1

Introduction

1.1 Introduction and Motivation

1.1.1 Networked Control Systems

Due to the flexible architecture and ease of installation and maintenance, communication networks are widely used in control systems, which result in networked control systems (NCSs), where the plants, actuators, sensors, and controllers are spatially distributed and interconnected by communication channels [Schenato et al., 2007, Hespanha et al.]. NCSs are ubiquitous in industry and daily life, such as teleoperation [Arcara and Melchiorri, 2002], power systems [Wang et al., 2012], and transportation systems [Seiler and Sengupta, 2001].

Even though NCSs have the advantages of low cost, easy implementation, and expansion to large-scale applications, they also introduce new challenging problems arising from the limited resources and unreliability of the communication networks used for information transmission (see Figure 1.1). For example, the time delay may occur in digital communication channels due to data processing and transmission [Tse and Viswanath, 2005, Goldsmith, 2005]. Notably, in wireless communication networks, communication channels naturally suffer from interference, fading, and transmission noises [Tse and Viswanath, 2005, Goldsmith, 2005]. There into, fading is the time variation of channel strengths and is usually caused by two factors: one is the shadowing from obstacles; the other one is the multipath propagation [Tse and Viswanath, 2005, Goldsmith, 2005]. Packet drops can also be modeled as a special case of channel fading. Take Figure 1.2 as an illustration. The wireless signal may transmit through the car and undergo several paths before arriving at the receiver. If the phases of the received signals from different paths are the same, the signal strength is enhanced. Otherwise, the signal strength is reduced as a result of the cancellation of radio waves. Besides, the signal strength at the receiver side might be reduced due to the shadowing from

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the car. Since control is often used in safety- or mission-critical applications, we must take the uncertainties in communication networks into consideration and investigate how they affect the stability and performance of control systems.

The classical control theory mainly deals with the systems with nearly perfect point-to-point connections and focuses on the design of control laws to achieve the given control performance. It can't be applied directly to the NCSs when the uncertainties in the communication network must be considered. A new control paradigm is required to deal with the interplay between control and communication. In this book, one of the main objectives is to study the stabilization, estimation, and optimal control of NCSs over channels with fading, packet drops, or delay.

1.1.2 Multi-Agent Systems

Motivated by the collective behavior in nature, such as schooling fish, flocking birds, and marching locusts, multi-agent systems (MASs) have attracted considerable research interest from the control community [Jadbabaie et al., 2003, Olfati-Saber and Murray, 2004, Olfati-Saber et al., 2007, Bliman and Ferrari-Trecate, 2008, Cao et al., 2008, Ren and Beard, 2008, You and Xie, 2010, Cao et al., 2012, Trentelman et al., 2013, Qi et al., 2016, Qiu et al., 2017, Xu et al., 2018, Zheng et al., 2018]. With the rapid development of wireless communication networks, MASs have been applied in many industrial and military applications. Such systems usually involve large numbers of autonomous agents (e.g. robots, unmanned aerial vehicles, satellites), which share information via local interactions and work together to achieve collective objectives.

For MASs, each agent can have the same or different system dynamics, resulting in different types of MASs, e.g. first- and second-order MASs, linear and nonlinear MASs, homogeneous and heterogeneous MASs. The interactions among the agents form the interaction topology, which can be fixed or time-varying. Then the cooperative control of MASs is based on the system dynamics and the interaction topology to design the control laws, which can be centralized or distributed, to fulfill a task. Typical cooperative control tasks include consensus, formation, swarming/flocking, rendezvous, etc. There into, the consensus problem, which requires all agents to agree on a certain quantity of common interests, builds the foundation of other cooperative tasks.

Existing research on consensus assumes that the communication networks among agents are perfect. However, as mentioned earlier, in practical applications, communication channels naturally suffer from fading, signal-to-noise ratio (SNR) constraints, time delay, etc. Hence, it is of great significance to study how

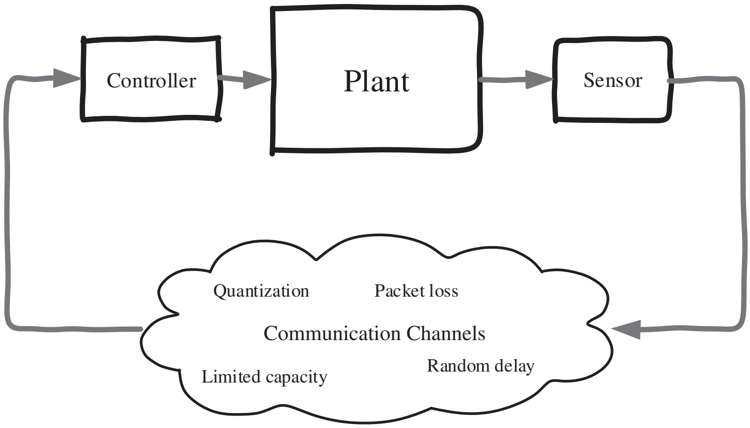

Figure 1.1 Networked control systems.

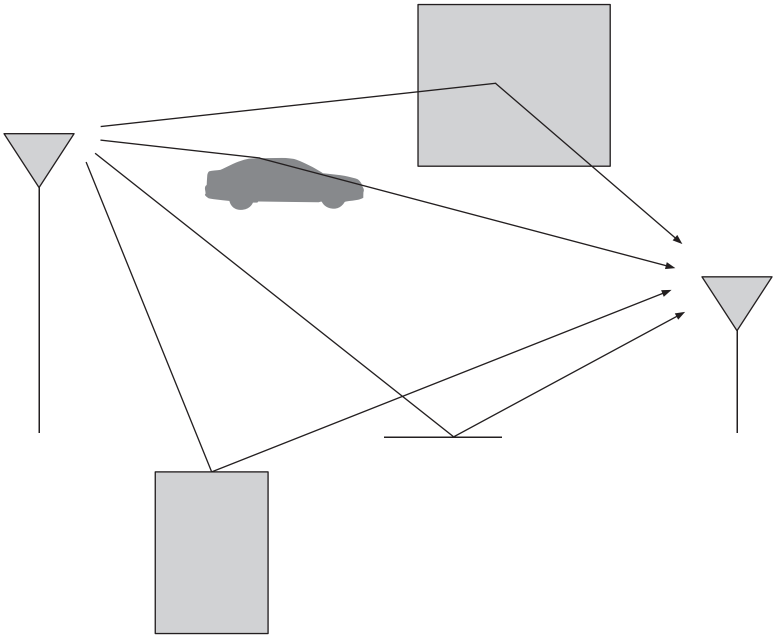

Figure 1.2 Fading phenomenon in wireless communications.

the uncertainties in communication networks influence the consensus of MASs. The other main objective of this book is to analyze the consensus problem of MASs over channels with fading, packet drops, and delay.

