CHAPTER 1

The Brain and Mathematics Learning

In the last few decades we have seen the emergence of technologies that have given researchers new access into the workings of the mind and brain. Now scientists can study children and adults working on math and watch their brain activity; they can look at brain growth and brain degeneration, and they can see the impact of different emotional conditions upon brain activity. One area that has emerged in recent years and stunned scientists concerns "brain plasticity." It used to be believed that the brains people were born with couldn't really be changed, but this idea has now been resoundingly disproved. Study after study has shown the incredible capacity of brains to grow and change within a really short period (Abiola & Dhindsa, 2011; Maguire, Woollett, & Spiers, 2006; Woollett & Maguire, 2011).

When we learn a new idea, one of three things happens in the brain (see Figure 1.1). The first possibility is that you start a new brain pathway. The more deeply you learn, the stronger the pathway becomes. The second possibility is that you strengthen a pathway you already had, and the third possibility is that you make connections between pathways. This brain development is taking place all the time, and the pathways you build, strengthen, or connect were not in your brain at birth; they are created by your learning experiences.

I wish all students knew this—when you are teaching them math, you are changing their brains! Neuroscientist Norman Doidge (2007) likes to share with his audiences that every day you wake up, your brain is different from the day before—that is the extent of brain growth and change that occur every day. If you learn something deeply, you form lasting brain pathways that you can revisit and use, but if you visit an idea only once or in a superficial way, the pathway can "wash away" like a path made in the sand. These brain connections form when learning takes place, but learning does not occur only in classrooms or when reading books; as we all know, we are forming brain connections when we have conversations, play games, or build with toys, and in the course of many, many other experiences.

The first research on what became known as neuroplasticity, which shocked the scientific world, came from studies of "Black Cab" drivers in London. I am from England, and I have traveled in taxicabs in London many times. I still have fond

FIGURE 1.1 Brains form, develop, and connect pathways.

FIGURE 1.2 The Black Cab of London
Source: Peter Fuchs/Shutterstock.

memories of the exciting day trips my family and I took to London when I was a child, from our home a few hours away. As an adult I studied and worked at King's College, London University, and had many more opportunities for trips around London in taxis. A number of different taxis work in the London area, but the queen bee of taxis in London is the Black Cab (see Figure 1.2).

For most of my rides through London in a Black Cab, I had no idea how highly qualified the drivers were. It turns out that to become a Black Cab driver in London, applicants need to study for four years or more, and during that time learn routes around 25,000 streets and 20,000 landmarks within a 25-mile radius of Charing Cross in London. Learning your way around the city of London is considerably more challenging than learning your way around most American cities, as London is not built on a grid structure and comprises thousands of interweaving, interconnected streets (see Figure 1.3).

At the end of their training period the Black Cab drivers take a test that is simply and elegantly called "The Knowledge." If you ride in a London Black Cab and ask your driver about "The Knowledge," they are usually happy to regale you with stories of the difficulty of the test and their training period. The Knowledge is known to be one of the world's most demanding courses, and applicants take the test an average of 12 times before passing.

In the early 2000s scientists chose to study London Black Cab drivers to look for brain changes as the drivers took years of complex spatial training, but the scientists were not expecting such dramatic results. Researchers found that at the end of the training period the hippocampus in the taxi drivers' brains had grown significantly (Maguire et al., 2006; Woollett & Maguire, 2011).

The hippocampus is the brain area specialized in acquiring and using spatial information (see Figure 1.4).

FIGURE 1.3 Map of London
Source: jason cox/Shutterstock.

In other studies, scientists compared the brain growth of Black Cab drivers to that of London bus drivers. Bus drivers learn only simple and singular routes, and the studies showed that they did not experience the same brain growth (Maguire et al., 2006). This confirmed the scientists' conclusion that the Black Cab drivers' unusually complex training was the reason for their dramatic brain growth. In a further study, scientists found that after Black Cab drivers retired, their hippocampus shrank back down again (Woollett & Maguire, 2011). This was not because of age but because of lack of using the brain pathways.

