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Introduction

1.1 Purpose

This book attempts to describe the discipline of spacecraft observation, as practiced at the Jet Propulsion Laboratory (JPL) since the late 1960s and more recently elsewhere. It is not intended to be a user's guide for JPL software, nor a mathematical models document describing the functionality of JPL software, nor a requirements document. Rather, it is meant to be more of a textbook treating all aspects of optical navigation: predicting the contents of pictures, analyzing the pictures to extract the observables, and introducing these observations into a solution for navigation parameters. Discussions of cameras and detectors are included as well, but only in enough depth that a navigation engineer can understand the properties of the hardware.

Chapter 2 contains the history of optical navigation, both at JPL and elsewhere. Chapter 3 discusses cameras, including their optics and detectors, from the point of view of an optical navigation engineer. Chapter 4 presents the mathematics required to predict where the image of some object or star will appear in a picture. Chapter 5, the longest and most diverse, discusses various methods of extracting observed image locations from the brightness levels in a received picture. Chapter 6 deals with the incorporation of optical data into an orbit determination solutions. Appendix treats the "overlapping plate method," which enables accurate camera calibration even though its original purpose was for the development of star catalogs. A glossary of terms and references for each chapter are also presented.

1.2 Definitions

Optical navigation is the use of pictures, taken by cameras aboard spacecraft, to help determine the trajectory of the spacecraft. A *camera* in this context means some instrument that projects incoming light onto a detector to form a *picture* of

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the scene being imaged. The picture consists of a rectangular array of *pixels*, each of which contains a *digital number* or *DN* value that ideally is linearly proportional to the number of photons impinging upon a corresponding rectangular region of the detector. The picture may therefore contain *images* of stars, natural bodies, or even other spacecraft. *Landmarks* are specific features or points on the surface of a body, and images of landmarks are also used in navigation.

The fundamental coordinate system is the International Celestial Reference Frame or ICRF, currently defined at optical wavelengths by the coordinates of stars in the HIPPARCOS catalog (European Space Agency 1997). This frame's x-axis is closely aligned with the direction of the Earth's mean vernal equinox at epoch January 1, 2000 12:00:00 barycentric dynamical time (TDB), and its z-axis is closely aligned with the Earth's mean north pole at the same epoch. The origin is the barycenter of the Solar System. Star catalogs and ephemerides of Solar System bodies used in optical navigation operations are presumed to be referred to the ICRF as well.

1.3 Notation

Throughout this book, scalars are set in *italic* type, vectors in **boldface roman**, and matrices in uppercase sans-serif. Terms specific to astronomy, optics, or optical navigation appear in *slanted* type at their first occurrence.

1.4 Rotations

Rotations are to be thought of as rotating a *coordinate system*, rather than as a change in the orientation of (say) a vector. A *rotation matrix* is a 3×3 matrix that, when multiplying a vector, transforms its rectangular coordinates from one coordinate system to another. An "elementary" rotation by angle θ about axis i is denoted as $\mathsf{R}_i(\theta)$. Written out in full,

$$\mathsf{R}_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}, \tag{1.1a}$$

$$\mathsf{R}_2(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{1.1b}$$

$$\mathsf{R}_3(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1.1c}$$

Figure 1.1 shows a typical right-handed Cartesian coordinate system, whose axes are denoted by x, y, and z. Figures 1.2–1.4 show the result after a rotation

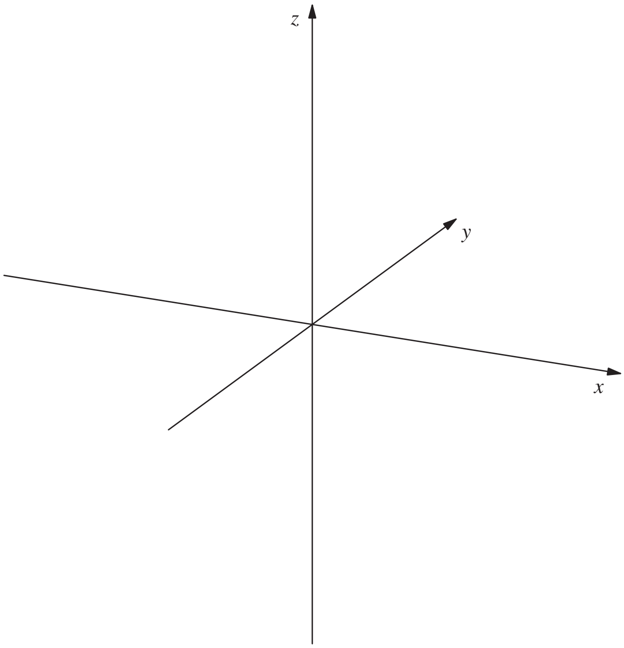

Figure 1.1 A right-handed coordinate system.