1.2 Literature Review

Control over communication channels/networks has been a hot research topic in the past decades [Matveev and Savkin, 2009, Como et al., 2014, You et al., 2015], motivated by the rapid developments of wireless communication technologies that enable the wide connection of geographically distributed devices and systems. However, the inclusion of wireless communication channels/networks also introduces challenges in the analysis and design of control systems due to constraints and uncertainties in wireless communications. We must take the communication channels/networks into consideration and study their impact on the stability and performance of control systems. This section briefly reviews existing results on the analysis and design of NCSs and MASs over imperfect communication channels.

1.2.1 Basics of Communication Theory

One of the main focuses of this book is to characterize the critical channel requirement such that the NCS can be mean-square stabilized. Since the communication channel is used to transmit information about the system state, as illustrated in Figure 1.1, it is expected that if the channel capacity is large enough, the feedback connected system can be mean-square stable. From this perspective, the communication channel capacity might be critical for the mean-square stabilization of control systems.

The channel capacity problem is fundamental in communication theory since it dictates the maximum data rates that can be transmitted over channels with asymptotically small error probability [Tse and Viswanath, 2005, Goldsmith, 2005]. In this subsection, we briefly review the communication channel capacity definitions and discuss why the communication theoretic channel capacity is not the critical characterization of the capacity required for controls. We only discuss discrete memoryless channels, and most of the definitions are borrowed from Cover and Thomas [2006].

A discrete memoryless channel consists of three parts: an input alphabet $\mathcal{X}$, an output alphabet $\mathcal{Y}$, and a probability transition matrix $p(y|x)$ that describes the probability of observing the output symbol y given the input symbol x. The channel is memoryless if the probability distribution of the current channel output conditioned on the current channel input is independent of previous channel inputs or outputs. The configuration of the point-to-point communication system

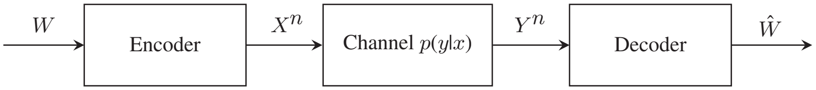

Figure 1.3 Point-to-point communication system.

is depicted in Figure 1.3. We want to transmit a message W reliably through the communication channel with appropriately designed channel encoders and decoders. The (M, n) code in a communication system is defined as follows.

Definition 1.1 ***((M, n) code)*** An (M, n) code for the channel $(\mathcal{X}, p(y|x), \mathcal{Y})$ consists of three parts:

1. A message index set $\{1, 2, \dots, M\}$.
2. An encoding function $X^n : \{1, 2, \dots, M\} \to \mathcal{X}^n$, generating codewords $x^n(1)$, $x^n(2)$, …, $x^n(M)$.
3. A decoding function $g : \mathcal{Y}^n \to \{1, 2, \dots, M\}$, generating an estimate for the transmitted message index.

The performance of the code is measured by the decoding error.

Definition 1.2 ***(Decoding error)*** The maximal probability of error for an (M, n) code is defined as $\lambda^{(n)} = \max_{i\in\{1,2,\dots,M\}} \Pr(g(Y^n) \neq i|X^n = x^n(i))$.

The communication channel capacity which measures the maximal capacity for reliably transmitting the information is defined below.

Definition 1.3 ***(Channel capacity)*** The rate R of the (M, n) code is defined as

$$R = \frac{\log M}{n} \text{ bits per transmission.}$$

A rate R is achievable if there exists a sequence of $(\lceil 2^{nR} \rceil, n)$ codes such that $\lambda^{(n)}$ tends to 0 as $n \to \infty$. The channel capacity C is then defined as the supremum of all achievable rates.

The channel capacity in Definition 1.3 is called the Shannon channel capacity since C. E. Shannon proved in the channel coding theorem that this channel capacity equals the mutual information of the channel maximized over all possible input distributions [Shannon, 2001, Cover and Thomas, 2006]:

$$C = \max_{p(x)} \mathcal{I}(X; Y),$$

where the mutual information $\mathscr{I}(X;Y)$ is defined as

$$\mathscr{I}(X;Y) = \sum_{x\in\mathcal{X}, y\in\mathcal{Y}} p(x,y) \log \frac{p(x,y)}{p(x)p(y)}.$$

The Shannon capacity of fading channels has been studied under various scenarios in Goldsmith and Varaiya [1997], Biglieri et al. [1998], Sadeghi et al. [2008], Abou-Faycal et al. [2001], and Caire et al. [1999]. For example it is proved in Goldsmith and Varaiya [1997] that if the channel state information is available at the receiver side, the Shannon channel capacity of a fading channel is

$$C = \int_0^\infty \frac{1}{2} \log \left(1 + \frac{\gamma^2 \mathcal{P}}{\sigma_\omega^2}\right) p(\gamma) \mathrm{d}\gamma,$$

where $p(\gamma)$ is the probability distribution function of the channel fading γ.

