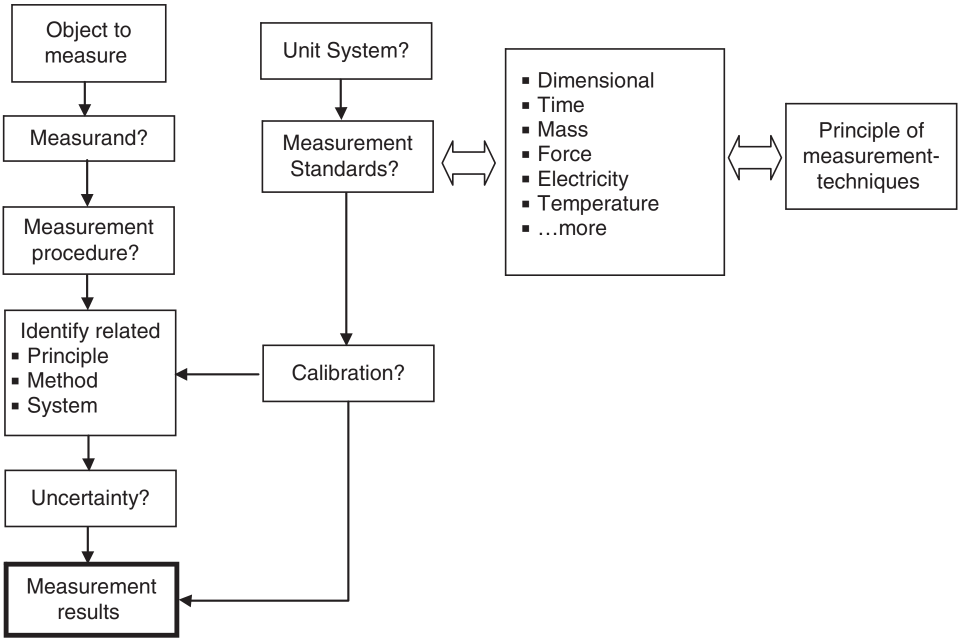

Figure 1.1 Simplified methodology producing measurement results.

Metrology also includes precision, repeatability, and accuracy, which refers to how accurate the measured value is. It establishes a well-known understanding of the measuring process and the related units that are critical in connecting various human activities and ensures that these measurements are linked to reference standards, which is commonly referred to as traceability. For as long as civilization has existed, measurements have been taken. It is necessary for a country's economic and social development. It provides precise measurements that have an impact on the economy, health, safety, and general well-being. It could also be a legal problem. As a result, the topic is always in demand.

This chapter will introduce the fundamental units and constants in metrology through conversions between units and systems. To put measurements into context, a complete methodology of the act of measurement beginning with the object to measure and ending with the result that constitutes the information needed for the object is required. The complete process is summarized in Figure 1.1. The figure depicts a simplified methodology for producing measurement results with minimal conditions such as the units to be known, the calibration of the measurement instruments, and the uncertainty of such measurements. This is to cast the majority of the aspects that engineers conducting measurements must be aware of. A dimension is a non-numerical measure of a physical variable. The unit is used to associate a quantity or measurement with a dimension.

Example 1.1 The mass of an object is a primary dimension, while 15 kg is associated with the quantity 15 of mass with the unit of kg. We need a comparison with some precise unit value to measure the quantity of anything. Body parts (Figure 1.2) and natural surroundings were used by early humans to provide suitable measuring instruments. Elementary measures became essential in the primitive human societies for tasks such as building dwellings, making clothing, bartering for food, and exchanging raw materials.