The studies conducted with Black Cab drivers, of which there have now been many (Maguire et al., 2006; Woollett & Maguire, 2011), showed a degree of brain flexibility, or plasticity, that stunned scientists. They had not previously thought that the extent of brain growth they measured was possible. This led to a shift in the scientific world in thinking about learning and "ability" and the possibility of the brain to change and grow.

FIGURE 1.4 The hippocampus
Source: decade3d/Shutterstock.

Around the time that the Black Cab studies were emerging, something happened that would further rock the scientific world. A nine-year old girl, Cameron Mott, had been having seizures that the doctors could not control. Her physician, Dr. George Jello, proposed something radical. He decided he should remove half of her brain, the entire left hemisphere. The operation was revolutionary—and ultimately successful. In the days following her operation, Cameron was paralyzed. Doctors expected her to be disabled for many years, as the left side of the brain controls physical movements. But as weeks and months passed, she stunned doctors by recovering function and movement that could mean only one thing—the right side of her brain was developing the connections it needed to perform the functions of the left side of the brain. Doctors attributed this to the incredible plasticity of the brain and could only conclude that the brain had, in effect, "regrown." The new brain growth had occurred faster than doctors imagined possible (http://www.today.com/id/36032653/ns/today-today_health/t/meet-girl-half-brain/#.UeGbixbfvCE).

This operation has now been performed on many different people. Christina Santhouse was eight when she underwent the operation to have half of her brain removed. Christina went on to many notable achievements, including making the honor roll at high school, earning a master's degree, and becoming a speech pathologist.

The new findings that brains can grow, adapt, and change shocked the scientific world and spawned new studies of the brain and learning, making use of ever-developing new technologies and brain scanning equipment. In one study, researchers at the National Institute for Mental Health gave people a 10-minute exercise to work on each day for three weeks. The researchers compared the brains of those receiving the training with those who did not. The results showed that the people who worked on an exercise for a few minutes each day experienced structural brain changes. The participants' brains "rewired" and grew in response to a 10-minute mental task performed daily over 15 weekdays (Karni et al., 1998).

In another study on mathematics learning in particular, Teresa Iuculano and her colleagues at Stanford's school of medicine uncovered critically important information. They brought into their labs two groups of students. One group had been diagnosed in schools as having mathematics learning disabilities; the other group comprised "regular" performers. The researchers looked at the students' brains as they worked on mathematics, using MRI scans. They found fascinating differences—the students identified as having learning disabilities had more brain regions lighting up when they worked on a mathematics question. This is counterintuitive for many people, who think that students with learning disabilities have less going on in their brains, not more. The researchers point out that success does not always come from *more* brain activity, but focused activity in certain areas. The research then became even more interesting. Both sets of students were given eight weeks of one-to-one tutoring. At the end of the eight-week period, the two sets of students had not only the same achievement but the exact same brain areas lighting up (Iuculano et al., 2015).

These and other results should prompt educators to abandon the traditional fixed ideas of the brain and learning that currently permeate schools—ideas that children are smart or dumb, quick or slow. If brains can change significantly in eight weeks, imagine what can happen in a year of maths class if students are given engaging tasks (more on that later) and receive positive messages about their potential and ability. Chapter Five will explain the nature of the very best mathematics tasks that students should be working on to experience this brain growth.

The new evidence from brain research tells us that everyone, with the right teaching and messages, can be successful in mathematics and that everyone can achieve at the highest levels in school. There are a few children who have severe special educational needs that make mathematics learning difficult, but for the vast majority of children—at least 95%—any level of school mathematics is within their reach. And the potential of the brain to grow and change is just as strong in children diagnosed as having special educational needs (see also Boaler & LaMar, 2019). This was clearly illustrated by a beautiful event that took place in Australia. As Nicholas Letchford was growing up, he was diagnosed as "learning disabled." In his first year of school, his parents were told that he had a "very low IQ," and teachers told his parents that "he was the worst child they had seen in twenty years of teaching." Nicholas found it difficult to focus, make connections, read, or write. But Nicholas's mother, Lois, refused to accept the labels placed on her son, and she worked with Nicholas, teaching him how to focus, connect, read, and write. In 2018, Nicholas graduated from Oxford University with a doctoral degree in applied mathematics—the highest level of achievement possible to attain (Letchford, 2018).

