1

Power systems operation

Learning outcomes

By the end of this chapter, the student will be able to:

- Identify problems related to power systems operation.
- Link mathematical optimization models to power systems operation problems.

1.1 Mathematical programming for power systems operation

Mathematical optimization is a fundamental tool for the electrical supply chain, from generation through transmission, distribution, and end-use. It may also be used, in different time frames, from a few milliseconds to several years. This book concentrates on optimization problems for power systems operation. These problems are usually continuous and have a time frame from several minutes to one day. Optimization problems with faster dynamics lie in the control and stability analysis, whereas problems with slower dynamics are planning problems.

Mathematical optimization problems associated with power system operation have existed since the beginning of operations research as an independent area, back in the middle of the 20th century. However, modern technologies such as renewable energies and electric vehicles; and current concepts, such as smart-grids, active distribution networks, and microgrids, have created a renewed interest in mathematical optimization applied to power systems. Smart-grids implicate a massive use of technologies such as power

Mathematical Programming for Power Systems Operation: From Theory to Applications in Python. First Edition. Alejandro Garcés.

electronics, communications, and advanced metering. However, the *smart* aspect of these grids comes from mathematical techniques such as mathematical optimization, that manage these technologies, in order to improve the efficiency, reliability, security, and resilience of the system.

Figure 1.1 depicts schematically four common types of mathematical optimization models. These are linear programming (LP), mixed-integer linear programming (MIP), non-linear programming (NLP), and mixed-integer non-linear programming (MINLP). Another classification is to separate them into convex and non-convex problems. The former include LPs and some NLPs; the latter is the rest of the problems. Convex problems are *well-behaved* in the sense that they have theoretical guarantees, such as global optimum and practical algorithms with fast convergence rate. The first part of the book presents these theoretical aspects. However, not all power systems operation problems are convex; therefore, we need to develop convex approximations for those problems, most of them based on conic optimization as presented in Chapter 5.

A power system is quite complex, and therefore, modeling and implementing mathematical optimization problems are equally complex. We need to gain experience in the complex art of modeling and solving mathematical optimization problems for power system applications. Our approach is to create *toy-models* for each problem. These are simplified models that allow us to understand the central issues and do numerical experiments. In the following sections, we briefly describe each of these toy-models, explained in detail in the second part of the book.

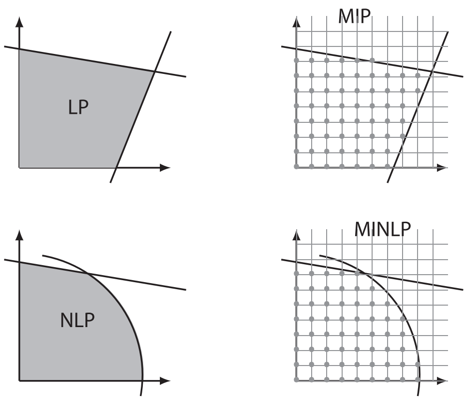

Figure 1.1 Types of optimization models.

1.2 Continuous models

1.2.1 Economic and environmental dispatch

The economic and environmental dispatch of thermal units is one of the most classic problems in power systems operation. It consists of minimizing the operating costs or the total CO_2 emissions, subject to physical constraints such as the power balance and the maximum generation capacity. For the economic dispatch, each generation unit has a cost function f_i, which is usually quadratic and depends on the power generated by each unit. Thus, the objective is to minimize the total cost (or emissions), subject to power balance, as presented below:

$$
\begin{aligned}
\min \sum_i & f_i(p_i) \\
& \sum_i p_i = d \\
& p_{\min} \leq p_i \leq p_{\max}
\end{aligned}
\tag{1.1}
$$

where p_i is the power generated by each thermal unit, and d is the total demand. Environmental dispatch introduces quadratic or exponential functions in the objective function, but the problem's structure is the same. Moreover, power flow constraints can be introduced into the model, although, in that case, it is more precise to name the problem as an optimal power flow (OPF). Chapter 7 presents the economic and environmental dispatches, while Chapter 10 presents the OPF problem.

