IN THIS CHAPTER

- Recognizing and using the rules of algebra
- Adding products to your list
- Raising your exponential power and taming radicals
- Looking at special products and factoring

Chapter 1

Beginning at the Beginning of Algebra

Algebra is a branch of mathematics that people study before they move on to other areas or branches of mathematics and science. For example, you use the processes and mechanics of algebra in calculus to complete the study of change; you use algebra in probability and statistics to study averages and expectations; and you use algebra in chemistry to solve equations involving chemicals.

Any study of science or mathematics involves rules and patterns. You approach the subject with the rules and patterns you already know, and you build on them with further study. And any discussion of algebra presumes that you're using the correct notation and terminology.

Going into a bit more detail, the basics of algebra include rules for dealing with equations, rules for using and combining terms with exponents, patterns to use when factoring expressions, and a general order for combining all of the above. In this chapter, I present these basics so you can further your study of algebra and feel confident in your algebraic ability. Refer to these rules whenever needed as you investigate the many advanced topics in algebra.

Following the Order of Operations and Other Properties

Mathematicians developed the rules and properties you use in algebra so that every student, researcher, curious scholar, and bored geek working on the same problem would get the same answer — regardless of the time or place. You don't want the rules changing on you every day; you want consistency and security, which you get from the strong algebra rules and properties presented in this section.

Ordering your operations

When mathematicians switched from words to symbols to describe mathematical processes, their goal was to make dealing with problems as simple as possible; however, at the same time, they wanted everyone to know what was meant by an expression and for everyone to get the same answer to the same problem. Along with the special notation came a special set of rules on how to handle more than one operation in an expression. For instance, if you do the problem $4+3^2-5\cdot 6+\sqrt{23-7}+\frac{14}{2}$, you have to decide when to add, subtract, multiply, divide, take the root, and deal with the exponent.

REMEMBER

The *order of operations* dictates that you follow this sequence:

1. **Perform all operations inside/on grouping symbols (parentheses, brackets, braces, fraction lines, and so on).**
2. **Raise to powers or find roots.**
3. **Multiply and divide, moving from left to right.**
4. **Add and subtract, moving from left to right.**

EXAMPLE

Q. Simplify: $4+3^2-5\cdot 6+\sqrt{23-7}+\frac{14}{2}$.

A. $\mathbf{4+9-30+4+7=-6}$. Follow the order of operations. The radical acts like a grouping symbol, so you subtract what's in the radical first: $4+3^2-5\cdot 6+\sqrt{16}+\frac{14}{2}$.

Raise the power and find the root: $4+9-5\cdot 6+4+\frac{14}{2}$.

Multiply and divide, working from left to right: $4+9-30+4+7$.

Add and subtract, moving from left to right: $4+9-30+4+7=-6$.

Q. Simplify: $\frac{-3 \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2 \cdot 2}$.

A. $\mathbf{\frac{1}{2}}$ **or** $\mathbf{-2}$.

Follow the order of operations: The fraction line serves as a grouping symbol, so you simplify the numerator and denominator separately before dividing. Also, the radical is a grouping symbol, so the value under the radical is determined before finding the square root. Under the radical, first square the –3, then multiply, and finally subtract. You can then take the root.

$$\frac{-3 \pm \sqrt{(-3)^2 - 4(2)(-2)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{9 - (-16)}}{2 \cdot 2} = \frac{-3 \pm \sqrt{25}}{2 \cdot 2} = \frac{-3 \pm 5}{2 \cdot 2}$$

Now multiply the two numbers in the denominator: $\frac{-3 \pm 5}{2 \cdot 2} = \frac{-3 \pm 5}{4}$.

Apply the two operations, and perform those operations before dividing: $\frac{-3+5}{4} = \frac{2}{4} = \frac{1}{2}$ or $\frac{-3-5}{4} = \frac{-8}{4} = -2$.

YOUR TURN

1 Simplify: $4(-3)^2 + 7 - \sqrt{25}$.

2 Simplify: $\frac{3(6-7)}{\sqrt[3]{5 \cdot 4 + 7}}$.

Examining various properties

Mathematics has a wonderful structure that you can depend on in every situation. Most importantly, it uses various properties, such as associative, commutative, distributive, and so on. These properties allow you to adjust mathematical statements and expressions without changing their values. You make them more usable without changing the outcome.

Property	Description	Math Statement
Commutative (of addition)	You can change the order of the values in an addition statement without changing the final result.	$a+b=b+a$
Commutative (of multiplication)	You can change the order of the values in a multiplication statement without changing the final result.	$a\cdot b=b\cdot a$
Associative (of addition)	You can change the grouping of operations in an addition statement without changing the final result.	$a+(b+c)=(a+b)+c$
Associative (of multiplication)	You can change the grouping of operations in a multiplication statement without changing the final result.	$a(b\cdot c)=(a\cdot b)c$
Distributive (multiplication over addition)	You can multiply each term in an expression within parentheses by the multiplier outside the parentheses and not change the value of the expression. It takes one operation, multiplication, and spreads it out over terms that you add to one another.	$a(b+c)=a\cdot b+a\cdot c$
Distributive (multiplication over subtraction)	You can multiply each term in an expression within parentheses by the multiplier outside the parentheses and not change the value of the expression. It takes one operation, multiplication, and spreads it out over terms that you subtract from one another.	$a(b-c)=a\cdot b-a\cdot c$
Identity (of addition)	The additive identity is zero. Adding zero to a number doesn't change that number; it keeps its identity.	$a+0=0+a=a$
Identity (of multiplication)	The multiplicative identity is 1. Multiplying a number by 1 doesn't change that number; it keeps its identity.	$a\cdot 1=1\cdot a=a$
Multiplication property of zero	The only way the product of two or more values can be zero is for at least one of the values to actually be zero.	$a\cdot b\cdot c\cdot d\cdot e\cdot f=0$ $\rightarrow a,b,c,d,e \text{ or } f=0$
Additive inverse	A number and its additive inverse add up to zero.	$a+(-a)=0$
Multiplicative inverse	A number and its multiplicative inverse have a product of 1. The only exception is that zero does not have a multiplicative inverse.	$a\cdot\left(\frac{1}{a}\right)=1, a\neq 0$

EXAMPLE

Q. Use the commutative and associative properties of addition to simplify the expression: $(16a+11)+5a$.

A. $\mathbf{11+21}\boldsymbol{a}$**.** First, reverse the order of the terms in the parentheses using the commutative property: $(11+16a)+5a$.

Next, reassociate the terms: $11+(16a+5a)$.

Finally, add the two terms in the parentheses: $11+21a$.

