CHAPTER 1

The Basic Physics of Ultrasound

SOUND WAVES

A sound wave is a fluctuating variation in pressure within a medium such as air, water, or solid material. Our ears are sensitive to such pressure changes in air, and we hear sounds all the time. The faster the changes in pressure take place, the higher the pitch or **frequency** of the sound we hear. Frequency is measured in hertz (Hz) and, for a young person, their hearing goes from 20 Hz to 20 kHz. Middle C on a piano is 261 Hz. A sound above 20 kHz is called **ultrasound** ('beyond sound'), in other words, we cannot hear it. Dogs can hear sounds of higher frequency than we can, and bats use ultrasound, up to 200 kHz, for echo location in the dark. The ultrasound we use for medical imaging is in the range of megahertz (MHz), far above anything we can hear. As we will see, the reason for going to such high frequencies is that we can then make narrow beams of ultrasound that we can point in a particular direction and which can produce high-resolution images showing fine details.

We can think of a medium like air or water as a collection of molecules with mass connected by springs that represent the forces between the molecules (Figure 1.1). If you push on one molecule, it will move closer to the adjacent molecule and exert a force so that it too begins to move. Adjacent molecules having been squashed together will then repel one another and recover to their resting position. They will keep moving beyond their resting position due to their momentum. The force holding them together then becomes an attractive force that pulls the molecules back towards

Ultrasound Technology for Clinical Practitioners, First Edition. Crispian Oates.

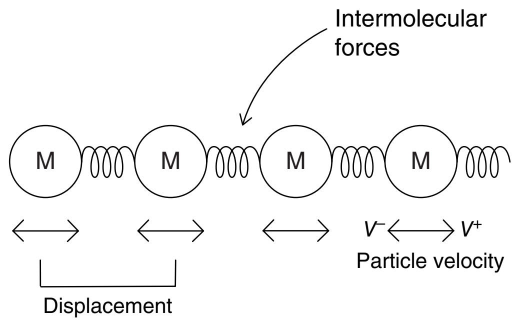

FIGURE 1.1 Ball and spring model of a sound wave travelling in some medium.

their resting position again as they are then stretched apart. That is, each molecule will move forwards then backwards in the direction of the applied force. This oscillating molecular motion within a material is the basis of a sound wave.

> **DEFINITION**
>
> A **sound wave** is a longitudinal pressure wave travelling at the **speed of sound** through a medium (e.g. air, water, and soft tissue).

The pressure is the force exerted from the excess density (number of molecules per unit volume) of molecules above or below the average density of the medium as the molecules get alternately squashed and pulled apart from one another. In other words, it is the excess pressure about the mean resting pressure.

A sound wave may be generated by a piston moving forwards and backwards. This is what a normal loudspeaker does (Figure 1.2). It has a diaphragm that moves forwards and backwards driven by an electric signal. This will alternately push, then move away, from the molecules in front of it. These molecules will then alternately push on to those in front of them and then pull on them, and so on, as described

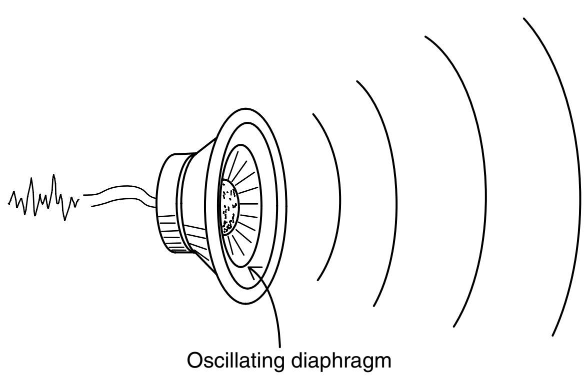

FIGURE 1.2 A loudspeaker driven by an electric signal acts as a piston on the air in front of it.

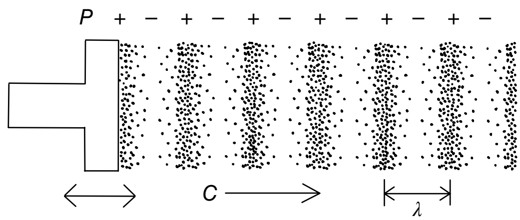

FIGURE 1.3 Oscillating piston producing a longitudinal pressure wave, a sound wave consisting of compression (+) followed by rarefaction (−). c is the speed of sound, and λ is the wavelength.

above. This will create a series of compressions and rarefactions in the molecules of the medium that move away from the source of movement (the piston), i.e. a sound wave as shown in Figure 1.3.

The speed at which they move away from the source is called the **speed of sound** (c). The speed of sound can be considered constant for each medium, so the speed of sound in air is 330 $m{\cdot}s^{-1}$, and if a plane goes faster than this it breaks the sound barrier. The speed of sound in water varies with temperature and at 20 °C is 1480 $m{\cdot}s^{-1}$. The average speed of sound in soft tissue is 1540 $m{\cdot}s^{-1}$, and, as will be seen, this is a key number for ultrasound imaging.

For ultrasound imaging, the ultrasound transducer acts exactly like the loudspeaker pushing and pulling the molecules of the medium in front of it.