Another problem closely related to the economic dispatch of thermal units is the unit commitment. This problem considers not only the operating costs but also the start-up and shut-down costs of thermal units. Therefore, the problem becomes binary and dynamic. This problem is studied in Chapter 8.

1.2.2 Hydrothermal dispatch

The economic dispatch problem may include hydroelectric power plants; said plants, generate two fundamental changes into the model. On the one hand, the model becomes dynamic since current operational decisions affect the future operation of the system. On the other hand, the problem becomes stochastic, because the inflows are usually random variables, especially in long-term models. The later aspect is usually solved by an accurate forecasting of the loads and the inflows; hence, it is possible to formulate a deterministic problem, called hydrothermal dispatch or hydrothermal coordination. The basic model has the following structure, namely:

$$
\begin{aligned}
\min \; & \sum_t \sum_i f_i(p_{it}) \\
& \sum_i p_{it} + \sum_j p_{jt} = d_t \;\; \forall t \in \mathcal{T} \\
& p_{jt} = g(q_{jt}, v_{jt}) \\
& v_{j(t+1)} = v_{jt} + \alpha(a_{jt} - q_{jt} - s_{jt}) \\
& p_{\min} \leq p_{it} \leq p_{\max} \\
& q_{\min} \leq q_{jt} \leq q_{\max} \\
& s_{\min} \leq s_{it} \leq s_{\max} \\
& v_{\min} \leq v_{it} \leq v_{\max}
\end{aligned}
\tag{1.2}
$$

Where i represents thermal units and j enumerates hydroelectric units; t represents the time, thus, p_{it} is the power generated by the thermal unit i at time t. The values of $a_{jt}, v_{jt}, p_{jt}, q_{jt}$, and s_{jt} are respectively, the inflow, volume, power, outflow, and spillage of the hydroelectric unit j at time t. Figure 1.2, which is self-explanatory, shows these variables.

In this model, g represents the relation between generated power, outflow, and water volume stored in the dam. Although the planning horizon $\mathcal{T}$ may be of short-term (1 day to 1 week), medium-term (1 month), or long-term (1 or more years), we are interested only in the short-term model. As aforementioned, the problem may be stochastic since power demands d_t and inflows a_{jt} are all random variables. However, a determinist model is suitable to understand the problem and its practical implementation. The situation becomes even more problematic when introducing other renewable energies, such as

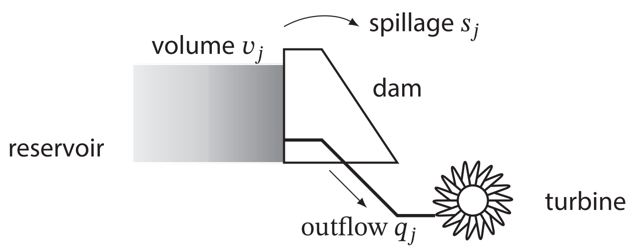

Figure 1.2 Schematic representation of the variables associated to a hydroelectric generation unit.

wind generation and photovoltaic solar generation. Chapter 9 presents the hydrothermal dispatch problem.

1.2.3 Effect of the grid constraints

Both the economic dispatch of thermal units and the hydrothermal dispatch are relatively simple problems. However, the transmission grid introduces additional constraints. Let us consider, for example, a power system with three operation areas, as shown in Figure 1.3. Each line has a maximum power flow capacity that introduces additional constraints in the model (both economic dispatch and hydrothermal dispatch). This constraint limits the flow among areas and modifies the results. These aspects are studied in Chapter 7 from a classic perspective and later, in Chapter 10, under a modern view based on conic optimization.

1.2.4 Optimal power flow

The power flow is one of the most important tools for the analysis of power systems. It allows to determine the state of the power system, knowing the magnitude of voltage and active power in all generating nodes and, active/reactive power demanded in the loads. This results in a non-linear system of equations in complex variables, as presented below:

$$s_k^* = \sum_{m \in \mathcal{N}} y_{km} v_k^* v_m \tag{1.3}$$

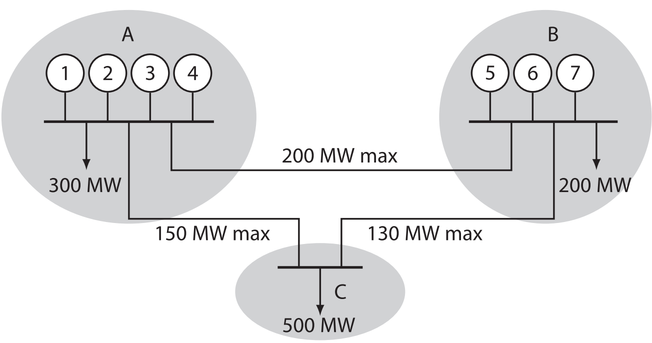

Figure 1.3 Economic dispatch by areas considering network constraints with the transport model.

where $s_k = p_k + q_k$ represents the active and reactive power in node k; $\mathcal{N}$ represents the set of nodes of the grid; y_{km} is the entry km of the Y_{bus} matrix; v_k and v_k are the voltages at nodes k and m, respectively, both represented as complex variables; and s_k^* and v_k^* are the complex conjugate of the respective variables. This representation on the complex number can be splitted into real and imaginary parts. However, as presented in Chapter 10, a complex representation is suitable both for modeling and implementation purposes.

These constraints can be introduced into the economic dispatch, as well as in an optimization model that minimizes total power loss (p_{loss}). In both cases, we named the problem as OPF. The basic model has the following structure:

$$\begin{aligned}
&\min_{v,s} P_{\text{loss}}(v) \\
&s_k^* = \sum_{m \in \mathcal{N}} y_{km} v_k^* v_m \\
&p_{\min} \leq p \leq p_{\max} \\
&\|s\| \leq s_{\max} \\
&v_{\min} \leq \|v_k\| \leq v_{\max} \\
&\text{angle}(v_0) = 0
\end{aligned} \tag{1.4}$$

This problem is highly complex due to the non-linear and non-convex nature of the power flow equations. Therefore, it is required to review different approximations that simplify the model. Chapter 10 presents three of these approximations. These are linearization, second-order cone approximation, and semidefinite programming approximation.

Linearization is, perhaps, the most straightforward way to solve the problem. Although the concept of linearization is well-known in real numbers, in this case, we do a linearization on the complex domain. This linearization uses Wirtinger's calculus since Equation (1.3) is non-holomorphic (i.e., it does not have a derivative in the complex numbers). Chapter 10 and Appendix B study this aspect in detail.

We also solve the optimal power flow using second-order cone and semidefinite programming. These approximations demand a basic understanding of conic optimization. Therefore, we present a general background of conic optimization in Chapter 5, and its application to the optimal power flow problem in Chapter 10, including a complete discussion about their advantages and disadvantages.

An optimal power flow may optimize both power systems and power distribution grids. However, the latter case presents some particularities that deserve an independent study. Moreover, both solar and wind energy require power electronic converters connected at power distribution level. These converters are capable of controlling reactive power, and therefore, it is possible to formulate an OPF wherein the decision variables are the power factor of each converter. The problem can be deterministic for real-time operation or stochastic for the day ahead planning. In both cases, the model has, at least, the same complexity as the basic OPF.

1.2.5 Hosting capacity

The concept of hosting capacity refers to the amount of solar photovoltaic generation (or wind) that can be hosted on a power distribution network, at a given time, without adversely impacting safety, power quality, reliability, or other operational features [1]. This analysis can use different performance criteria, including transient and stationary state analysis. The latter alludes to the maximum amount of generated power possible to host without creating over-voltage problems and maintaining operational limits on the distribution lines.

A hosting capacity analysis requires considering the stochastic nature of solar radiation and power demand [2]. Furthermore, it needs to consider the nonlinear characteristics related to the power flow equations. Therefore, it is a problem as complex as the optimal power flow (Chapter 11 investigates this problem).

1.2.6 Demand-side management

Demand-side management constitutes a paradigm shift in the context of smart grids, where loads are active components subject to optimization. The role of demand-side management is crucial to decrease CO_2 emissions, reduce the bottleneck in the transmission system, diminish operational cost, and improve efficiency. In order to attain these objectives, it is required to apply mechanisms of electrical load management with static and dynamic techniques. Static techniques involve administrative measures as policies and activities to incentive the end-users to change their energy demand pattern; dynamic techniques include actions to reduce the electricity consumption, such as peak-clipping, valley filling, and load shifting, among others. Information and communication technologies (ICT), as well as the concept of the Internet of the Things (IoT), allow to control the loads and integrate this control in a centralized optimization model.