The Shannon channel capacity in Definition 1.3 assumes that the capacity-achieving code can be sufficiently long, which would inevitably result in a large delay. Since delay is critical in control systems, we may expect that the communication theoretic Shannon channel capacity is not the right choice for controls. This has been confirmed by Sahai and Mitter [2006], where another kind of channel capacity is defined, named the anytime capacity, and showed that the anytime capacity should be the critical characterization of channel capacities for controls when moment stability is concerned. However, there is no systemic method to calculate the anytime capacity. In the following, we will briefly review the existing results on the control and estimation of NCSs, and the consensus of MASs over communication networks.

1.2.2 Stabilization of NCSs

1.2.2.1 Control over Noiseless Digital Channels

For control systems, with components connected through noiseless digital communication channels, the celebrated data rate theorem [Nair and Evans, 2004] is an important result in the past decades. The data rate theorem states that to keep the state of a scalar unstable discrete-time linear system

$$x_{t+1} = \lambda x_t + u_t + w_t \tag{1.1}$$

mean-square bounded, the data rate R for the digital communication channel that connects the sensor to the controller should satisfy that

$$R > \log |\lambda|. \tag{1.2}$$

Intuitively, this result has the following explanation, see Figure 1.4. The controller wants to compensate for the expansion of uncertainties in the state estimation during the communication process. To ensure the boundedness of the system state, $\lambda^2/2^{2R}$ should be smaller than one, which gives the data rate theorem.

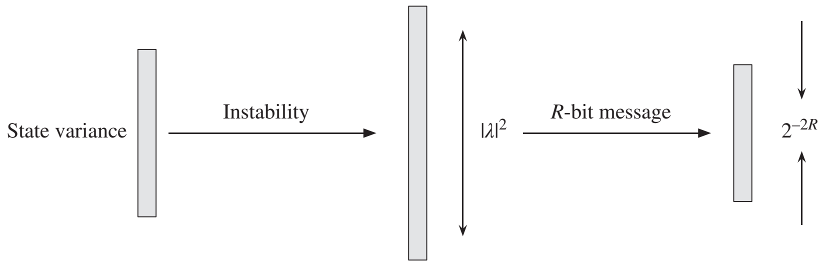

Figure 1.4 Intuitive explanations of the data rate theorem.

The result in (1.2) resembles the Shannon's source-channel coding theorem [Cover and Thomas, 2006], with the left-hand side being the Shannon channel capacity and the right-hand side the source's uncertainty measure. Indeed, the right-hand side of (1.2) denotes the information generating speed of the linear time-invariant (lti) system [Elia, 2004, Nair et al., 2004], which is generating information about the unknown initial system state. This resemblance also motivates the researchers to study the control systems from the perspective of information theory, e.g. see Touchette and Lloyd [2000], Zang and Iglesias [2003], Martins et al. [2007], Martins and Dahleh [2008], Nair [2013], Silva [2013], Ranade and Sahai [2011, 2013, 2015], Ramnarayan et al. [2014], and Ranade [2015].

1.2.2.2 Control over Stochastic Digital Channels

For noisy channels, the stability problem is more complex because different stability definitions require different channel capacities. Matveev and Savkin [2007] prove that for almost sure stability, the Shannon capacity in relation to the unstable dynamics of the system establishes the critical condition for its stabilizability. For moment stability, Sahai and Mitter [2006] show that the Shannon capacity is too optimistic, while the zero-error capacity is too pessimistic, and the anytime capacity is introduced to characterize the stabilizability conditions. Essentially, to keep the η-moment of the state of an unstable scalar plant bounded, it is necessary and sufficient for the feedback channel's anytime capacity corresponding to anytime-reliability $\alpha = \eta \log|\lambda|$ to be greater than $\log|\lambda|$, where λ is the unstable eigenvalue of the plant. The anytime capacity has a more stringent reliability requirement than the Shannon capacity. However, it is worth noting that there exists no systematic method to calculate the anytime capacities of channels. In the control community, the anytime capacity is usually studied under the mean-square stability requirement and is also named the mean-square capacity. In the following, we survey the related results that aim to determine the requirements on noisy channels to ensure that the feedback-connected linear systems can be mean-square stabilized, which, on the other hand, reveals the mean-square capacities for the channels studied.

One important kind of communication channel is the time-varying digital channel. Minero et al. [2009] assumes that the data rate R_t of the time-varying digital channel under consideration is stochastic and iid and gives the mean-square stabilizability condition for a connected discrete-time lti system. For the scalar systems to ensure the mean-square stabilizability, the following condition should be satisfied:

$$\mathbb{E}\left\{\frac{\lambda^2}{2^{2R_t}}\right\} < 1. \tag{1.3}$$

Similar to the explanation of the data rate theorem for noiseless channels, the inequality (1.3) intuitively implies that to ensure mean-square stabilizability, it is necessary and sufficient for the average expanding factor of the system state during one iteration to be smaller than one. For vector systems, necessary and sufficient conditions are provided in the form of stability regions or characterized by rate vectors [Minero et al., 2009].

For a stochastic rate-limited channel, You and Xie [2010] further show that the minimum data rate for the stabilization of a single-input vector system is explicitly given in terms of unstable eigenvalues of the open-loop matrix and the packet dropout rate, which reveals the amount of the additional bit rate required to counter the effect of packet dropouts on stabilization. Sufficient data rate conditions for mean-square stabilization of multiple-input vector systems are also derived there. When the packet drop is correlated over time, the problem becomes much more complicated. You and Xie [2011b] study mean-square stabilization of linear systems over networks with Markovian packet drops. Since the sojourn time of the time-homogeneous Markovian process that models the two-state packet drop process is iid [Xie and Xie, 2009], a randomly sampled system approach is developed in You and Xie [2011b] to derive the mean-square stabilizability condition. The same method is also adopted when deriving the data rate theorem with the additional consideration of system uncertainties in Okano and Ishii [2014]. Borrowing results from Markov jump linear systems, the mean-square stabilizability results for a more general n-state Markovian packet drop process are given in Minero et al. [2013], which contains the two-state Markovian packet drop process as a special case. The existing results in Minero et al. [2009], You and Xie [2010, 2011b], and Minero et al. [2013] are both necessary and sufficient for scalar systems. However, for vector systems, generally, there exists a gap between the derived sufficient conditions and necessary conditions. The main difficulty for deriving conditions that are both necessary and sufficient is how to optimally allocate the bits to each unstable subsystem.