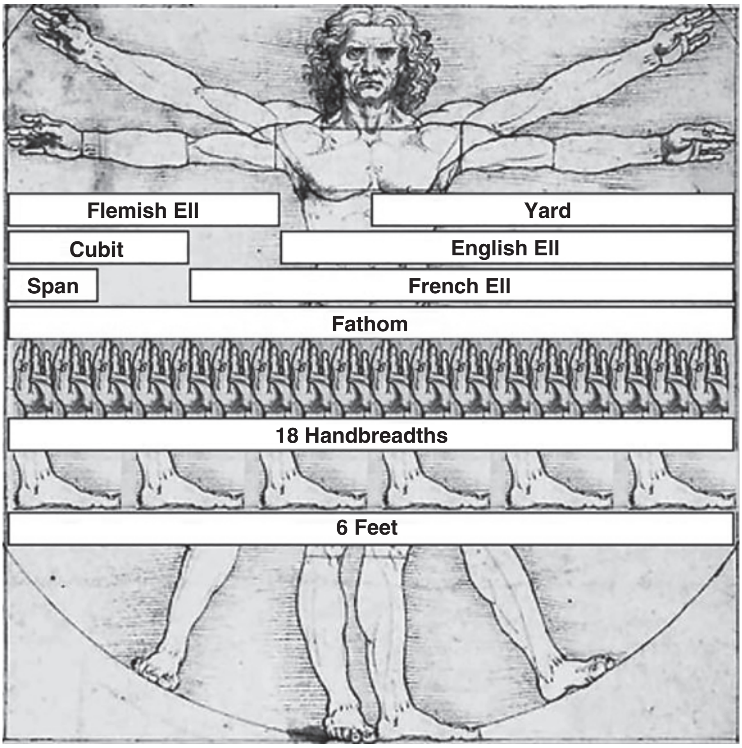

Figure 1.2 Vitruvian man by Leonardo da Vinci showing nine historical units of measurement. *Source:* Wikimedia Commons

- According to early Babylonian and Egyptian transcripts, length was first measured with the forearm (cubit), hand (palm and span), and finger (digit).
- The cycles of the celestial bodies such as the sun, moon, and others were used for time measurements.
- Plant seeds were used for the sake of establishing volume measurement, while with the expansion of scales for weighing, seeds and stones became standards. As sample, the carob seed was the base measure for the carat, which is still used as a mass unit in the gemstone industry.

As trade and commerce expanded, it became necessary to standardize measurement systems across many countries. This decreased the possibility of disagreements arising from measurement system misunderstandings.

The international system of units, known as the SI (from French “Système International”) unit system, distinguishes physical units into two classes as shown below:

1) Base or primary units; and
2) Derived units.

These two categories cover the most commonly used units, such as time, temperature, length, mass, pressure, and flow rate. The National Institute of Standards and Technology (NIST) [2] introduced the SI units, which can be found at this hyperlink: **SI units** (https://physics.nist.gov/cuu/Units/). For more information on the SI units, visit the website of the international standards organization known as the Bureau International des Poids et Mesures (BIPM).

Table 1.1 Primary units.

Measurement	Units	Symbol	Description
Unit of length	meter	m	One meter is equal to the length of the path travelled by light in vacuum during a time interval of 1/299792458 of a second.
Unit of mass	kilogram	kg	One kilogram is equal to the unit of mass presented by the international prototype of the kilogram in Figure 1.2. Since 2019, the new definition based on Planck's constant has been used.
Unit of time	second	s	One second is equal to the duration of 9192631770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.
Unit of electric current	ampere	A	One ampere is defined as follows: the constant current if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, will produce between these conductors a force equal to 2×10^{-7} newton per meter of length.
Unit of thermodynamic temperature	kelvin	K	One kelvin is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.
Unit of amount of substance	mole	mol	One mole is the amount of substance of a system containing as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. When the mole is used, the elementary entities must be specified and may be atoms, molecules, ions, electrons, other particles, or specified groups of such particles.
Unit of luminous intensity	candela	cd	One candela is the luminous intensity within one direction, of a source that emits monochromatic radiation of frequency 540 × 1012 hertz and having a radiant intensity of 1/683 watt per steradian in that direction.