Q. Use the distributive property to simplify the expression: $12\left(\frac{1}{2}+\frac{2}{3}-\frac{3}{4}\right)$.

A. **5.**

$$12\left(\frac{1}{2}+\frac{2}{3}-\frac{3}{4}\right)=12\cdot\frac{1}{2}+12\cdot\frac{2}{3}-12\cdot\frac{3}{4}={}^{6}\cancel{12}\cdot\frac{1}{\cancel{2}_1}+{}^{4}\cancel{12}\cdot\frac{2}{\cancel{3}_1}-{}^{3}\cancel{12}\cdot\frac{3}{\cancel{4}_1}=6+8-9=5$$

Q. Use the multiplicative identity to find a common denominator and add the terms: $\frac{3}{8}+\frac{5}{6}$.

A. $\mathbf{1\frac{5}{24}}$**.** Multiply the first fraction by 1 in the form of the fraction $\frac{3}{3}$, and the second fraction by $\frac{4}{4}$:

$$\frac{3}{8}+\frac{5}{6}=\frac{3}{8}\cdot\frac{3}{3}+\frac{5}{6}\cdot\frac{4}{4}=\frac{9}{24}+\frac{20}{24}=\frac{29}{24}=1\frac{5}{24}$$

Q. Use an additive inverse to solve the equation $3x+y=112$ for y.

A. $\boldsymbol{y = 112 - 3x}$**.** Add $-3x$ to each side of the equation:

$$3x-3x+y=112-3x \rightarrow y=112-3x$$

Q. Apply the multiplication property of zero to solve the equation: $x(x+5)(x-4)=0$.

A. **−5, 4, 0.** At least one of the factors has to equal 0. So, if $x=0$, then the product is 0. Or, if $x=-5$, then the second factor is 0 and the product is 0. Likewise if $x=4$, then the third factor is 0 and the product is 0.

Q. Use the additive inverse to add and subtract a number that makes the expression x^2-10x factorable as the square of a binomial.

A. $\boldsymbol{(x-5)^2-25}$**.** Adding and subtracting 25 will do the trick:

$$x^2-10x+25-25=\left(x^2-10x+25\right)-25=\left(x-5\right)^2-25$$

YOUR TURN

3 Use the associative and commutative properties of multiplication to simplify $\left(\frac{4}{11}\right)(-15)\left(\frac{11}{8}\right)$.

4 Use the distributive property to simplify $10x\left(x-\frac{3}{5}\right)$.

5 Use a multiplicative inverse to solve the equation $-4x=100$ for the value of x.

THE BIRTH OF NEGATIVE NUMBERS

In the early days of algebra, negative numbers weren't an accepted entity. Mathematicians had a hard time explaining exactly what the numbers illustrated; it was too tough to come up with concrete examples. One of the first mathematicians to accept negative numbers was Fibonacci, an Italian mathematician. When he was working on a financial problem, he saw that he needed what amounted to a negative number to finish the problem. He described it as a loss and proclaimed, "I have shown this to be insoluble unless it is conceded that the man had a debt."

Specializing in Products and "FOIL"

Multiplying numbers and variables is a pretty standard procedure in algebra, and you can take advantage of some of the rules, such as distributing and associating, to solve problems. Another timesaver is to use special situations. You don't want to slug through a messy multiplication process when there's an easier and more efficient way (and one that doesn't encourage little errors).

You've already seen the distributive property at work in the previous section. Distributing means to multiply each term in the grouping symbol by the designated factor. But what if there is more than one term to multiply by, or if there are more than two factors? This is where some nice rules step in to assist you.

First-Outer-Inner-Last

When multiplying binomials, a very nice method to use is called FOIL (First, Outer, Inner, Last). This describes the order in which you perform multiplication of the terms in the binomials before simplifying for the final answer.

When multiplying the two binomials $(a+b)(c+d)$, *ac* is the product of the first terms in each binomial; *ad* is the product of the two outer terms; *bc* is the product of the two inner terms; and *bd* is the product of the two last terms.

EXAMPLE

Q. Find the product of $(x-7)(x+11)$.

A. $\boldsymbol{x^2+4x-77}$**.** Using FOIL: F $=x^2$, O = $11x$, I = $-7x$, L = -77. Writing them together, you get $x^2+11x-7x-77$, which simplifies to $x^2+4x-77$.

Q. Find the product of $(2y+5)(3y+2)$.

A. $\boldsymbol{6y^2+19y+10}$**.** Using FOIL: F = $6y^2$, O = $4y$, I = $15y$, L = 10. Writing them together, you get $6y^2+4y+15y+10$, which simplifies to $6y^2+19y+10$.

YOUR TURN

6 Find the product: $(x+5)(x-8)$.

7 Find the product: $(2z-7)(3z+7)$.

8 Find the product: $(x+4y)(3x-5y)$.

Raising binomials to powers

A *binomial* is an algebraic expression that has two terms. The terms can both be variables, or they can be a variable and a number. You usually don't see two numbers, because then you could add them and you wouldn't have a binomial anymore. A common task is to raise a binomial to a power. For example:

$$(a+2)^2 = a^2+4a+4 \qquad (x+y)^3 = x^3+3x^2y+3xy^2+y^3 \qquad (z+1)^4 = z^4+4z^3+6z^2+4z+1$$

A big help in determining these products (powers) is Pascal's Triangle. This triangle gives you the coefficients when raising the binomial $(x+y)$ to a power. The first row gives the coefficient of $(x+y)^0$; the second row gives the coefficient of $(x+y)^1$; the third row gives the coefficient of $(x+y)^3$; and so on.

$$\begin{array}{ccccccccccccc}
&&&&&&1&&&&&&\\
&&&&&1&&1&&&&&\\
&&&&1&&2&&1&&&&\\
&&&1&&3&&3&&1&&&\\
&&1&&4&&6&&4&&1&&\\
&1&&5&&10&&10&&5&&1&\\
1&&6&&15&&20&&15&&6&&1
\end{array}$$

EXAMPLE

Q. Use Pascal's Triangle to expand $(x+y)^4$.

A. $\boldsymbol{x^4+4x^3y+6x^2y^2+4xy^3+y^4}$**.** Go to the row with the second number 4. Write down the coefficients, leaving spaces for the variables: 1 4 6 4 1

Beginning with the first coefficient, multiply it by x^4, and then decrease the exponent by 1 as you go down the line of coefficients, ending with x^0 (or 1):

$1x^4 \quad 4x^3 \quad 6x^2 \quad 4x^1 \quad 1x^0.$

Next, beginning at the right end, multiply each term by a power of y, starting with y^4 and decreasing as you move to the left:

$1x^4y^0 \quad 4x^3y^1 \quad 6x^2y^2 \quad 4x^1y^3 \quad 1\cdot x^0y^4.$

Finally, simplify the terms and write the expression as the sum of the terms.

