

## IN THIS CHAPTER

- » Finding out how numbers were invented
- » Looking at a few familiar number sequences
- » Examining the number line
- » Understanding four important sets of numbers

# Chapter 1

# Playing the Numbers Game

One useful characteristic of numbers is that they're *conceptual*, which means that, in an important sense, they're all in your head. (This fact probably won't get you out of having to know about them, though — nice try!)

For example, you can picture three of anything: three cats, three baseballs, three tigers, three planets. But just try to picture the concept of three all by itself, and you find it's impossible. Oh, sure, you can picture the numeral 3, but *threeness* itself — much like love or beauty or honor — is beyond direct understanding. But when you understand the *concept* of three (or four, or a million), you have access to an incredibly powerful system for understanding the world: mathematics.

In this chapter, I give you a brief history of how numbers likely came into being. I discuss a few common *number sequences* and show you how these connect with simple math *operations* like addition, subtraction, multiplication, and division.

After that, I describe how some of these ideas come together with a simple yet powerful tool: the *number line*. I discuss how numbers are arranged on the number line, and I also show you how to use the number line as a calculator for simple arithmetic. Finally, I describe how the *counting numbers* (1, 2, 3, . . .) sparked the invention of more unusual types of numbers, such as *negative numbers*, *fractions*, and *irrational numbers*. I also show you how these *sets of numbers* are *nested* — that is, how one set of numbers fits inside another, which fits inside another.

# Inventing Numbers

Historians believe that the first written number systems came into being at the same time as agriculture and commerce. Before that, people in prehistoric, hunter-gatherer societies were pretty much content to identify bunches of things as “a lot” or “a little.” They may have had concepts of small numbers, probably less than five or ten, but lacked a coherent way to think about, for example, the number 42.

Throughout the ages, the Babylonians, Egyptians, Greeks, Hindus, Romans, Mayans, Arabs, and Chinese (to name just a few) all developed their own systems of writing numbers.

Although Roman numerals gained wide currency as the Roman Empire expanded throughout Europe and parts of Asia and Africa, the more advanced system that was invented in India and adapted by the Arabs turned out to be more useful. Our own number system, the Hindu-Arabic numbers (also called decimal numbers), is mainly derived from these earlier number systems.

# Understanding Number Sequences

Although humans invented numbers for counting commodities, as I explain in the preceding section, they soon put them to use in a wide range of applications. Numbers were useful for measuring distances, counting money, amassing armies, levying taxes, building pyramids, and lots more.

But beyond their many uses for understanding the external world, numbers have an internal order all their own. So numbers are not only an *invention*, but equally a *discovery*: a landscape reflecting fundamental truths about nature, and how humans think about it, that seems to exist independently, with its own structure, mysteries, and even perils.

One path into this new and often strange world is the *number sequence*: an arrangement of numbers according to a rule. In the following sections, I introduce you to a variety of number sequences that are useful for making sense of numbers.

## Evening the odds

One of the first facts you probably heard about numbers is that all of them are either even or odd. For example, you can split an even number of marbles *evenly* into two equal piles. But when you try to divide an odd number of marbles the same way, you always have one *odd*, left-over marble. Here are the first few even numbers:

2   4   6   8   10   12   14   16 ...

You can easily keep the sequence of even numbers going as long as you like. Starting with the number 2, keep adding 2 to get the next number.

Similarly, here are the first few odd numbers:

1 3 5 7 9 11 13 15 ...

The sequence of odd numbers is just as simple to generate. Starting with the number 1, keep adding 2 to get the next number.

Patterns of even or odd numbers are the simplest number patterns around, which is why kids often figure out the difference between even and odd numbers soon after learning to count.

## Counting by threes, fours, fives, and so on

When you get used to the concept of counting by numbers greater than 1, you can run with it. For example, here's what counting by threes, fours, and fives looks like:

Threes:	3	6	9	12	15	18	21	24 ...
Fours:	4	8	12	16	20	24	28	32 ...
Fives:	5	10	15	20	25	30	35	40 ...



TIP

Counting by a given number is a good way to begin learning the multiplication table for that number, especially for the numbers you're kind of sketchy on. (In general, people seem to have the most trouble multiplying by 7, but 8 and 9 are also unpopular.)

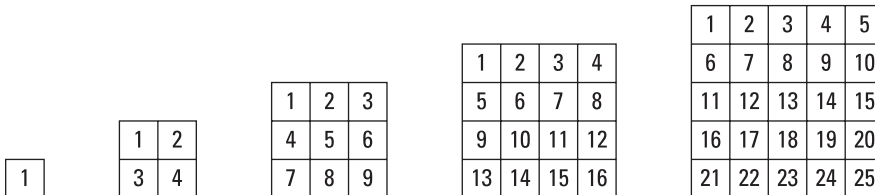
These types of sequences are also useful for understanding factors and multiples, which you get a look at in Chapter 9.