As will be demonstrated later, each measurement unit has a primary quantity that is used by convention. Each primary quantity has only one primary unit. As a result, every primary unit can be decomposed or recomposed further. Table 1.1 shows primary units of different kinds of physical quantities, symbols, and their descriptions. Figure 1.3 depicts the kilogram prototype safely conserved in Paris as a reference unit of kg kept constant in quantity for comparison. The following section discusses derived units, which are shown in Table 1.2.

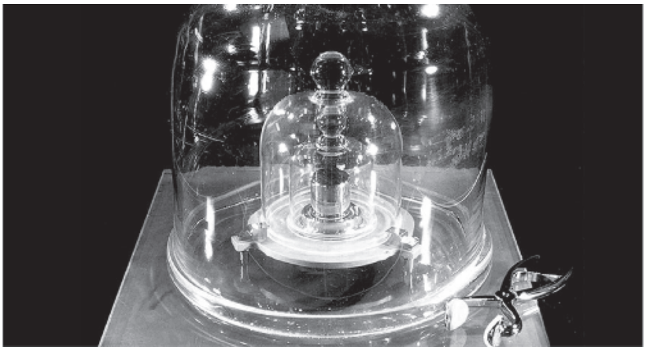

Figure 1.3 The standard kilogram for mass.

Table 1.2 Derived units.

Derived quantity	Si derived new unit	Symbol	SI units	SI base units
Force	newton	N		$mkgs^{-2}$
Pressure, stress	pascal	Pa	N/m^2	$m^{-1}kgs^{-2}$
Energy, work, quantity of heat	joule	J	Nm	m^2kgs^{-2}
Power	watt	W	J/s	m^2kgs^{-2}
Electric charge	coulomb	C		sA
Electromotive force	volt	V		$m^2kgs^{-3}A^{-1}$
Electric capacitance	farad	F	C/V	$m^{-2}kg^{-1}s^4A^2$
Electric resistance	ohm	Ω	V/A	$m^2kgs^{-3}A^{-2}$
Electric conductance	siemens	S	A/V	$m^{-2}kg^{-1}s^3A^2$
Velocity	meter per second		m/s	
Angular velocity	radian per second		1/s	
Mass flow rate	kilogram per second		kg/s	
Flow rate	liter per second		l/s	

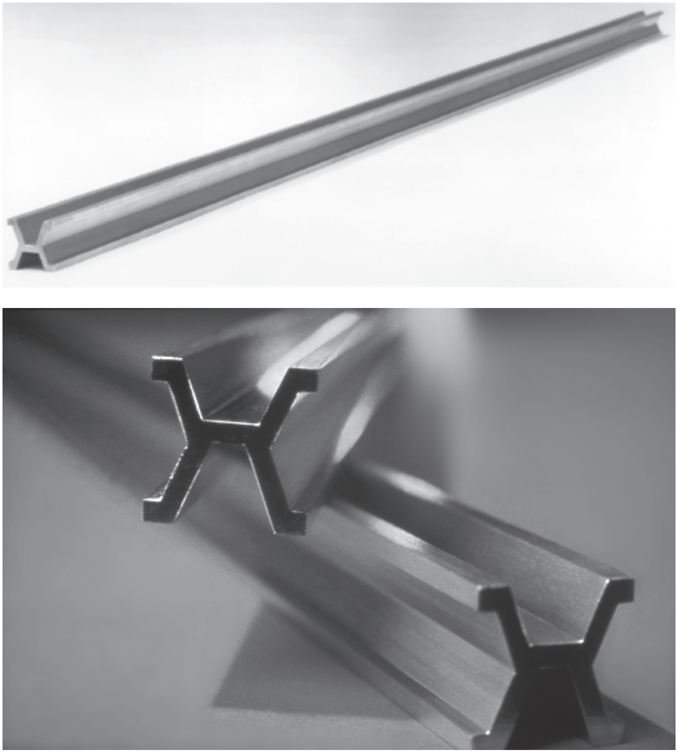

Figure 1.4 The platinum-iridium meter bar reference. *Source:* Wikimedia Commons

The International Prototype Meter bar, shown in Figure 1.4, is made of 90% platinum and 10% iridium alloy and served as the SI (metric system) standard of length from 1889 until 1960, when the SI system switched to a new definition of length based on the wavelength of light emitted by krypton-86. The practical length of the meter was defined by the distance between two fine lines ruled on the central rib of the bar near the ends measured at the freezing temperature of water.