$x^4+4x^3y+6x^2y^2+4xy^3+y^4$

Q. Use Pascal's Triangle to expand $(x-3)^5$.

A. $\boldsymbol{x^5-15x^4+90x^3-270x^2+405x-243}$**.** Go to the row with the second number 5. Write down the coefficients, leaving spaces for the variables: 1 5 10 10 5 1

Beginning with the first coefficient, multiply it by x^5, and then decrease the exponent by 1 as you go down the line of coefficients, ending with x^0 (or 1).

$1x^5 \quad 5x^4 \quad 10x^3 \quad 10x^2 \quad 5x^1 \quad 1x^0$

Next, beginning at the right end, multiply each term by a power of –3, starting with $(-3)^5$ and decreasing as you move to the left.

$1x^5(-3)^0 \quad 5x^4(-3)^1 \quad 10x^3(-3)^2 \quad 10x^2(-3)^3 \quad 5x^1(-3)^4 \quad 1x^0(-3)^5$

Finally, simplify the terms and write the expression as the sum of the terms.

$x^5-15x^4+90x^3-270x^2+405x-243$

YOUR TURN

9 Expand: $(z-1)^6$.

10 Expand: $(2z+3)^3$.

Expounding on Exponential Rules

Several hundred years ago, mathematicians introduced powers of variables and numbers called exponents. Exponents weren't immediately popular, however. Scholars around the world had to be convinced of their value; eventually, the quick, slick notation of exponents won over, and you benefit from their use today. Instead of writing *xxxxxxxx*, you use the exponent 8 by writing x^8. This form is easier to read and much quicker.

The expression a^n is an exponential expression with a base of a and an exponent of n. The n tells you how many times you multiply the a by itself.

You use radicals to show roots. When you see $\sqrt{16}$, you know that you're looking for the number that multiplies itself to give you 16. The answer? Four, of course. If you put a small superscript in front of the radical, you denote a cube root, a fourth root, and so on. For instance, $\sqrt[4]{81} = 3$, because the number 3 multiplied by itself four times is 81. You can also replace radicals with fractional exponents — terms that make them easier to combine. Instead of using the radical with the root 4, you can write the expression as $81^{1/4} = 3$. This use of exponents is very systematic and workable — thanks to the mathematicians who came before us.

Multiplying and dividing exponents

When two numbers or variables have the same base, you can multiply or divide those numbers or variables by adding or subtracting their exponents.

- $a^m \cdot a^n = a^{m+n}$: When multiplying numbers with the same base, you add the exponents.
- $\dfrac{a^m}{a^n} = a^{m-n}$: When dividing numbers with the same base, you subtract the exponents (numerator – denominator). And, in this case, $a \neq 0$.

REMEMBER

You must be sure that the bases of the expressions are the same. You can multiply 3^2 and 3^4, but you can't use the rule of exponents when multiplying 3^2 and 4^3.

EXAMPLE

Q. Multiply $x^4 \cdot x^5$.

A. $\boldsymbol{x^9}$. Add the exponents: $x^{4+5} = x^9$.

Q. Divide $\dfrac{x^8}{x^5}$.

A. $\boldsymbol{x^3}$. Subtract the exponents: $\dfrac{x^8}{x^5} = x^{8-5} = x^3$.

Getting to the roots of exponents

Radical expressions — such as square roots, cube roots, fourth roots, and so on — appear with a radical to show the root. Another way you can write these values is by using fractional exponents. You'll have an easier time combining variables with the same base if they have fractional exponents in place of radical forms.

- $\sqrt[n]{x} = x^{1/n}$: The root goes in the denominator of the fractional exponent.
- $\sqrt[n]{x^m} = x^{m/n}$: The root goes in the denominator of the fractional exponent, and the power goes in the numerator.

So, you can say $\sqrt{x} = x^{1/2}$, $\sqrt[3]{x} = x^{1/3}$, $\sqrt[4]{x} = x^{1/4}$, and so on, along with $\sqrt[5]{x^3} = x^{3/5}$.

EXAMPLE

Q. Simplify $\sqrt{x} \cdot \sqrt[4]{x}$.

A. $\boldsymbol{\sqrt[4]{x^3}}$. First, change the radicals to an exponential form. Then add the exponents.

$$\sqrt{x} \cdot \sqrt[4]{x} = x^{1/2} \cdot x^{1/4} = x^{1/2+1/4} = x^{3/4} = \sqrt[4]{x^3}$$

Q. Simplify the radical expression $\dfrac{\sqrt[4]{x}\sqrt[6]{x^{11}}}{\sqrt[2]{x^3}}$.

A. $\boldsymbol{\sqrt[12]{x^7}}$. Change the radicals to exponents and apply the rules for multiplication and division of values with the same base:

$$\frac{\sqrt[4]{x}\sqrt[6]{x^{11}}}{\sqrt[2]{x^3}} = \frac{x^{1/4} \cdot x^{11/6}}{x^{3/2}} = \frac{x^{1/4+11/6}}{x^{3/2}} = \frac{x^{3/12+22/12}}{x^{18/12}} = \frac{x^{25/12}}{x^{18/12}} = x^{25/12-18/12} = x^{7/12} = \sqrt[12]{x^7}$$

Raising or lowering the roof with exponents

You can raise numbers or variables with exponents to higher powers or reduce them to lower powers by taking roots. When raising a power to a power, you multiply the exponents. When taking the root of a power, you divide the exponents.

- $(a^m)^n = a^{m \cdot n}$: Raise a power to a power by multiplying the exponents.
- $\sqrt[m]{a^n} = \left(a^n\right)^{1/m} = a^{n/m}$: Reduce the power when taking a root by dividing the exponents.

The second rule may look familiar — it's one of the rules that govern changing from radicals to fractional exponents.

EXAMPLE

Q. Simplify $\left(x^4\right)^2$.

A. $\boldsymbol{\left(x^4\right)^2 = x^8}$

Q. Apply the two rules to simplify $\sqrt[3]{\left(x^4\right)^6 \cdot x^9}$.

A. $\boldsymbol{x^{11}}$. $\sqrt[3]{\left(x^4\right)^6 \cdot x^9} = \sqrt[3]{x^{24} \cdot x^9} = \sqrt[3]{x^{33}} = x^{33/3} = x^{11}$

Making nice with negative exponents

You use negative exponents to indicate that a number or variable belongs in the denominator of the term:

$$\frac{1}{a}=a^{-1} \text{ and } \frac{1}{a^n}=a^{-n}$$

Writing variables with negative exponents allows you to combine those variables with other factors that share the same base.