Parents and teachers need to know this information from neuroscience and from records of people. When I share both forms of evidence with teachers in workshops and presentations, most of them are encouraged and inspired, but not all of them. I was with a group of teachers recently, and one high school math teacher was clearly troubled by the idea. He said, "You aren't telling me, are you, that *any* of the sixth graders in my school could take calculus in twelfth grade?" When I said, "That is exactly what I am saying," I could tell he was genuinely troubled by the idea—although, to his credit, he was not rejecting it outright. Some teachers find the idea that anyone can learn math to high levels difficult to accept, especially if they have spent many years deciding who can and who can't do math and teaching them accordingly. Of course, sixth graders have had many experiences and messages since birth that have held some of them back, and some students may come to sixth grade with significantly less mathematical knowledge than others, but this doesn't mean they cannot accelerate and reach the highest levels—they can, if they receive the high-quality teaching and support that all children deserve.

I am often asked whether I am saying that everyone is born with the same brain. I am not. What I am saying is that any brain differences children are born with are nowhere near as important as the brain growth experiences they have throughout life. People hold very strong views that the way we are born determines our potential; they point to well-known people who were considered geniuses—such as Albert Einstein or Ludwig van Beethoven. But scientists now know that any brain differences present at birth are eclipsed by the learning experiences we have from birth onward (Wexler in Thompson, 2014). Every second of the day, brain pathways are forming, connecting, and strengthening, and students given encouraging messages and rich mathematics experiences are capable of anything. Brain differences can give some people a head start, but infinitesimally small numbers of people have the sort of head start that gives them advantages over time. And those people who are heralded as natural geniuses are the same people who often stress the hard work they have put in and the number of mistakes they made. Einstein, probably the most well known of those thought to be a genius, did not learn to read until he was nine and spoke often about his achievements coming from the number of mistakes he had made and the persistence he had shown. He tried hard, and when he made mistakes he tried harder. He approached work and life with the attitude of someone with a growth mindset. A lot of scientific evidence suggests that the difference between those who succeed and those who don't is not the brains they were born with, but their approach to life, the messages they receive about their potential, and the opportunities they have to learn. The very best opportunities to learn come about when students believe in themselves. For far too many students in school, their learning is hampered by the messages they have received about their own potential, making them believe they are not as good as others, that they don't have the potential of others. This book provides the information you need, whether you are a teacher or parent, to give students the self-belief they need and should have; to set them on a pathway that leads to a mathematical mindset, whatever their prior experiences. This new pathway involves a change in the way students consider themselves and also a change in the way they approach the subject of mathematics, as the rest of the book will describe.

Although I am not saying that everyone is born with the same brain, I *am* saying that there is no such thing as a "math brain" or a "math gift," as many believe. No one is born knowing math, and no one is born lacking the ability to learn math. Unfortunately, ideas of giftedness are widespread. Researchers recently investigated the extent to which college professors held ideas about giftedness in their subject, and they found something remarkable (Leslie, Cimpian, Meyer, & Freeland, 2015). Math was the subject whose professors were found to hold the most fixed ideas about who could learn. Additionally, researchers found that the more a field values giftedness, the fewer female PhDs there were in the field and that field-specific beliefs were correlated with female representation across all 30 fields they investigated. The reason that there are fewer women in fields where professors believe that only the "gifted" can achieve is that stereotypical beliefs still prevail about who really belongs, as Chapter Six describes. It is imperative for our society that we move to a more equitable and informed view of mathematics learning in our conversations and work with students. Schools should seriously consider the new science of the brain and communicate to all students that everyone can learn mathematics to high levels. This could well be the key to unlocking a different future—one in which math trauma is a thing of the past and students from all backgrounds are given access to high-quality mathematics learning opportunities.

In studies by Carol Dweck and her colleagues, about 40% of the children were found to hold a damaging fixed mindset, believing that intelligence is a gift that you either have or you don't. Another 40% of the students had a growth mindset. The remaining 20% wavered between the two mindsets (Dweck, 2006b). Students with a fixed mindset are more likely to give up easily, whereas students with a growth mindset keep going even when work is hard and are persistent, displaying what Angela Duckworth has termed "grit" (Duckworth & Quinn, 2009). In one study, seventh-grade students were given a survey to measure their mindset, then researchers followed the students over two years to monitor their mathematics achievement. The results were dramatic, as the achievement of the students with a fixed mindset stayed constant, but the achievement of those with a growth mindset went onward and upward (Blackwell et al., 2007) (see Figure 1.5).