The bar was given an X (Tresca) cross-sectional shape to increase its stiffness-to-weight ratio and improve its thermal accommodation time so the graduation lines could be located on the "neutral" axis of the bar where the change in length with flexure is minimum. The prototype was made in 1889, its length made equal to the previous French standard "Meter of the Archives." At the same time, twenty-nine identical copies were made, which were calibrated against the prototype and distributed to nations to serve as national standards and possibly for comparison after a few years.

1.2 Current Definitions of the Main SI Units

The current definition of the base and primary units are shown in Table 1.1.

1.3 New Definition of Seven Base Units of the SI

Seven base units of the SI are known to be the second, meter, kilogram, ampere, kelvin, mole, and candela. Some have been based on physical constants for a long time. Since 1983, the meter has been defined as the length of the path traveled by light in vacuum over a time interval of 1/299 792 458 s. However, the four that metrologists have agreed to redefine recently were previously based on something—i.e., an object, experiment, or phenomenon—implying that their value is not universal.

As a result of this decision [3], all seven SI units are currently defined in terms of physical constants.

The meter, symbol m, is the SI unit of length. It is defined by taking the fixed numerical value of the speed of light in vacuum, c, to be 299792458 when expressed in the unit $\mathrm{m{\cdot}s^{-1}}$, where the second is defined in terms of the cesium frequency $\Delta\nu_{\mathrm{Cs}}$. The meter may be expressed directly in terms of the defining constants (Eq.(1.1)):

$$1\,\mathrm{m} = \frac{9\,192\,631\,770}{299\,792\,458}\frac{c}{\Delta\nu_{\mathrm{Cs}}} \tag{1.1}$$

Previously, one meter was defined as the length traveled by light in 3.335641×10^{-9} s (based on the speed of light in a vacuum). It was also defined as 1,650,763.73 wavelengths in vacuum of the orange red line of the spectrum of krypton-86.

Most affected is the kilogram, which is currently fixed by a 143-year-old platinum alloy cylinder known as the "Le Grand K" and kept at the International Bureau of Weights and Measures (BIPM) in Paris. The kilogram is now defined by Planck's constant, h, recently measured with extraordinary precision. Its agreed value is set at $6.626\,070\,15 \times 10^{-34}$ kg m^2 s^{-1} when expressed in the unit J s, which is equal to kg m^2 s^{-1}, the meter and second being defined in terms of c and $\Delta\nu$. This means that the kilogram is defined in terms of Planck's constant instead of the mass of a cylinder of metal called International Prototype Kilogram.

Meanwhile, the ampere is determined by the elementary electric charge, e, which is given as $1.602\,176\,634 \times 10^{-19}$ when expressed in coulombs. The kelvin is determined by the fixed numerical value of Boltzmann's constant, k, which is $1.380\,649 \times 10^{-23}$ when expressed in units of J K^{-1}, and the mole is determined by Avogadro's constant (N_{A}), which contains exactly $6.02\,214\,076 \times 10^{23}$

atoms or molecules. This number is the fixed numerical value of the Avogadro constant, N_A, when expressed in units of mol^{-1}.

1.4 Derived International System (SI) Units

A derived SI unit is a measurement unit that is devised for a derived quantity different from primary units shown previously. Derived units combine different base units as described in Table 1.2. These derived units are obtained by simple mathematical transformations.

The Imperial unit system now includes the customary units of the United States in North America. The British Weights and Measures Act of 1824 established the Imperial unit system. Following that, the system was made official throughout the United Kingdom. It should be noted that some units are used in the United States but not in the United Kingdom, and vice versa. The differences are found in the following:

i) British fluid ounce = 0.961 US fluid ounce;
ii) US fluid ounce = 1.041 British fluid ounces;
iii) British Imperial gallon = 1.201 US gallons;
iv) US gallon = 0.833 British Imperial gallon.