NOTE

The individual molecules oscillate backwards and forwards about a mean position, but the pressure disturbance (p) propagates forward at the speed of sound (c). It is the moving disturbance that is the sound wave.

A typical sine wave plot of a sound wave is seen if we plot the change in pressure (p) at a given point against time, or if we plot the change in pressure versus distance away from the piston.

Looking at one position in space versus time (t), we see the pressure increasing and decreasing as the sound wave passes by (Figure 1.4a).

The **frequency** (f) is the number of cycles (peaks) passing a given point in one second.

The **period** (T) is the time taken to complete 1 cycle. The relationship between period and frequency is

$$f = \frac{1}{T}$$

Looking at one instant of time versus distance (x) away from the sound source (Figure 1.4b), we see the pressure increases and decreases as we move away from the transducer.

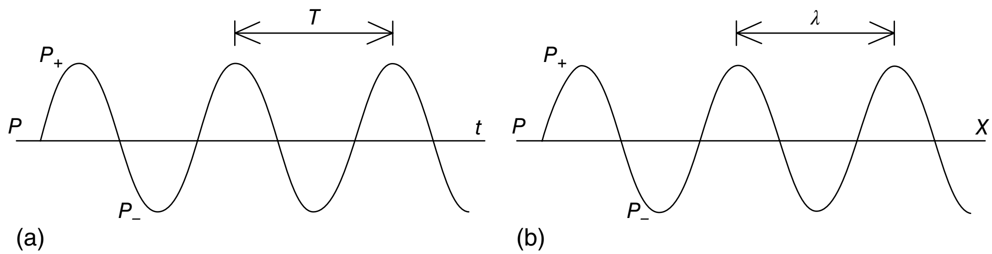

FIGURE 1.4 Illustration of the change in pressure with time at one point in space (a) and the change in pressure with distance *x* from the sound source (b).

The **wavelength** (λ) of the sound is then defined as the distance in space between two successive peaks on the wave.

THE SPEED OF SOUND EQUATION

The relationship between speed of sound (c), frequency (f), and wavelength (λ) is

$$c = f.\lambda$$

Relationship Between Pressure, Particle Velocity and Particle Displacement

We can also plot the change in particle velocity (v) versus time and the particle displacement (s), from its resting position, versus time, to give similar graphs (Figure 1.5).

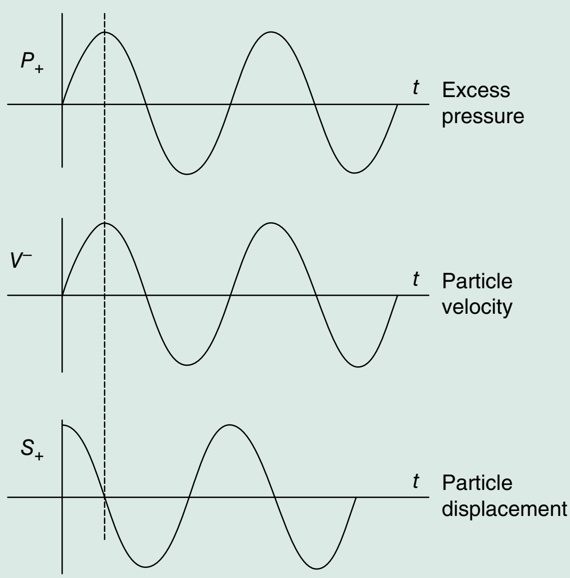

FIGURE 1.5 The sine waves produced by plotting the change in pressure, particle velocity, and displacement associated with a sound wave.

Comparing these waves, we see that the pressure is greatest when the particle velocity is greatest and the particle displacement is greatest when the rising pressure passes through its mean zero level.

When we simply talk about a 'sound wave,' we usually mean the (excess) **pressure wave** – also known as **acoustic pressure**. It is acoustic pressure that our ultrasound transducers are sensitive to and detect.

NOTE

Do not confuse particle velocity with the speed of sound. Particle velocity is movement at a molecular level, whereas the speed of sound is the speed at which the sound wave propagates through the medium.

A **transducer** is anything that converts one form of energy into another form. The loudspeaker and the ultrasound transducer both convert electrical energy into sound energy and so are transducers.

NOTES

- As a sound wave propagates its frequency remains constant at the same frequency as the transducer 'piston' oscillates.
- For a given medium, the speed of sound is constant (it may vary with temperature). This means that if the transmitted frequency increases, the wavelength must get shorter to balance the speed of the sound equation.
- **Key Concept**: 'High frequencies give short wavelengths'.

DESCRIBING WAVES

The **amplitude of a wave** is the difference between the peak value and the mean zero value. We can also define the **peak-to-peak amplitude** A_+ to A_- as shown in Figure 1.6.

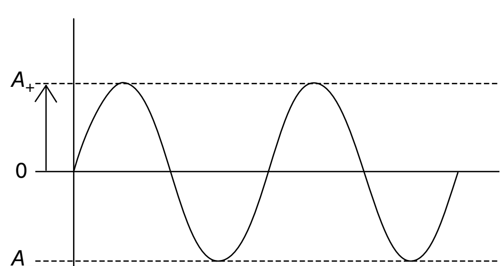

FIGURE 1.6 Definition of wave amplitude A.

