

Introduction

CHAPTER ROAD MAP Our purpose is to provide a path so that any student can learn the main ideas of fluid mechanics. This path begins in this chapter in which we present (a) introductory fluid mechanics topics and (b) descriptions of engineering and professional skills. Of course, the primary reason for learning fluid mechanics is so that you can solve problems associated with technology; see Fig. 1.1.

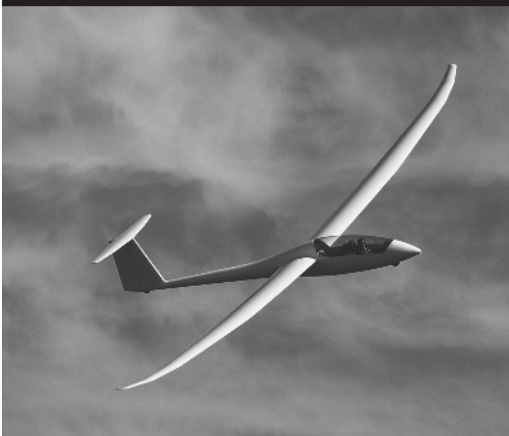


FIGURE 1.1

As engineers, we get to design fascinating systems like this glider. This is exciting! (© Ben Blankenburg/Corbis RF/Age Fotostock America, Inc.)

LEARNING OUTCOMES

ENGINEERING FLUID MECHANICS (§1.1*)

- Define engineering.
- Define fluid mechanics.

MATERIAL SCIENCE TOPICS (§1.3)

- Explain material behaviors using either a microscopic or a macroscopic approach or both.
- Know the main characteristics of liquids, gases, and fluids.
- Understand the concepts of body, material particle, body-as-a-particle, and the continuum assumption.

DENSITY AND SPECIFIC WEIGHT (§1.6)

- Know the main ideas about $W = mg$.
- Know the main ideas about density and specific weight.

THE IDEAL GAS LAW (IGL) (§1.7)

- Describe an ideal gas and a real gas.
- Convert temperature, pressure, and mole/mass units.
- Apply the IGL equations.

PREREQUISITE ENGINEERING SKILLS (§1.1, §1.3, §1.4, §1.5, §1.8, §1.9)

- Apply problem-solving methods to fluid mechanics problems.
- Apply critical thinking to fluid mechanics problems.
- Describe modeling.
- Make estimates when solving fluid mechanics problems.
- Apply ideas from calculus to fluid mechanics.
- Carry and cancel units when doing calculations.
- Check that an equation is DH (dimensionally homogeneous).

*The symbol § means “section”; for example, the notation “§1.1” means Section 1.1.

1.1 Engineering Fluid Mechanics

In this section, we (a) explain what engineering fluid mechanics means, (b) introduce *critical thinking* (CT), a method that is at the heart of doing engineering well, and (c) introduce a method for explaining what a term means.

The Meaning of Fluid Mechanics

Fluid mechanics is a **subject within mechanics** that equips a person to solve engineering problems when the situation involves a flowing or stationary gas or liquid.

The subject of **mechanics** involves loads, energy, motion, deformation, failure, and so on. Mechanics is comprised of the subjects of statics, dynamics, strengths of materials, fluid mechanics, and advanced topics. Engineers organize mechanics into solid mechanics and fluid mechanics, where solid mechanics focuses on matter in the solid phase and fluid mechanics focuses on matter in the liquid and gas phases. Subjects that are prerequisite to mechanics include calculus, chemistry, and physics.

Engineering is a subject that equips a person to solve problems that involve technology as in the (a) invention of a new technology, (b) design of products that requires application of a technology, (c) research to create new knowledge relevant to a technology, (d) application of a technology, and so forth.

Regarding examples of the application of fluid mechanics, you can look at your everyday world and see too many examples to count. For instance, fluids mechanics is applied to create and improve the following:

1. Systems to supply safe drinking water to towns and cities
2. Airplanes that fly safely
3. Dams that resist enormous hydrostatic loads
4. Automobiles that have low aerodynamic drag
5. Machines that force liquid plastic into injection molds
6. Pipe systems that transport oil and gas
7. Power plants that generate electricity
8. Systems to transport and treat sewer
9. Heating, ventilating, and air conditioning systems
10. Systems to pump and transport diesel through the fuel line in a semi truck

In summary, the subject of fluid mechanics is useful to those individuals who want to invent, or apply, or improve technology that benefits people and communities.

Critical Thinking (CT)

Because critical thinking (CT) is needed for solving fluid mechanics problems, we next present a brief summary.

Critical thinking (CT) is the **subject** that equips an individual to make good decisions instead of bad decisions by using facts together with a purposeful and systematic organization of these facts in ways that justify a claim.

Since CT is a subject, it can be taught, learned, and applied just like algebra, cooking, chemistry, or driving a car. In the CT nomenclature—that is, the CT naming scheme—the person making a claim is called the **arguer**. The purpose of CT is to avoid making or agreeing with bad decisions.

1. A **good decision** is a decision that leads to the maximum or the near maximum benefits to the arguer and to those stakeholders who are impacted by the decision.
2. A **bad decision** is a decision that does not tend to maximize the benefits.

To apply CT, an arguer gives facts. A **fact** is a statement that has been or can easily be proven to be true. Five examples of facts are as follows:

1. The density of water at 4°C and 101 kPa is 1000 kg/m³.
2. The sum of the angles of a triangle is 180°, which is π radians.
3. Pressure is the ratio of normal force to the area at a point.
4. For a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.
5. A manometer is a device that uses one or more liquid columns to measure differential pressure.

In the CT nomenclature, a statement that might be true or false is called a **premise**. In classical logic, the truth value of a premise is either true or false. In general, when an arguer states a premise, they believe that the premise is true.

In the CT nomenclature, a decision is called a claim. A **claim** is a statement that the arguer proposes or asserts as the truth, the best solution, the recommendation, the right answer, the findings, and so forth. Examples of claims are as follows:

1. A 2.5 kW pump is the best option for the proposed system.
2. The given statement is false.
3. Carrying and canceling units will save time and lead to fewer mistakes.
4. The energy equation can be applied to the present situation.
5. The head loss for the proposed penstock is 7.4 m.

A subclaim is a subordinate claim that supports a main claim. Both a main claim and a subclaim are represented by this symbol $\therefore$, which means therefore or thus. For example, the statement

$$\therefore F = 7.4 \text{ N}$$

means “therefore, the force is 7.4 newtons.”

In professional fields such as mathematics, science, engineering, business, and law, a claim must be justified to be accepted. There are two ways to justify a claim:

1. Prove that the claim must be true.
2. Provide evidence that shows that there is a high probability that the claim is true.

In all cases, the justification is called the reasoning. **Reasoning** itself is the list of facts, definitions, and subclaims that collectively justify the central claim or claims being made by the arguer.

Three common types of reasoning are (a) deductive, (b) inductive, and (c) abductive. The frame that follows gives one example of each.

EXAMPLES OF COMMON TYPES OF REASONING

Deductive. It is true that $A = B$. It is also true that $B = C$. Thus, $A = C$.

Inductive. The sun rose today and yesterday. The sun has risen every day that I can remember. Thus, the sun will rise tomorrow and each day after that.

Abductive. The car has stopped running. Because it has been many miles since we refueled, the vehicle is out of fuel.

In deductive reasoning, the arguer begins with statements known to be true and then uses these premises to prove that the claim must be true. An arguer applies deductive reasoning for a mathematical proof. Similarly, an arguer uses deductive reasoning to solve most textbook problems in engineering classes.

In inductive reasoning, the arguer begins with facts from specific cases and then generalizes these facts to make a claim. Inductive reasoning is applied to justify most scientific laws such as the ideal gas law, Ohm's law, and $\Sigma F = ma$.

In abductive reasoning, the arguer uses facts plus a theory to justify a claim. In the example just presented, the theory involves the knowledge that a car needs fuel to operate.

In the CT nomenclature, an **argument** is a claim plus the reasoning that supports the claim. Notice that this definition differs from the more familiar meaning which is that an argument is an exchange of diverging views, typically with elevated levels of voice, frustration, anger, and so on.

If you are interested in learning more about CT, then we recommend *Logic* by Hurley and *Critical Thinking* by Moore and Parker.

Defining a Term

To learn fluid mechanics, or any other subject, it is critical to learn the vocabulary of the subject. To this end, the following structure is useful:

THE STANDARD STRUCTURE OF A DEFINITION

The concise and logical way to define a term is to

1. List the term (word or phrase) to be defined
2. List the class to which the term belongs
3. List the characteristics that distinguish this term from all other members of its class

Example: Water (term) is a molecule (class) made up of atoms of hydrogen and oxygen in the ratio of 2 to 1 (distinguishing characteristics).

To reveal the parts of a definition, we annotate like this: A **gas** is a **phase of matter** in which the atomic particles are far apart and the atomic particles are free to move about. The key is

1. The term being defined is shown in a **bold font**.
2. The class is also shown in a **bold font**.
3. Each distinguishing characteristic is underlined.

Example: **Weight** is a **property of a body** that characterizes the pull of a nearby planet, typically Earth, in units of force as predicted by Newton's law of universal gravitation (NLUG).

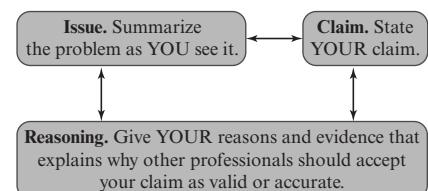
For two more examples of the standard definition, see *fluid mechanics* and *critical thinking* that were presented earlier in this section.

Regarding “how to do critical thinking,” we teach and we apply the **Standard Structure of CT** (Fig. 1.2), which involves three methods:

1. **Issue.** Define the problem you are trying to solve so that is clear and unambiguous. Note that you will often need to rewrite or paraphrase the issue or question.
2. **Reasoning.** List the reasons that explain why professionals should accept your claim (i.e., your answer, your explanation, your conclusion, or your recommendation). To create your reasoning, take actions such as stating facts, citing references, defining terms, applying deductive logic, applying inductive logic, and building subconclusions.
3. **Claim.** State your claim. Make sure your claim addresses the issue. Recognize that a claim can be presented in multiple ways, such as an answer, a recommendation, or your stance on a controversial issue.

FIGURE 1.2

The Standard Structure of Critical Thinking (SSCT).



1.2 Modeling in Fluid Mechanics and Engineering

Modeling in Fluid Mechanics

In engineering subjects—such as fluid mechanics—modeling is everywhere present. For example, most of the problems in this text are related to modeling of a complex system such the Glenn Canyon Dam (Fig. 1.3).



FIGURE 1.3

The Glenn Canyon Dam.
(Photo by Donald Elger.)

For solving an engineering problem, the Glen Canyon dam might be modeled like this:

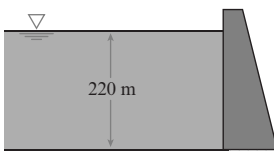


FIGURE 1.4

A model of the Glenn Canyon Dam.

where the engineering sketch shown in Fig. 1.4 communicates that the dam is modeled as a trapezoidal prism, and the water is modeled as a stationary reservoir that is 220 m deep. In this example, the symbol ∇ represents an interface between air and water, and the shaded region $\blacksquare$ represents water.