EXAMPLE

Q. Simplify $\left(x^{-5}\right)^2$.

A. $\boldsymbol{\frac{1}{x^{10}}}$. Raise the power and then apply the negative exponent rule: $\left(x^{-5}\right)^2=x^{-10}=\frac{1}{x^{10}}$.

Q. Simplify $\frac{1}{x^4}\cdot x^7\cdot\frac{3}{x}$.

A. $\boldsymbol{3x^2}$. Rewrite the fractions by using negative exponents, and then simplify by using the rules for multiplying factors with the same base:

$$\frac{1}{x^4}\cdot x^7\cdot\frac{3}{x}=x^{-4}\cdot x^7\cdot 3x^{-1}=3x^{-4+7+(-1)}=3x^2$$

YOUR TURN

Simplify the following expressions.

11 $\frac{8y^7}{2y^4}$

12 $\frac{18x^2y^3}{6x^4y^2}$

13 $x^6\left(x^2\right)^2$

14 $\sqrt[3]{4x^9y^6z^3}$

Taking On Special Operators

The operations you first learned were addition, subtraction, multiplication, and division. These are all called *binary operations*, because it takes two numbers to perform the operation. Then you learned about powers and roots, which only require one number in order to be performed. Some special operations that are important in mathematics are absolute value, greatest integer, and factorial.

Working with absolute value

The *absolute value* function takes a number and reports back its distance from 0. Basically, this means that it leaves all positive numbers positive and changes negative numbers to positive numbers. The formal rule is stated as follows:

$$|n| = \begin{cases} n, n \geq 0 \\ -n, n < 0 \end{cases}$$

EXAMPLE

Q. $|-7| =$

A. **7.** Change the negative number to a positive number. This gives you the distance from –7 to 0 on the number line: $|-7| = 7$.

Q. $|6^2 - 11| =$

A. **25.** First, perform the operations in the absolute value. Then find that absolute value.
$|6^2 - 11| = |36 - 11| = |25| = 25$

Doing great with the greatest integer function

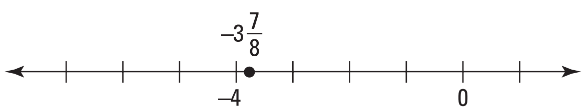
The *greatest integer function* reports which integer the number being assessed is greater than or equal to. It is used frequently on fractions and decimals. Another way of looking at the function is to say, "On the number line, what integer is directly to the left of the number?" This is if the number isn't already an integer.

EXAMPLE

Q. $[5.8] =$

A. $[\mathbf{5.8}] = \mathbf{5}$**.** The number 5.8 is larger than 5. The integer 5 is to the left of 5.8 on the number line.

Q. $\left[-3\frac{7}{8}\right] =$



A. $\left[\mathbf{-3\frac{7}{8}}\right] = \mathbf{-4}$**.** Refer to the number line. The integer immediately to the left of $-3\frac{7}{8}$ is –4.

Getting the facts with factorial

The factorial function is indicated with an exclamation mark. This function tells you to take the number being operated upon and multiply it by every positive integer smaller than that number.

$$n! = n \cdot (n-1) \cdot (n-2) \cdot (n-3) \ldots 3 \cdot 2 \cdot 1$$

And there's one special rule for 0!: it has been declared that $0! = 1$.

EXAMPLE

Q. $5! =$

A. **120.** $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

Q. $\frac{8!}{6!} =$

A. **56.** $\frac{8!}{6!} = \frac{8 \cdot 7 \cdot 6!}{6!} = \frac{8 \cdot 7 \cdot \cancel{6!}}{\cancel{6!}} = \frac{56}{1} = 56$

YOUR TURN

Evaluate each of the following.

15 $|-3.17| =$

16 $[15.9] =$

17 $\frac{4!}{0!} =$

Simplifying Radical Expressions

Making a radical expression simpler makes it more usable. When a number in a radical is simplified, it's easier to estimate the value of the expression (if the number isn't already a power of the root). Simplifying algebraic expressions in radicals makes them easier to factor or reduce when using the expression. Basically, you try to find some power of the root as a factor of the expression in the radical.

EXAMPLE

Q. Simplify $\sqrt{18}$.

A. $\mathbf{3\sqrt{2}}$**.** First, factor the 18. Choose the factorization that gives you a perfect square as one of the choices: $\sqrt{18} = \sqrt{9 \cdot 2} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$.

Q. Simplify $\sqrt{24x^3y^4}$.

A. $\boldsymbol{2xy^2\sqrt{6x}}$**.** Write each factor under the radical as a product of a perfect square and another factor: $\sqrt{24x^3y^4} = \sqrt{4 \cdot 6 \cdot x^2 \cdot x \cdot y^4}$. Next, write a product of the roots of the perfect square factors times the remaining factors: $\sqrt{4 \cdot 6 \cdot x^2 \cdot x \cdot y^4} = \sqrt{4x^2y^4}\sqrt{6x}$.

Finally, simplify: $\sqrt{4x^2y^4}\sqrt{6x} = 2xy^2\sqrt{6x}$.

YOUR TURN

Simplify each of the following radical expressions.

18 $\sqrt{60x^{11}}$

19 $\sqrt{400y}$

20 $\dfrac{\sqrt{80x^3y^7}}{\sqrt{40xy}}$

Factoring in Some Factoring Techniques

When you factor an algebraic expression, you rewrite the sums and differences of the terms as a product. For instance, you write the three terms $x^2 - x - 42$ in factored form as $(x-7)(x+6)$. The expression changes from three terms to one big, multiplied-together term. You can factor two terms, three terms, four terms, and so on for many different purposes. Factorization comes in handy when you set the factored forms equal to zero to solve an equation. Factored numerators and denominators in fractions also make it possible to reduce the fractions.

You can think of factoring as the opposite of distributing. You have good reasons to distribute or multiply through by a value; for example, the process allows you to combine like terms and simplify expressions. Factoring out a common factor also has its purposes for solving equations and combining fractions. The different formats are equivalent; they just have different uses.

Factoring two terms

When an algebraic expression has two terms, you have four different choices for its factorization — if you can factor the expression at all. If you try the following four methods and none of them work, you can stop your attempt; you just can't factor the expression.

$ax + ay = a(x+y)$	Greatest common factor
$x^2 - a^2 = (x-a)(x+a)$	Difference of two perfect squares
$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$	Difference of two perfect cubes
$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$	Sum of two perfect cubes

In general, you check for a greatest common factor before attempting any of the other methods. By taking out the common factor, you often make the numbers smaller and more manageable, which helps you see clearly whether any other factoring is necessary.