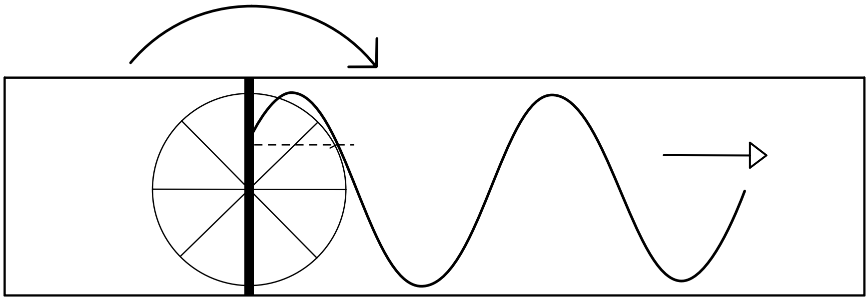

FIGURE 1.7 Illustration of a spinning bicycle wheel over a moving roll of paper.

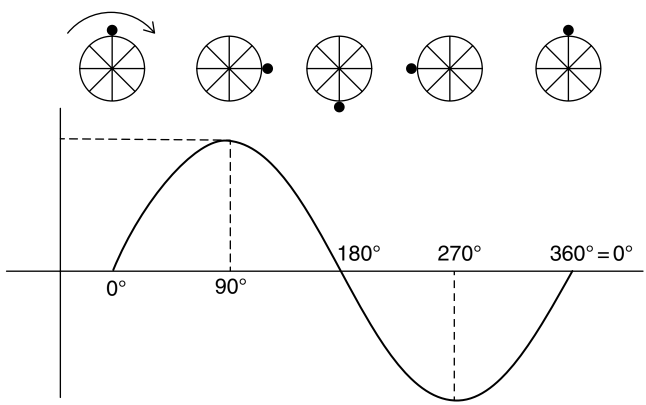

FIGURE 1.8 Definition of the phase angle of a sound wave.

The **phase of a wave** is a point along the course of one period of the wave expressed as an angle.

Think of a spinning bicycle wheel mounted above a moving roll of paper as shown in Figure 1.7. On the paper, we mark the vertical distance of the valve away from the hub at each moment as the wheel spins round. What results is a sine wave drawn on the paper. By the time, the wheel has gone round once, you would have drawn one period of the sine wave and the valve would have travelled round 360°. So, by measuring the angle of the valve as it goes round, we can mark the phase angle along the sine wave as shown in Figure 1.8. One cycle is equal to 360° (hence, frequency is equal to 'cycles' per second).

This gives us a very useful way to compare two sine waves. If one wave has a phase angle of 45° at the same time another sine wave has a phase angle of 0°, we know where the peak of one wave is compared to the other (Figure 1.9). If we know the frequency, amplitude, and phase of a wave, we know everything about it. The wavelength will depend on the speed of sound of the medium the sound wave travels through.

A special case of sound wave transmission is seen if we look at a single frequency sound wave propagating in one direction through a uniform medium from an infinitely large sound source. This type of wave is called a **plane wave**. The **wavefronts**

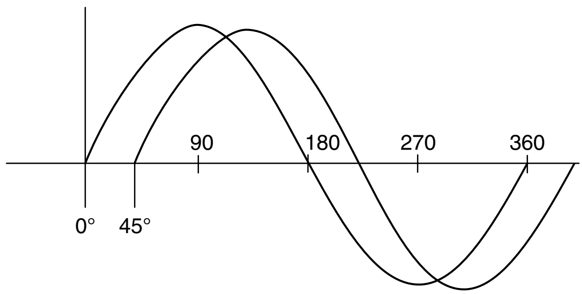

FIGURE 1.9 Two sine waves with a phase difference of 45°.

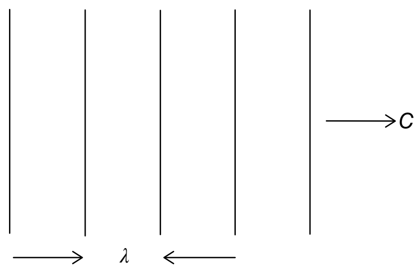

FIGURE 1.10 A plane wave showing the wavefronts parallel to one another.

(wave peaks) are then parallel to one another (Figure 1.10). Sea waves coming on to a straight beach closely approximate this situation.

Plane Waves

If we compare the pressure wave with the particle displacement and particle velocity waves for a plane wave, they all have the same frequency. Looking at Figure 1.5 and comparing the phase difference between them, we find that the excess pressure and particle velocity waves are in phase with each other (i.e. peaks occur at the same time), whilst the particle displacement wave is 90° ahead of the pressure wave. That is, its peak occurs a quarter of a cycle before that of the pressure and particle velocity waves. Their relative amplitudes will depend on the medium they are travelling through.