1.2.2.3 Control over Analog Channels

The above results focus on digital channels. As to analog channels, Elia [2005] considers the mean-square stabilization problem over a pure multiplicative noise channel and derives the mean-square capacity of such channels. Since the iid packet drop channel is one special kind of pure multiplicative noise channels, the results obtained in Elia [2005] can be easily used to derive the results for iid packet drop channels. Xiao et al. [2012] further derive sufficient and necessary conditions for mean-square stabilization of multi-input-multi-output (MIMO) systems controlled over parallel multiplicative noise channels. Qiu et al. [2013] propose a channel/controller codesign approach with channel resource allocations to stabilize lti systems controlled with imperfect input channels when the total input channel capacity is fixed. When the subchannel capacities are fixed a priori, Chen et al. [2014] derive the stabilizability condition with a majorization approach. The joint effect of the quantization and multiplicative noise on the mean-square stabilizability is studied in Gu et al. [2015], whereas the case of both time-delay and multiplicative noise is studied in Qi et al. [2017], Su et al. [2017], and Tan et al. [2015]. Chiuso et al. [2014] consider linear quadratic Gaussian (LQG)-like control of scalar systems over communication channels suffering from data losses, delays, and SNR limitations. The stability of the closed-loop system depends on a tradeoff among the snr constraint, packet loss probability, and time delay.

Braslavsky et al. [2007] study the mean-square stabilization problem over an additive white Gaussian noise (awgn) channel and characterize the critical capacity to ensure mean-square stabilizability. To ensure the mean-square stabilization of a networked scalar system, the channel parameters should satisfy the following relation:

$$\log|\lambda| < \frac{1}{2}\log\left(1+\frac{\mathcal{P}}{\sigma_\omega^2}\right) \tag{1.4}$$

with $\mathcal{P}/\sigma_\omega^2$ denoting the SNR of the awgn channel. They also show that for the output feedback case, the capacity required for the awgn channel is generally larger than that of the state feedback case, unless the plant is minimum phase. They further show that the extension from linear encoders/decoders to more general causal encoders/decoders cannot provide additional benefits of increasing the channel capacity [Freudenberg et al., 2010].

Specifically, the results stated above deal with multiplicative noise channels or awgn channels separately. While in wireless communications, it is practical to consider them as a whole. Xiao and Xie [2011] have derived the necessary and sufficient conditions for such kinds of channels to ensure the mean-square stabilizability under a linear encoder/decoder. It is still unknown whether we can achieve a larger stabilizability region with a more general causal encoder/decoder. We provide a positive answer to this question in Chapters 2 and 3.

1.2.3 LQ Optimal Control of NCSs over Fading Channels

As one of the most fundamental problems in control theory, linear-quadratic (LQ) optimal control has attracted great attention and has been extensively studied for deterministic and stochastic linear systems. See Dragan et al. [2010], Fragoso et al. [1998], Freiling and Hochhaus [2003], Kalman [1960], Wonham [1968], and Zhou et al. [1996] and the references therein. It aims to design an optimal state-feedback controller such that the closed-loop system is stable, and the quadratic cost function is minimized. When the channel fading is characterized by a random process, the associated NCS can be treated as a stochastic system and the LQ optimal control problem over fading channels can be tackled in virtue of some results from the stochastic case. Motivated by the control of some macroeconomic models, Katayama [1976] considers the LQ optimal control for LTI systems with only a scalar gain. In Huang et al. [2006], the LQ optimal control for a general stochastic system with both state- and control-dependent scalar noise is studied. The optimal control law was obtained under certain assumptions of mean-square stabilizability and exact observability. Much of the research treats the LQ optimal control as part of the LQG control problem. The LQG control of a MIMO system with a single packet dropping input channel and a single packet dropping output channel is considered in Imer et al. [2006] and Sinopoli et al. [2005]

It is well known that the solvability of a deterministic LQ optimal control problem depends on the existence of a stabilizing solution to the associated algebraic Riccati equation (ARE). Not surprisingly, as shown in the aforementioned literature, a class of modified algebraic Riccati equations (MAREs) play a vital role in these stochastic optimal control problems, as well as the stochastic optimal filtering problem. Specifically, the solvability of the stochastic LQ/LQG optimal control boils down to the existence problem of a mean-square stabilizing solution to the associated MARE. However, unlike AREs, which have been studied extensively (see Lancaster and Rodman [1995], Qiu [1999], and Zhou et al. [1996] and the references therein), the solutions and properties of MAREs are still under investigation. In most of the existing research, e.g. Wonham [1968], Katayama [1976], Freiling and Hochhaus [2003], Imer et al. [2006], Huang et al. [2008], and Garone et al. [2012], only sufficient conditions are obtained to ensure the existence of a mean-square stabilizing solution. In Garone et al. [2012], a sufficient condition for the existence of a stabilizing solution to a MARE, which is associated with LQG control over erasure channels with perfect acknowledgment, is given in terms of the loss probabilities as well as the classical stabilizability and detectability. A similar condition is provided by Wonham [1968]. In Zhang et al. [2008], by assuming stabilizability and exact detectability, a MARE is shown to have a stabilizing solution. The sufficient condition provided by Freiling and Hochhaus [2003] is given in terms of mean-square stabilizability and another definition of detectability, which is dual to the mean-square stabilizability. Note that the stabilizability of stochastic systems or NCSs over fading channels is usually defined in the mean-square sense, while there are several ways to define the stochastic observability and detectability from different perspectives in these papers. As seen above, detectability for stochastic systems is always assumed. Is it necessary? Does there exist a necessary and sufficient condition to ensure the existence of a mean-square stabilizing solution? In Dragan et al. [2010], one numerical necessary and sufficient condition for a discrete-time MARE is given in terms of the feasibility of some linear matrix inequalities (LMIs). However, such a condition does not have explicit interpretations with respect to the dynamical properties of the underlying stochastic system. It is of great significance to obtain an explicit necessary and sufficient condition for the existence of a mean-square stabilizing solution to the MAREs and thus solve the LQ optimal control problem for NCSs over fading channels. We will thoroughly investigate the solvability of MAREs in Chapter 4.