In general, controllable loads are introduced in a model very similar to the economic dispatch. Its basic structure is presented below:

$$\begin{aligned} \min \; & \sum_{t}\sum_{i} c_{it} p_{it} \\ & \sum_{t} (\bar{p}_{it} - p_{it}) \geq d_t \\ & 0 \leq p_{it} \leq \bar{p}_{it} \end{aligned} \tag{1.5}$$

where $\bar{p}_{it}$ is the power required by the load i at time t; p_{it} is the amount of power that is reduced due to the demand-side management model; c_{it} is the cost of disconnecting one unit of power; and d_t is the minimum demand. This is only the basic optimization model, which can be modified, in order to include more type of loads and other aspects of the operation of the system.

Some loads can be moved in time, for example, the washing machine in a residential user. These loads, known as shifting loads, can be optimized by defining the load's optimal starting time. This optimization model is binary but tractable as presented in Chapter 13.

A demand-side management model can also include a model for tertiary control in microgrids or a model for charging electric vehicles. The latter is usually called vehicle-to-grid or V2G. In these cases, the optimization model requires to be executed in real-time by an aggregator as depicted in Figure 1.4.

An aggregator is a crucial component in modern smart distribution networks. This device receives information of the final users – in this case, the electric vehicles – and gives the control actions in order to obtain a *smart* operation. However, the intelligent part of this system is not in the hardware but in the optimization required to solve the problem efficiently and in real-time; therein lies the importance of understanding the optimization model.

A V2G strategy can be unidirectional or bidirectional. In unidirectional V2G, an aggregator controls the electric vehicles' charge similarly as shifting loads.

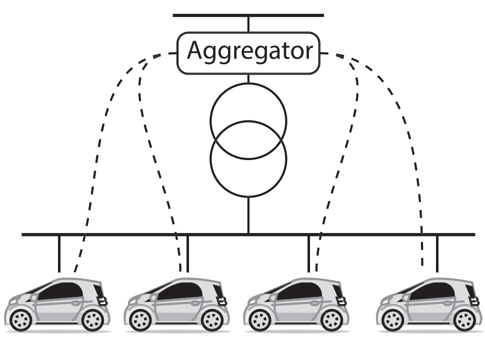

Figure 1.4 Vehicle-to-grid concept with an aggregator that centralizes control actions. Dashed lines represent a communication architecture with the aggregator.

In bidirectional, the electric vehicle can inject power into the grid if required for improving the operation. In any case, the model can become stochastic since the state of charge of the vehicles can be unknown, and the aggregator does not control the arrival/departing time of the vehicles[1]. The aggregator can also incorporate economic dispatch and OPF models to manage other distributed resources such as local batteries, solar panels, and wind turbines. Chapter 11 examines these problems.

1.2.7 Energy storage management

Modern power systems can integrate renewable energy and energy storage devices through a virtual power plant (VPP), an entity that group and centralize the operation of distributed resources to be dispatched by the power system operator. A VPP can encompass an entire region with different renewable sources and energy storage devices. It can also group other microgrids along a distribution feeder.

There are at least two moments where optimization models are required: day-ahead dispatch and real-time operation. Day-ahead dispatch corresponds to the optimization model executed the day before the operation as an economic dispatch model (see Section 1.2.1). This model must include the availability of generation and consider a forecast of the primary resource (inflows, wind, and solar radiance). Moreover, it gives the value power that the VPP operator undertakes on the day of the operation. During the operation, the VPP requires satisfying operative constraints and correcting errors in forecasting the primary resource. Again, a real-time algorithm is necessary for energy storage management.

1.2.8 State estimation and grid identification

The problem of state estimation is classic in power systems. It is also a key component in Supervisory Control And Data Acquisition (SCADA) systems. The problem consists in determining the most probable state of the system from redundant measurements and knowledge of the topology and electrical relations of the grid. When the variables to be measured are active and reactive powers, a non-convex problem is obtained with the same degree of complexity as the load flow. Modern technologies such as the phasor measurement units (PMUs) allow to include direct measures of voltages and angles.

