

PART I

LEARNING OBJECTIVES,
SUMMARY OVERVIEW,
AND PROBLEMS

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CHAPTER 1

THE TIME VALUE OF MONEY

LEARNING OUTCOMES

The candidate should be able to:

- interpret interest rates as required rates of return, discount rates, or opportunity costs;
- explain an interest rate as the sum of a real risk-free rate and premiums that compensate investors for bearing distinct types of risk;
- calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding;
- solve time value of money problems for different frequencies of compounding;
- calculate and interpret the future value (FV) and present value (PV) of a single sum of money, an ordinary annuity, an annuity due, a perpetuity (PV only), and a series of unequal cash flows;
- demonstrate the use of a time line in modeling and solving time value of money problems.

SUMMARY

In this chapter, we have explored a foundation topic in investment mathematics, the time value of money. We have developed and reviewed the following concepts for use in financial applications:

- The interest rate, r , is the required rate of return; r is also called the discount rate or opportunity cost.
- An interest rate can be viewed as the sum of the real risk-free interest rate and a set of premiums that compensate lenders for risk: an inflation premium, a default risk premium, a liquidity premium, and a maturity premium.
- The future value, FV, is the present value, PV, times the future value factor, $(1 + r)^N$.
- The interest rate, r , makes current and future currency amounts equivalent based on their time value.
- The stated annual interest rate is a quoted interest rate that does not account for compounding within the year.

- The periodic rate is the quoted interest rate per period; it equals the stated annual interest rate divided by the number of compounding periods per year.
- The effective annual rate is the amount by which a unit of currency will grow in a year with interest on interest included.
- An annuity is a finite set of level sequential cash flows.
- There are two types of annuities, the annuity due and the ordinary annuity. The annuity due has a first cash flow that occurs immediately; the ordinary annuity has a first cash flow that occurs one period from the present (indexed at $t = 1$).
- On a time line, we can index the present as 0 and then display equally spaced hash marks to represent a number of periods into the future. This representation allows us to index how many periods away each cash flow will be paid.
- Annuities may be handled in a similar approach as single payments if we use annuity factors rather than single-payment factors.
- The present value, PV, is the future value, FV, times the present value factor, $(1 + r)^{-N}$.
- The present value of a perpetuity is A/r , where A is the periodic payment to be received forever.
- It is possible to calculate an unknown variable, given the other relevant variables in time value of money problems.
- The cash flow additivity principle can be used to solve problems with uneven cash flows by combining single payments and annuities.

PRACTICE PROBLEMS

1. The table below gives current information on the interest rates for two two-year and two eight-year maturity investments. The table also gives the maturity, liquidity, and default risk characteristics of a new investment possibility (Investment 3). All investments promise only a single payment (a payment at maturity). Assume that premiums relating to inflation, liquidity, and default risk are constant across all time horizons.

Investment	Maturity (in Years)	Liquidity	Default Risk	Interest Rate (%)
1	2	High	Low	2.0
2	2	Low	Low	2.5
3	7	Low	Low	r_3
4	8	High	Low	4.0
5	8	Low	High	6.5

Based on the information in the above table, address the following:

- A. Explain the difference between the interest rates on Investment 1 and Investment 2.
 - B. Estimate the default risk premium.
 - C. Calculate upper and lower limits for the interest rate on Investment 3, r_3 .
2. A couple plans to set aside \$20,000 per year in a conservative portfolio projected to earn 7 percent a year. If they make their first savings contribution one year from now, how much will they have at the end of 20 years?
 3. Two years from now, a client will receive the first of three annual payments of \$20,000 from a small business project. If she can earn 9 percent annually on her investments and

- plans to retire in six years, how much will the three business project payments be worth at the time of her retirement?
4. To cover the first year's total college tuition payments for his two children, a father will make a \$75,000 payment five years from now. How much will he need to invest today to meet his first tuition goal if the investment earns 6 percent annually?
 5. A client can choose between receiving 10 annual \$100,000 retirement payments, starting one year from today, or receiving a lump sum today. Knowing that he can invest at a rate of 5 percent annually, he has decided to take the lump sum. What lump sum today will be equivalent to the future annual payments?
 6. You are considering investing in two different instruments. The first instrument will pay nothing for three years, but then it will pay \$20,000 per year for four years. The second instrument will pay \$20,000 for three years and \$30,000 in the fourth year. All payments are made at year-end. If your required rate of return on these investments is 8 percent annually, what should you be willing to pay for:
 - A. The first instrument?
 - B. The second instrument (use the formula for a four-year annuity)?
 7. Suppose you plan to send your daughter to college in three years. You expect her to earn two-thirds of her tuition payment in scholarship money, so you estimate that your payments will be \$10,000 a year for four years. To estimate whether you have set aside enough money, you ignore possible inflation in tuition payments and assume that you can earn 8 percent annually on your investments. How much should you set aside now to cover these payments?
 8. A client plans to send a child to college for four years starting 18 years from now. Having set aside money for tuition, she decides to plan for room and board also. She estimates these costs at \$20,000 per year, payable at the beginning of each year, by the time her child goes to college. If she starts next year and makes 17 payments into a savings account paying 5 percent annually, what annual payments must she make?
 9. A couple plans to pay their child's college tuition for 4 years starting 18 years from now. The current annual cost of college is C\$7,000, and they expect this cost to rise at an annual rate of 5 percent. In their planning, they assume that they can earn 6 percent annually. How much must they put aside each year, starting next year, if they plan to make 17 equal payments?
 10. The nominal risk-free rate is *best* described as the sum of the real risk-free rate and a premium for:
 - A. maturity.
 - B. liquidity.
 - C. expected inflation.
 11. Which of the following risk premiums is most relevant in explaining the difference in yields between 30-year bonds issued by the US Treasury and 30-year bonds issued by a small private issuer?
 - A. Inflation
 - B. Maturity
 - C. Liquidity