1.2.4 Estimation of NCSs with Intermittent Communication

In NCSs, intermittent communication is frequently involved, which may result from unreliable channels or the stochastic manner of data transmission. The intermittent communication may influence the performance of the components in a NCS; for example if the estimator receives measurements from the sensors intermittently, it may be unstable. In this subsection, we summarize existing results on remote state estimation with intermittent communication. We mainly focus on the stability issue of Kalman filtering over intermittent observations and the optimal state estimator design problem in the presence of packet drops and sensor scheduling.

1.2.4.1 Stability of Kalman Filtering with Intermittent Observations

The stability issue of remote state estimation caused by intermittent measurements has been investigated by many researchers. Particularly, the stability problem of Kalman filtering caused by intermittent measurements from one single sensor is well studied in Sinopoli et al. [2004]. They show the existence of a critical arrival rate below which the estimation error may diverge; they also provide lower and upper bounds of the critical arrival rate. This result is further developed by Mo and Sinopoli [2008, 2012]. Researchers also work on the stability issue of multisensor cases. Liu and Goldsmith [2004] consider a system with two sensors and provide a form of the lower bound of the expected estimation error covariance. Rong [2012] extends the result of the lower bound in Liu and Goldsmith [2004] to the system with multiple sensors and also proposes an explicit form of the upper bound of the critical arrival rate for the system with a single sensor, which is an improvement of Sinopoli et al. [2004]. When just one sensor is chosen at each time, the upper bound of the expected estimation error covariance is proposed in Gupta et al. [2006]. In Chapter 5, a comprehensive study of the stability of multisensor Kalman filtering with intermittent observations is studied, which provides lower and upper bounds of the expected estimation error covariance and gives conditions on the divergence and convergence of them.

1.2.4.2 Remote State Estimation with Sensor Scheduling

In a distributed sensor network, a large number of sensors are deployed on a vast terrain and take measurements of some processes of interest and then transmit the sensed information to a remote center. The sensors are often supplied by battery power. Since it is usually difficult to replace the battery when a sensor runs out of power, to make sufficient use of each sensor and prolong its lifetime, one needs to design an appropriate sensor communication schedule. Sensor scheduling algorithms can be roughly categorized as off-line schedules and event-triggered schedules. The off-line schedules are designed based on the communication frequency requirement and the statistics of the systems [Yang and Shi, 2011, Shi et al., 2011, Mo et al., 2014]. Compared with off-line schedules, event-triggered schedules depend on both the statistics and the realization of the system, which is expected to achieve better performance than off-line ones. Many triggering rules have been proposed in the literature based on the conditions that the estimation error [Xia et al., 2017], error in predicated output [Trimpe, 2014], functions of the estimation error [Wu et al., 2013, Han et al., 2015], or the error covariance [Trimpe and D'Andrea, 2014], exceed a given threshold. For example, a measurement innovation-based, event-triggered sensor-scheduling scheme is proposed to reduce the communication rate in the remote state estimation problem in Wu et al. [2013]. However, since the innovation is not Gaussian, only suboptimal estimators can be obtained. Stochastic event-triggered sensor-scheduling algorithms are further proposed in Han et al. [2015] to handle the non-Gaussian problem. Both open-loop and closed-loop schedules are proposed, and it is shown that the conditional distributions of the system state are Gaussian. As a result, closed-form minimum mean-square error (MMSE) estimators are obtained. A similar non-Gaussianity phenomenon could appear when transmit power control is used in sensor networks [Li et al., 2018]. To overcome the problem, a transmit power controller based on a specific quadratic form of measurement innovations is carefully designed in Li et al. [2018] to preserve the Gaussianity of posterior state distribution, which facilities the MMSE estimator design and performance analysis.

Wireless communications are mostly utilized in sensor networks, and packet drops are inevitable in wireless communications. Therefore, it is necessary to study how packet drops affect sensor-scheduling algorithms [Leong et al., 2017, Mo et al., 2014]. It should be noted that, for off-line schedulers and estimation error covariance-based event-triggered schedulers, there is no need to distinguish between the channel loss event and the hold of transmission event when designing estimators. As long as the estimator receives the packet, it can conduct the measurement update to improve the estimate and vice versa. However, the case is different for the stochastic event-triggered sensor-scheduling algorithms in Han et al. [2015] where the sensor measurement is used as the trigger criterion and the hold of transmission event contains information about the sensor measurement. In the presence of possible channel losses, the estimator cannot decide whether the nonreception of the packet can be attributed to the sensor measurement or the channel loss. If it is due to that the sensor measurement lies below the given threshold, then this information can be leveraged to improve the estimate. However, if it is caused by the channel loss, the estimator will have no information about the sensor measurement and no update will be carried out. This fact complicates the optimal estimator design. Furthermore, it is proved in Kung et al. [2017] that, in the presence of channel losses, the Gaussian properties with the stochastic event-triggered sensor-scheduling algorithms in Han et al. [2015] no longer hold. Chapter 6 is devoted to providing solutions to the remote state estimation problem of LTI systems with stochastic event-triggered sensor schedules in the presence of packet drops.