---

1 We incorporate uncertainty in the models using robust optimization. Chapter 6 is dedicated to this aspect.

The problem can be also formulated in power distribution networks and microgrids, both AC and DC. Figure 1.5 shows, for example, a microgrid with a centralized control. Each active element of the network can have both voltage and current measurement. We can use these measurements in order to find the most likely state of the system based on the least squares model as shown below:

$$\begin{aligned} \min_{I,V} \; & (I-J)^\top M(i-j) + (v-u)^\top N(v-u) \\ & I = YV \\ & I_{\min} \le I \le I_{\max} \\ & V_{\min} \le V \le V_{\max} \end{aligned} \tag{1.6}$$

where J, U are measurements of current and voltage, respectively; I, V are the corresponding estimations and M, N are diagonal matrices that represent the weight of each measurement. The state estimation problem is closely related to the optimal power flow. In fact, some authors call this problem as the inverse power flow problem. The problem is studied in more detail in the second part of the book (Chapter 12).

Another operation problem, closely linked to the state estimation, is the identification of the network. In this case, we have measurements of both voltages and currents at different operating points. Our goal is to estimate the value of the nodal admittance matrix from these measurements. In this case, the optimization model is the following:

$$\min_{Y} f(Y) = \frac{1}{2} \|I - YV\|_F^2 \tag{1.7}$$

The decision variable is the nodal admittance matrix Y, and the objective function is the norm of error between measurements and estimations[2].

The model can include information about the structure of the matrix Y. For example, we already know that the matrix is symmetric and, some of its entries are zero. In that case, the optimization model is the following:

$$\begin{aligned} \min_{Y} f(Y) = \frac{1}{2} \|I - YV\|_F \\ y_{ij} = y_{ji}, \; \forall i, j \\ y_{ij} = 0, \; \text{if } i \text{ is not connected to} j \end{aligned} \tag{1.8}$$

2 It is usual to consider the Frobenius norm as explained in Chapter 12.

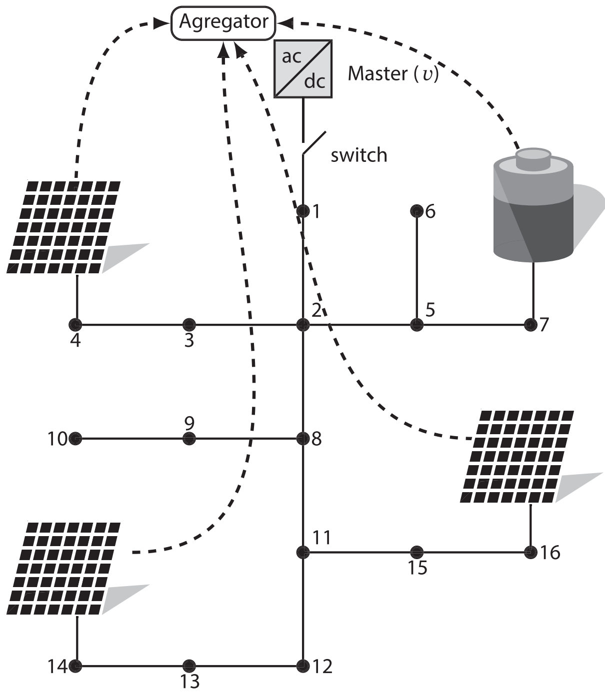

Figure 1.5 Example of a microgrid with a centralized control/measurement in the aggregator.

Both AC and DC grids may handle this type of estimation. In this case, we only presented the DC case because it is easier to develop. The entire model must be implemented in an aggregator structure, as depicted in Figure 1.5.