## Getting square with square numbers

When you study math, sooner or later, you probably want to use visual aids to help you see what numbers are telling you. (Later in this book, I show you how one picture can be worth a thousand numbers when I discuss geometry in Chapter 19 and graphing in Chapter 25.)

The tastiest visual aids you'll ever find are those little square cheese-flavored crackers. (You probably have a box sitting somewhere in the pantry. If not, saltine crackers or any other square food works just as well.) Shake a bunch out of a box and place the little squares together to make bigger squares. Figure 1-1 shows the first few.

**FIGURE 1-1:**  
Square  
numbers.



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Voilà! The square numbers:

1   4   9   16   25   36   49   64 ...



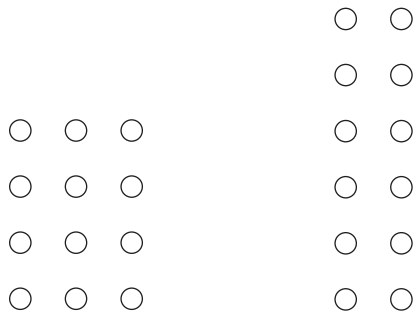
TIP

You get a *square number* by multiplying a number by itself, so knowing the square numbers is another handy way to remember part of the multiplication table. Although you probably remember without help that  $2 \times 2 = 4$ , you may be sketchy on some of the higher numbers, such as  $7 \times 7 = 49$ . Knowing the square numbers gives you another way to etch that multiplication table forever into your brain.

Square numbers are also a great first step on the way to understanding exponents, which I introduce later in this chapter and explain in more detail in Chapter 5.

## Composing yourself with composite numbers

Some numbers can be placed in rectangular patterns. Mathematicians probably should call numbers like these “rectangular numbers,” but instead they chose the term *composite numbers*. For example, 12 is a composite number because you can place 12 objects in rectangles of two different shapes, as in Figure 1-2.



**FIGURE 1-2:**  
The number 12  
laid out in two  
rectangular  
patterns.

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As with square numbers, arranging numbers in visual patterns like this tells you something about how multiplication works. In this case, by counting the sides of both rectangles, you find out the following:

$$3 \times 4 = 12$$

$$2 \times 6 = 12$$

Similarly, other numbers such as 8 and 15 can also be arranged in rectangles, as in Figure 1-3.

As you can see, both these numbers are quite happy being placed in boxes with at least two rows and two columns. And these visual patterns show this:

$$2 \times 4 = 8$$

$$3 \times 5 = 15$$

**FIGURE 1-3:**  
Composite  
numbers, such  
as 8 and 15,  
can form  
rectangles.



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The word *composite* means that these numbers are *composed of* smaller numbers. For example, the number 15 is composed of 3 and 5 — that is, when you multiply these two smaller numbers, you get 15. Here are all the composite numbers from 1 to 16:

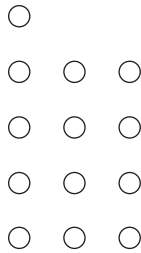
4    6    8    9    10    12    14    15    16

Notice that all the square numbers (see the section, “Getting square with square numbers”) also count as composite numbers because you can arrange them in boxes with at least two rows and two columns. Additionally, a lot of other non-square numbers are also composite numbers.

## Stepping out of the box with prime numbers

Some numbers are stubborn. Like certain people you may know, these numbers — called *prime numbers* — resist being placed in any sort of a box. Look at how Figure 1-4 depicts the number 13, for example.

**FIGURE 1-4:**  
Unlucky 13,  
a prime  
example of a  
number that  
refuses to fit in  
a box.



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Try as you may, you just can’t make a rectangle out of 13 objects. (That fact may be one reason why the number 13 got a bad reputation as unlucky.) Here are all the prime numbers less than 20:

2    3    5    7    11    13    17    19

As you can see, the list of prime numbers fills the gaps left by the composite numbers (see the preceding section). Therefore, every counting number is either prime or composite. The only exception is the number 1, which is neither prime nor composite. In Chapter 8, I give you a lot more information about prime numbers and show you how to *decompose* a number — that is, break down a composite number into its prime factors.

## Multiplying quickly with exponents

Here's an old question whose answer may surprise you: Suppose you took a job that paid you just 1 penny the first day, 2 pennies the second day, 4 pennies the third day, and so on, doubling the amount every day, like this:

1    2    4    8    16    32    64    128    256    512 ...