1.2.5 Distributed Consensus of MASs

In many applications, single-agent systems are incapable of dealing with complex tasks. Cooperation among mass becomes necessary, and thus they have attracted great attention from various research fields, ranging from biology [Zhu et al., 2017], to robotics [Hu et al., 2013], control theory to applied mathematics, among many others. Among various cooperative tasks, consensus, which requires all agents to reach an agreement on a certain quantity of common interest, builds the foundation of others [Olfati-Saber et al., 2007, Ren and Beard, 2008, Cao et al., 2012]. One question arises before control synthesis: whether there exist distributed controllers such that the mas can achieve consensus. This problem is usually referred to as the consensusability of mass. Several important results have been derived to answer this question, under an undirected/directed communication topology [Ma and Zhang, 2010, You and Xie, 2011a, Li et al., 2010, Trentelman et al., 2013]. In Ma and Zhang [2010], it is shown that to ensure the consensus of a continuous-time linear mas, the lti dynamics should be stabilizable and detectable, and the undirected communication topology should be connected. Furthermore, You and Xie [2011a] and Gu et al. [2012] show that for a discrete-time linear mas, the product of the unstable eigenvalues of the system matrix should additionally be upper bounded by a function of the eigenratio of the undirected graph. Extensions to directed graphs and robust consensus can be found in Li et al. [2010] and Trentelman et al. [2013]. Most of the consensusability results discussed above are derived assuming perfect communications. However, this is not the case in practical applications, where communication channels naturally suffer from limited data rate constraints, SNR constraints, time delay, and so on. Therefore, it is necessary to study the consensusability problem of mass under communication channel constraints.

Li et al. [2011] consider the average consensus problem for discrete-time first-order mass over rate-limited channels with undirected graphs. A distributed consensus protocol based on dynamic encoding and decoding is proposed, and the average consensus can be achieved with only one-bit information exchange between each pair of adjacent agents at every time step. The extensions to the case with bounded time-delay and time-varying graphs for first-order mass can be found in Liu et al. [2011] and Li and Xie [2011], respectively. Li and Xie [2012] consider the distributed coordination problem of second-order MASs with partially measurable states under rate-limited communication channels. A quantized-observer-based encoding-decoding scheme and a distributed coordinated control law are proposed. The two bits quantization is sufficient for the asymptotic synchronization of agent states. Determining the critical data rate for distributed consensus of general nth order mass can be challenging. Only limited results exist for special kinds of nth order systems; see Qiu et al. [2016, 2017].

Consensusability of MASs over communication channels with time delays has been studied in Olfati-Saber and Murray [2004], Bliman and Ferrari-Trecate [2008], Yu et al. [2010], Zhou and Lin [2014], and Qi et al. [2013, 2016, 2014]. Specifically, a necessary and sufficient condition for consensusability under an undirected communication graph with constant time delays is given in Olfati-Saber and Murray [2004], while sufficient conditions for the existence of average consensus under an undirected communication graph with bounded communication delays are provided in Bliman and Ferrari-Trecate [2008]. A truncated predictor feedback approach is established in Zhou and Lin [2014] to solve the consensus problem with time delays in both the communication network and control inputs. Utilizing techniques from robust control, Qi et al. [2013, 2014, 2016] characterize the maximal tolerable time-delay for the existence of a linear distributed consensus controller for discrete-time linear mass over undirected graphs and directed graphs, respectively. The results show that the consensusability is related to the time delay, unstable poles, and nonminimum phase zeros of the system dynamics.

There are also results on the consensusability of MASs over communication channels corrupted by packet dropouts. In Hatano and Mesbahi [2005], consensusability with probability 1 is addressed under the assumptions that each link has an equal packet dropout rate and the mean of the random graph is complete. Moreover, sufficient conditions based on the second smallest eigenvalue of the mean Laplacian matrix are provided in Kar and Moura [2009] for mean-square consensusability in undirected networks with nonuniform packet dropout rates.

In Tahbaz-Salehi and Jadbabaie [2008], a necessary and sufficient condition for almost sure asymptotic consensus over directed networks with stochastic i.i.d. weight matrices is obtained.

In this book, we are interested in the consensusability problems of discrete-time linear mass over fading networks and delay. The framework considered in Xiao et al. [2014] deals with identical fading networks with undirected communication topologies only. It is still unknown how the directed communication topology and nonidentical fading networks affect the consensusability of mass, and this problem will be analyzed in Chapters 7, 8 and 10. For the consensus of discrete-time MASs with delay and packet dropouts, the approaches given in Zong et al. [2016] depend heavily on the order and structure of the system matrices and cannot be extended to general MASs. We will discuss the consensus of general discrete-time MASs with constant communication delay and packet dropouts in Chapter 9. The synchronization problem of a special class of MASs, named Vicsek model, with delay is also studied in Chapter 11.

1.3 Preview of the Book

The rest of this book is organized as follows.

In Chapters 2–4, we study the stabilization and optimal control problem for NCSs.

In Chapter 2, we consider the mean-square stabilization problem of discrete-time LTI systems over a power-constrained fading channel. Fundamental limitations on the mean-square stabilizability are obtained via information-theoretic arguments. For scalar and two-dimensional systems, necessary and sufficient conditions for the mean-square stabilizability are provided, respectively. Moreover, an adaptive time division multiple access (TDMA) communication scheme is designed for high-dimensional systems, which achieves a larger stabilizability region than the conventional TDMA communication scheme and is proved to be optimal under certain situations.

Chapter 3 studies the mean-square stabilization problem of discrete-time LTI systems over Gaussian finite-state Markov channels, which suffer from both SNR constraint and correlated channel fading modeled by a Markov process. The existence of a fundamental limitation for mean-square stabilization is first established. Then, sufficient stabilization conditions under a TDMA communication scheme are derived in terms of the stability of a Markov jump linear system. Moreover, we present a necessary and sufficient condition for mean-square stabilization of two-dimensional systems controlled over power-constrained Markov lossy channels. Furthermore, improved sufficient stabilization conditions are derived based on an adaptive TDMA communication scheme for general high-dimensional systems, which achieve a larger stabilization region than the TDMA communication scheme.