1.3 Binary problems in power systems operation

Some of the problems previously described are binary. These problems appear in electrical systems, both in the planning and the operation stages. In the planning stage, the problems of transmission expansion planning and distribution planning are typical. Heuristic techniques usually solve these problems together with mixed-integer programming approximations [3]. Although mixed-integer problems are non-convex, they can be solved efficiently using methodologies such as the branch-and-bound algorithm. We present a brief introduction of this method in Chapter 4. Besides, several modules can be called from Python to solve these types of problems. We used Gurobi and Mosek for this task, although there are plenty of options available. Binary operation problems include unit commitment and phase balancing in power distribution networks.

1.3.1 Unit commitment

As aforementioned, the unit commitment problem consists in determining the order of starting and stopping of the thermal units, taking into account the costs of turning-ON and turning-OFF, as well as the starting ramps. The optimization model is similar to the economic dispatch, but in this case, there are binary variables s_k related to the state ON or OFF of each thermal unit. The most simplified model is presented below:

$$
\begin{aligned}
\min \; & \sum f(p_k) + \sum h(s_k) \\
& \sum p_k = p_D \\
& r(s_k) \leq 0 \\
& p_{\min} s_k \leq p_k \leq p_{\max} s_k \\
& s_k \in \{0, 1\}
\end{aligned}
\tag{1.9}
$$

where f represents operative costs, h are costs of turning-ON and turning-OFF, r are starting ramps, and $p_{\min}$, $p_{\max}$ are the operation limits of each unit. Binary problems are usually difficult; however, it is possible to find either convex approximation or MILP equivalents as presented in Chapter 5.

1.3.2 Optimal placement of distributed generation and capacitors

We can use the same convex approximations of the optimal power flow problem in problems such as the optimal placement of distributed generation in power distribution networks. The problem consists of determining the placement and capacity of distributed generation, subject to physical constraints, such as voltage regulation and transmission lines' capacity. Besides, the problem includes binary constraints related to the placement of the distributed generation. The objective function is usually total power loss, although other objectives, such as reliability and optimal costs, are also suitable.

Another binary problem closely related to the OPF is the optimal placement of capacitors. In this problem, fixed or variable capacitors are placed along the primary feeder to minimize power loss. The capacitors' location and the number of fixed capacitors in each node are binary variables considered in the model. Chapter 11 examines the optimal placement of capacitors and the optimal placement of distributed generation.

1.3.3 Primary feeder reconfiguration and topology identification

The topology of a grid is not constant, especially in power distribution networks. There are several sectionalizing switches along with the primary feeders that permit transferring load among circuits. A feeder reconfiguration is an operating model that determines the optimal topology to minimize power loss. This model is non-linear, non-convex, and binary. In addition, it has a constraint related to the connectivity and radiality of the graph that is tricky to represent in equations. All these aspects are studied in Chapter 11.

Like the state estimation is the inverse of the optimal power problem, there is a problem that can be considered as the inverse of the primary feeder reconfiguration. It is the topology identification in power distribution grids, which takes measurements of current and voltages in different parts of the grid and determines the state of each sectionalizing switch. This problem is studied in Chapter 12.

1.3.4 Phase balancing

Phase balancing is a combinatorial problem that consists of phase swapping of the loads and generators to reduce power loss. Despite being a classic problem, it is still relevant since the unbalance is a common phenomenon in microgrids, especially when single-phase photovoltaic units are included. Because it is a combinatorial problem, phase balancing requires heuristic algorithms with high computational effort. However, it is possible to generate simplified instances of the problem, as presented in Chapter 13.

Each three-phase node has six possible configurations as depicted in Figure 1.6. The problem consists of defining the phase in which each load or generator is connected in order to reduce the grid's total losses. Therefore, the problem is combinatorial since there are 6^n possible configurations, where n is the number of three-phase nodes. The problem is nontrivial even in small

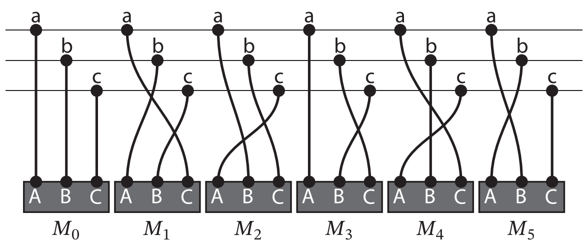

Figure 1.6 Set of possible configurations in a three-phase node.

networks; for example, a microgrid with 10 nodes will have 6×10^7 possible configurations.