Some of the symbols used in drawings are presented in this table.

Example of symbols that are used in engineering drawings

Description	Symbol
The interface between two fluids	∇
The center of mass	$\odot$
The center of pressure	$\oplus$
A liquid; e.g., water, oil, or kerosene	$\blacksquare$

Modeling in Engineering Practice

A **model** is a simplified version of reality that provides utility to people by capturing essential features while excluding nonessential features. For example, a map of Crater Lake National Park in Oregon, USA, shows the essential features of the geography in the vicinity of the lake. For example, an architect's drawing of a building shows the essential features of what the architect proposes to build.

Some examples of the types of models used by engineers are as follows:

- A *physical model* of a spillway for a proposed dam modification
- A *math model* that predicts the speed and fuel consumption for a proposed new airplane
- An *how-things-work model* that describes how a piston pump works

Engineers use models because modeling (a) saves time, (b) reduces costs, (c) helps us to understand the physical world, (d) helps us do things in safe ways, and so forth.

1.3 Modeling of Materials

To model materials, engineers and scientists have developed the ideas that are presented in this section.

The Microscopic and Macroscopic Descriptions

Engineers strive to understand things. For example, an engineer might ask, why does steel alloy #1 fail given that steel alloy #2 does not fail in the same application? Or, an engineer might ask, why does water boil? Why does this boiling sometimes damage materials, as in cavitation? To address questions about materials, engineers often apply the following ideas:

- **Microscopic Description.** Explain something about a material by describing what is happening at the atomic level (i.e., describing the atoms, molecules, electrons, etc.).
- **Macroscopic Description.** Explain something about a material without resorting to descriptions at the atomic level.

Forces between Molecules

One of the best ways to understand materials is to apply the idea that molecules attract one another if they are close together and repel if they are too close[†] (Fig. 1.5).

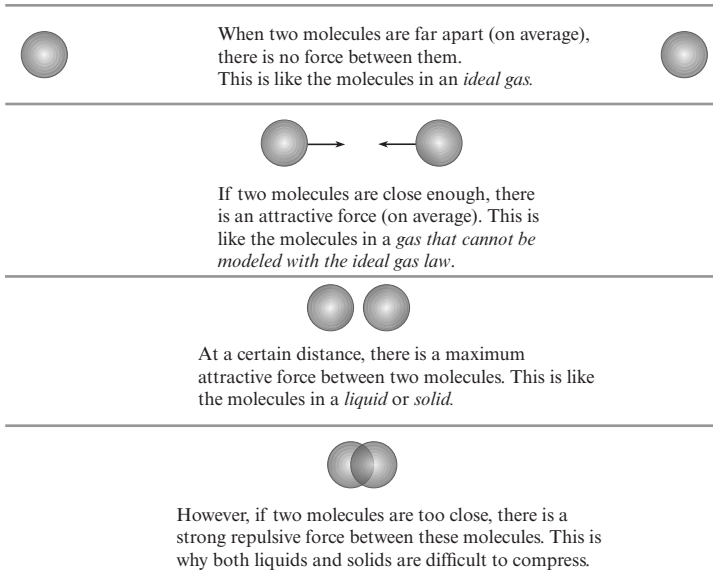
Defining the Liquid, Gas, and Fluid

In science, there are four states of matter: gas, liquid, solid, and plasma. A **gas** is a state of matter in which the molecules are on average far apart so that the forces between molecules (or atoms) are typically very small or zero. Consequently, a gas lacks a fixed shape, and it also lacks a fixed volume, because a gas will expand to fill its container.

A **liquid** is a state of matter in which the molecules are on average close together so that the forces between molecules (or atoms) are strong. In addition, the molecules are relatively free to move around. In comparison, when a material is in the solid state, atoms tend to be

*Cavitation is explained in §5.5.

[†]Dr. Richard Feynman, who won the Nobel Prize in Physics, calls this the *single most important idea* in science. See the *Feynman Lectures on Physics*, Vol. 1, p. 2.

**FIGURE 1.5**

A description* of the forces between molecules.

fixed in place—for example, in a crystalline lattice. Thus, a liquid flows easily as compared to a solid. Due to the strong forces between molecules, a liquid has a fixed volume but not a fixed shape.

The term **fluid** refers to both a liquid and a gas and is generally defined as a state of matter in which the material flows freely under the action of a shear stress.[†]

Table 1.1 provides additional facts about solids, liquids, and gases. Notice that many features in this table can be explained by applying the ideas in Fig. 1.5. **Example.** The density of a liquid or a solid is much higher than the density of a gas because the strong attractive forces in a liquid or solid act to bring the molecules closer together. **Example.** A liquid is difficult to compress because the molecules will have strong repulsive forces if they are brought close together. In contrast, a gas is easy to compress because there are no forces (on average) between the molecules.

The Body, the Material Particle, the Body-as-a-Particle

Engineers have invented terms that can be used to describe *any material*. Learning this vocabulary will help you learn engineering.

In engineering, the term “body” or “material body” has a special meaning. Examples. A coffee cup can be a *body*. The air inside a basketball can be a *body*. A jet airplane can be a *body*. **Body** or **material body** is a label to identify objects or matter that exist in the real world, without specifying any specific object. It is like applying the term “sports” to identify many activities (e.g., soccer, tennis, golf, or swimming) without specifying a particular sport.

A **material particle** is a small region of matter within a *material body* (Fig. 1.6). Some useful facts about material particles are as follows:

- A material particle is often imagined to be *infinitesimal* in the calculus sense.
- A material particle can be selected or visualized so that it has any shape (e.g., spherical, cubical, cylindrical, or amorphous[‡]).
- The term “fluid particle” refers to a material particle that is comprised of a liquid or a gas.

*For additional details about forces between molecules, consult an expert source, such as a chemistry text or a professor who teaches material science.

[†]Shear stress is explained in §2.4.

[‡]“Amorphous” means without a clearly defined shape or form.

TABLE 1.1 Comparison of Solids, Liquids, and Gases

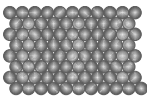
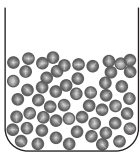
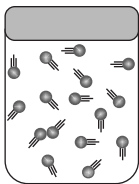
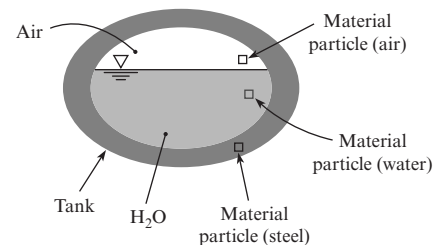
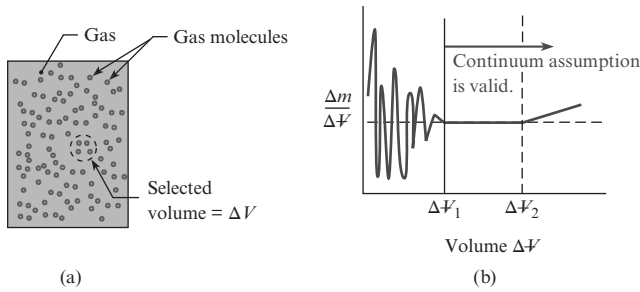
Attribute	Solid	Liquid	Gas
Typical Visualization			
Description	Solids hold their shape; no need for a container	Liquids take the shape of the container and will stay in an open container	Gases expand to fill a closed container
Mobility of Molecules	Molecules have low mobility because they are bound in a structure by strong intermolecular forces	Molecules move around freely even though there are strong intermolecular forces between the molecules	Molecules move around freely with little interaction except during collisions; this is why gases expand to fill their container
Typical Density	Often high; e.g., the density of steel is 7700 kg/m^3	Medium; e.g., the density of water is 1000 kg/m^3	Small; e.g., the density of air at sea level is 1.2 kg/m^3
Molecular Spacing	Small—molecules are close together	Small—molecules are held close together by intermolecular forces	Large—on average, molecules are far apart
Effect of Shear Stress	Produces deformation	Produces flow	Produces flow
Effect of Normal Stress	Produces deformation that may associate with volume change; can cause failure	Produces deformation associated with volume change	Produces deformation associated with volume change
Viscosity	NA	High; decreases as temperature increases	Low; increases as temperature increases
Compressibility	Difficult to compress; bulk modulus of steel is $160 \times 10^9 \text{ Pa}$	Difficult to compress; bulk modulus of liquid water is $2.2 \times 10^9 \text{ Pa}$	Easy to compress; bulk modulus of a gas at room conditions is about $1.0 \times 10^5 \text{ Pa}$

FIGURE 1.6

To find examples of material particles: (1) Select any body; for example, we selected a steel tank filled with water and air. (2) Select a small amount of matter and define this small chunk of matter as a material particle. This figure shows a material particle comprised of air, a material particle comprised of water, and a material particle comprised of steel.



There is another way that engineers use the term “particle.” For example, to model the motion of an airplane, an engineer can idealize the airplane as a *particle*. A physics book might state that Newton’s second law of motion only applies to a *particle*. In this context, the term has a different meaning than *material particle*. This alternative concept is that the **particle (the body-as-a-particle)** is a way to idealize a material body as if all the mass of the body is concentrated at a point and the dimensions of the body are negligible.

**FIGURE 1.7**

When a measuring volume ΔV is large enough for random molecular effects to average out, the continuum assumption is valid.

Summary. There are two distinct concepts used in engineering: the *material particle* and the *body-as-a-particle*. However, it is common for the label “particle” to be used for both of these ideas. Engineers typically figure out which idea is meant by the context in which the term is being used.

The Continuum Assumption

Because a body of fluid is comprised of molecules, properties are due to average molecular behavior. That is, a fluid usually behaves as if it were comprised of continuous matter that is infinitely divisible into smaller and smaller parts. This idea is called the **continuum assumption**.

When the continuum assumption is valid, engineers can apply limit concepts from differential calculus. A limit concept typically involves letting a length, an area, or a volume approach zero. Because of the continuum assumption, fluid properties such as density and velocity can be considered continuous functions of position with a value at each point in space.

To gain insight into the validity of the continuum assumption, consider a hypothetical experiment to find density. Fig. 1.7a shows a container of gas in which a volume ΔV has been identified. The idea is to find the mass of the molecules Δm inside the volume and then to calculate density by

$$\rho = \frac{\Delta m}{\Delta V}$$

The calculated density is plotted in Fig. 1.7b. When the measuring volume ΔV is very small (approaching zero), the number of molecules in the volume will vary with time because of the random nature of molecular motion. Thus, the density will vary as shown by the wiggles in the plotted line. As volume increases, the variations in calculated density will decrease until the calculated density is independent of the measuring volume. This condition corresponds to the vertical line at ΔV_1 . If the volume is too large, as shown by ΔV_2 , then the value of density may change due to spatial variations.