ENERGY IN A SOUND WAVE

In order to move the sound source piston backwards and forwards, work must be done and this requires energy. The disturbance created in the molecules can then make molecules some distance away from the piston move, so the sound wave must be transporting energy through the medium. The average pressure in a sound wave is zero as the amplitude oscillates equally above and below the mean. But the amplitude squared

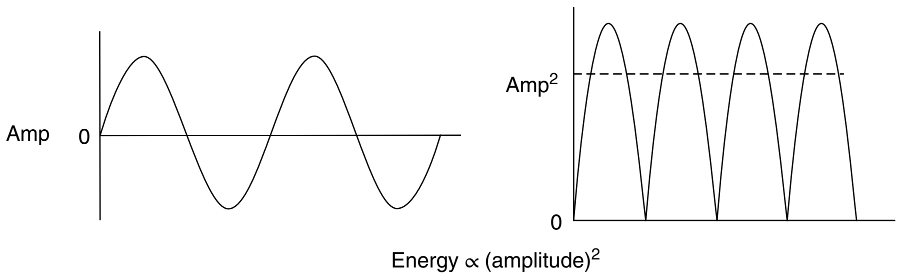

FIGURE 1.11 The relationship between the amplitude of a sound wave and the energy carried by the sound wave.

always has a positive value (think of $(-1) \times (-1) = +1$) (Figure 1.11). The **energy** carried by a sound wave is proportional to the pressure amplitude squared $(\text{amp})^2$.

$$\text{So, for the acoustic pressure } p, \text{energy } E \propto p^2.$$

The energy carried by a sound wave is important when considering the safety of ultrasound exposure in the body. The ultrasound is depositing energy in tissue, and it can cause damage if too much energy is deposited in one place.

We will consider the energy and safety of ultrasound further in Chapter 11.

ULTRASOUND PULSES

A pulse is a very short burst of sound. Most of the imaging we do uses very short pulses of sound, so we can tell where the echoes are coming from at any one time.

If we consider a pulse in time, it will have a shape and duration. A typical imaging pulse might have a large amplitude at first then die away (Figure 1.12).

- Peak amplitude – typically, 1 atmosphere = 10^5 Pa
- Particle velocity – typically, peak amplitude value ~ 10 cm s^{-1}
- Particle displacement – typically, peak amplitude value ~ 20 nm (1 nm = 10^{-9} m)
- Pulse duration – three—four cycles for B-mode ultrasound at 5 MHz
- Pulse duration – $0.2\,\mu s \times 4 = 0.8\,\mu s$

We can also consider the shape of the pulse in space at one instant in time. Note the reversal of wave shape – peak amplitude comes first in time and in space (Figure 1.13).

- Pulse length in tissue – three—four cycles
- For B-mode ultrasound at 5 MHz in soft tissue
  $c = 1540\,\text{m·s}^{-1}\ \lambda = 0.3\,\text{mm} \times 4$ to give a pulse length of 1.2 mm

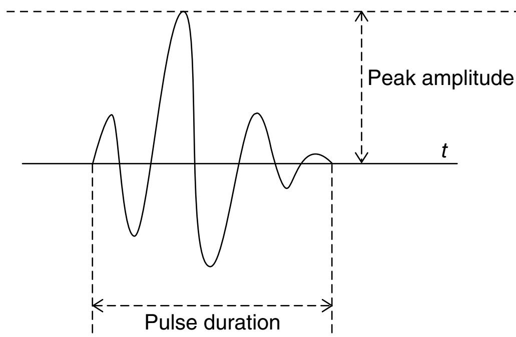

FIGURE 1.12 Typical pulse shape of a short pulse of ultrasound.

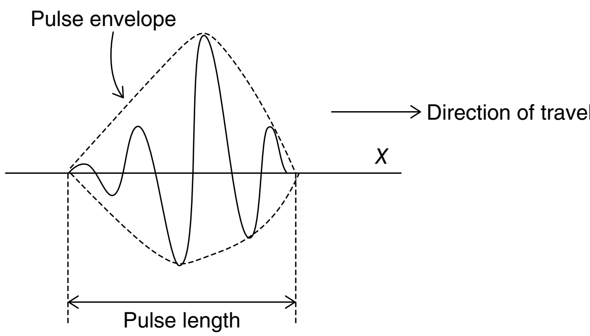

FIGURE 1.13 The pulse envelope of an ultrasound pulse in space.

We can characterise the shape of the pulse by drawing a line through successive peaks to show the **pulse envelope,** as shown by the dashed line.

ENERGY SPECTRUM OF A PULSE

DEFINITION

A **spectrum** is a plot of energy against frequency.

- For example, in a light spectrum, as seen in a rainbow, red has a lower frequency and blue has a higher frequency. The energy is seen in the brightness of the colours.

The rate at which energy is produced or transmitted is called **power,** which is equal to energy per unit time ('**rate**' means 'per unit time').

For a sound wave, the **power spectrum** is a plot of the power in the sound wave versus the frequency of the wave. Figure 1.14 shows what the spectrum of a simple sine wave of frequency f_0 looks like.

