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# Chapter 1 Looking at Numbers Scientifically



Like any other kind of scientist, a chemist tests hypotheses by doing experiments. Better tests require more reliable measurements, and better measurements are those that have more accuracy and precision. Accurate and precise calculations are essential to successful experiments, so a large chunk of chemistry centers on ways to report and describe measurements.

How do chemists report their precious measurements? What's the difference between accuracy and precision? And how do chemists do math with measurements? These questions may not keep you awake at night, but knowing the answers to them will keep you from making mistakes in chemistry.

## Using Exponential and Scientific Notation to Report Measurements

Because chemistry concerns itself with ridiculously tiny things like atoms and molecules, chemists often find themselves dealing with extraordinarily small or extraordinarily large numbers. Numbers describing the distance between two atoms joined by a bond, for example, run in the ten-billionths of a meter. Numbers describing how many water molecules populate a drop of water run into the trillions of trillions.

To make working with such extreme numbers easier, chemists turn to scientific notation, which is a special kind of exponential notation. In *exponential notation,* a number is represented as a value raised to a power of 10. The decimal point can be located anywhere within the number as long as the power of 10 is correct.

Suppose that you have an object that's 0.00125 meters in length. Express it in a variety of exponential forms:

$$
\begin{aligned}
0.00125\ \text{m} = {} & 0.0125\times10^{-1}\ \text{m, or}\\
& 0.125\times10^{-2}\ \text{m, or}\\
& 1.25\times10^{-3}\ \text{m, or}\\
& 12.5\times10^{-4}\ \text{m, and so on}
\end{aligned}
$$

All these forms are mathematically correct as numbers expressed in exponential notation. But in scientific notation the decimal point is placed so that only one digit other than zero is to the left of the decimal point. In the preceding example, the number expressed in scientific notation is $1.25\times10^{-3}$ m. Most scientists express numbers in scientific notation.

In scientific notation, every number is written as the product of two numbers, a coefficient and a power of 10. In plain old exponential notation, a coefficient can be any value of a number multiplied by a power with a base of 10 (such as $10^4$). But scientists have rules for coefficients in scientific notation. In *scientific notation,* the coefficient is always at least 1 and always less than 10. For example, the coefficient could be 7, 3.48, or 6.0001.

TIP

To convert a very large or very small number to scientific notation, move the decimal point so it falls between the first and second digits. Count how many places you moved the decimal point to the right or left, and that's the power of 10. If you moved the decimal point to the left, the exponent on the 10 is positive; to the right, it's negative. (Here's another easy way to remember the sign on the exponent: If the initial number value is greater than 1, the exponent will be positive; if the initial number value is between 0 and 1, the exponent will be negative.)

To convert a number written in scientific notation back into decimal form, just multiply the coefficient by the accompanying power of 10.

In many cases, chemistry teachers refer to powers of 10 using scientific notation instead of their decimal form. With that in mind, here's a quick chart showing you the most common powers of 10 used in chemistry, along with their corresponding scientific notation.

EXAMPLE

**Q.** Convert 47,000 to scientific notation.

**A.** $47{,}000 = 4.7\times10^4$. First, imagine the number as a decimal:

47,000.

Next, move the decimal point so it comes between the first two digits:

4.7000

Then count how many places to the left you moved the decimal (four, in this case) and write that as a power of 10: $4.7\times10^4$.

**Q.** Convert 0.007345 to scientific notation.

**A.** $0.007345 = 7.345 \times 10^{-3}$. First, put the decimal point between the first two nonzero digits:

7.345

Then count how many places to the right you moved the decimal (three, in this case) and write that as a power of 10: $0.007345 = 7.345 \times 10^{-3}$.

**YOUR TURN**

1. Convert 200,000 to scientific notation.

2. Convert 80,736 to scientific notation.

3. Convert 0.00002 to scientific notation.

4. Convert $6.903 \times 10^{2}$ from scientific notation to decimal form.

# Multiplying and Dividing in Scientific Notation

A major benefit of presenting numbers in scientific notation is that it simplifies common arithmetic operations. The simplifying abilities of scientific notation are most evident in multiplication and division. (As we note in the next section, addition and subtraction benefit from exponential notation but not necessarily from strict scientific notation.)