In most applications, the continuum assumption is valid, as shown by the next example.

EXAMPLE. Probability theory shows that including 10^6 molecules in a volume will allow the determination of density to within 1%. Thus, a cube that contains 10^6 molecules should be large enough to accurately estimate macroscopic properties such as density and velocity. Find the length of a cube that contains 10^6 molecules. Assume room conditions. Do calculations for (a) water and (b) air.

Solution. (a) The number of moles of water is $10^6 / 6.02 \times 10^{23} = 1.66 \times 10^{-18}$ mol. The mass of the water is $(1.66 \times 10^{-18} \text{ mol})(0.0180 \text{ kg/mol}) = 2.99 \times 10^{-20} \text{ kg}$. The volume of the cube is $(2.99 \times 10^{-20} \text{ kg}) / (999 \text{ kg/m}^3) = 2.99 \times 10^{-23} \text{ m}^3$. Thus, the length of the side of a cube is $3.1 \times 10^{-8} \text{ m}$. (b) Repeating this calculation with air gives a length of $3.5 \times 10^{-7} \text{ m}$.

Review. For the continuum assumption to apply, the object being analyzed would need to be larger than the lengths calculated in the solution. If we select 100 times larger as our criteria, then the *continuum assumption applies* to objects with

- Length (L) $> 3.1 \times 10^{-6}$ m (for liquid water at room conditions)
- Length (L) $> 3.5 \times 10^{-5}$ m (for air at room conditions)

Given the two length scales just calculated, it is apparent that the *continuum assumption applies to most problems of engineering importance*. However, there are a few situations where the problem length scales are too small.

EXAMPLE. When air is in motion at a very low density, such as when a spacecraft enters the Earth's atmosphere, then the spacing between molecules is significant in comparison to the size of the spacecraft.

EXAMPLE. When a fluid flows through the tiny passages in nanotechnology devices, then the spacing between molecules is significant compared to the size of these passageways.

1.4 Weight, Mass, and Newton's Law of Gravitation

This section reviews weight and mass and also introduces ideas (called the “voice of the engineer”) that will help you learn fluid mechanics better.

Voice of the Engineer. *Build working knowledge in every subject that you learn.* Working knowledge is defined as knowledge that you have firmly locked into your brain (no need to look up anything) that is useful for engineering tasks. **Rationale.** Working knowledge is essential for estimation and validation, and these two skills are essential for doing engineering well. Examples of working knowledge are as follows:

- 1.0 pound of force (i.e., 1.0 lbf) is about 4.5 newtons.
- 1.0 horsepower is about 750 watts.
- The weight of water at typical room conditions is about 10,000 newtons for each cubic meter.

Voice of the Engineer. Learn the meaning of main concepts such as mass and force. **Rationale.** Understanding concepts and the relationships between these concepts is needed if you want to apply your knowledge.

Defining Mass

The *mass* of 1.0 liter of liquid water at room conditions is 1.0 kilogram. A body with a *mass* of 2.0 slugs has a *mass* of 29 kilograms. In Newton's second law, the sum-of-forces-vector is exactly balanced by the product of the *mass* and the acceleration. **Mass** is a property of a *body* that provides a measure of the amount of matter in the body. For example, Body A, which has a mass of 20 grams, has more matter than Body B, which has a mass of 5 grams.

Recommended working knowledge. Know four mass units (kilograms, grams, slugs, and pounds mass) and be able to convert between these units without the need of a calculator.* Regarding conversion formulas, see Table F.1, which is located on the inside cover of this text.

Defining Force

When water falls in a waterfall, we can say that the Earth is pulling on the water with a force that is called the *gravity force*. When wind blows on a stop sign, we can say that the air is exerting a *drag force* on the sign. When water behind a dam pushes on the dam, we can say that the water is exerting a *hydrostatic force* on the face of the dam.

*Your accuracy should be typical of an engineering estimate—for example, within 10% of the number you would get if you used a calculator.

Some facts about force are as follows:

- Every force can be thought of a push or a pull of one body on another.
- Force is a vector. In this text, we use a bold face roman font (e.g., **F**) to represent a vector. To represent the magnitude of a vector, we use an italic font (e.g., *F*).
- *Recommended working knowledge.* Know two force units: pounds-force (lbf) and newtons (N). Be able to convert units (i.e., make estimates) without the need of a calculator.
- Forces classify into two categories:
 - A **surface force** is any force that requires the two bodies to be touching. Most forces are surface forces. Some books use the term *contact force*.
 - A **body force** is any force that does not require the two bodies to be touching. There are only a few types of body forces (e.g., the *gravity force*, the *electrostatic force*, and the *magnetic force*).
- Another way to describe forces is to talk about *action forces* (a force that acts to cause a body to accelerate) and *reaction forces* (a force that acts to prevent a body from accelerating; typically a force from a support). We do not use the concepts of action and reaction forces in this textbook.

In summary, a **force** is a push or pull between two bodies. A push or pull is an action that will cause a body to accelerate if the sum-of-forces vector in Newton's second law of motion is nonzero.

Equation Literacy

Voice of the Engineer. Build equation literacy in all your engineering subjects. **Rationale.** Equation literacy is essential for building math models, and building math models is arguably the most important skill of the engineering method.

You have **equation literacy** for equation XYZ if you can do the following tasks: (1) You can explain how the equation was derived or where the equation came from. (2) You can explain the main ideas—that is, the physical interpretation—of the equation. (3) You can list the common equational forms, define each variable, and state the units and dimensions. (4) You can describe the assumptions and limitations of the equation and make correct choices about when to apply this equation or when to avoid applying this equation. (5) You have a systematic method for applying the equation correctly.

Newton's Law of Universal Gravitation (NLUG)

Newton's Law of Universal Gravitation (NLUG) reveals that any two bodies will attract each other with a force **F**, which is called the gravitational force (Fig. 1.8). Because this idea applies to any two bodies located anywhere in the universe, the equation is *universal* (hence the name).

The magnitude of the gravitational force *F* is given by

$$F = G \frac{m_1 m_2}{R^2} \quad (1.1)$$

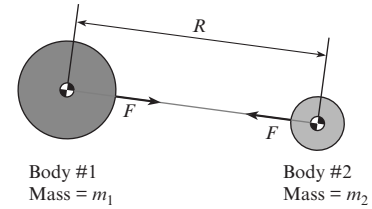
where the term $G = 6.674 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$ is called the *gravitational constant*, m_1 is the mass of Body #1, m_2 is the mass of Body #2, and R is the distance between the center of mass of each body.

The law of gravity, like nearly all scientific laws, was developed by *inductive reasoning*. In particular, Newton examined data on planetary motion and found that the data were fit by Eq. (1.1). Newton concluded that the equation must be true in general.

To apply Eq. (1.1) on the Earth, start with Fig. 1.8 and let Body #1 represent the Earth and Body #2 represent a body that is on or near the surface of the Earth. Now, G and m_1

FIGURE 1.8

Any two bodies will attract each other. The corresponding force is called the **gravitational force**. Note that the magnitude of the gravitational force on Body #1 equals the magnitude of the gravitational force on Body #2.



are constant and R is very nearly constant. Thus, define a new constant g that is given by $g \equiv Gm_E/R_E^2$, in which the subscript E denotes the Earth. Also, rename the gravitational force F to be the weight of the body W . Then, Eq. (1.1) simplifies to

$$W = mg \quad (1.2)$$

where W is the weight of a body on a planet (typically Earth), m is the mass of the body, and g is a constant.

Useful Facts and Information

- The constant g is called **gravitational acceleration**. On the Earth, this parameter varies slightly with altitude; however, engineers commonly use the standard value, which is $g = 9.80665 \text{ m/s}^2 = 32.1740 \text{ ft/s}^2$.
- Gravitational acceleration (g) has a useful physical interpretation; g is the vertical component of acceleration that results when the vertical component of the sum-of-forces-vector in Newton's second law of motion is exactly equal to the weight vector.
- In general, a falling body will not accelerate at a rate g because of the presence of additional forces, such as the lift force, the drag force, or the buoyant force.
- It is common for people to state that $W = mg$ is derived from $\Sigma \mathbf{F} = m\mathbf{a}$. However, it is more correct to say that $W = mg$ is derived from NLUG.
- **Weight** is the gravitational force acting on a body from a planet (typically Earth).
- Weight and mass are different concepts. For example, the mass of a body is the same at any location, whereas the weight can change with location. For example, if a body weighs 60 newtons on Earth, the same body will weigh about 10 newtons on the Moon. Also, recognize that it is common (but incorrect) to report a weight using mass units. For example, to say that a body weighs 10 g or that a body weighs 60 kg is incorrect.

Relating Force and Mass Units

We wrote this section because we have seen many mistakes involving force and mass units. Three useful ideas about units are (1) units were invented by people, (2) units are related to each other by equations, and (3) the definition of a given unit can be looked up.

The definition of a newton is “one newton of force is the quantity of force that will give one kilogram of mass an acceleration of one meter per second squared.”

To relate force and mass units, engineers start with Newton's second law of motion ($\Sigma \mathbf{F} = m\mathbf{a}$). Next, apply the definition of the newton to conclude that it must be true that

$$(1.0 \text{ N}) \equiv (1.0 \text{ kg})(1.0 \text{ m/s}^2) \quad (1.3)$$

Since Eq. (1.3) is true, it must also be true (by algebra) that

$$1.0 = \left(\frac{\text{kg} \cdot \text{m}}{\text{N} \cdot \text{s}^2} \right) \quad (1.4)$$

Thus, the weight of a 2.0 kg body must be 19.6 N because of the analysis shown in Eq. (1.5).

$$W = mg = \frac{2.0 \text{ kg}}{1} \left| \frac{9.81 \text{ m}}{\text{s}^2} \right| \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} = \boxed{19.6 \text{ N}} \quad (1.5)$$

Do you see the logic? Eq. (1.5) must be true because it is based on correct facts that are applied in a correct way. The main issue that we want to address is that many people become confused with English units. However, with English units you can apply the same logic. In particular, start with the definition of the pound-force (lbf). One pound of force is the amount of force that will accelerate one pound of mass (lbm) at a rate of 32.2 ft/s^2 . Thus, it is true that

$$(1.0 \text{ lbf}) \equiv (1.0 \text{ lbm})(32.2 \text{ ft/s}^2) \quad (1.6)$$

Since Eq. (1.6) is true, it must also be true (by algebra) that

$$1.0 = \left(\frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} \right) \quad (1.7)$$

Thus, the weight of a 2.0 lbm body must be 2.0 lbf because of the analysis shown in Eq. (1.8).

$$W = mg = \frac{2.0 \text{ lbm}}{1} \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| \frac{\text{lbf} \cdot \text{s}^2}{32.2 \text{ lbm} \cdot \text{ft}} = \boxed{2.0 \text{ lbf}} \quad (1.8)$$

Eq. (1.8) shows that the magnitude of the weight (2.0) is the same as the magnitude of the mass (2.0). This occurs because of the way that English units are defined. It is correct to say that a body that has a mass of 2.0 lbm will have a weight of 2.0 lbf on Earth. However, avoid generalizing this. For example, a body with a mass of 2.0 lbm will have a weight about 0.33 lbf on the moon. Also, avoid saying that 2.0 lbm equals 2.0 lbf because mass and weight are different concepts.