1.5 SI Conversion

Converting SI units is very common when considering the SI unit and its related prefix described in Table 1.3.

This system comprises 7 base quantities (common) and 16 prefixes that designate the amount. The base unit and prefixes can be combined to produce the desired result.

Example 1.2 A car's weight can be written as 2000 kg, but it is better expressed in tons. It is no longer appropriate to write the results in grams. The possibilities for combining are limitless. It is

Table 1.3 SI Units and prefixes.

Prefix	Abbreviation	Meaning	Example
tera	T	10^{12}	1 terameter (Tm) = 10^{12} m
giga	G	10^{9}	1 gigameter (Gm) = 10^{9} m
mega	M	10^{6}	1 megameter (Mm) = 10^{6} m
kilo	k	10^{3}	1 kilometer (km) = 10^{3} m
deci	d	10^{-1}	1 decimeter (dm) = 10^{-1} m
centi	c	10^{-2}	1 centimeter (cm) = 10^{-2} m
milli	m	10^{-3}	1 millimeter (mm) = 10^{-3} m
micro	μ	10^{-6}	1 micrometer (μm) = 10^{-6} m
nano	n	10^{-9}	1 nanometer (nm) = 10^{-9} m
angstrom	Å	10^{-10}	1 angstrom (Å) = 10^{-10}m
pico	p	10^{-12}	1 picometer (pm) = 10^{-12} m
femto	f	10^{-15}	1 femtometer (fm) = 10^{-15} m

critical to present the measurement results in a clear and easy-to-understand figure. Based on the previous table:

1 g = 0.001 kg, which can be better presented as 1 g = 10^{-3} kg;
1 nm = 0,000000001 m, which can be better written as 1 nm = 10^{-9} m.

Or 1 000 000 mm = 1 km.

When converting using SI units, the prefix is very important if the user knows the ranking right away.

Example 1.3

567 m = 0.567 km (dividing by 1000 since 1 km = 1000 m)
30 s = 0.5 min (since 1 min = 60 s).

1.6 Fundamental Constants

As a general definition, a fundamental constant refers to a dimensionless physical constant. They are usually assumed to be universal and have constant quantitative values. The numbers are constant and do not involve any physical measurement.

a) ***The gravitation constant***

This is an empirical constant involving gravitational effects and used in Newton's law of universal gravitation, which states that all objects attract each other with a force that is proportional to the product of their masses (m_1 and m_2) and inversely proportional to the square of their distance, as shown in Eq.(1.2).

$$F_1 = F_2 = G\,(m_1 \times m_2)/r^2. \tag{1.2}$$

Where $G = 6.67430(15) \times 10^{-11}$ with the unit $m^3\ kg^{-1}\ s^{-2}$ in SI units.

b) ***The speed of light***

The speed of light is a constant, denoted by *c*, and is equal to 299 792 458 m/s (approximately 300,000 km/s, or 186,000 mi/s) in **vacuum**. It is defined as a universal **physical constant** that is important in many areas of **physics**. This constant is exact since the international agreement on the **meter** was defined as the length of the path traveled by light in the vacuum during a time interval of 1/299 792 458 **s**. This constant also features in Einstein's equation of mass-energy equivalence, $E = mc^2$.

c) ***Planck's constant***

Planck's constant, *h*, can be found in problems classified as quantum physics. It is a **physical constant** representing the **quantum** of electromagnetic action, relating the energy carried by a **photon** to its frequency. The product of Planck's constant by the frequency of a photon gives its energy.

In quantum mechanics, Planck's constant is of fundamental importance. It serves to define the kilogram in metrology. The value of Planck's constant is exact, with no uncertainty and is given as $h = 6.626\ 070 \times 10^{-34}$ J s (or J Hz^{-1}). Planck's constant may be used in the SI unit of frequency, and hence the so-called reduced Planck's constant is used instead, defined as $\hbar = h/2\pi$ ($\hbar$ is pronounced "h-bar").