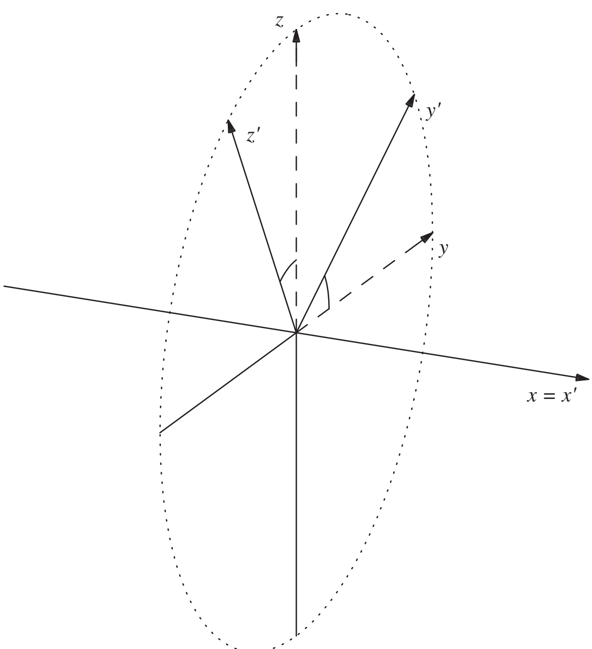

Figure 1.2 Rotation about the *x*-axis, by +30°.

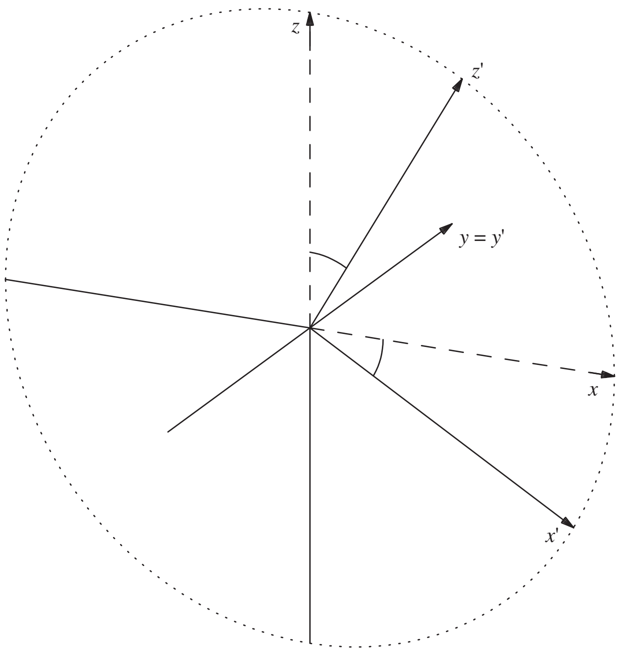

Figure 1.3 Rotation about the *y*-axis, by +30°.

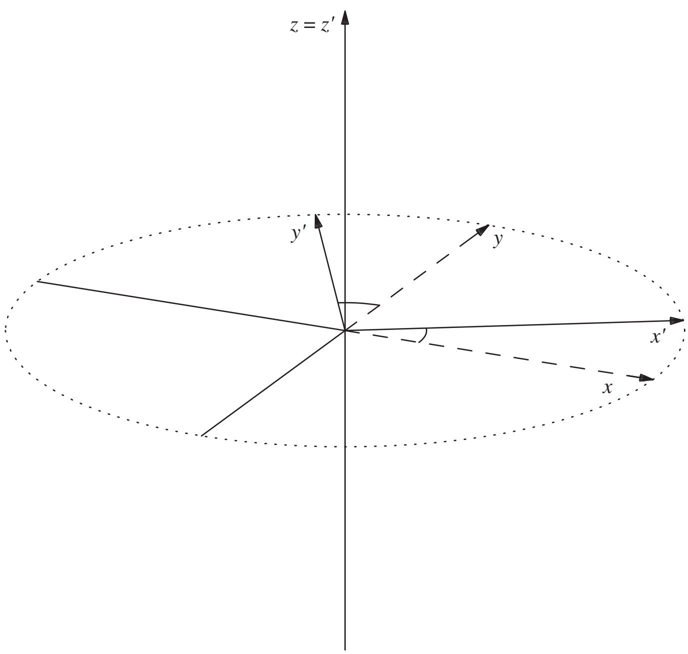

Figure 1.4 Rotation about the *z*-axis, by +30°.

of +30° about each of the three axes. The dotted circle in each of these figures represents the plane in which the rotation takes place—in other words, the plane which is perpendicular to the axis of rotation.