The General Equation

A **general equation** is an equation that applies to many or all problems. A **special-case equation** is an equation that is derived from a general equation but is more limited in scope because there are assumptions that must to be met in order to apply the special-case equation.

Voice of the Engineer. *Learn the general equations and then derive each special-case equation on an as needed basis.* **Rationale.** (1) Given that there are only a few general equations, this approach will make your learning simpler. (2) You are less likely to make mistakes because general equations, by definition, apply more often than special-case equations. Examples of general and special-case equations follow.

- NLUG is a general equation, and $W = mg$ is a special-case equation that is derived from NLUG.
- Newton's second law of motion, $\Sigma \mathbf{F} = m\mathbf{a}$, is a general equation; note that this is a vector equation. Some special-case equations that can be derived from this equation are $\Sigma F_x = ma_x$ (a scalar equation) and $\Sigma F_z = 0$ (also a scalar equation).
- The general equation that defines mechanical work W is the line integral of the force vector dotted with the displacement vector $W = \oint_{x_1}^{x_2} \mathbf{F} \cdot d\mathbf{x}$. One associated special-case equation is $W = Fd$, where W is work, F is force, and d is displacement.

1.5 Essential Math Topics

Estimates

Voice of the Engineer. *Become skilled with pencil/paper estimates.* A **pencil/paper estimate** is defined as an estimate that you can do using only your brain, a pencil, and a sheet of paper (i.e., no books, calculators, or computers needed). **Rationale:** (1) All engineering calculations are estimates anyway; learning pencil/paper estimation skills will give you great insight into the nature of engineering estimates. (2) In the process of learning how to do pencil/paper estimates, you will acquire a great deal of practical knowledge. (3) You will save yourself huge amounts of time because you will do calculations much faster. (4) You will have strong methods for validating your technical work. (5) It is fun to figure out clever ways to estimate things.

Four Tips for Representing Numbers

To represent your numerical results in simple and effective ways, we have four recommendations:

1. Represent your result so that the digit term is between 0.1 and 1000; this makes your result easier to understand and remember. For example, 645798 can be represented as 646E3 or as 64.6E4 or as 6.46E5.
2. Use scientific or engineering notation to represent large and small numbers.
3. Use metric prefixes to represent numbers; for example, 142,711 pascals can be represented as 143 kPa.
4. Use a maximum of three significant figures to represent your final answers (unless you can justify more significant figures).

Scientific notation is a method of writing a number as a product of two numbers: a digit term and an exponential term. For example, the number 7600 is written as the product of 7.6 (the digit term) and 10^3 (the exponential term) to give 7.6×10^3 . **Fact.** There are three common forms of scientific notation, which are as follows: $7.6 \times 10^3 = 7.6\text{E}3$ (upper case “E”) = $7.6\text{e}3$ (lower case “e”). Avoid mixing up the “e” that is used in scientific notation with Euler’s number, which is $e = 2.718$.

Engineering notation is a version of scientific notation in which the powers of 10 are written as multiples of three. **Example.** $0.000475 = 4.75\text{E}-4$ (scientific notation) = $0.475\text{E}-3$ (engineering notation) = $475\text{E}-6$ (engineering notation). **Example.** $692000 = 6.92\text{E}5$ (scientific notation) = $0.692\text{e}6$ (engineering notation).

Unit prefixes (Metric System). In the SI system, it is common to use prefixes on units to multiply or divide by powers of 10. **Example.** $0.001 \text{ newton} = 1.0 \text{ mN}$. **Example.** $0.000475 \text{ m} = 0.475 \text{ mm} = 475 \text{ }\mu\text{m}$. **Example.** $1000 \text{ pascals} = 1.0 \text{ kPa}$.

Significant figures. When a number is reported with three significant figures (e.g., 1.97), this means that two of the digits are known with precision (i.e., the 1 and the 9), and one of the digits (i.e., the 7) is an approximation. The rationale for significant figures is that values in engineering (e.g., the density of water is about 998 kg/m^3) ultimately come from measurements, and measurements can only provide certain levels of precision. In this text, we report answers with three significant figures. Of course, during intermediate calculations, you should carry more than three significant digits to prevent rounding errors.

Thinking with the Derivative

We have seen many mistakes because the main idea of the derivative was not in place. Thus, we wrote this subsection to explain this idea in detail.

To describe a common mistake, we’ll give an example of this mistake. Suppose you were asked to answer the following true/false question (T/F). If a car has traveled in a straight line for $\Delta x = 10.0$ meters during a time interval of $\Delta t = 2.5$ seconds, then its speed at the end of the time interval is $(10.0 \text{ m})/(2.5 \text{ s}) = 4.0 \text{ m/s}$.

It seems like one could answer this question as true, because $V = (\Delta x)/(\Delta t) = (10.0 \text{ m})/(2.5 \text{ s}) = 4.0 \text{ m/s}$. However, this answer is only valid if the speed of the car was constant with time. A better answer is to say false because there is not enough information to reach the conclusion that the car is traveling at 4 m/s at the end of the time interval. The problem we are illustrating is the difference between *average speed* and *instantaneous speed*. The best way to think about speed is to apply the definition of the derivative. In words, speed is the ratio of the distance traveled to the amount of time in the limit as the amount of time goes to zero. In equation form (more compact), speed V is defined by

$$V = \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} \quad (1.9)$$

If speed is constant, then Eq. (1.9) will automatically simplify to give the equation for average speed. If speed is varying with time, then Eq. (1.9) will give a correct value of speed. Of course, Eq. (1.9) is based on the definition of the derivative. Regarding this definition, calculus books give the definition in three ways:

$$\frac{dy}{dx} = \lim_{h \rightarrow 0} \frac{y(x+h) - y(x)}{h} \quad (1.10)$$

$$= \lim_{\Delta x \rightarrow 0} \frac{y(x+\Delta x) - y(x)}{\Delta x} \quad (1.11)$$

$$= \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} \quad (1.12)$$

We apply the last definition, Eq. (1.12), in multiple places in this text. This definition shows that the derivative means the ratio of Δy to Δx in the limit as Δx goes to zero. Note that the delta symbol (i.e., the triangle) preceding the variable y denotes an amount or quantity of the variable y .

Thinking with the Integral

The integral was invented to solve problems in which rates change with time. To build up the definition of the integral, we note that it is tempting to state that the distance a car travels (Δx) is given by $\Delta x = V\Delta t$, where V is the speed and Δt is the time that the car has been traveling. The problem with this formula is that it does not apply in general, because speed can be changing. To modify the formula so that it is more general, one can do the following:

$$\Delta x = \sum_{i=1}^N V_i \Delta t_i \quad (1.13)$$

where the motion has been divided into time intervals. Here, Δt_i is a small time interval, V_i is the speed during this time interval, and N is the number of time intervals. To make this formula more accurate, we can let $N \rightarrow \infty$, and we arrive at a general formula for distance traveled:

$$\Delta x = \lim_{N \rightarrow \infty} \sum_{i=1}^N V_i \Delta t_i \quad (1.14)$$

Now, the summation on the right-hand side of Eq. (1.14) can be modified by applying the definition of the integral to give

$$\Delta x = \int_0^{t_f} V dt \quad (1.15)$$

In calculus texts, you will find the following definition of the integral:

$$\int_a^b f(x) dx = \lim_{N \rightarrow \infty} \sum_{i=1}^N f(x_i) \Delta x_i \quad (1.16)$$

Thus, the integral is an infinite sum of small terms that is applied when a dependent variable f is changing in response to changes in the independent variable x .

Summary: Derivatives and Integrals

For parameters that involve the derivative, the facts are

$$z = \lim_{\Delta y \rightarrow 0} \frac{\Delta x}{\Delta y} \quad (\text{always true})$$

$$z = \frac{\Delta x}{\Delta y} \quad (\text{sometimes true})$$

$$\bar{z} = \frac{\Delta x}{\Delta y} \quad (\text{always true})$$

The terms z , x , and y are parameters such as density, mass, and time. The overbar as in $\bar{z}$ means the average value of a parameter; for example, the average density.

For parameters that involve the integral, the facts are

$$z = \int y dx \quad (\text{always true})$$

$$z = \sum_i^n (y_i \Delta x_i) \quad (\text{useful approximation})$$

$$z = y \Delta x \quad (\text{sometimes true})$$

$$z = \bar{y} \Delta x \quad (\text{always true})$$

EXAMPLE: UNDERSTANDING THE DERIVATIVE

(T/F) In general, pressure p is given by F_n/A , where F_n is a normal force that acts on an area A .

Claim: The best choice is F.

Reasoning: While $p = F_n/A$ is sometimes true, there are many situations for which this formula gives average pressure instead of pressure at a point. Pressure is best defined as a

derivative like this: $p = \lim_{\Delta A \rightarrow 0} \frac{\Delta F_n}{\Delta A}$.

EXAMPLE: UNDERSTANDING THE INTEGRAL

(T/F) In general, the force due to pressure F_p is given by $F_p = pA$, where p is pressure and A is area.

Claim: The best choice is F.

Reasoning: The general formula for the pressure force is $F_p = \int_A p dA$. The formula

$F_p = pA$ is only true when average pressure is used or when pressure is uniform.

1.6 Density and Specific Weight

Solving most problems in fluids requires calculation of mass or weight. These calculations involve the properties of density and specific weight, which are presented in this section.

Defining Density

For a simple problem, density (ρ) can be found by taking the ratio of mass (Δm) to volume (ΔV) as in

$$\rho = \frac{\Delta m}{\Delta V} \quad (1.17)$$

For example, if you took 1.0 liter of water at room conditions and measured the mass, the amount of mass would be (Δm) $\approx$ 1000 grams, and so the density would be

$$\rho = \Delta m / \Delta V = (1000 \text{ grams}) / (1.0 \text{ liter}) = 1.0 \text{ kg/L}$$

EXAMPLE. What is the mass of 2.5 liters of water? **Reasoning.** (1) The mass is given by $\Delta m = \rho(\Delta V)$. (2) The density of water at room conditions is about 1.0 kg/L. (3) Thus, the mass is $\Delta m = (1.0 \text{ kg/L})(2.5 \text{ L}) = 2.5 \text{ kg}$.

Eq. (1.17) defines average density, not the density at a point. To build a more general definition of density, apply the concept of the derivative (see §1.5). In general, density is defined using the derivative as shown in Eq. (1.18).

$$\rho \equiv \lim_{\Delta V \rightarrow 0} \frac{\Delta m}{\Delta V} \quad (1.18)$$

where ΔV denotes the volume of a tiny region of material surrounding a point (e.g., an x, y, z location) and Δm is the corresponding amount of mass that is contained within this region. Thus, density can be defined as the ratio of mass to volume at a point.