EXAMPLE

Q. Factor $12x^2 - 15x^3$.

A. $\mathbf{3x^2(4-5x)}$. The greatest common factor of the two terms is $3x^2$:
$12x^2 - 15x^3 = 3x^2(4-5x)$.

Q. Factor $100 - y^2$.

A. $\mathbf{(10-y)(10+y)}$. This is the difference of two perfect squares: $100 - y^2 = (10-y)(10+y)$.

Q. Factor $6x^4 - 6x$.

A. $\mathbf{6x(x-1)(x^2+x+1)}$. First, factor out the common factor, 6*x*, and then use the pattern for the difference of two perfect cubes:

$$6x^4 - 6x = 6x(x^3 - 1) = 6x(x-1)(x^2+x+1).$$

Q. Factor $z^4 - 81$.

A. $\mathbf{(z-3)(z+3)(z^2+9)}$**.** The expression $z^4 - 81$ is the difference of two perfect squares. When you factor it, you get $z^4 - 81 = (z^2 - 9)(z^2 + 9)$. Notice that the first factor is also the difference of two squares — you can factor again. The second term, however, is the sum of squares — you can't factor it. With perfect cubes, you can factor both differences and sums, but not with the squares. So, the factorization of $z^4 - 81$ is $(z-3)(z+3)(z^2+9)$.

YOUR TURN

21 Factor $48x^3y^2 - 300x^3$.

22 Factor $8a^3 + 27b^6$.

Taking on three terms

A quadratic trinomial is a three-term polynomial with a term raised to the second power. When you see something like $x^2 + x + 1$, you often run through the possibilities of factoring it into the product of two binomials. In this case, you can just stop. These trinomials that crop up when factoring cubes just don't cooperate.

When a quadratic expression has three terms, making it a trinomial, you have two different ways to factor it. One method is factoring out a greatest common factor, and the other is finding two binomials whose product is identical to those three terms.

$ax + ay + az = a(x + y + z)$	Greatest common factor
$x^{2n} + (a+b)x^n + ab = (x^n + a)(x^n + b)$	Two binomials

You can often spot the greatest common factor with ease; you see a multiple of some number or variable in each term. With the product of two binomials, you either find the product or become satisfied that it doesn't exist.

Trinomials that factor into the product of two binomials have related powers on the variables in two of the terms. The relationship between the powers is that one is twice the other. When factoring a trinomial into the product of two binomials, you first look to see if you have a special product: a perfect square trinomial. If you don't, you can proceed to unFOIL. The acronym FOIL helps you multiply two binomials (First, Outer, Inner, Last); unFOIL helps you factor the product of those binomials. You find tips and techniques for factoring these products in the following sections.

Q. Factor $6x^3 - 15x^2y + 24xy^2$.

EXAMPLE

A. $\mathbf{3x(2x^2 - 5xy + 8y^2)}$**.** The greatest common factor of the three terms is $3x$: $6x^3 - 15x^2y + 24xy^2 = 3x(2x^2 - 5xy + 8y^2)$.

Q. Factor $44x^2 + 99y + 121$.

A. $\mathbf{11\left(4x^2 + 9x + 11\right)}$**.** The greatest common factor of the three terms is 11: $44x^2 + 99y + 121 = 11\left(4x^2 + 9x + 11\right)$. It doesn't look very hopeful that this trinomial can be factored, but the following sections will help you determine if that's the case.

Factoring perfect square trinomials

A perfect square trinomial is an expression of three terms that results from the squaring of a binomial — multiplying it times itself. Perfect square trinomials are fairly easy to spot — their first and last terms are perfect squares, and the middle term is twice the product of the roots of the first and last terms:

$$a^2 + 2ab + b^2 = \left(a + b\right)^2$$
$$a^2 - 2ab + b^2 = \left(a - b\right)^2$$

EXAMPLE

Q. Factor $x^2 - 20x + 100$.

A. $\mathbf{(x - 10)^2}$**.** The first and third terms are both perfect squares. The middle term, $20x$, is twice the product of the root of x^2 and the root of 100; therefore, the factorization is $(x - 10)^2$.

Q. Factor $25y^2 + 30y + 9$.

A. $\mathbf{(5y + 3)^2}$**.** You can see that the first and last terms are perfect squares. The root of $25y^2$ is $5y$, and the root of 9 is 3. The middle term, $30y$, is twice the product of $5y$ and 3, so you have a perfect square trinomial that factors into $(5y + 3)^2$.

Resorting to unFOIL

When you factor a trinomial that results from multiplying two binomials, you have to play detective and piece together the parts of the puzzle. Look at the following generalized product of binomials and the pattern that appears:

$$\left(ax + b\right)\left(cx + d\right) = acx^2 + adx + bcx + bd = acx^2 + \left(ad + bc\right)x + bd$$

So, where does FOIL come in? You need to FOIL before you can unFOIL, don't ya think? Just go back to the earlier section, "Specializing in Products and FOIL."

Now, think of every quadratic trinomial as being of the form $acx^2 + (ad + bc)x + bd$. The coefficient of the x^2 term, ac, is the product of the coefficients of the two x terms in the parentheses; the last term, bd, is the product of the two second terms in the parentheses; and the coefficient of the middle term is the sum of the outer and inner products. To factor these trinomials into the product of two binomials, you can use the opposite of FOIL, which I call *unFOIL.*

Here are the basic steps you take to unFOIL the trinomial $acx^2 + (ad + bc)x + bd$:

1. **Determine all the ways you can multiply two numbers to get ac, the coefficient of the squared term.**
2. **Determine all the ways you can multiply two numbers to get bd, the constant term.**
3. **If the last term is positive, find the combination of factors from Steps 1 and 2 whose sum is that middle term; if the last term is negative, you want the combination of factors to be a difference.**
4. **Arrange your choices as binomials so that the factors line up correctly.**
5. **Insert the + and – signs to finish off the factoring and make the sign of the middle term come out right.**

EXAMPLE

Q. Factor $x^2 + 9x + 20$.

A. $\boldsymbol{(x+4)(x+5)}$**.** Find two terms whose product is 20 and whose sum is 9. The coefficient of the squared term is 1, so you don't have to take any other factors into consideration. You can produce the number 20 with $1 \cdot 20$, $2 \cdot 10$, or $4 \cdot 5$. The last pair is your choice, because $4 + 5 = 9$. Arranging the factors and *x*'s into two binomials, you get $x^2 + 9x + 20 = (x+4)(x+5)$.

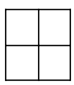
Q. Factor $4y^2 + 5y - 6$.