In Chapter 4, LQ optimal control problem for a discrete-time LTI system with random input gains is studied. The finite-horizon case can be solved by dynamic programming, while the solvability of the infinite-horizon case is equivalent to the existence of a mean-square stabilizing solution to the associated MARE. By virtue of the theory of cone-invariant operators, some properties of the associated MARE are obtained and an explicit necessary and sufficient condition ensuring the existence of the mean-square stabilizing solution to the MARE is derived. Such a condition is compatible with the one ensuring the stabilizing solution to the standard ARE, and it indicates that the common condition of observability or detectability of certain stochastic systems is unnecessary. With this result, the LQ optimal control problem of NCSs with random input gains can be well solved under the framework of channel/controller codesign.

In Chapters 5 and 6, we study the remote state estimation problem with intermittent communication.

In Chapter 5, we extend the stability theory on Kalman filtering with intermittent measurements from the scenario of one single sensor to one of multiple sensors. Based on the measurements received intermittently from a group of sensors, the estimator computes the estimates of the process states by multisensor Kalman filtering. Because of the intermittent measurements, the estimator may be unstable. A notion of transmission capacity, which is related to the communication rates of sensors, is proposed. It is shown that the expected estimation error covariance diverges for all feasible communication rates collections of the sensors when the transmission capacity is below a certain value; meanwhile, when the transmission capacity is above another certain value, there exists a feasible communication rates collection such that the expected estimation error covariance is bounded.

In Chapter 6, we study the remote state estimation problem of linear time-invariant systems with stochastic event-triggered sensor schedules in the presence of packet drops between the sensor and the estimator. Due to the existence of packet drops, the Gaussianity at the estimator side no longer holds. It is proved that the system state conditioned on the available information at the estimator side is Gaussian mixture distributed. The MMSE estimator can be obtained from a bank of Kalman filters. Since the optimal estimators require exponentially increasing computation and memory with time, suboptimal estimators to reduce computational complexities by limiting the length and number of hypotheses are further provided. In the end, simulations are conducted to illustrate the performance of the optimal and suboptimal estimators.

Starting with Chapter 7, we turn the attention from NCSs to MASs.

In Chapter 7, we investigate the consensus problem of MASs over undirected fading networks. That is, each agent can only receive corrupted information about its neighborhoods' states, which make it difficult to reach consensus. How the channel fading affects the consensus of MASs over undirected networks is the essential problem to address in this chapter. For consensus over identical fading networks, a decomposition method is used and the mean-square consensus problem is transformed to a simultaneous mean-square stabilization problem. For consensus over nonidentical fading networks, the edge Laplacian defined for undirected graphs is introduced to model the consensus error dynamics. Then sufficient consensus conditions are derived, which demonstrate how the system dynamics, the communication quality, and the network topological structure interplay with each other to allow the existence of a linear distributed consensus controller.

In Chapter 8, the mean-square consensus problem of discrete-time linear MASs over analog fading networks with directed graphs is further studied. However, since the graph Laplacian for directed graphs may contain complex eigenvalues, and there is no appropriate definition of edge Laplacian for directed graphs, the method used in Chapter 7 for the consensus of MASs over undirected fading networks cannot be applied to directed graph cases, which complicate the consensusability analysis due to the coupling between the channel fading and the network topology. In this chapter, we introduce the definitions of compressed in-incidence matrix (CIIM), compressed incidence matrix (CIM), and compressed edge Laplacian (CEL) to facilitate the modeling and consensus analysis. It is then shown that the mean-square consensusability is solely determined by the edge state dynamics on a directed spanning tree. As a result, sufficient conditions are provided for mean-square consensus over fading networks with directed graphs in terms of fading parameters, the network topology, and the agent dynamics.

In Chapter 9, a more practical and complex scenario is taken into account, i.e. the multi-agents communicate through the networks with both delay and packet dropouts. The approaches given in Chapter 8 cannot be directly applied to the case when both communication delays and packet dropouts exist in the communication channels. By proposing some novel mean-square stability criteria of NCSs with delay and multiplicative noise and employing the notion of CEL from Chapter 8 to build the dynamics over edges to separate the packet dropouts from the network topology, a sufficient consensusability condition in terms of the communication delay, the packet dropout rates, the communication network topology, and the agent dynamics is provided. Closed-form consensusability conditions can be obtained under some specific configurations.

In Chapters 7, 8, and 9, the fading or packet dropouts are assumed to be i.i.d., which cannot capture the temporal correlation of channel fadings. In Chapter 10, it is assumed that the multi-agents communicate with each other through undirected Markovian packet loss channels. For the case with identical Markovian packet losses, a necessary and sufficient consensus condition is derived based on the stability of Markov jump linear systems, together with a numerically verifiable consensus criterion in terms of the feasibility of LMIs. For the case with nonidentical packet loss, the edge Laplacian is used to model the consensus error on edges rather than on vertexes and a sufficient consensus condition in terms of LMIs is given.

In Chapter 11, the synchronization problem of a special class of MASs, named Vicsek model, is studied. Simulation results show that all agents, using only the local rule, might eventually move in the same direction, exhibiting the synchronization behavior. A theoretical analysis of the synchronization of both linear and nonlinear Vicsek models with bounded time-varying delays is given in this chapter. Some delay-dependent synchronization conditions, imposed only on the initial state, are derived. Compared to the synchronization conditions based on some connectivity conditions on the dynamically changing neighbor graphs in the literature, the derived results can be easily checked.

1.4 Preliminaries

1.4.1 Graph Theory

In this subsection, we introduce the basis of graph theory used to model the interactions among agents in MASs. For detailed reference to graph theory, please refer to Godsil and Royle [2001] and Mesbahi and Egerstedt [2010].

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to characterize the interaction among agents, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the node set representing N agents and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set with ordered pairs of nodes denoting the information transmission among agents. An edge $(i, j) \in \mathcal{E}$ means that the ith agent can send information to the jth agent, where node i and node j are called the initial node and terminal node of this edge, respectively. Self-edge (i, i) is not allowed. The neighborhood set $\mathcal{N}_i$ of agent i is defined as $\mathcal{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. A sequence of edges $\{(i_{j-1}, i_j)\}_{j=1}^{l}$ with $(i_{j-1}, i_j) \in \mathcal{E}$ and $\{i_j\}_{j=0}^{l}$ being distinct is called a directed path from agent i_0 to agent i_l. A directed cycle is a directed path starting and ending at the same node. A graph contains a directed spanning tree if it has at least one node with directed paths to all other nodes. A graph is called strongly connected if, for any two agents i and j, there is a path in each direction between them. A graph is complete if there exists an edge for each pair of nodes. The underlying graph of $\mathcal{G}$ is the graph obtained by treating the edges of $\mathcal{G}$ as unordered pairs.