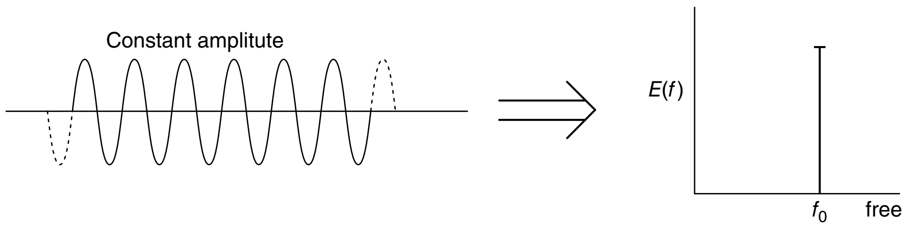

FIGURE 1.14 The single line power spectrum of a continuous sound wave of a single frequency, f_0.

It has all of its energy (E_0) at a single frequency (f_0).
For a pulse, we can ask the following questions:

- What is the frequency of a pulse?
- What does the spectrum of a single pulse look like?

BANDWIDTH

Looking at the pulse we saw in Figure 1.12 or the pulse in Figure 1.15, we see that no two cycles are the same, unlike the pure sine wave where every cycle is identical. On the other hand, the waveform for most of the pulse itself does look like a sine wave. So, what is its frequency?

The answer is that the pulse has a spectrum that centres around f_0, but extra frequencies need to be included on either side to account for the sine wave stopping at the ends of the pulse. Figure 1.16 shows the spectrum of the pulse has a **bandwidth** or range of frequencies centred about f_0. We talk about the '**full width at half maximum'**, or **FWHM**, as a measure of the bandwidth. This is the width of the spectrum, in hertz, measured at the half height of the peak around the centre frequency.

We can then look at the shape a pulse has and we can consider the energy that the pulse carries. The energy of a pulse becomes an important factor when we look at the safety of ultrasound.

When talking about the shape of a pulse over time, we talk about the **time domain** and when talking about the power spectrum of the pulse, we talk about the **frequency domain.** Figure 1.17 shows what the energy spectrum is for various pulses, i.e. the frequency content of a single pulse.

Notice how the very long pulse looks more like the sine wave than the very short pulse. Similarly, its spectral bandwidth is narrow and close to that of the sine wave frequency, f_0. The shorter the pulse, the wider the bandwidth of the spectrum, until with an infinitesimally short pulse the bandwidth is infinite and there is no associated single frequency, f_o. All frequencies are equally present. As an example of an infinitesimally short pulse, in order to study how all the frequencies respond at once in a concert hall, sound engineers use a gunshot from a starting pistol. A sharp hand clap also has a very wide range of frequencies emitted.

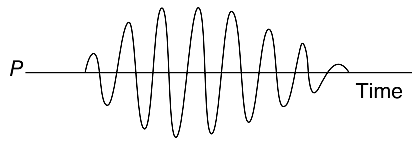

FIGURE 1.15 A 7-cycle pulse.

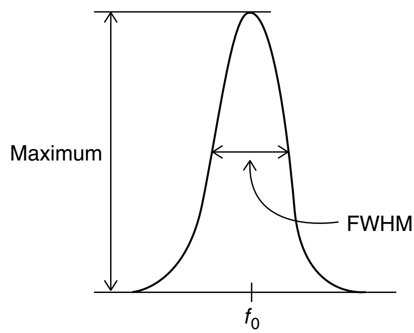

FIGURE 1.16 The full width at half maximum or FWHM bandwidth of an ultrasound pulse with centre frequency, f_0.

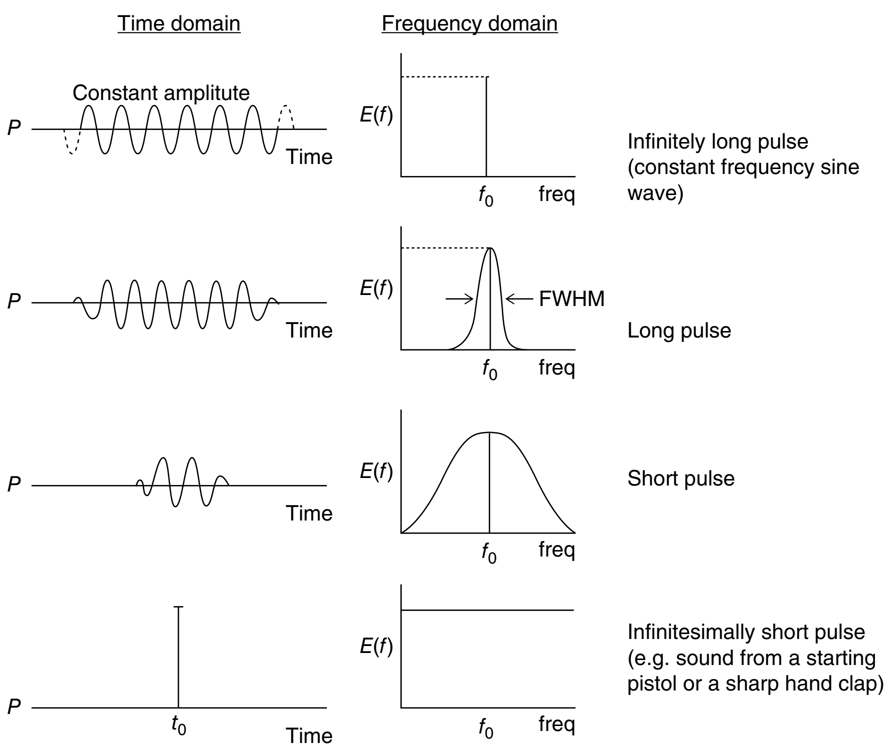

FIGURE 1.17 The relationship between the time domain picture and the frequency domain picture for various pulse shapes.