Some useful facts about density are as follows:

- You can look up density values in the front of the book (Tables F.4–F.6) and in the appendices (Tables A.2–A.5).
- In general, the value of the density will vary with the pressure and temperature of the material. For a liquid, the variation with pressure is usually negligible.
- The density of a gas is often calculated by applying the *density form* of the ideal gas law: $p = \rho RT$.
- To calculate the amount of mass in a given volume, it is tempting to apply: $\Delta m = \rho \Delta V$. However, this equation is a special-case equation, not a general equation. The general equation that accounts for the fact that density can vary with position is

$$m = \int_V \rho dV \quad (1.19)$$

- *Recommended working knowledge.* Know the density of liquid water at typical room conditions in common units: $\rho = 1000 \text{ kg/m}^3 = 1.0 \text{ g/mL} = 1.0 \text{ kg/L} = 62.4 \text{ lbm/ft}^3 = 1.94 \text{ slug/ft}^3$. Know the density of air at atmospheric pressure and 20°C: $\rho = 1.2 \text{ kg/m}^3 = 1.2 \text{ g/L}$.

Defining Specific Weight

Specific weight is the ratio of weight to volume at a point:

$$\gamma \equiv \lim_{\Delta V \rightarrow 0} \frac{\Delta W}{\Delta V} \quad (1.20)$$

where ΔV denotes the volume of a tiny region of material surrounding a point (e.g., an x, y, z location) and ΔW is the corresponding weight of the matter that is contained within this region.

Average specific weight of a body is defined by $\gamma = \Delta W / \Delta V$, where ΔW is the weight of the body and ΔV is the volume of the body.

Specific weight and density are related by:

$$\gamma = \rho g \quad (1.21)$$

Thus, if you know one property, you can easily calculate the other. **Example.** The specific weight corresponding to a density of 800 kg/m^3 is $\gamma = (800 \text{ kg/m}^3)(9.81 \text{ m/s}^2) = 7.85 \text{ kN/m}^3$.

The reasoning to show that Eq. (1.21) is true involves the following steps. (1) On Earth, NLUG simplifies to $W = mg$. (2) Divide $W = mg$ by volume to give $(\Delta W / \Delta V) = (\Delta m / \Delta V)g$. (3) Take the limit as volume goes to zero. (4) Apply the definitions of γ and ρ to give $\gamma = \rho g$.

Some useful facts about specific weight are as follows:

- You can look values of γ in the front of the book (Tables F.4–F.6) and in the back of the book (Tables A.3–A.5).
- Since ρ and γ are related via Eq. (1.21), γ varies with temperature and pressure in a similar fashion to density.
- Specific weight is commonly used for liquids, but not commonly used for gases.
- *Recommended working knowledge.* Know the specific weight of liquid water at typical room conditions: $\gamma = 9800 \text{ N/m}^3 = 9.80 \text{ N/L} = 62.4 \text{ lbf/ft}^3$.

1.7 The Ideal Gas Law (IGL)

The IGL is commonly applied in fluid mechanics. For example, you will likely apply the IGL when you are designing products such as air bags, shock absorbers, combustion systems, and aircraft.

The IGL, the Ideal Gas, and the Real Gas

The IGL was developed by the logical method called induction. *Induction* involves making many experimental observations and then concluding that something is always true because every experiment indicates this truth. For example, if a person concludes that the sun will rise tomorrow because it has risen every day in the past, this is an example of inductive reasoning.

The IGL was developed by combining three empirical equations that had been discovered previously. The first of these equations, called Boyle's law, states that when temperature T is held constant, the pressure p and volume V of a fixed quantity of gas are related by

$$pV = \text{constant} \quad (\text{Boyle's law}) \quad (1.22)$$

The second equation, Charles's law, states that when pressure is held constant, the temperature and volume V of a fixed quantity of gas are related by

$$\frac{V}{T} = \text{constant} \quad (\text{Charles's law}) \quad (1.23)$$

The third equation was derived by a hypothesis formulated by Avogadro: *Equal volumes of gases at the same temperature and pressure contain equal number of molecules.* When Boyle's law, Charles's law, and Avogadro's law are combined, the result is the ideal gas equation in this form:

$$pV = nR_u T \quad (1.24)$$

where n is the amount of gas measured in units of moles.

Eq. (1.24) is called the pVT form or the *mole form* of the IGL. **Tip.** There is no need to remember Charles' law or Boyle's law, because they are both special cases of the IGL.

The ideal gas and the real gas can be defined as follows:

- An **ideal gas** refers to a gas that can be modeled using the ideal gas equation, Eq. (1.24), with an acceptable degree of accuracy; for example, calculations have less than a 5% deviation

from the true values. Another way to define an ideal gas is to state that an ideal gas is any gas in which the molecules do not interact except during collisions.

- A **real gas** refers to a gas that cannot be modeled using the ideal gas equation, Eq. (1.24), with an acceptable degree of accuracy because the molecules are close enough together (on average) that there are forces between the molecules. Although real gas behavior can be modeled, the equations are more complex than the IGL. Thus, the IGL is the preferred model if it provides an acceptable level of accuracy.

For every problem that we (the authors) have solved, the IGL has provided a valid model for gas behavior; that is, we have never needed to apply the equations used to model real gas behavior. However, there are a few instances in which you should be careful about applying the IGL:

- When a gas is very cold or under very high pressure, then the molecules can move close enough together to invalidate the IGL.
- When both the liquid phase and the gas phase are present (e.g., propane in a tank used for a barbecue), you might want to be careful about applying the IGL to the gas phase.
- When a gas is very hot, such as the exhaust stream of a rocket, then the gas can ionize or disassociate. Both of these effects can invalidate the ideal gas law.

Also, the IGL works well for modeling a mixture of gases. The classic example is air, which is a mixture of nitrogen, oxygen, and other gases.

Units in the IGL

Because we have seen many mistakes with units, we wrote this subsection to give you the essential facts so that you can avoid most of these mistakes and also save time.

Temperature in the IGL must be expressed using *absolute temperature*. Absolute temperature is measured relative to a temperature of absolute zero, which is the temperature at which (theoretically) all molecular motion ceases. The SI unit of absolute temperature is Kelvin (K with no degree symbol, as in 300 K). A temperature given in Celsius ($^{\circ}\text{C}$) can be converted to Kelvin using this equation: $T(\text{K}) = T(^{\circ}\text{C}) + 273.15$. For example, a temperature of 15°C will convert to $15^{\circ}\text{C} + 273 = 288 \text{ K}$. The English unit of absolute temperature is Rankine; for example, a temperature of 70°F is the same as a temperature of 530°R . A temperature given in Fahrenheit ($^{\circ}\text{F}$) may be converted to Rankine using this equation: $T(^{\circ}\text{R}) = T(^{\circ}\text{F}) + 459.67$. For example, a temperature of 65°F will convert to $65^{\circ}\text{F} + 460 = 525^{\circ}\text{R}$.

Pressure in the IGL must be expressed using *absolute pressure*. Absolute pressure is measured relative to a perfect vacuum, such as outer space. Now, it is common in engineering to give a value of pressure that is measured relative to local atmospheric pressure; this is called *gage pressure*. To convert a gage pressure to absolute pressure, add the value of local atmospheric pressure. For example, if the gage pressure is 20 kPa and the local atmospheric pressure is 100 kPa, then the absolute pressure will be $100 \text{ kPa} + 20 \text{ kPa} = 120 \text{ kPa}$. If the local atmospheric pressure is unavailable, then use the standard value of atmospheric pressure, which is 101.325 kPa (14.696 psi or 2116.2 psf). More details about pressure are presented in §3.1.

The IGL also uses the **mole**, defined as the amount of material that has the same number of “entities” (atoms, molecules, ions, etc.) as there are atoms in 12 g of carbon 12 (C^{12}). Think of the mole as a way to count *how many*. By analogy, the dozen is also a unit for counting how many; for example, three dozen donuts is a way of specifying 36 donuts. The number of atoms in 12.0 grams of carbon 12 is equal to one mole of atoms. This number, called **Avogadro’s number**, is 6.022×10^{23} entities. There are three different mole units in use:

- A gram mole (mol) has 6.022×10^{23} entities (atoms, molecules, etc.).
- A kilogram mole (kg-mol) has $(6.022\text{E}2)(1000 \text{ g/kg}) = 6.022\text{E}26$ entities.
- A pound-mass mole (lbm-mol) has $(454.3 \text{ g/lbm})(6.022 \times 10^{23}) = 2.732\text{E}26$ entities.

Another unit issue arises because the amount of matter can be characterized by using either moles or by using mass. Moles and mass units are related by using the **molar mass**, which is defined by

$$M = \frac{\text{amount of mass}}{\text{number of moles}} = \frac{m}{n} \quad (1.25)$$

Values of molar mass can be looked up on the Internet. Some typical values are also listed in Table 1.2.

EXAMPLE. What is the mass (in kg) of 2.7 moles of air?

Solution. $m = nM = (28.97\text{E-}3 \text{ kg/mol})(2.7 \text{ mol}) = 78.2\text{E-}3 \text{ kg}$.

The Universal and Specific Gas Constant (R_u and R)

In the IGL, there are two gas constants: the universal gas constant and the specific gas constant. When you write the IGL like this, $pV = nR_uT$, the term R_u is called the universal gas constant. The word “universal” means that this gas constant is the same for every gas. The value of R_u in SI units is $R_u = 8.314462 \text{ J/mol}\cdot\text{K}$. The value of R_u in traditional units is $R_u = 1545.349 \text{ ft}\cdot\text{lbf/lbm}\cdot\text{mol}\cdot^\circ\text{R}$.

Often, engineers prefer to work with mass units instead of mole units. In this case, the IGL can be modified like this: (1) Start with Eq. (1.24) and substitute $n = m/M$ to give $pV = m(R_u/M)T$. (2) Define the specific gas constant (R) using this equation: $R \equiv R_u/M$. **Conclusion.** An alternative way to write the IGL is $pV = mRT$, where R is the *specific gas constant*.

Summary. Anytime you are using the IGL, figure out whether you need to use R or R_u . As needed, you can relate R and R_u using this equation:

$$R = \frac{R_u}{M} \quad (1.26)$$

Also, you can find values of the specific gas constant (R) in Table A.2.

EXAMPLE. If 3.0 moles of a gas has a mass of 66 grams, what is the specific gas constant for this gas (SI units)?

Reasoning. (1) Since molar mass is the ratio of mass/moles, $M = (0.066 \text{ kg})/(3 \text{ mol}) = 0.022 \text{ kg/mol}$. (2) Now that M is known, $R = R_u/M = (8.314 \text{ J/mol}\cdot\text{K})/(0.022 \text{ kg/mol}) = 378 \text{ J/kg}\cdot\text{K}$.

Claim. $R = 378 \text{ J/kg}\cdot\text{K}$.