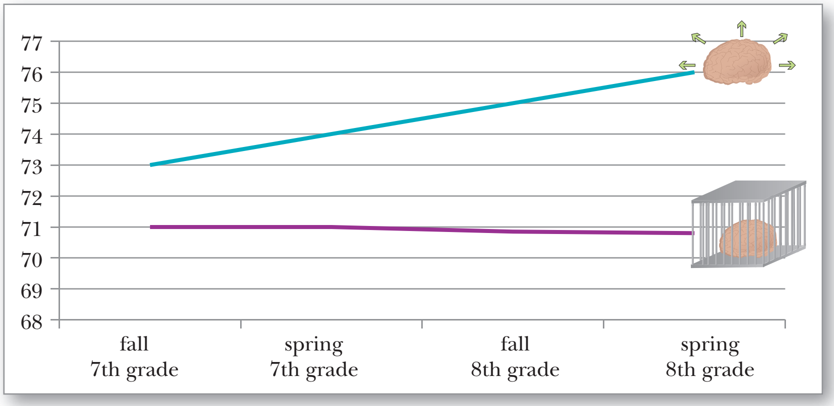

FIGURE 1.5 Students with a growth mindset outperform those with a fixed mindset in mathematics
Source: Modified from Blackwell et al., 2007.

In other studies, researchers have shown that students' (and adults') mindsets can change from fixed to growth, and when that happens their learning approach becomes significantly more positive and successful (Blackwell et al., 2007). We also have new evidence, that I review in Chapter Two, that students with a growth mindset have more positive brain activity when they make mistakes, with more brain regions lighting up and more attention to and correcting of errors (Moser, Schroder, Heeter, Moran, & Lee, 2011).

I didn't need more evidence of the importance of helping students—and adults—develop a growth mindset in relation to math in particular, but recently I found myself sitting with the Program for International Student Assessment (PISA) team at the Organisation for Economic Co-operation and Development (OECD) in Paris, exploring with them their incredible data set of 13 million students worldwide. The PISA team gives international tests every four years, and the results are reported in news outlets across the globe. The test scores often start alarm bells ringing around the United States, and for good reason. In the latest tests, the United States ranked 36th out of 65 OECD countries in math performance (OECD, 2018)—a result that speaks, as do many others, to the incredible need to reform mathematics teaching and learning in the United States. But the PISA team not only administer math tests; they also survey students to collect their ideas and beliefs about mathematics and their mindsets. I was invited to work with the PISA team after some of the group took one of my online classes (www.youcubed.org/resource/online-courses-for-teachers/). One of them was Pablo Zoido, a soft-spoken Spaniard who thinks deeply about math learning and has considerable expertise in working with giant data sets. Pablo, was, at the time, an analyst for PISA, and as he and I explored the data, we saw something amazing—that the highest-achieving students in the world are those with a growth mindset, and they outrank the other students by the equivalent of more than a year of mathematics (see Figure 1.6).

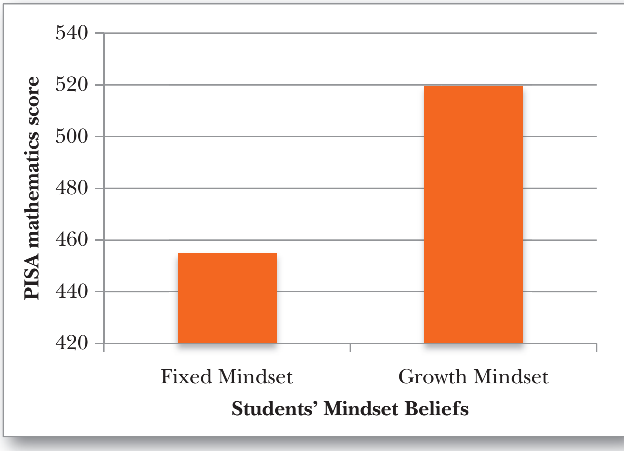

FIGURE 1.6 Mindset and mathematics
Source: Based on PISA, 2012.