Phase-balancing problems appear in many applications such as aircraft electric systems [4], and in power distribution grids with high penetration of electric vehicles [5]. Due to its combinatorial nature, the problem necessitates the use of heuristics [6], and meta-heuristics [7] as well as expert systems [8]. Modern approaches include the uncertainty associated to the load and generator [9].

1.4 Real-time implementation

A receding horizon control can implement most of the optimization algorithms presented in this book. Figure 1.7 depicts the main architecture for this simple but powerful strategy, for real-time implementation of operation models. These optimization models may be the optimal power flow, economic dispatch, energy storage management, or a mixed model that includes multiple models. An unbiased forecast predicts variables such as wind speed, solar radiation, and power demand. Moreover, a state estimator gives accurate measurements of the system variables.

Equation (1.10) represents the optimization model, where x is the vector of decision variables for each time t, and α are the parameters predicted by the forecast module. Of course, this forecast may change since renewable resources and loads may be highly variable in modern power systems. Therefore, the optimization model must be continuously executed and the solution updated.

$$\begin{aligned} &\min f(x, \alpha) \\ &\quad x \in \Omega \end{aligned} \tag{1.10}$$

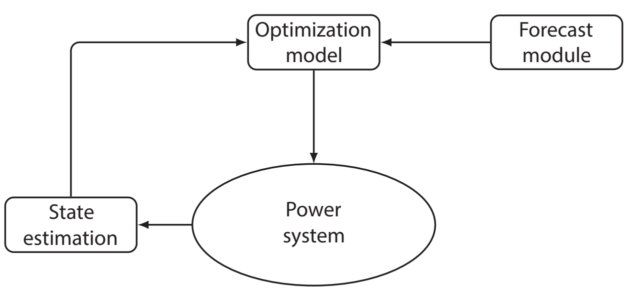

Figure 1.7 A possible architecture for implementing an optimization model for power systems operation.

In many cases, the forecast has an error that introduces uncertainty in the model. Either stochastic or robust optimization is a suitable option to face this uncertainty. Chapter 6 presents the latter option.

1.5 Using Python

Python is a general programming language that is gaining attention in power systems optimization. Although it is neither a mathematical software nor an algebraic modeling language, it has many free tools for solving optimization problems. Moreover, there are many other tools for data manipulation, plotting, and integration with other software. Hence, it is a convenient platform for solving practical problems and integrate different resources[3].

We use a module named CvxPy [10] that allows to solve convex and mixed-integer convex optimization problems; this module, together with NumPy, MatplotLib, and Pandas, forms a robust platform for solving all types of optimization problems in power systems applications. Let us consider, for instance, the following optimization problem:

$$\begin{aligned} \min\ & c^{\top}x \\ & \sum_i x_i = 1 \\ & x_i \geq 0 \end{aligned} \tag{1.11}$$

where $x \in \mathbb{R}^6$ and $c = (5, 3, 2, 4, 8, 7)^{\top}$. A model in CvxPy for this problem looks as follows:

```
import numpy as np
import cvxpy as cvx
c = np.array([5,3,2,4,8,7])
x = cvx.Variable(6)
objective = cvx.Minimize(c.T * x)
constraints = [ sum(x) == 1, x >= 0]
Model = cvx.Problem(objective,constraints)
Model.solve()
```

Without much effort, we can identify variables, the objective function, and constraints. This neat code feature is an essential aspect of Python and CvxPy. After the problem is solved, we can make additional analyses using the same platform. This combination of tools is, of course, a great advantage; however, we must avoid any fanaticism for software. There are many programming

3 See appendix C for a brief introduction to Python.

languages and many modules for mathematical optimization. What is learned in this book may be translated to any other language. The problem is the same, although its implementation may change from one platform to another.

We made a great effort in making the examples as simple as possible (we call them *toy-models*). This approach allows us to understand each problem individually and do numerical experiments. Real operation models may include different aspects of these toy-models; for instance, they may combine economic dispatch, optimal power flow, and state estimation. These models are highly involved with thousands of variables and constraints. Nevertheless, they can be solved using the same paradigm presented in this book.