The adjacency matrix A_{adj} is defined as $[A_{\text{adj}}]_{ii} = 0$, $[A_{\text{adj}}]_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $[A_{\text{adj}}]_{ij} = 0$, otherwise. When A_{adj} is symmetric, $\mathcal{G}$ is called an undirected graph.

An undirected graph is connected if there is a path between every pair of distinct nodes. The in-degree and out-degree of agent i are given by $\deg_i = \sum_{j=1}^{N} [A_{\text{adj}}]_{ij}$ and $\deg_i^{\text{out}} = \sum_{j=1}^{N} [A_{\text{adj}}]_{ji}$, respectively. A graph $\mathcal{G}$ is called balanced if and only if $\deg_i = \deg_i^{\text{out}}$ for any $i = 1, \dots, N$. Denote the Laplacian matrix of $\mathcal{G}$ by $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}\{\deg_1, \dots, \deg_N\}$. The eigenvalues of $\mathcal{L}$ are denoted by $\lambda_i \in \mathbb{C}, i = 1, \dots, N$ with an ascending order in magnitude, i.e. $0 = |\lambda_1| \leq |\lambda_2| \leq \cdots \leq |\lambda_N|$. Then the graph Laplacian $\mathcal{L}$ has the following property.

Lemma 1.1 ***(Ren and Beard [2008])***

1. *For a directed graph $\mathcal{G}$, all the eigenvalues of $\mathcal{L}$ have nonnegative real parts. Then $\mathcal{G}$ contains a directed spanning tree if and only if zero is a simple eigenvalue of $\mathcal{L}$ with a right eigenvector* $\mathbf{1}$*, equivalently to say, $\lambda_2 \neq 0$.*
2. *For an undirected graph $\mathcal{G}$, $\lambda_i \geq 0$ for all $i \in \mathcal{N}$. Then $\mathcal{G}$ is connected if and only if $\lambda_2 > 0$. Moreover, $\mathcal{G}$ is complete if and only if $\lambda_2 = \lambda_N$.*

1.4.2 Hadamard Product and Kronecker Product

The Hadamard product (also known as Schur product) refers to the component-wise multiplication of matrices with the same dimension. That is, given matrices $A, B \in \mathbb{C}^{m\times n}$, the Hadamard product, denoted by $A \odot B \in \mathbb{C}^{m\times n}$, is defined as

$$[A \odot B]_{ij} = [A]_{ij}[B]_{ij}.$$

For example, the Hadamard product of a 2×2 matrix A and a 2×2 matrix B is

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \odot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11}b_{11} & a_{12}b_{12} \\ a_{21}b_{21} & a_{22}b_{22} \end{bmatrix}.$$

The Hadamard product is commutative, associative, and distributive over addition. That is, given matrices A, B, and C with the same size, and a scalar k, it holds that

$$A \odot B = B \odot A,$$
$$(kA) \odot B = A \circ (kB) = k(A \odot B),$$
$$A \odot (B \odot C) = (A \odot B) \odot C,$$
$$A \odot (B + C) = A \odot B + A \odot C.$$

If D and E are diagonal matrices, then

$$D(A \odot B)E = (DAE) \odot B = (DA) \odot (BE) = (AE) \odot (DB) = A \odot (DBE).$$

The Kronecker product is an operation that maps two matrices of arbitrary size into a larger block matrix. That is, given matrices $A \in \mathbb{C}^{m\times n}$ and $B \in \mathbb{C}^{p\times q}$

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}, \quad B = \begin{bmatrix} b_{11} & \cdots & b_{1q} \\ \vdots & \ddots & \vdots \\ b_{p1} & \cdots & a_{pq} \end{bmatrix},$$

the Kronecker product, denoted by $A \otimes B \in \mathbb{C}^{mp\times nq}$, is defined as

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}.$$

The Kronecker product is associative and distributive over addition. That is, given matrices A, B, and C, it holds that

$$A \otimes (B + C) = A \otimes B + A \otimes C,$$
$$(A \otimes B) \otimes C = A \otimes (B \otimes C).$$

Unlike the matrix product or Hadamard product, it is in general noncommutative. That is for arbitrary matrices A and B, usually $A \otimes B \neq B \otimes A$.

When $A \in \mathbb{C}^{n\times n}$ and $B \in \mathbb{C}^{m\times m}$, let $\lambda_1, \dots, \lambda_n$ be the eigenvalues of A and $\mu_1, \dots, \mu_m$ be those of B. Then the eigenvalues of $A \otimes B$ are $\{\lambda_i\mu_j, i = 1, \dots, n, j = 1, \dots, m\}$. It also holds that $A \otimes B$ is invertible if and only if both A and B are invertible, i.e. $(A \otimes B)^{-1} = A^{-1} \otimes B^{-1}$.

If A and C are matrices of the same size, B and D are matrices of the same size, then the following mixed-product property holds

$$(A \otimes B) \odot (C \otimes D) = (A \odot C) \otimes (B \odot D).$$

The Kronecker product is also closely related to the vectorization operator (denoted by vec(·)), which stacks the column vectors of a matrix into one column vector. To be specific,

$$\left(B' \otimes A\right) \ \mathrm{vec}(X) = \mathrm{vec}(AXB) = \mathrm{vec}(C).$$

Thus, by using the Kronecker product, we can see that the equation $AXB = C$, where A, B, and C are given matrices, has a unique solution X, if and only if A and B are invertible.

For a detailed reference to Hadamard product and Kronecker product, please refer to Horn and Johnson [1985, 1991].

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