The fixed mindset thinking that is so damaging—a mindset in which students believe they either are smart or are not—cuts across the achievement spectrum, and some of the students most damaged by these beliefs are high-achieving girls (Dweck, 2006a). It turns out that even believing you *are* smart—one of the fixed mindset messages—is damaging, as students with this fixed mindset are less willing to try more challenging work or subjects because they are afraid of slipping up and no longer being seen as smart. Students with a growth mindset take on hard work, and they view mistakes as a challenge and motivation to do more. The high incidence of fixed mindset thinking among girls is one reason that girls opt out of STEM subjects—science, technology, mathematics, and engineering. This not only reduces their own life chances but also impoverishes the STEM disciplines that need the thinking and perspectives that girls and women bring (Boaler, 2014a).

One reason so many students in the United States have fixed mindsets is the praise they are given by parents and teachers. When students are given fixed praise—for example, being told they are smart when they do something well—they may feel good at first, but when they fail or struggle later (and everyone does), they think that means they are not so smart after all. In an important study, researchers found that the praise parents gave their babies between birth and age three predicted their mindsets five years later (Gunderson et al., 2013). The impact of the praise students receive can be so strong that it affects their behavior immediately. In one of Carol Dweck's studies, researchers asked 400 fifth graders to take an easy short test, on which almost all performed well. Half the children were then praised for "being really smart." The other half were complimented on "having worked really hard." The children were then asked to take a second test and choose between one that was pretty simple, that they would do well on, or one that was more challenging, that they might make mistakes on. Ninety percent of those who were praised for their effort chose the harder test. Of those praised for being smart, the majority chose the easy test (Mueller & Dweck, 1998).

Praise feels good, but when people are praised for who they are as a person ("You are so smart") rather than what they did ("That is amazing thinking and creativity"), they get the idea that they have a fixed amount of ability. Telling students they are smart sets them up for problems later. As students go through school and life, failing at many tasks—which, again, is perfectly natural—they evaluate themselves, deciding how smart or not smart this means they really are. Instead of praising students for being smart, or any other personal attribute, it's better to say things like: "It is great that you have learned that," and "You have thought really deeply about this."

Our education systems have been pervaded with the traditional notion that some students are not developmentally ready for certain levels of mathematics. A group of high school math teachers in a school I recently encountered had, shockingly, written to the school board arguing that some students could never pass algebra 2. They particularly cited minority students from low-income homes; they argued that these students could not learn algebra unless the teachers watered down the curriculum. Such deficit and racist thinking needs to be banished from schools. The letter written by the teachers was published in local newspapers and ended up being used in the state legislature as an example of the need for charter schools (Noguchi, 2012). The letter shocked many people, but unfortunately this idea that some students cannot learn high-level mathematics is shared by many. Deficit thinking can take all sorts of forms and is sometimes used with genuine concern for students—many people believe there is a developmental stage students must go through before they are ready for certain mathematics topics. But these ideas are also outdated, as students are as ready as the experiences they have had, and if students are not ready, they can easily become so with the right experiences, high expectations from others, and a growth mindset. There is no preordained pace at which students need to learn mathematics, meaning it is *not* true that if they have not attained a certain age or emotional maturity they cannot learn some mathematics. Students may be unready for some mathematics because they still need to learn some foundational, prerequisite mathematics they have not yet learned, but *not* because their brain cannot develop the connections because of their age or maturity. When students need new connections, they can learn them.