TABLE 1.2 Selected Values of Molar Mass

Substance	Molar Mass (grams/mole)
Hydrogen	1.0079
Helium	4.0026
Carbon	12.0107
Nitrogen N_2	14.0067
Oxygen O_2	15.9994
Dry air	28.97

TABLE 1.3 The Ideal Gas Law (IGL) and Related Equations

Description	Equation	Variables
Density form of the IGL	$p = \rho RT$	p = pressure (Pa) (use absolute pressure, not gage or vacuum pressure) ρ = density (kg/m ³) R = specific gas constant (J/(kg·K)) (look up R in Table A.2) T = temperature (K) (use absolute temperature)
Mass form of the IGL	$pV = mRT$	V = volume (m ³) m = mass (kg)
Mole form of the IGL, also called the pVT form	$pV = nR_u T$	n = number of moles R_u = universal gas constant $(R_u = 8.314 \text{ J/(mol·K)} = 1545 \text{ (ft·lbf)/(lbmol·°R)})$
Apply this equation to relate R and R_u	$R = \frac{R_u}{M}$	M = molar mass (kg/mol)
Apply this equation to relate mass and moles	$M = m/n$	

The IGL (Working Equations)

The purpose of this subsection is to (a) present three equations that are commonly used to represent the IGL and (b) explain the meaning of a working equation. Before we do this, we want to share an idea that we have found to be useful.

Voice of the Engineer. *Become skillful with the working equations in each engineering subject you study.* A **working equation** is defined as an equation that is often used in application. The benefit of using working equations is simplicity; in particular, each engineering subject has about 15 working equations. If you know these equations well, then you know a great deal about the subject. It is true that most engineering textbooks have hundreds of equations in them. This is because the authors are using these equations to explain things, but you do not need to remember most of these equations.

The working equations associated with the IGL are summarized in Table 1.3. Notice that there are three common IGL equations called the density form, the mass form, and the mole form. These equations are equivalent because you can start with one of these equations and derive the other two. Notice that the last column in Table 1.3 provides SI units and tips for application.

1.8 Quantity, Units, and Dimensions

Because engineers must have the ability to understand and work with units—this might be called “unit literacy”—this section presents the foundational concepts associated with unit practices.

Definition of Quantity

A **quantity** is a specified amount, measure, rate, and so forth of an entity—like mass, force, speed, flow rate, stress, or temperature—for describing how much or the extent of the entity.

A quantity involves a number multiplied by one or more units. For example, the quantity (3 apples) is the number (3) multiplied by the unit (apples). For example, the quantity (9 m/s) is the number (9) multiplied by the units (m and s⁻¹). The number itself is called the *magnitude*. In summary:

$$(\text{quantity}) = (\text{magnitude})(\text{unit(s)})$$

EXAMPLES:

- The quantity 7.4 J is comprised of a magnitude (7.4) times a unit (joule).
- The quantity 9.81 m/s² is comprised of a magnitude (9.81) times several units (m/s²).
- The quantity 15°C is comprised of a magnitude (15) times a unit (degree Celsius).

When the unit(s) of a quantity change, the magnitude must also change such that the quantity itself is unchanged. For example, the quantity of 11 N is the same quantity as 2.5 lbf.

The Grid Method

Of the various methods for carrying and canceling units, the *grid method* (Fig. 1.9) is the best method that we have seen. To learn how to apply the grid method, see the method and examples presented in Table 1.4.

The essence of the grid method is to multiply the right side of the equation by 1.0 (i.e., the *multiplicative identity*) over and over until the units cancel in a way that gives you the desired unit. For example, in Fig 1.9, the right side of the equation was multiplied by 1.0 three times:

$$\begin{aligned} 1.0 &= \frac{1.0 \text{ m/s}}{2.237 \text{ mph}} \text{ (first time)} \\ 1.0 &= \frac{1.0 \text{ N}}{0.2248 \text{ lbf}} \text{ (second time)} \\ 1.0 &= \frac{1.0 \text{ W} \cdot \text{s}}{\text{N} \cdot \text{m}} \text{ (third time)} \end{aligned}$$

As shown in the above three examples, a **conversion ratio** is an equation involving numbers and units that can be arranged so that the number 1.0 appears on one side of the equation. **Example.** 100 cm = 1.0 m is a conversion ratio because this equation can be written as 1.0 = (100 cm)/(1.0 m).

FIGURE 1.9

The *grid method*. This example shows a calculation of the power P required to ride a bicycle at a speed of $V = 20 \text{ mph}$ when the force to move against wind drag is $F = 4.0 \text{ lbf}$.

$$\begin{aligned} P = FV &= \frac{4.0 \text{ lbf}}{1} \left| \frac{20 \text{ mph}}{1} \right| \left| \frac{1.0 \text{ m/s}}{2.237 \text{ mph}} \right| \left| \frac{1.0 \text{ N}}{0.2248 \text{ lbf}} \right| \left| \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} \right| \\ &= \boxed{159 \text{ W}} \end{aligned}$$

TABLE 1.4 Applying the Grid Method (Two Examples)

Step	Example 1	Example 2
Problem Statement $\Rightarrow$	Situation: Convert a pressure of 2.00 psi to pascals.	Situation: Find the force in newtons that is needed to accelerate a mass of 10 g at a rate of 15 ft/s ² .
Step 1. Write the equation down.	Not applicable	$F = ma$
Step 2. Insert numbers and units.	$p = 2.00 \text{ psi}$	$F = ma = (0.01 \text{ kg})(15 \text{ ft/s}^2)$
Step 3. Look up conversion ratios (see Table F.1).	$1.0 = \frac{1 \text{ Pa}}{1.45 \times 10^{-4} \text{ psi}}$	$1.0 = \frac{1.0 \text{ m}}{3.281 \text{ ft}} \quad 1.0 = \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$
Step 4. Multiply terms and cancel units.	$p = [2.00 \text{ psi}] \left[\frac{1 \text{ Pa}}{1.45 \times 10^{-4} \text{ psi}} \right]$	$F = [0.01 \text{ kg}] \left[\frac{15 \text{ ft}}{\text{s}^2} \right] \left[\frac{1.0 \text{ m}}{3.281 \text{ ft}} \right] \left[\frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}} \right]$
Step 5. Do calculations.	$p = 13.8 \text{ kPa}$	$F = 0.0457 \text{ N}$

We recommend four methods for finding conversion ratios.

- **Method #1.** Derive the conversion ratio as shown in the following example:

1. Power is defined as

$$\text{power} = \frac{\text{work}}{\text{time}}$$

2. Substituting SI units shows that

$$1.0 \text{ W} = \frac{1.0 \text{ N} \cdot \text{m}}{1.0 \text{ s}}$$

3. Algebra shows that

$$1.0 = \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}}$$

- **Method #2.** Derive the conversion ratio using data from Table F.1 (front of book). **Example.** To relate the speed units of m/s and mph, find the row labeled “Speed” in Table F.1 and extract the data that $1.0 \text{ m/s} = 2.237 \text{ mph}$. Then, do algebra to show that $1.0 = (1.0 \text{ m/s})/(2.237 \text{ mph})$. **Example.** To relate the pressure units of kPa and torr, find the row labeled “Pressure/Shear Stress” and extract the data that $6.895 \text{ kPa} = 51.71 \text{ torr}$. Then, do algebra to show that $1.0 = (6.895 \text{ kPa})/(51.71 \text{ torr}) = (1.0 \text{ kPa})/(7.50 \text{ torr})$.
- **Method #3.** Apply a fact; for example, because there are 30.48 cm in 1.0 foot, the conversion ratio from meters to feet is $1.0 = (0.3048 \text{ m})/(1.0 \text{ ft})$.
- **Method #4.** Use Web resources. We recommend Google and www.onlineconversion.com. **Example.** A common way to measure the volume of water in hydrology is to use the unit of acre-feet. However, this unit is not in this textbook. Thus, go to Google, and type in “acre-feet to cubic meters,” and Google will output “1 acre-foot = 1233.48184 cubic meters.” Then, do algebra to show that $1.0 = (1233 \text{ m}^3)/(1.0 \text{ acre-foot})$.

Consistent Units

Voice of the Engineer. *Before you solve a problem, convert all your units to consistent units (SI preferred), do your analysis, and then report your answer in the units that are the most useful for your context.* We call this idea the **Consistent Unit Rule**. The rationale is that this will save you a lot of time, keep your documentation shorter and neater, and eliminate mistakes.

Consistent units are defined as any set of units for which the conversion factors only contain the number 1.0. This means, for example, that

- $(1.0 \text{ unit of force}) = (1.0 \text{ unit of mass})(1.0 \text{ unit of acceleration})$
- $(1.0 \text{ unit of power}) = (1.0 \text{ unit of work})/(1.0 \text{ unit of time})$
- $(1.0 \text{ unit of speed}) = (1.0 \text{ unit of distance})/(1.0 \text{ units of time})$

EXAMPLE. If length is measured in millimeters and force in newtons, then what is the consistent unit of pressure? **Reasoning.** (1) The definition of consistent units means that $(1.0 \text{ unit of pressure}) = (1.0 \text{ unit of force})/(1.0 \text{ unit of area})$. (2) The unit of area in this case is millimeters squared. (3) Combining steps 1 and 2 gives $(1.0 \text{ unit of pressure}) = (1.0 \text{ N})/(1.0 \text{ mm}^2) = \text{N/mm}^2$. (4) Because the unit of N/mm^2 is uncommon, it is best to convert this to more familiar units like this: $(1.0 \text{ N/mm}^2) = (1.0 \text{ N})/[(10^{-3})^2(1.0 \text{ m})^2] = 10^6 \text{ N/m}^2 = 1.0 \text{ MPa}$.

Conclusion. The consistent unit of pressure for the given units is MPa (mega pascals).

EXAMPLE. Is the *given set* of units consistent (*given set*: force is in units of pounds-force (lbf), mass in lbm, and acceleration in ft/s^2)? **Reasoning.** (1) By definition, $(1.0 \text{ lbf}) = (1.0 \text{ lbm})$

(32.2 ft/s²). (2) By the definition of consistent units, the only number that can appear is the number 1.0. (3) The number 32.2 is not the number 1.0. (4) Thus, the given set of units cannot be consistent. **Conclusion.** The given set of units is not consistent.

In principle, there are an infinite number of sets of consistent units. Fortunately, people before us have figured out an optimum set—that is, the SI unit system. The best method for using consistent units is to *convert all your units to SI units* (Fig. 1.10).

We recommend that do all your technical work in SI units. However, we also recommend that you become skilled with English units. This is like being able to speak two languages, as in I speak “SI units” and I speak “English units,” but making one of the languages (i.e., SI units) your language of choice. Regarding English units, there are actually two systems of units in use. In this text, we combine these two systems and call them “traditional units” or “English units.”