A. $\boldsymbol{(y+2)(4y-3)}$**.** This is a little more complicated, because the coefficient of the y^2 term isn't 1. And this time you need a difference rather than a sum. So look at the possible factors of both that first term and the last term. For the factor 4, you can use $1 \cdot 4$ or $2 \cdot 2$. For the 6, you can use $1 \cdot 6$ or $2 \cdot 3$. You need to try out some combinations of the factors of each so that the difference between the products is 5. After some playing around, you see that if you use the $1 \cdot 4$ and the $2 \cdot 3$, you can create a product of 8 with the 4 and 2 and a product of 3 with the 1 and 3. Voila! You have a difference of 5. Start by setting up the binomials without signs: $(y \quad 2)(4y \quad 3)$. Then, to create $+5y$, you assign the + to the first binomial: $(y+2)(4y-3) = 4y^2 - 3y + 8y - 6 = 4y^2 + 5y - 6$.

Employing the "Box Method"

A very handy technique to use when factoring trinomials is called the *Box Method*. It helps you arrange the terms and their factors in such a way that the terms in the factorization just pop into view! Let me show you this method in the factorization of $3x^2 + 5x - 12$:

1. **Create a 2-by-2 box.**



2. **Enter the first and last terms of the trinomial in the upper-left and lower-right areas.**

$3x^2$	
	-12

3. **Find the product of the two terms you entered.**

$$\left(3x^2\right)\left(-12\right) = -36x^2$$

4. **Find two factors of this product whose sum results in the middle term, 5x.**

$$(9x)(-4x) = -36x^2$$
$$9x + (-4x) = 5x$$

5. **Place the two factors in the open areas.**

$3x^2$	$9x$
$-4x$	-12

6. **Find the GCF of each row and column.**

$3x^2$	$9x$	$3x$
$-4x$	-12	-4
x	3	

7. **The terms along the right side and across the bottom are the terms in the factorization.**

$$\left(3x-4\right)\left(x+3\right) = 3x^2 + 5x - 12$$

YOUR TURN

23 Factor $x^2 - 8x + 15$.

24 Factor $12x^2 - 7x - 10$.

25 Factor $6x^2 - x - 35$.

26 Factor $x^4 + 4x^2 - 12$.

Factoring four or more terms by grouping

When four or more terms come together to form an expression, you have bigger challenges in the factoring. If you can't find a factor common to all the terms at the same time, another option is grouping. To group, you take the terms two at a time and look for common factors in each of the pairs on an individual basis. After factoring, you see if the new groupings have a common factor.

EXAMPLE

Q. Factor $x^3 - 4x^2 + 3x - 12$.

A. $\boldsymbol{(x-4)(x^3+3)}$**.** The four terms $x^3 - 4x^2 + 3x - 12$ don't have any common factors. However, the first two terms have a common factor of x^2, and the last two terms have a common factor of 3: $x^3 - 4x^2 + 3x - 12 = x^2(x-4) + 3(x-4)$. Now there are two terms, each with the common factor $(x-4)$. Factor that out of the two terms: $x^2(x-4) + 3(x-4) = (x-4)(x^3+3)$.

Q. Factor $xy^2 - 2y^2 - 5xy + 10y - 6x + 12$.

A. $\boldsymbol{(x-2)(y-6)(y+1)}$**.** The six terms don't have a common factor, but, taking them two at a time, you can pull out the factors y^2, $-5y$, and -6. Factoring by grouping, you get the following: $xy^2 - 2y^2 - 5xy + 10y - 6x + 12 = y^2(x-2) - 5y(x-2) - 6(x-2)$. The three new terms have a common factor of $(x-2)$, so the factorization becomes $(x-2)(y^2 - 5y - 6)$. The trinomial that you create also factors (see the previous section): $(x-2)(y^2 - 5y - 6) = (x-2)(y-6)(y+1)$. Factored, and ready to go!

YOUR TURN

27 Factor $2x^3 + 5x^2 - 8x - 20$.

28 Factor $x^3y - x^3 - 5x^2y + 5x^2 - 33xy + 33x - 27y + 27$.

Practice Questions Answers and Explanations

1 **38.** First, evaluate the two powers. Then multiply. And, finally, add and subtract, moving left to right: $4(-3)^2+7-\sqrt{25}=4(9)+7-5=36+7-5=38$.

2 **–1.** Simplifying the numerator, first subtract in the parentheses, and then multiply by 3. In the denominator, first multiply and add under the radical, and then take the cube root. Finally, simplify the resulting fraction:

$$\frac{3(6-7)}{\sqrt[3]{5\cdot 4+7}}=\frac{3(-1)}{\sqrt[3]{5\cdot 4+7}}=\frac{-3}{\sqrt[3]{5\cdot 4+7}}=\frac{-3}{\sqrt[3]{20+7}}=\frac{-3}{\sqrt[3]{27}}=\frac{-3}{3}=-1$$

3 $\mathbf{\frac{-15}{2}}$. Reverse the order of the last two factors. Then reduce the fractions and multiply.

$$\left(\frac{4}{11}\right)(-15)\left(\frac{11}{8}\right)=\left(\frac{4}{11}\right)\left(\frac{11}{8}\right)(-15)=\left(\frac{\cancel{4}}{\cancel{11}}\cdot\frac{\cancel{11}}{\cancel{8}_2}\right)(-15)=\left(\frac{1}{2}\right)(-15)=\frac{-15}{2}$$

4 $\mathbf{10x^2-6x}$. Multiply each term in the parentheses by $10x$.

$$10x\left(x-\frac{3}{5}\right)=10x\cdot x-10x\cdot\frac{3}{5}=10x^2-\cancel{10}^{2}x\cdot\frac{3}{\cancel{5}}=10x^2-6x$$

5 **–25.** Multiply each side of the equation by $-\frac{1}{4}$.

$$-\frac{1}{4}\cdot(-4)x=-\frac{1}{4}\cdot 100\rightarrow-\frac{1}{\cancel{4}}\cdot(-\cancel{4})x=-\frac{1}{\cancel{4}}\cdot\cancel{100}^{25}\rightarrow x=-25$$

6 $\mathbf{x^2-3x-40}$. Using FOIL: $(x+5)(x-8)=x^2-8x+5x-40=x^2-3x-40$.

7 $\mathbf{6z^2-7z-49}$. Using FOIL: $(2z-7)(3z+7)=6z^2+14z-21z-49=6z^2-7z-49$.

8 $\mathbf{3x^2+7xy-20y^2}$. Using FOIL: $(x+4y)(3x-5y)=3x^2-5xy+12xy-20y^2=3x^2+7xy-20y^2$.