For many of us, appreciating the importance of mathematical mindsets and developing the perspective and strategies to change students' mindsets involves some careful thinking about our own learning and relationship with mathematics. Many of the elementary teachers I have worked with, some of whom took my online class, have told me that the ideas I gave them on the brain, on potential, and on growth mindsets has been life-changing for them. It caused them to develop a growth mindset in mathematics, to approach mathematics with confidence and enthusiasm and to pass that on to their students. This is often particularly important for elementary teachers, because many have, at some point in their own learning, been told *they* cannot do mathematics or that mathematics is not for them. Many teach mathematics with their own fear of the subject. The research I shared with them helped banish that fear and put them on a different mathematical journey. In an important study, Sian Beilock and colleagues found that the extent of negative emotions elementary teachers held about mathematics predicted the achievement of girls in their classes, but not boys (Beilock, Gunderson, Ramirez, & Levine, 2009). This gender difference probably comes about because girls identify with their female teachers, particularly in elementary school. Girls quickly pick up on teachers' negative messages about math—the sort that are often given out of kindness, such as: "I know this is really hard, but let's try and do it" or "I was bad at math at school" or "I never liked math." This study also highlights the link between the messages teachers give and the achievement of their students.

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Neuroscience has also made huge contributions to our understanding of mathematics learning in other areas, beyond neuroplasticity. I am fortunate in being able to work with neuroscientists at Stanford, and two I have collaborated with in recent years have been instrumental in this work: Vinod Menon and Lang Chen. They have studied the interacting networks in the brain, particularly focusing on the ways the brain learns mathematics. They have found that whenever we work on mathematics, there are five different brain areas involved; two of them are visual pathways. The dorsal visual pathway is the main brain area for representing quantity. These researchers, and others, have also found that communication between the different brain areas enhances learning and achievement (see also Park & Brannon, 2013). For example, if a student sees a calculation with numbers, as well as a visual representation, this will encourage brain connections—communication between brain areas.

Other incredible neuroscience has shown that fingers are particularly critical to mathematics learning (Boaler & Chen, 2016). The work with first graders I mentioned in the preface was a project that involved educators (myself and my Youcubed team), neuroscientists, and engineers, building on the research on the importance of fingers to mathematics learning. We created a robotic device that allowed students to answer questions with their fingers, and feel vibrations in their fingers, to really help them associate fingers with numbers. It was a fascinating experimental study which showed that the students who worked with the robotic finger device learned significantly more mathematics over a short period of time. But finger work does not need to be so high-tech or to involve robotics. We share paper resources on youcubed.org that can be similarly helpful for students to develop what is known as "finger perception" (see www.youcubed.org/resource/visual-mathematics/).

We can learn mathematics with numbers, but we can also learn through words, visuals, models, algorithms, tables, and graphs, and from moving and touching. Experiencing mathematics in these multidimensional ways encourages brain communication and understanding. Lang Chen illustrates the different brain areas involved in mathematical thinking in Figure 1.7.

As I shall explain in later chapters, I like to create mathematical activities that give students a connected experience of mathematics, with different parts of their brains firing and communicating with one another, as they consider mathematics in different ways, as illustrated in Figure 1.8.

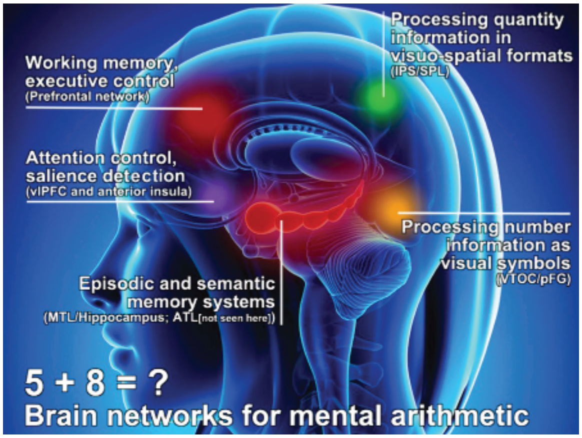

FIGURE 1.7 The brain areas involved in mathematical thought
Source: Lang Chen.

FIGURE 1.8 A connected mathematical experience
Source: pixologic/Depositphotos.

I hope that you enjoy the different ideas for helping students develop brain connections as they learn mathematics. Wherever you are on your own mathematical mindset journey, whether these ideas are new to you or you are a mathematical mindset expert, I hope that the data and ideas I share in this book will help you and your students see mathematics—any level of mathematics—as both reachable and enjoyable. In the next chapters, Two through Eight, I will share the many strategies I have collected over years of research and teaching for encouraging a mathematical mindset experience for students.