Consistent units for both the SI system and the English system are listed in Table 1.5. The way to use this table is to convert all variables in your problem so that they are expressed using only the units listed in Table 1.5. **Example.** Convert the following values so that they have consistent units: $\rho = 50 \text{ lbm/ft}^3$, $V = 200 \text{ ft/min}$, $D = 12 \text{ in}$. **Reasoning.** The method is to convert the given units so that they match the units specified in Table 1.5. The conversions are straightforward, so we do not show these. **Conclusion.** Use $\rho = 1.55 \text{ slug/ft}^3$, $V = 3.33 \text{ ft/s}$, and $D = 1.0 \text{ ft}$.

The Dimension: A Way to Organize Units

Because there are thousands of units, this section will show you a way to organize units into categories called *dimensions*. Dimensions will be used throughout this book and will be featured in Chapter 8, in which a powerful method of analysis (called dimensional analysis) is introduced.

FIGURE 1.10

This example shows how to apply the *Consistent Unit Rule*.

Calculate the power P (in watts) required to ride a bicycle at a speed of $V = 20 \text{ mph}$ when the force to move against wind drag is $F = 4.0 \text{ lbf}$.

$$F = (4.0 \text{ lbf}) \left(\frac{4.45 \text{ N}}{\text{lbf}} \right) = 17.8 \text{ N}$$

$$V = \frac{20 \text{ mph}}{10/6 \text{ mp}} \left| \frac{0.447 \text{ m/s}}{10/6 \text{ mp}} \right| = 8.94 \text{ m/s}$$

$$P = FV = \frac{17.8 \text{ N}}{1} \left| \frac{8.94 \text{ m}}{\text{s}} \right| \left| \frac{\text{W} \cdot \text{s}}{\text{N} \cdot \text{m}} \right| = 159 \text{ W}$$

$$P = 159 \text{ W}$$

To apply the consistent unit rule to this problem, take the three steps that follow.

First, convert the given variables to consistent units (SI units).

Second, do your calculations with consistent units.

Third, report your answer with the most appropriate units for your context.

TABLE 1.5 Consistent Units

Dimension	SI System	English (Traditional) Units
Length	meter (m)	foot (ft)
Mass	kilogram (kg)	slug (slug)
Time	second (s)	second (s)
Force	newton (N)	pounds-force (lbf)
Pressure	pascal (Pa)	pounds-force per square foot (psf)
Density	kilogram per meter cubed (kg/m ³)	slug per foot cubed (slug/ft ³)
Volume	cubic meters (m ³)	cubic feet (ft ³)
Power	watt (W)	foot pounds-force per second (ft·lbf/s)

Mass is an example of a *dimension*. To describe the amount of mass, engineers apply various units (e.g., slug, gram, kilogram, ounce, pound-mass, etc.). *Time* is an example of a dimension. To describe the amount of time, you can apply various units (seconds, minutes, hours, days, weeks, months, years, centuries, etc.). Other examples of dimensions are speed, volume, and energy. Each dimension has associated with it many possible units, but the dimension itself does not have a specified unit. As these examples show, a **dimension** is an entity that is measured using units. The relationship between dimensions and units is shown in Fig. 1.11. Notice that dimensions can be identified by asking this question: *What are we interested in measuring?* For example, engineers are generally interested in measuring force, power, energy, and time. Each of these entities is a dimension.

EXAMPLE. Is temperature a dimension? **Reasoning.** (1) A dimension is an entity that is measured and quantified with units. (2) Temperature is something (i.e., an entity) that is measured and quantified with units such as Kelvin, Celsius, and Fahrenheit. (3) Thus, temperature aligns with the definition of dimension. **Claim.** Temperature is a dimension.

Dimensions can be related by using equations. For example, Newton's second law, $F = ma$, relates the dimensions of force, mass, and acceleration. Because dimensions can be related, engineers and scientists can express dimensions using a limited set of dimensions that are called **primary dimensions** (Table 1.6).

A **secondary dimension** is any dimension that can be expressed using primary dimensions. For example, the secondary dimension “force” is expressed in primary dimensions by using $\Sigma F = ma$. The primary dimensions of acceleration are L/T^2 , so

$$[F] = [ma] = M \frac{L}{T^2} = \frac{ML}{T^2} \quad (1.27)$$

In Eq. (1.27), the square brackets means “dimensions of.” Thus, $[F]$ means “the dimension of force.” Similarly, $[ma]$ means “the dimensions of mass times acceleration.” Notice that primary dimensions are not enclosed in brackets. For example, ML/T^2 is not enclosed in brackets.

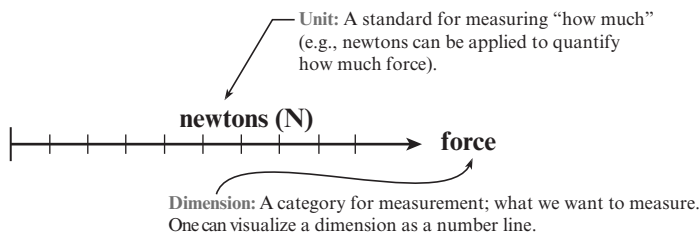


FIGURE 1.11

Dimensions describe *what is measured*. Units provide the method by which quantification is possible.

TABLE 1.6 Primary Dimensions

Dimension	Symbol	Unit (SI)
Length	L	meter (m)
Mass	M	kilogram (kg)
Time	T	second (s)
Temperature	θ	kelvin (K)
Electric current	i	ampere (A)
Amount of light	C	candela (cd)
Amount of matter	N	mole (mol)

To find the primary dimensions, we recommend two methods:

Method #1 (Primary Method). Figure out the primary dimensions by applying fundamental definitions on physical quantities.

Method #2 (Secondary Method). Look up the primary dimensions in Table F.1 (front of book) or in other engineering references. We recommend that you only use this method if you have not yet had enough practice to use Method #1.

EXAMPLE. If work is given the symbol W , what are $[W]$?

Reasoning.

1. The symbol $[W]$ means “the primary dimensions of work.” Thus, the question is asking *what are the primary dimensions of work?*
2. The definition of mechanical work reveals that (work) = (force)(distance).
3. Thus, $[W] = [F][d] = (ML/T^2)(L) = ML^2/T^2$.

Conclusion. $[W] = ML^2/T^2$.

In some engineering literature, the dimensions of force are given the symbol F as if force were a primary dimension, which it is not. The rationale for F is that this symbol simplifies notation in certain cases. For example, this equation

$$[p] = \frac{F}{L^2}$$

means that the dimension of pressure is the dimension of force divided by the dimension of area.

Dimensional Homogeneity (DH)

Voice of the Engineer. Routinely check each equation you encounter for dimensional homogeneity. **Reasoning.** (1) You can recognize and fix mistakes in equations. (2) This skill will help you make sense out of each equation you encounter and also make equations easier to remember.

An equation is **dimensionally homogenous** if each term in the equation has the same primary dimensions. The method for checking an equation for DH is to find the primary dimensions on each term and then check to see if each term has the same primary dimensions.* This method is illustrated in the next example.

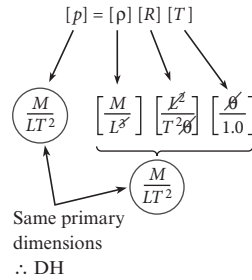
EXAMPLE. Show that the IGL (density form) is DH.

Reasoning.

1. The density form of the IGL is $p = \rho RT$.
2. The secondary dimensions of pressure are $[p] = [\text{force}]/[\text{area}]$.
Thus, the primary dimensions are $[p] = M/LT^2$.
3. The SI units of the specific gas constant R are J/kg·K.
Thus, the secondary dimensions are $[R] = [\text{energy}]/([\text{mass}][\text{temperature}])$.
Thus, the primary dimensions are $[R] = L^2/T^2\theta$.

*Of course, one can also use secondary dimensions or units. However, we recommend using primary dimensions because this builds knowledge that is useful when you learn dimensional analysis in Chapter 8.

4. The IGL can be analyzed as follows:



Claim. The density form of the IGL is dimensionally homogeneous, as shown by the analysis just presented.

The π -Group (Dimensionless Group)

In fluid mechanics, it is common to arrange variables so that the primary dimensions cancel out. This group of variables is called a dimensionless group or a π -group. The reason for the use of pi (i.e., π) in the label is that the main theorem used in analysis is called the Buckingham Π theorem. This topic is presented in Chapter 8.

A common example of a π -group is the Reynolds number (Re_D). One equation for the Reynolds number is $Re_D = (\rho VD)/\mu$, where ρ = fluid density, V = velocity, D = pipe diameter, and μ = fluid viscosity. Analysis of the Re_D (Fig. 1.12) shows that the primary dimensions cancel out.

$$[Re_D] = \left[\frac{VD\rho}{\mu} \right] = \frac{\mathcal{L}}{\mathcal{T}} \frac{\mathcal{L}}{\mathcal{T}} \frac{M}{\mathcal{L}^3} \frac{\mathcal{L} \cdot \mathcal{T}}{M}$$

Note $[\mu] = \frac{M}{\mathcal{L} \cdot \mathcal{T}}$

$$\therefore [Re_D] = [-]$$

The symbol $[-]$ means that the primary dimensions cancel out.

FIGURE 1.12

This example shows how to analyze the Reynolds number to establish that the primary dimensions cancel out.

1.9 Problem Solving

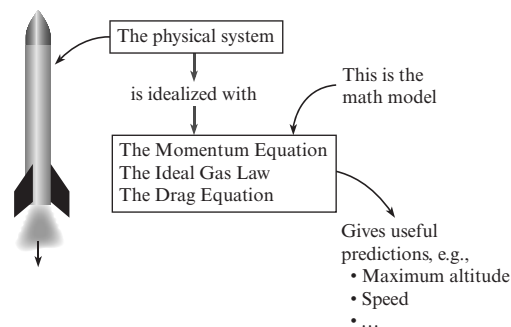
Although people solve problems every day, not everyone is equally skilled at problem solving. To illustrate this idea, consider the game of golf. Nearly anyone can strike a golf ball with a golf club, but only a tiny percentage of the population can do this well. Golfers who have a high level of skill owe their abilities to many years of practicing. Problem solving is like this as well, but the good news is that the number of skills you need to master is small. These skills are explained in this section. We hope you practice these skills (they are fun!) and that over time you develop into a great problem solver.

Defining Problem Solving

A **problem** is a situation that you need to resolve, especially when you have no clear idea of how to effectively resolve the problem. Given that a problem is the situation that needs to be resolved, then **problem solving** is a label for the methods that empower you to solve problems. A person who is skilled at problem solving can create great solutions with minimal amounts of time, effort, and cost while also greatly enjoying the experience. In addition, the process of problem solving nearly always results in meaningful learning.

FIGURE 1.13

Example of a math model of a rocket.



Applying Problem Solving to Building Math Models

There are general methods for solving problems. In this section, we explain how to apply these methods in the context of engineering classes. Our logic can be explained by using an analogy: If you are going to spend a lot of time practicing the guitar, then you should apply methods that will help you develop as a great guitar player. In the same way, if you are going to spend a lot of time in engineering school doing calculation problems, then you should apply methods that will help you excel at problem solving in general and building math models in particular.

A math model (Fig. 1.13) is comprised of equations plus a method of solving these equations. The purpose of the math model is to help you predict variables that are useful for engineering a system.