12. A bank quotes a stated annual interest rate of 4.00%. If that rate is equal to an effective annual rate of 4.08%, then the bank is compounding interest:
 - A. daily.
 - B. quarterly.
 - C. semiannually.
13. The value in six years of \$75,000 invested today at a stated annual interest rate of 7% compounded quarterly is *closest* to:
 - A. \$112,555.
 - B. \$113,330.
 - C. \$113,733.
14. A client requires £100,000 one year from now. If the stated annual rate is 2.50% compounded weekly, the deposit needed today is *closest* to:
 - A. £97,500.
 - B. £97,532.
 - C. £97,561.
15. For a lump sum investment of ¥250,000 invested at a stated annual rate of 3% compounded daily, the number of months needed to grow the sum to ¥1,000,000 is *closest* to:
 - A. 555.
 - B. 563.
 - C. 576.
16. Given a €1,000,000 investment for four years with a stated annual rate of 3% compounded continuously, the difference in its interest earnings compared with the same investment compounded daily is *closest* to:
 - A. €1.
 - B. €6.
 - C. €455.
17. An investment pays €300 annually for five years, with the first payment occurring today. The present value (PV) of the investment discounted at a 4% annual rate is *closest* to:
 - A. €1,336.
 - B. €1,389.
 - C. €1,625.
18. A perpetual preferred stock makes its first quarterly dividend payment of \$2.00 in five quarters. If the required annual rate of return is 6% compounded quarterly, the stock's present value is *closest* to:
 - A. \$31.
 - B. \$126.
 - C. \$133.
19. A saver deposits the following amounts in an account paying a stated annual rate of 4%, compounded semiannually:

Year	End-of-Year Deposits (\$)
1	4,000
2	8,000
3	7,000
4	10,000

- At the end of Year 4, the value of the account is *closest* to:
- \$30,432
 - \$30,447
 - \$31,677
- An investment of €500,000 today that grows to €800,000 after six years has a stated annual interest rate *closest* to:
 - 7.5% compounded continuously.
 - 7.7% compounded daily.
 - 8.0% compounded semiannually.
 - A sweepstakes winner may select either a perpetuity of £2,000 a month beginning with the first payment in one month or an immediate lump sum payment of £350,000. If the annual discount rate is 6% compounded monthly, the present value of the perpetuity is:
 - less than the lump sum.
 - equal to the lump sum.
 - greater than the lump sum.
 - At a 5% interest rate per year compounded annually, the present value (PV) of a 10-year ordinary annuity with annual payments of \$2,000 is \$15,443.47. The PV of a 10-year annuity due with the same interest rate and payments is *closest* to:
 - \$14,708.
 - \$16,216.
 - \$17,443.
 - Grandparents are funding a newborn's future university tuition costs, estimated at \$50,000/year for four years, with the first payment due as a lump sum in 18 years. Assuming a 6% effective annual rate, the required deposit today is *closest* to:
 - \$60,699.
 - \$64,341.
 - \$68,201.
 - The present value (PV) of an investment with the following year-end cash flows (CF) and a 12% required annual rate of return is *closest* to:

Year	Cash Flow (€)
1	100,000
2	150,000
5	-10,000

- €201,747.
 - €203,191.
 - €227,573.
- A sports car, purchased for £200,000, is financed for five years at an annual rate of 6% compounded monthly. If the first payment is due in one month, the monthly payment is *closest* to:
 - £3,847.
 - £3,867.
 - £3,957.

26. Given a stated annual interest rate of 6% compounded quarterly, the level amount that, deposited quarterly, will grow to £25,000 at the end of 10 years is *closest* to:
- A. £461.
 - B. £474.
 - C. £836.
27. Given the following time line and a discount rate of 4% a year compounded annually, the present value (PV), as of the end of Year 5 (PV_5), of the cash flow received at the end of Year 20 is *closest* to:
- A. \$22,819.
 - B. \$27,763.
 - C. \$28,873.
28. A client invests €20,000 in a four-year certificate of deposit (CD) that annually pays interest of 3.5%. The annual CD interest payments are automatically reinvested in a separate savings account at a stated annual interest rate of 2% compounded monthly. At maturity, the value of the combined asset is *closest* to:
- A. €21,670.
 - B. €22,890.
 - C. €22,950.