SUMMARY

There is a reciprocal relationship between the time domain and the frequency domain. An infinitely long sine wave has a single frequency whilst a single instantaneous sound has infinite bandwidth.

The importance of this relationship for medical ultrasound is that in order to have very short pulses and to give high-resolution images, we must use transducers that have a wide bandwidth so they can transmit and receive across a wide range of frequencies. For this reason, manufacturers will emphasise the fact that they have **wideband transducers**. Typical pulse shapes for B-mode and Doppler pulses are shown in Figure 1.18.

An approximation of the bandwidth of a given pulse length is given by

$$\text{Bandwidth} \sim \pm 2 \times \frac{1}{\text{Pulse length}}$$

So, a short pulse gives a wider bandwidth. For a typical diagnostic pulse of three cycles for B-mode, FWHM bandwidth, $\approx f_0$, where f_0 is the centre frequency.

For example, a 5 MHz pulse has an FWHM bandwidth of ±2.5 MHz

Quality Factor

Some texts refer to the quality factor Q of a resonator such as an ultrasound transducer (not to be confused with Q for flow volume!)

$$Q = \frac{\text{Centre frequency}, f_0}{\text{Bandwidth}}$$

So, a wideband transducer has a low Q, and a transducer with a narrow bandwidth has a high Q.

SPEED OF SOUND (C)

(Note: do not confuse the speed of sound with particle velocity!)

Returning to the ball and spring model of a material, Figure 1.19, we see that the model consists of a series of masses representing the molecules, connected together by springs of a certain stiffness representing the forces between the molecules. In

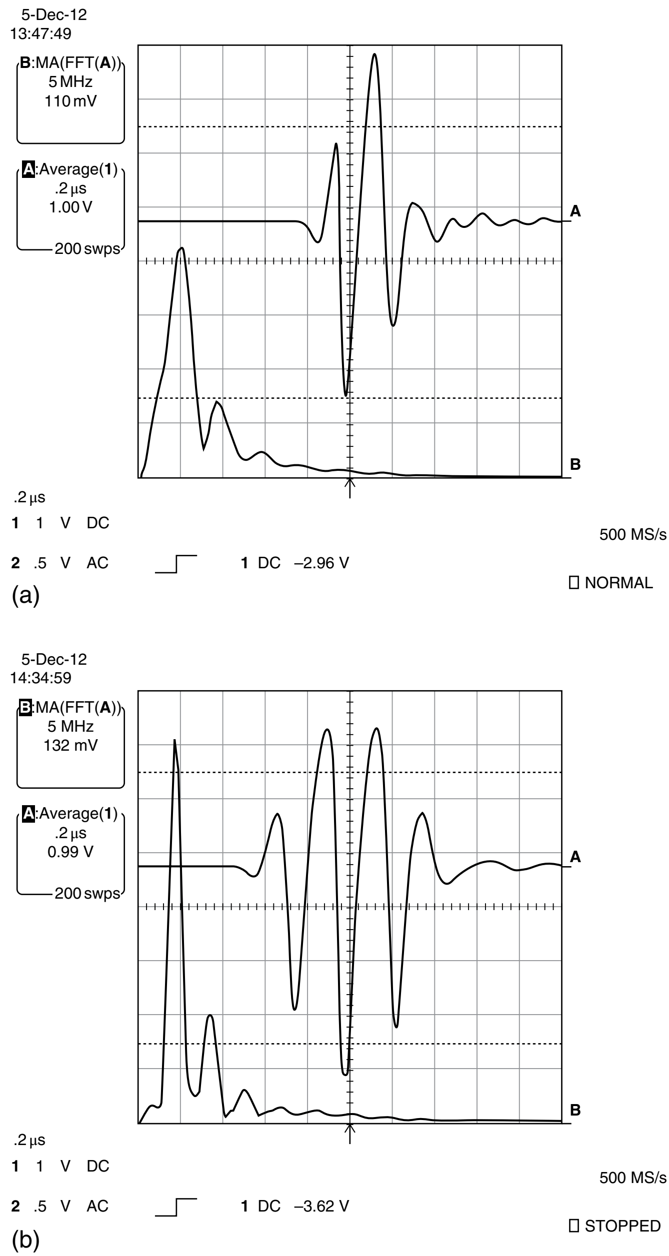

FIGURE 1.18 (a) A B-mode pulse and (b) a pulse wave Doppler pulse. The lower left corner of each image shows the spectrum of each pulse. Note that the longer pulse in (b) has a narrower bandwidth. *Source:* Courtesy: Jie Tong.

order to get a sound wave to propagate through the medium we need to get a mass moving and that moving mass must transmit its movement onto the next molecule through the spring. The lighter the mass, the easier it will be to get it moving, and the

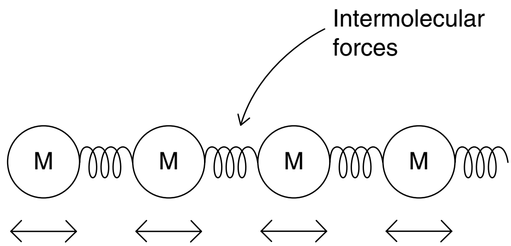

FIGURE 1.19 Ball and spring model of the molecules of a material showing the inter-molecular forces between the oscillating molecular masses.