The Planck length, denoted ℓ_P, is a unit of **length** describing the distance traveled by light in one unit of **Planck time** in a perfect vacuum. The Planck length ℓ_P is defined as ℓ_P = sqrt

($\hbar G/c^3$). This has been considered as the approximate equivalent value of this unit with respect to the meter:

$$\ell_P = 1.616229(38) \times 10^{-35} \text{m}, \tag{1.3}$$

where c is the **speed of light** in a vacuum, G is the **gravitational constant**, h is the **reduced Planck's constant**, and the two digits enclosed by parentheses are the standard uncertainty. This length is about 10^{-20} times the diameter of a proton.

The Planck mass, denoted by m_P, is the unit of **mass** in the system of **natural units** of **Planck units**. It is roughly equivalent to 0.021 milligrams (mg). For example, it is roughly the size of a flea egg. It is of the order of 10^{15} (a quadrillion) times larger than the highest energy available to contemporary **particle accelerators**. It is defined as: $m_p = \text{sqrt}(\hbar c/G)$, where c is the **speed of light** in a vacuum, G is the **gravitational constant**, and $\hbar$ is the **reduced Planck's constant**.

$$1 \text{ m}_p = 2.176435(24) \times 10^{-8} \text{ kg}. \tag{1.4}$$

The Planck time (t_P) is the **unit of time** in the system of **Planck units** in **quantum mechanics** as expressed in Equation 1.4. A Planck time unit is the **time** needed for **light** to travel a distance of one **Planck length** in a **vacuum**. This time is approximated as 5.39×10^{-44} s.

$$tp = \text{sqrt}(\hbar.G/c^5), \tag{1.5}$$

Where $\hbar = h/2\pi$ is the **reduced Planck's constant** (sometimes h is used instead of $\hbar$ in the definition), G = **gravitational constant**, and c = **speed of light** in **vacuum**.

Many other fundamental constants are discussed in their related areas toward the end of this book.

d) ***The standard acceleration for gravity***

The standard acceleration for gravity, known as g, varies depending on location and is equal to 9.809 m s^{-2} in USA.

e) ***Avogadro's number***

Avogadro's number refers to the number of units in one mole of any substance, which is also known as the molecular weight in grams. It is defined as $L = 6.02214199 \times 10^{23}$. The unit of this depends on the nature of the substance. It can be electrons, atoms, or molecules.

1.7 Common Measurements

The International System of units (SI) is used as a comparison framework for the most commonly used measurements in inspection and testing. It establishes seven fundamental units:

i) Meter [m] - length;
ii) Second [s] – time;
iii) Kilogram [kg] - mass;
iv) Ampere [A] - current;
v) Candela [cd] - light;
vi) Kelvin [K] - temperature;
vii) Mole [mole] – amount of substance.

Measurements are carried out in laboratories, outdoor and in situ in plants. Proper equipment is used to measure with a condition that has been previously calibrated.

Indirect measurements can be carried out using equations, with the outcomes being the results of the execution of these equations.

Example 1.4

- voltage [V] = resistance [ohm] × current [A]; hence, current = voltage/resistance.
- area [m^2]= length [m] × length [m]
- pressure [Pa] = force [N] / area [m^2].

Accuracy in measurements is required in many fields, and because all measurements are close approximations, great care must be taken when taking measurements.

Example 1.5 When calibrating, you must generate a known amount of the variable to be measured as well as the SI unit under test.

1.8 Principles and Practices of Traceability

The objective as introduced in this book is to learn and understand measurements and their related calibration and standards, as well as principles and practice of traceability. This is a short introduction.

1.8.1 Definition of Traceability

It is defined as the ability to link the results of the calibration and measurements to the related standard and/or reference through an unbroken chain of comparisons.