9 $\mathbf{z^6-6z^5+15z^4-20z^3+15z^2-6z+1}$. Find the coefficients in Pascal's Triangle. Then insert decreasing powers of z moving from left to right and decreasing powers of –1 moving from right to left. Simplify the resulting terms.

$$\begin{array}{ccccccc} 1 & 6 & 15 & 20 & 15 & 6 & 1\\ 1z^6 & 6z^5 & 15z^4 & 20z^3 & 15z^2 & 6z^1 & 1z^0\\ 1z^6(-1)^0 & 6z^5(-1)^1 & 15z^4(-1)^2 & 20z^3(-1)^3 & 15z^2(-1)^4 & 6z^1(-1)^5 & 1z^0(-1)^6 \end{array}$$

$$z^6-6z^5+15z^4-20z^3+15z^2-6z+1$$

10 $\mathbf{8z^3+36z^2+54z+27}$. Find the coefficients in Pascal's Triangle. Then insert decreasing powers of $2z$ moving from left to right, and decreasing powers of 3 moving from right to left. Simplify the resulting terms.

$$\begin{array}{cccc} 1 & 3 & 3 & 1\\ 1(2z)^3 & 3(2z)^2 & 3(2z)^1 & 1(2z)^0\\ 1(2z)^3\cdot 3^0 & 3(2z)^2\cdot 3^1 & 3(2z)^1\cdot 3^2 & 1(2z)^0\cdot 3^3 \end{array}$$

$$8z^3+36z^2+54z+27$$

(11) $\mathbf{4y^3}$**.** Reduce the powers of y by subtracting: $\frac{8y^7}{2y^4} = \frac{\cancel{8}^4 y^{7-4}}{\cancel{2}} = 4y^3$.

(12) $\mathbf{\frac{3y}{x^2}}$**.** Reduce the powers of x and y by subtracting: $\frac{18x^2y^3}{6x^4y^2} = \frac{\cancel{18}^3 x^{2-4}y^{3-2}}{\cancel{6}} = 3x^{-2}y^1$. Rewrite the power of x as a positive number in the denominator: $3x^{-2}y^1 = \frac{3y}{x^2}$.

(13) $\mathbf{x^{10}}$**.** First, raise the power to a power by multiplying the exponents. Then find the product of the two factors by adding the exponents: $x^6\left(x^2\right)^2 = x^6\left(x^4\right) = x^{10}$.

(14) $\mathbf{4^{1/3}x^3y^2z}$**.** Change the cube root to a power of $\frac{1}{3}$. Then raise the variable factors to that power by multiplying the exponents: $\sqrt[3]{4x^9y^6z^3} = \left(4x^9y^6z^3\right)^{1/3} = (4)^{1/3}\left(x^9\right)^{1/3}\left(y^6\right)^{1/3}\left(z^3\right)^{1/3} = 4^{1/3}x^3y^2z^1$.

(15) **3.17.** Change the number to a positive value.

(16) **15.** The largest integer smaller than 15.9 is 15.

(17) **24.** Zero factorial is equal to 1: $\frac{4!}{0!} = \frac{4\cdot 3\cdot 2\cdot 1}{1} = 24$.

(18) $\mathbf{2x^5\sqrt{15x}}$**.** Factor the number and variable, separating perfect square factors from the others. Then simplify, finding the square roots: $\sqrt{60x^{11}} = \sqrt{4\cdot 15\cdot x^{10}\cdot x} = \sqrt{4x^{10}}\cdot\sqrt{15x} = 2x^5\sqrt{15x}$.

(19) $\mathbf{20\sqrt{y}}$**.** Write the number and variable separately under radicals, and simplify:
$\sqrt{400y} = \sqrt{400}\cdot\sqrt{y} = 20\sqrt{y}$.

(20) $\mathbf{xy^3\sqrt{2}}$**.** Write the fraction under one radical. Reduce the fraction and simplify the result:
$\frac{\sqrt{80x^3y^7}}{\sqrt{40xy}} = \sqrt{\frac{80x^3y^7}{40xy}} = \sqrt{\frac{\cancel{80}^2x^{3-1}y^{7-1}}{\cancel{40}}} = \sqrt{2x^2y^6} = xy^3\sqrt{2}$

(21) $\mathbf{12x^3(2y-5)(2y+5)}$**.** First, factor out the common factor $12x^3$:
$48x^3y^2 - 300x^3 = 12x^3\left(4y^2-25\right)$. Now factor the binomial as the difference of perfect squares:
$12x^3\left(4y^2-25\right) = 12x^3(2y-5)(2y+5)$.

(22) $\mathbf{\left(2a+3b^2\right)\left(4a^2-6ab^2+9b^4\right)}$**.** Use the pattern for the sum of perfect cubes:
$8a^3+27b^6 = \left(2a+3b^2\right)\left(4a^2-6ab^2+9b^4\right)$.

(23) $\mathbf{(x-3)(x-5)}$**.** Two factors of 15 are 3 and 5, and their sum is 8. They are both negative.
$x^2-8x+15 = (x\quad 3)(x\quad 5) = (x-3)(x-5)$

(24) $\mathbf{(3x+2)(4x-5)}$**.** Using the Box Method, the two factors of $-120x^2$ that give you a difference of $-7x$ are $-15x$ and $8x$.

$12x^2$	$-15x$	$3x$
$8x$	-10	2
$4x$	-5	

(25) $\mathbf{(3x+7)(2x-5)}$**.** Two factors of 35 are 5 and 7. Two factors of 6 are 3 and 2. You want a difference of $-1x$: $6x^2-x-35 = (3x\quad 7)(2x\quad 5) = (3x+7)(2x-5)$.

26 $\mathbf{(x^2+6)(x^2-2)}$. Treat the trinomial like the quadratic trinomial $y^2+4y-12$: $x^4+4x^2-12=(x^2+6)(x^2-2)$.

27 $\mathbf{(2x+5)(x-2)(x+2)}$. First, factor by grouping: $2x^3+5x^2-8x-20=x^2(2x+5)-4(2x+5)=(2x+5)(x^2-4)$. And now factor the difference of squares: $(2x+5)(x^2-4)=(2x+5)(x-2)(x+2)$.

28 $\mathbf{(y-1)(x^3-5x^2-33x-27)}$. Factor by grouping the four pairs of terms.

$$\begin{aligned}x^3y-x^3-5x^2y+5x^2-33xy+33x-27y+27&=x^3(y-1)-5x^2(y-1)-33x(y-1)-27(y-1)\\&=(y-1)(x^3-5x^2-33x-27)\end{aligned}$$

The expression in the parentheses can be factored even further. See Chapter 3, for more information.

Whaddya Know? Chapter 1 Quiz

Quiz time! Complete each problem to test your knowledge on the various topics covered in this chapter. You can then find the solutions and explanations in the next section.