- A positive rotation about the x-axis moves the y-axis toward $+z$ and the z-axis toward $-y$.
- A positive rotation about the y-axis moves the z-axis toward $+x$ and the x-axis toward $-z$.
- A positive rotation about the z-axis moves the x-axis toward $+y$ and the y-axis toward $-x$.

If a vector $\mathbf{r}^A$ has rectangular coordinates that are expressed in some coordinate system A, the rotation matrix T_A^B will transform the coordinates into those of some other coordinate system B by

$$\mathbf{r}^B = \mathsf{T}_A^B\, \mathbf{r}^A. \tag{1.2}$$

The physical direction of the vector does not change under this operation. Its rectangular components, however, will in general change as a result of the change of coordinate system. The length of the vector is also unchanged, as the determinant of a rotation matrix is 1.

Rotation matrices may be strung together, one multiplying the next, to produce a new matrix which will also be a rotation. Three elementary rotations suffice to generate any arbitrary rotation. Matrix multiplication is associative but not commutative, and so the order in which the rotations are carried out is important. Figures 1.5–1.7 provide an example of a “3–1–3” sequence of rotations. The first rotation establishes the line of intersection between the initial and final x–y planes, the second gives the dihedral angle between these two planes (thereby establishing the final z-axis), and the third locates the final x- and y-axes. For example, if the final plane is the plane of an orbit, the three angles would be, in order, the longitude or right ascension[1] of the ascending node, the inclination, and the argument of pericenter.

Another rotation scheme in common use is 1–2–3, or some permutation thereof. This approach is particularly appropriate if the three rotation angles are very small, as is often the case for camera misalignment angles. If so, then the three rotations very nearly commute, and one can solve easily and simultaneously for the three angles.

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1 These two terms are not synonymous. The *right ascension* of the ascending node is used when the fundamental plane is the earth’s equator; the *longitude* of the ascending node is used not only when the fundamental plane is the ecliptic (as for planetary ephemeris theory) but also in general when the central body is not the earth or sun.

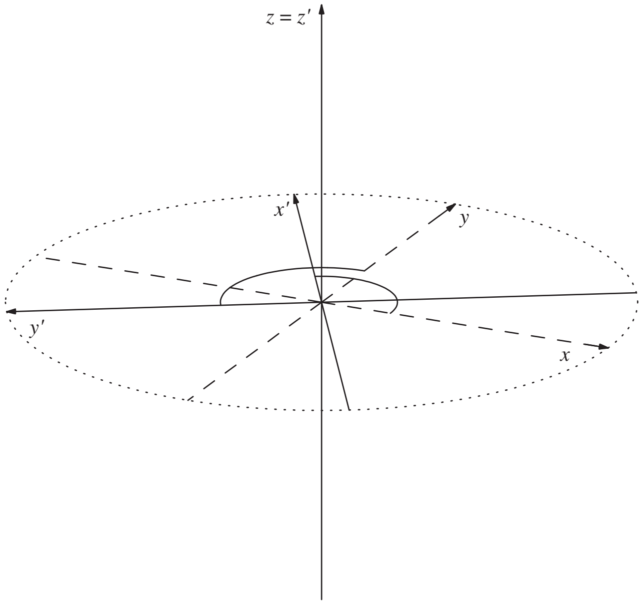

Figure 1.5 First rotation in a 3–1–3 sequence, by 120° about the z-axis. The dashed lines represent the original x- and y-axes.

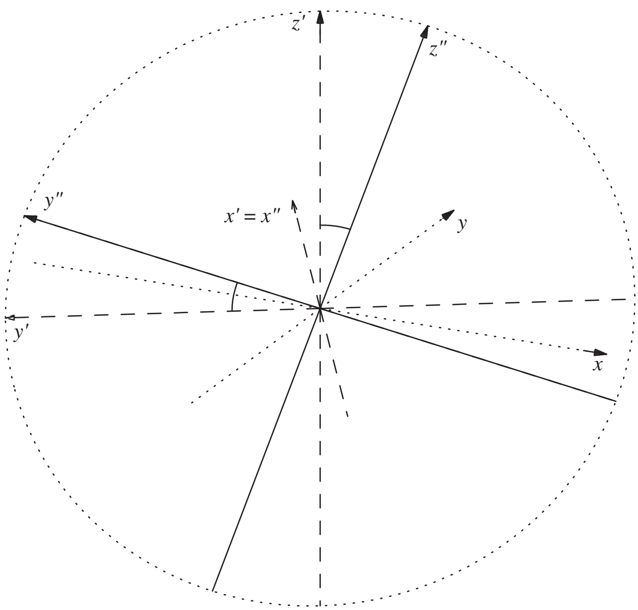

Figure 1.6 Second rotation in a 3–1–3 sequence by 20°, about the new x-axis. The original x- and y-axes now appear as dotted lines; the original z-axis and the y'-axis (i.e. after the first rotation) are dashed lines. Note that as the x-axis points into the page, a positive rotation appears to be clockwise from the reader's point of view.