As you can see, in the first ten days of work, you would've earned a little more than \$10 (actually, \$10.23 — but who's counting?). How much would you earn in 30 days? Your answer may well be, "I wouldn't take a lousy job like that in the first place." At first glance, this looks like a good answer, but here's a glimpse at your second ten days' earnings:

... 1,024   2,048   4,096   8,192   16,384   32,768   65,536   131,072   262,144   524,288 ...

By the end of the second 10 days, when you add it all up, your total earnings would be over \$10,000. And by the end of 30 days, your earnings would top out around \$10,000,000! How does this happen? Through the magic of exponents (also called *powers*). Each new number in the sequence is obtained by multiplying the previous number by 2:

$$2^1 = 2$$

$$2^2 = 2 \times 2 = 4$$

$$2^3 = 2 \times 2 \times 2 = 8$$

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

As you can see, the notation  $2^4$  means *multiply 2 by itself 4 times*.

You can use exponents on numbers other than 2. Here's another sequence you may be familiar with:

1   10   100   1,000   10,000   100,000   1,000,000 ...

In this sequence, every number is 10 times greater than the number before it. You can also generate these numbers using exponents:

$$10^1 = 10$$

$$10^2 = 10 \times 10 = 100$$

$$10^3 = 10 \times 10 \times 10 = 1,000$$

$$10^4 = 10 \times 10 \times 10 \times 10 = 10,000$$



REMEMBER

This sequence is important for defining *place value*, the basis of the decimal number system, which I discuss in Chapter 3. It also shows up when I discuss decimals in Chapter 13 and scientific notation in Chapter 17. You find out more about exponents in Chapter 5.

# Four Important Sets of Numbers

In the preceding section, you see how a variety of number sequences extend infinitely. In this section, I provide a quick tour of how numbers fit together as a set of nested systems, one inside the other.



TIP

When I talk about a set of numbers, I'm really just talking about a group of numbers. You can use the number line to deal with four important sets of numbers.

- »» **Counting numbers (also called natural numbers):** The set of numbers beginning 1, 2, 3, 4 and going on infinitely
- »» **Integers:** The set of counting numbers, zero, and negative counting numbers
- »» **Rational numbers:** The set of integers and fractions
- »» **Real numbers:** The set of rational and irrational numbers

The sets of counting numbers, integers, rational, and real numbers are nested, one inside another. This nesting of one set inside another is similar to the way that a city (for example, Boston) is inside a state (Massachusetts), which is inside a country (the United States), which is inside a continent (North America). The set of counting numbers is inside the set of integers, which is inside the set of rational numbers, which is inside the set of real numbers.

## Counting on the counting numbers

The set of *counting numbers* is the set of numbers you first count with, starting with 1. Because they seem to arise naturally from observing the world, they're also called the *natural numbers*:

1 2 3 4 5 6 7 8 9...

The counting numbers are infinite, which means they go on forever.



REMEMBER

When you add two counting numbers, the answer is always another counting number. Similarly, when you multiply two counting numbers, the answer is always a counting number. Another way of saying this is that the set of counting numbers is *closed* under both addition and multiplication.

## Introducing integers

The set of *integers* arises when you try to subtract a larger number from a smaller one. For example,  $4 - 6 = -2$ . The set of integers includes the following:

- »» The counting numbers
- »» Zero
- »» The negative counting numbers

Here's a partial list of the integers:

... -4   -3   -2   -1   0   1   2   3   4 ...

Like the counting numbers, the integers are closed under addition and multiplication. Similarly, when you subtract one integer from another, the answer is always an integer. That is, the integers are also closed under subtraction.

## Staying rational

Here's the set of *rational numbers*:

- » Integers (which include the counting numbers, zero, and the negative counting numbers)
- » Fractions

Like the integers, the rational numbers are closed under addition, subtraction, and multiplication. Furthermore, when you divide one rational number by another, the answer is always a rational number. Another way to say this is that the rational numbers are closed under division.

## Getting real

Even if you filled in all the rational numbers, you'd still have points left unlabeled on the number line. These points are the irrational numbers.

An *irrational number* is a number that's neither a whole number nor a fraction. In fact, an irrational number can only be approximated as a *non-repeating decimal*. In other words, no matter how many decimal places you write down, you can always write down more; furthermore, the digits in this decimal never become repetitive or fall into any pattern. (For more on repeating decimals, see Chapter 13.)

The most famous irrational number is  $\pi$  (you find out more about  $\pi$  when I discuss the geometry of circles in Chapter 19):

$$\pi = 3.14159265358979323846264338327950288419716939937510\dots$$

Together, the rational and irrational numbers make up the *real numbers*, which comprise every point on the number line. In this book, I don't spend too much time on irrational numbers, but just remember that they're there for future reference.