1. Simplify, applying the Order of Operations: $\frac{2(3-4\cdot 2)}{5} =$
2. Find the product: $(3x+10)(5x-11)$
3. Simplify: $\sqrt{200x^5y^6}$
4. Simplify, applying the Order of Operations: $-3(-2)^2+4(-2)^1+5(-2)^0 =$
5. Evaluate: $\frac{9!}{7!} =$
6. Simplify: $\left(\frac{3}{4}\right)^3 =$
7. Factor: $13x^2y-26xy^2$
8. Expand: $(x-2)^4$
9. Simplify: $\left(-\frac{1}{2}\right)^{-4} =$
10. Factor: $64-x^3$
11. Simplify: $25^{1/2} =$
12. Factor: $x^3+6x^2-2x-12$
13. Simplify: $64^{2/3} =$
14. Find the product: $(2x-z)(4x+5z)$
15. Simplify: $\frac{-27a^2b^3c}{-3ab^5c}$
16. Factor: $12x^2-31x+7$
17. Simplify: $\left(3^{-3}\right)^{-1} =$
18. Evaluate: $|7-11| =$
19. Simplify: $|6(2)-5|-|4+3(-5)| =$
20. Evaluate: $\left[-2\frac{1}{8}\right] =$
21. Factor: $64y^3-y^5$

Answers to Chapter 1 Quiz

1. **−2.** First, perform the multiplication in the parentheses and then subtract the result from 3: $\frac{2(3-4\cdot 2)}{5}=\frac{2(3-8)}{5}=\frac{2(-5)}{5}$. You can either reduce the fraction by dividing by 5 or multiply the two numbers in the numerator and then divide by 5. The result is –2.

2. $\mathbf{15x^2+17x-110}$. Using FOIL: $(3x+10)(5x-11)=15x^2-33x+50x-110=15x^2+17x-110$.

3. $\mathbf{10x^2y^3\sqrt{2x}}$. Factor the number and variables to create perfect square factors. Then write a product of two radicals and simplify:

   $\sqrt{200x^5y^6}=\sqrt{100\cdot 2\cdot x^4\cdot x\cdot y^6}=\sqrt{100x^4y^6}\cdot\sqrt{2x}=10x^2y^3\sqrt{2x}$.

4. **−15.** First, raise 2 to the indicated powers: $-3(-2)^2+4(-2)^1+5(-2)^0=-3(4)+4(-2)+5(1)$.

   Next, perform the multiplications: $=-12-8+5$. Now subtract and add, moving from left to right: $=-20+5=-15$.

5. **72.** Reduce the fraction: $\frac{9!}{7!}=\frac{9\cdot 8\cdot 7!}{7!}=\frac{9\cdot 8\cdot \cancel{7!}}{\cancel{7!}}=72$

6. $\mathbf{\frac{27}{64}}$. Raise both the numerator and denominator to the third power: $\left(\frac{3}{4}\right)^3=\frac{3^3}{4^3}=\frac{27}{64}$

7. $\mathbf{13xy(x-2y)}$. Factor out the greatest common factor, 13*xy*: $13x^2y-26xy^2=13xy(x-2y)$.

8. $\mathbf{x^4-8x^3+24x^2-32x+16}$. Use Pascal's Triangle and the descending powers of the two terms. Simplify the terms.

   $$\begin{array}{ccccc} 1 & 4 & 6 & 4 & 1 \\ 1x^4 & 4x^3 & 6x^2 & 4x^1 & 1x^0 \\ 1x^4\cdot(-2)^0 & 4x^3\cdot(-2)^1 & 6x^2\cdot(-2)^2 & 4x^1\cdot(-2)^3 & 1x^0\cdot(-2)^4 \end{array}$$

   $$x^4-8x^3+24x^2-32x+16$$

9. **16.** Change the negative exponent to a positive exponent when you write the reciprocal of the fraction: $\left(-\frac{1}{2}\right)^{-4}=\left(-\frac{2}{1}\right)^4=(-2)^4$. When raising the –2 to the fourth power, the result is a positive number: $(-2)^4=16$.

10. $\mathbf{(4-x)(16+4x+x^2)}$. Use the pattern for the difference of cubes: $64-x^3=(4-x)(16+4x+x^2)$.

11. **5.** An exponent of $1/2$ is equivalent to the square root of the number: $25^{1/2}=\sqrt{25}=5$.

12. $\mathbf{(x+6)(x^2-2)}$. Factor by grouping: $x^3+6x^2-2x-12=x^2(x+6)-2(x+6)=(x+6)(x^2-2)$.

13. **16.** The exponent of $2/3$ says to first find the cube root of the number and then square the result.

    $64^{2/3}=\left(\sqrt[3]{64}\right)^2=(4)^2=16$

14 $\mathbf{8x^2 + 6xz - 5z^2}$. Use FOIL to find the product:
$(2x - z)(4x + 5z) = 8x^2 + 10xz - 4xz - 5z^2 = 8x^2 + 6xz - 5z^2$

15 $\mathbf{\dfrac{9a}{b^2}}$. Divide the numbers. Use division on the variables by subtracting the exponents:
$\dfrac{-27a^2b^3c}{-3ab^5c} = 9a^{2-1}b^{3-5}c^{1-1} = 9a^1b^{-2}c^0$. Now rewrite the result, placing the variable with the negative exponent in the denominator and simplifying the other exponents: $9a^1b^{-2}c^0 = \dfrac{9a}{b^2}$

16 $\mathbf{(4x-1)(3x-7)}$. Using the Box Method, the two factors of $84x^2$ that give you a sum of $-31x$ are $-28x$ and $-3x$.

$12x^2$	$-28x$	$4x$
$-3x$	7	1
$3x$	7	

Both binomials will have a difference: $12x^2 - 31x + 7 = (4x-1)(3x-7)$.

17 **27.** When you raise a power to a power, you multiply the exponents: $\left(3^{-3}\right)^{-1} = 3^3 = 27$.

18 **4.** First, perform the subtraction, and then apply the absolute value to the result:
$|7-11| = |-4| = 4$.

19 **−4.** First, perform the multiplications in the absolute value terms. Then perform the subtraction and addition and apply absolute value to the results:
$|6(2)-5| - |4+3(-5)| = |12-5| - |4-15| = |7| - |-11| = 7 - 11$. Finally, perform the last subtraction:
$7 - 11 = -4$.

20 **−3.** This is the Greatest Integer Function. Put $-2\frac{1}{8}$ on a number line, and you see that the greatest integer (the integer immediately to the left) is −3.

21 $\mathbf{y^3(8-y)(8+y)}$. First, factor out the greatest common factor. Then factor the difference of squares: $64y^3 - y^5 = y^3(64 - y^2) = y^3(8-y)(8+y)$.