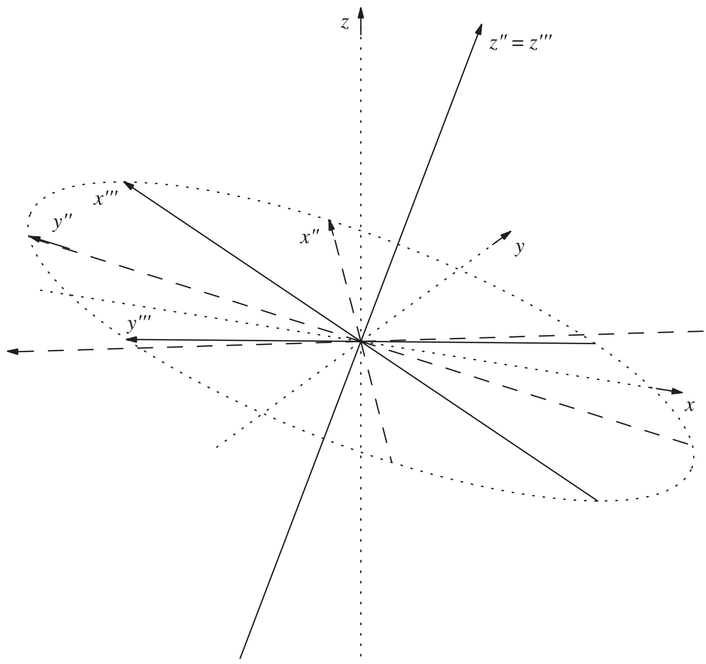

Figure 1.7 Third rotation in a 3–1–3 sequence, by 40° about the final z-axis.

1.5 Left-handed Coordinate Systems

Although the engineering world tends to use only right-handed coordinate systems, astronomers use three left-handed coordinate systems. Two are used primarily at observatories, but the third, treating planetary latitude and longitude, is definitely of concern to optical navigators. These three coordinate systems are:

1. *Topocentric azimuth and altitude.* The azimuth of a celestial object is measured from zero at due north, positive eastward. The altitude of a celestial object is its angular distance above the local horizon. (Some people incorrectly call this the "elevation," but for astronomers the term *elevation* is the observer's distance above sea level.) These two definitions imply that in the topocentric coordinate system, the x- and y-axes are horizontal, with $+x$ pointing north and $+y$ pointing east. The $+z$-axis of course must point to the zenith. This convention produces a left-handed system.
2. *Topocentric hour angle and declination.* The hour angle measures how far an object lies east or west of the meridian. As the earth rotates eastward, stars and other celestial objects appear to move westward. (It's not only the sun that rises

in the east and sets in the west.) As angles traditionally increase with time, the hour angle must be negative for objects to the east of the meridian and positive for those to the west. The corresponding coordinate system must therefore have the $+x$-axis pointing to the intersection of the celestial equator and the meridian, the y-axis pointing to the west point on the horizon (azimuth 270°) and the $+z$-axis pointing to the celestial north pole. This is therefore a left-handed system.

3. *Latitude and longitude for planets in direct rotation.* The International Astronomical Union has decreed that "the north pole is that pole of rotation that lies on the north side of the invariable plane of the Solar System" (Archinal *et al.* 2010) and further that the longitude of the "central meridian" of a planet should *increase with time*, as seen by an external observer. As these planets rotate from west to east, like the Earth, the longitude must accordingly increase to the west. Therefore the $+z$-axis points to the planet's north pole, the $+x$-axis lies at the intersection of the equator and prime meridian (however defined), and the $+y$-axis must point to 90° *west* longitude. Again, this is a left-handed system. Note, however, that Venus and Uranus, in retrograde rotation, will have a right-handed system. Note also that dwarf planets and minor planets are not covered by this definition; rather, their coordinate systems are always right-handed, even if (as for Pluto) their rotational angular momentum vector points south of the invariable plane.