On most engineering problems, a math model is useful. For example, suppose you are designing a pump and the associated piping system to deliver water from a lake to a building located 100 m higher than the lake. A math model gives you the ability to predict useful parameters such as the optimum pipe diameter as well as the size and power requirements for the pump. If you did not have a math model, you would have to take a guess on sizing, then build something and take data. Then, you would repeat your steps until you had an acceptable design figured out. However, this guess, build, and repeat method is expensive and time-consuming.

In general, a **math model** can be defined as a collection of equations that you solve to give you values of parameters that are useful in the context of solving real-world problems. The main reason that a math model is useful is that it significantly reduces the cost and time you need to solve your problem.

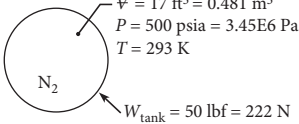
The method that we use and that we teach is called the Wales–Woods Model (WWM), because it is based on the research of Professor Charles Wales and his colleagues (Anni Nardi and Robert Stager) and also based on the research of Professor Donald Woods (1–9).

If you apply the WWM, then you will learn more effectively and you will grow your problem-solving skills. There are several reasons that we say this: (a) These methods work for us and we can attest to their benefits; (b) we have observed many students become better problem solvers and have had many students report that these methods benefited them; and (c) the methods are backed up with research data* that show the WWM is effective. In particular, Wales (3) analyzed five years of data and found that when students were taught the methods as freshmen, the graduation rate increased by 32% and the average grade point average increased by 25%, as compared to the control group, the members of which were not taught these skills. Based on 20 years of data, Woods (9) reports that students who were taught problem-solving skills, as compared to control groups, showed significant gains in confidence, problem-solving ability, attitude toward lifetime learning, self-assessment, and recruiter response.

*The gains reported in the literature are far above the gains reported for other educational methods that we know of.

The WWM is explained in Table 1.7. Skills that are the most useful are marked with a one or more check marks (✓). The best way to learn the WWM is to practice one or two skills at a time until you become good at them. Then, add a few more skills.

TABLE 1.7 The WWM for Problem Solving

Example. This column lists a sample problem and then shows how the Wales–Woods Model of problem solving might be applied to this sample problem.	Explanation. This column describes the actions you can take to apply the problem-solving model. Check marks (✓) indicate how useful each action is in the context of an engineering course. <i>More check marks means that an item is more useful.</i>
Problem Statement Find the total weight of a 17 ft³ tank of nitrogen if the nitrogen is pressurized to 500 psia, the tank itself weighs 50 lbf, and the temperature is 20°C. Work in SI units. Define the Situation A tank holds compressed N₂.  $V = 17 \text{ ft}^3 = 0.481 \text{ m}^3$ $P = 500 \text{ psia} = 3.45\text{E}6 \text{ Pa}$ $T = 293 \text{ K}$ $W_{\text{tank}} = 50 \text{ lbf} = 222 \text{ N}$ Assume: IGL applies N₂ (A2): $R_{\text{N}_2} = 297 \text{ J/kg} \cdot \text{K}$	Figure out what you are being asked (while reading the problem): • (✓✓) Interpret the given problem statement. • (✓) Look up unfamiliar terms. • (✓✓) Figure out how the given system works. • (✓✓) Visualize the system as it might exist in the real world. • (✓✓✓) Identify ideas or equations that might apply. Document your interpretation of the problem: • Summarize the situation in one to two sentences. • (✓✓✓) Sketch a <i>system diagram</i>. • (✓) List values of known parameters with their units. • (✓) Convert units to <i>consistent units</i>. • List main assumptions. • List properties and other relevant data.
State the Goal $W_T(\text{N}) \leftarrow$ Weight total (nitrogen + tank)	Describe your goal in a way that is unambiguous (the goal should be so clear that there will be no question about whether or not the goal is attained).
Generate Ideas 1. Weight total $\boxed{?} \quad \checkmark \quad ?$ $W_T = W_{\text{tank}} + W_{\text{N}_2} \quad (\text{a})$ 2. Newton's Law of Universal Gravity (applied to Earth) $\boxed{?} \quad ? \quad \checkmark$ $W_{\text{N}_2} = (m_{\text{N}_2})g \quad (\text{b})$ 3. The IGL (mass form) $\checkmark \checkmark \quad \boxed{?} \quad \checkmark \checkmark$ $pV = m_{\text{N}_2}RT \quad (\text{c})$	Apply the GENI method as developed by Charles Wales (1). 1. (✓✓✓✓✓) Identify an equation that contains your goal. Mark your goal with a boxed question mark. Mark known variables with a check mark and unknown variables with a question mark (e.g., see line a). 2. (✓✓✓✓✓) Make any unknown variable(s) your new goal. Repeat the marking process using checks and question marks (e.g., see lines b and c). 3. (✓✓✓✓✓) Repeat steps 1 and 2 until the number of equations is equal to the number of unknowns. At this point, the problem is solvable (we say that <i>the problem is cracked</i>, which means it is now figured out). In this example, the <i>problem is cracked</i> because there are three equations (a, b, and c) and three unknown variables (weight of nitrogen, mass of nitrogen, and total weight of the tank).

Execution	Make a Plan: 1. Calculate the mass of nitrogen using Eq. (c). 2. Calculate the weight of nitrogen using Eq. (b). 3. Calculate the total weight using Eq. (a).	As needed, list the set of steps that you can follow to reach your goal state (<i>most of the time, you can skip writing anything down</i>). In our examples, we often write out the plan steps so you can see what “making a plan” looks like.
	Take Action (Execute the Plan) 1. IGL $m_{N_2} = \frac{pV}{RT} = \frac{3.45 \text{ E6 N}}{\text{m}^2} \times \frac{0.481 \text{ m}^3}{\text{m}^2} \times \frac{\text{kg} \cdot \text{K}}{297 \text{ N} \cdot \text{m}} \times \frac{1}{293 \text{ K}}$ = 19.1 kg 2. NLUG $W_{N_2} = (m_{N_2})(g) = \frac{19.1 \text{ kg}}{\text{s}^2} \times \frac{9.81 \text{ m}}{\text{s}^2} \times \frac{\text{N} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$ = 181 N 3. Total Weight $W_T = W_{\text{tank}} + W_{N_2} = (222 + 187) \text{ N}$ = 409 N	Build your solution: • Do the calculations. • (✓✓✓✓) Apply the grid method. • Report your answer(s) with three significant figures. • Box or mark your answer(s).
	Review the Solution and the Process 1. When mass is the goal, the mass form of the IGL is the best equation to select. 2. To check the IGL assumption, the compressibility factor was calculated, and the IGL was accurate to about 98%. 4. For this problem, the weight of the gas is significant as compared to the weight of the tank.	Review your solution and your methods: • (✓✓) Validate your solution. • Figure out what recommendations you might make. • (✓✓✓) Review your problem-solving methods: • What actions worked well for you? • What actions might you take in the future? • Identify knowledge that was especially useful to you. • Identify significant aspects of the solution.

1.10 Summarizing Key Knowledge

Engineering Fluid Mechanics

- *Engineering* is the body of knowledge that equips individuals to solve problems by innovating, designing, applying, and improving technology.
- The *engineering method* is a label for the methods used to do engineering. The engineering method involves sub-methods such as critical thinking, building math models, the application of scientific experiments, and the application of existing technologies.
- *Mechanics* is the branch of science that deals with motion and with the forces that produce this motion. Mechanics is organized into two main categories: *solid mechanics* (materials in the solid state) and *fluid mechanics* (materials in the gas or liquid state).
- *Engineering fluid mechanics* is engineering when the project involves substantial knowledge from the discipline of fluid mechanics.

Fluids: Liquids and Gases

- Both liquids and gases are classified as fluids. A fluid is defined as a material that deforms continuously under the action of a shear stress.
- A significant difference between gases and liquids is that the molecules in a liquid experience strong inter-molecular forces, whereas the molecules in a gas move about freely with little or no interactions except during collisions.
- Liquids and gases differ in many important respects. Example #1: Gases expand to fill their containers, whereas liquids will occupy a fixed volume. Example #2: Gases have much smaller values of density than liquids.

Ideas for Modeling Materials

- A *microscopic viewpoint* involves understanding a material by understanding what the molecules are doing. A

macroscopic viewpoint involves understanding a material without the need to consider what the molecules are doing.

- Much of material behavior can be explained by understanding the *forces between molecules*. Molecules far apart do not attract one another, but molecules close together have strong attractive forces between them. However, when molecules are too close, there is a very strong repulsive force.
- “Body” is a label to identify objects or matter that exists in the real world, without specifying any specific object.
- The term “particle” is used in two ways:
 - A *material particle* is a small chunk of a body.
 - A *body-as-a-particle* involves idealizing a body as if all the mass is concentrated at a single point and the dimensions of the body are not relevant. For example, to analyze an airplane, we can idealize the airplane as a particle.
- In the continuum assumption, matter is idealized as consisting of continuous material that can be broken into smaller and smaller parts. The continuum assumption applies to most problems that involve flowing fluids.

Weight and Mass

- *Mass* is a material property that characterizes the amount of matter of a body. *Weight* is a property that characterizes the gravitational force on a body from a nearby planet (e.g., Earth).
- Weight and mass are related to each other by Newton’s Law of Universal Gravitation (NLUG). This law tells us that any two bodies anywhere in the universe will attract each other. The force of attraction depends on the mass of each body and inversely on the distance squared between the centers of mass of each body. In equation form, NLUG is $F = (Gm_1m_2)/R^2$. On Earth, NLUG simplifies to $W = mg$.

Density and Specific Weight

- Density is a material property that characterizes the ratio of mass/volume at a point; for example, the density of liquid water at room conditions is about $\rho = 1.0 \text{ kg/L} = 1000 \text{ kg/m}^3$.
- Specific weight is a material property that characterizes the ratio of weight/volume at a point; for example, the specific weight of liquid water at room conditions is about $\gamma = 9.8 \text{ N/L} = 9800 \text{ N/m}^3$.
- In general, ρ and γ vary with temperature and pressure. For liquids, ρ and γ are usually assumed to be constant with pressure but variable with temperature.

The Ideal Gas Law (IGL)

- An engineer models a gas as either (a) a real gas or (b) an ideal gas. The properties of an ideal gas can be accurately predicted with the IGL. The properties of a real gas cannot be accurately predicted with the IGL. In most situations, the IGL is an accurate model of a gas.
- To apply the IGL, use correct temperature and pressure units.
 - Temperature must be in *absolute temperature* (Kelvin or Rankine), not Celsius or Fahrenheit.
 - Pressure must be in *absolute pressure*, not gage or vacuum pressure.
- Moles and mass are related by $m = nM$; similarly, the specific gas constant and the universal gas constant are related by $R = R_u/M$. The molar mass M has dimensions of (mass)/(mole).
- There are multiple ways to write equations that represent the IGL. Three of the most useful equations are $p = \rho RT$, $pV = mRT$, and $pV = nR_uT$.

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