The international vocabulary of metrology (VIM) defines traceability as the property of the result of a measurement or the value of a standard that can be related to stated references, usually national or international standards, by an unbroken chain of comparisons, all with stated uncertainties [4]. The unbroken chain of comparisons is called the "traceability chain."

The latter is composed of a number of instruments linked together to supply measurement. The competence and uncertainty are essential elements in the traceability according to ISO 17025 section 5.6.

Because there is always a difference in measurement between the output of the instrument and the true value of the measurand, measurement uncertainty is used to evaluate a quantitative statistical estimate of the limits of that difference. This will be discussed in chapters 3 and 4. VIM defines the measurement uncertainty as a parameter associated with the results of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand.

The calibration is typically performed by measuring a test unit against a known standard or reference. National measurement institutes across countries are typically a source of official approvals and verification for the work performed of various types of measurements, such as NIST (USA), NPL (UK), and BNM (France). The traceability has three essential components described as follows:

- Traceable calibration requiring comparisons with traceable standards or reference materials;
- Traceable calibrations can be performed only by competent laboratories with accreditation to ISO 17025;
- A traceable calibration certificate must contain an estimate of the uncertainty associated with the calibration.

Example of Traceability of Measurement in a Coordinate Measuring Machine (CMM)
Traceability guarantees the proper use of a CMM [5] in a quality management system. It constitutes the most fundamental and important aspects in the operations of a CMM. The aspects of concern are the actual physical chains by which measurement may be related to the SI unit of length. The physical chain of traceability is described as follows:

1) Laser wavelength ref. to atomic clock frequency ($4/10^{14}$);
2) Laser displacement interferometer ($2.5/10^{11}$);
3) Calibrated line scale or step gage ($2–5/10^{8}$);
4) CMM ($1/10^{6}$).

1.8.2 Accreditation and Conformity Assessment

According to the International Organization for Standardization, or ISO, [6]: "Conformity assessment is the term given to different techniques that ensure a product, process, service, management system, person or organization fulfils specified requirements. This section describes the main conformity assessment techniques and explains how to combine them to form conformity assessment schemes. The icons below depict the main conformity assessment techniques and their most common applications. In some instances, one conformity assessment technique may encompass another, e.g. an inspection can include a test technique, or a product evaluation may take into account a test report or an inspection report. How these conformity assessment techniques are used and interrelated is often prescribed in a specific mandatory or voluntary **conformity assessment scheme.**"

Products and equipment must meet specifications, service, and process requirements, especially when sold globally; thus, this business in the global market and world trade necessitates *conformity assessment*. It primarily contains standard measurements based on well-known calibration and references with certifications. As a result, we recognize the significance of international standards in addressing the conformity assessment services provided by ISO.

- The conformity assessment needs are shown in Table 1.4, where their use under ISO standards by first parties that are the suppliers;
- The second parties are defined as the customers, regulators, and trade organizations;
- Third parties are represented by bodies independent from both suppliers and customers.

With the increasing use of these conformity assessment tools, assurance of the competence of conformity assessment bodies (CABs) [7] becomes important, with recognition of this as CABs *accreditation*.

The International Laboratory Accreditation Cooperation (ILAC) [8] is the world's principal international forum for the development of laboratory accreditation practices and procedures. ILAC

Table 1.4 Standards of conformity assessment tools.

Conformity assessment	First parties	Second parties	Third parties	Corresponding ISO standards
Declaration of supplier	✓			ISO/IEC 17050
The calibration and testing	✓	✓	✓	ISO/IEC 17025
The inspection	✓	✓	✓	ISO/IEC 17020
The certification			✓	ISO 17021

promotes laboratory accreditation as a tool for trade facilitation, as well as the recognition of competent calibration and testing facilities worldwide. Table 1.4 shows the conformity assessment tool standards.

Multiple Choice Questions of this Chapter

Multiple Choice Questions are given for each chapter with solutions in an online extension of this book. Please use link: www.wiley.com\go\mekid\metrologyandinstrumentation\

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