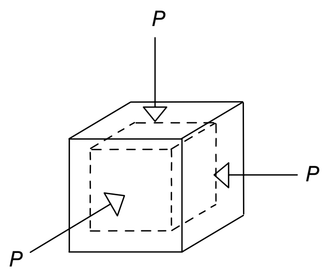

FIGURE 1.20 Definition of the bulk modulus showing the decrease in volume with an increase in applied pressure, *P*.

stiffer the spring, the faster it will transmit that movement on to the next molecule. We may, therefore, expect that light stiff materials will have a high speed of sound, and dense soft materials have a low speed of sound. For example, aluminium, a light rigid metal, has a speed of sound 5100 $m \cdot s^{-1}$, whilst in air the molecules are also light but are very poorly connected, and the speed of sound is only 330 $m \cdot s^{-1}$.

In terms of what we can measure, the average mass per unit volume of material is called the **density ρ** ($kg \cdot m^{-3}$), and the **stiffness** is a measure of the forces between the molecules

For a volume of a material, its resistance to such forces is called the **bulk modulus** *K*.

The bulk modulus relates the fractional change in volume $\Delta V/V$ to the applied pressure ΔP (Figure 1.20).

$$\text{Bulk modulus, } K\left(\mathrm{K}\right) \qquad K = -V\frac{\Delta P}{\Delta V}\left(\mathrm{kg}\cdot\mathrm{m}^{-1}\cdot\mathrm{s}^{-2}\right)$$

The minus sign occurs because the volume decreases with an increase in pressure.

Speed of sound in a material, $c = \sqrt{\dfrac{K}{\rho}}$

> **NOTES**
> - At a given temperature, the speed of sound is constant.
> - Speed of sound is independent of frequency (for most purposes in medical ultrasound).
> - Materials differ more in their stiffness than in their density. Therefore, stiffness is a better guide to predicting *c* than density.

Looking at Table 1.1, we see that the speeds of sound for different soft tissues are quite close together. The average speed of sound in soft tissue in the body is 1540 $m \cdot s^{-1}$ ± 5%. This is close enough to 1540 $m \cdot s^{-1}$ for us to use this value in most circumstances in clinical ultrasound. The value **1540 $m \cdot s^{-1}$ is a key number to remember when considering how ultrasound propagates through soft tissues**. The variation in speed of sound from 1540 $m \cdot s^{-1}$ will produce artefacts in the image (see Chapter 7). However, sometimes we need to apply a more accurate value to our measurements, for example when making measurements of the eye in ophthalmology. In Chapter 13, we will see how such variations in speed of sound may be corrected to improve image quality.

TABLE 1.1 Showing the Speed of Sound for Various Materials

Speed of sound table		
Material	**Speed of Sound ($m \cdot s^{-1}$)**	
Air	330	
Water	1480	
Plastic (Perspex)	2730	
PZT transducer	3741	
Fat	1450	Soft tissue average 1540 ± 5% (Fat to Muscle)
Brain	1546	
Liver	1550	
Kidney	1560	
Blood	1570	
Muscle	1580	
Bone (cortical)	3500	

CHARACTERISTIC ACOUSTIC IMPEDANCE, Z_0

QUESTION

How easily does a molecule in the medium move in response to a given change in acoustic pressure?

The answer is given in a quantity called the **characteristic acoustic impedance,** Z_0. This is a measure of how easily a molecule in the medium moves in response to the excess pressure (p) of the sound wave – assuming the sound wave is a plane wave.

Z_0 is constant for a given medium.

Characteristic Acoustic Impedance

$$\text{At a molecular level, acoustic impedance } Z_0 = \frac{p}{v} = \frac{\text{Excess pressure}}{\text{Particle velocity}}$$

Recall that for a plane wave, the pressure wave and the particle velocity are in phase with one another. The ratio of their amplitudes will therefore be constant at all points in the wave and equal to Z_0 as shown in Figure 1.21. This ratio is the **acoustic impedance** of the material carrying the sound wave.

Z_0 is constant for a given medium.

Think of it like the gears on a bike:

- The force exerted on the pedal ≡ pressure
- Speed of the pedals ≡ particle velocity

High gear Large pressure and low response to pedal push
Low gear Small pressure gives big response to pedal push

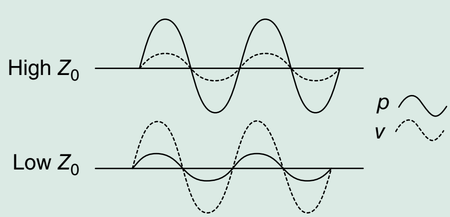

FIGURE 1.21 The relationship between pressure amplitude and the particle velocity for two materials with a large and small acoustic impedance Z_0.

KEY CONCEPT

Acoustic impedance Z_0 is important because **it is the difference in Z_0 between different materials that determines the size of the echo at an interface.**

Table 1.2 shows the value of Z_0 for a range of materials.

We cannot measure the excess pressure and the particle velocity of molecules in a material very easily. In terms of what we can measure, Z_0 depends on the stiffness (bulk modulus) K and density ρ of the medium and can be calculated by measuring the density and speed of sound, c.

$$Z_0 = \rho . c = \rho \sqrt{\frac{K}{\rho}} = \sqrt{\rho K}$$

A dense, rigid material will have a large Z_0, whilst a less dense non-rigid material will have a low $Z_{0.}$

NOTE

Many texts refer to acoustic impedance as the 'resistance' to the sound wave by the medium. Resistance is usually associated with energy loss, for example, electrical resistance causing heating of a wire. However, when considering echoes, acoustic impedance is not associated with any energy loss. Understanding that acoustic impedance is the 'response of the molecules' to the sound wave is much to be preferred as it is then not confused with the energy losses that do occur as sound travels through a medium. The energy losses are due to attenuation of ultrasound which will be discussed in Chapter 2 (in electrical terms, acoustic impedance is the analogue of reactance).

TABLE 1.2 Speed of Sound, Density, and Acoustic Impedance Z_0 for Several Tissues and Materials

Material	Speed of Sound ($m \cdot s^{-1}$)	Density ($kg \cdot m^{-3}$)	Z_0 ($kg \cdot m^2 \cdot s^{-1} \times 10^6$)
Air	330	1.3	0.0004
Water	1480	998	1.48
Plastic (Perspex)	2730	1180	3.22
PZT transducer	4560	7750	35.34
Fat	1450	911	1.32
Liver	1550	1079	1.67
Kidney	1560	1066	1.66
Blood	1570	1057	1.66
Muscle	1580	1090	1.72
Bone (cortical)	3500	1908	6.68

ENERGY IN A SOUND WAVE

Energy (joule J = kg m^2s^{-2})   (recall: energy ∝ p^2, where p is acoustic pressure)

Power (watt, W or J·s^{-1}) is the rate at which energy is transferred.

Total transmitted power is the total sound energy emitted by the transducer summed over all directions.

Recall: The sound source is a piston doing work moving backwards and forwards. It makes the molecules in front move, and energy is propagated away from the transducer as a sound wave.

Energy is not generally transmitted equally in all directions. For example, we want a narrow beam of ultrasound for imaging. We, therefore, need to know how much energy is being transferred in each direction away from the transducer. For this, we need to know the **intensity** of ultrasound.

Intensity (W·m^{-2} or mW·m^{-2} or J·s^{-1}·m^{-2} or kg·s^{-3}) is the power per unit area. It is the energy flowing across an imaginary surface cutting across the ultrasound beam as shown in Figure 1.22.

One way of thinking about intensity is to think of how we might distribute a brush full of paint. The paint on the brush is the total power. We might spread the paint very thinly over a large area or we might put a thick blob of paint in one spot and the rest of the area has no paint at all. The thickness of the paint at each point is the intensity at that point.

Intensity will vary as we measure it at various points in the **sound field**.

Power = energy per second (radiated by transducer)

Intensity = power crossing 1 cm^2 (e.g. at a sample point in the beam)

As the intensity is the energy crossing an area perpendicular to the beam axis, its value will also be proportional to the acoustic pressure squared.

i.e. $I \propto p_0^{\,2}$

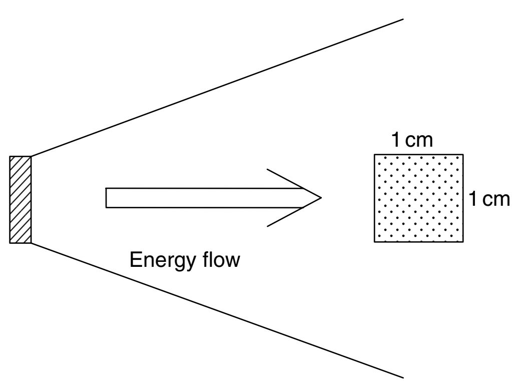

FIGURE 1.22 The relationship between power and intensity in an ultrasound beam. Intensity is the energy flowing across a unit area in the beam.

NOTE

Intensity is a measure of the power delivered to a point, e.g. in tissue. It is, therefore, the intensity that has most bearing on safety issues relating to medical ultrasound (see Chapter 11).

DECIBELS

It is often useful to express the change in energy, intensity, or power from one point to another as a ratio. As these changes can vary over a very large range of values, for example, the echo signal from the blood may be 10 000 less than that from a liver–fat interface; it is easier to use a logarithmic scale to compare values. The decibel scale (dB) is such a logarithmic measure. It is fully explained in Appendix 2.

A useful way to remember what the ratio is from the value in decibels is to remember

- A doubling = 3 dB and 10× = 10 dB
- A negative decibel value means a fraction, so −3 dB = a half and −10 dB = a tenth.