REMEMBER

To multiply two numbers written in scientific notation, multiply the coefficients and then add the exponents. To divide two numbers, simply divide the coefficients and then subtract the exponent of the *denominator* (the bottom number) from the exponent of the *numerator* (the top number).

EXAMPLE

**Q.** Multiply using the shortcuts of scientific notation: $\left(1.4\times10^{2}\right)\times\left(2.0\times10^{-5}\right)$.

**A.** $2.8\times10^{-3}$. First, multiply the coefficients:

$1.4\times2.0=2.8$

Next, add the exponents of the powers of 10:

$10^{2}\times10^{-5}=10^{2+(-5)}=10^{-3}$

Finally, join your new coefficient to your new power of 10:

$2.8\times10^{-3}$

**Q.** Divide using the shortcuts of scientific notation: $\frac{3.6\times10^{-3}}{1.8\times10^{4}}$.

**A.** $2.0\times10^{-7}$. First, divide the coefficients:

$\frac{3.6}{1.8}=2.0$

Next, subtract the exponent in the denominator from the exponent in the numerator:

$\frac{10^{-3}}{10^{4}}=10^{-3-4}=10^{-7}$

Then join your new coefficient to your new power of 10:

$2.0\times10^{-7}$

YOUR TURN

5 Multiply $\left(2.2\times10^{9}\right)\times\left(5.0\times10^{-4}\right)$.

6 Divide $\frac{9.3\times10^{-5}}{3.1\times10^{2}}$.

| | |
|---|---|
| **7** Using scientific notation, multiply $52 \times 0.035$. | **8** Using scientific notation, divide $\frac{0.00809}{20.3}$. |

# Using Scientific Notation to Add and Subtract

Addition or subtraction gets easier when you express your numbers as coefficients of identical powers of 10. To wrestle your numbers into this form, you may need to use coefficients less than 1 or greater than 10. So scientific notation is a bit too strict for addition and subtraction, but exponential notation still serves you well.

REMEMBER

To add two numbers easily by using exponential notation, first express each number as a coefficient and a power of 10, making sure that 10 has the same exponent in each number. Then add the coefficients. To subtract numbers in exponential notation, follow the same steps but subtract the coefficients.

EXAMPLE

**Q.** Use exponential notation to add these numbers: $3{,}710 + \left(2.4 \times 10^2\right)$.

**A.** $39.5 \times 10^2$. First, write both numbers with the same power of 10:

$37.1 \times 10^2$ and $2.4 \times 10^2$

Next, add the coefficients:

$37.1 + 2.4 = 39.5$

Finally, join your new coefficient to the shared power of 10:

$39.5 \times 10^2$

**Q.** Use exponential notation to subtract: $0.0743 - 0.0022$.

**A.** $7.21 \times 10^{-2}$. First, convert both numbers to the same power of 10. We've chosen $10^{-2}$:

$7.43 \times 10^{-2}$ and $0.22 \times 10^{-2}$

Next, subtract the coefficients:

$7.43 - 0.22 = 7.21$

Then join your new coefficient to the shared power of 10:

$7.21 \times 10^{-2}$

YOUR TURN

9 Add $\left(398 \times 10^{-6}\right) + \left(147 \times 10^{-6}\right)$.

10 Subtract $\left(7.685 \times 10^{5}\right) - \left(1.283 \times 10^{5}\right)$.

11 Use exponential notation to add $0.00206 + 0.0381$.

12 Use exponential notation to subtract $9{,}352 - 431$.

# Distinguishing between Accuracy and Precision

Whenever you make measurements, you must consider two factors, accuracy and precision. *Accuracy* is how well the measurement agrees with the accepted or true value. *Precision* is how well a set of measurements agree with each other. In chemistry, measurements should be *reproducible;* that is, they must have a high degree of precision. Most of the time chemists make several measurements and average them. The closer these measurements are to each other, the more confidence chemists have in their measurements. Of course, you also want the measurements to be accurate, very close to the correct answer. However, many times you don't know beforehand anything about the correct answer; therefore, you have to rely on precision as your guide.

Suppose you ask four lab students to make three measurements of the length of the same object. Their data follows.

| | Student 1 | Student 2 | Student 3 | Student 4 |
|---|---|---|---|---|
| Trial 1 | 27.77 cm | 27.30 cm | 27.55 cm | 27.30 cm |
| Trial 2 | 27.30 cm | 27.60 cm | 27.55 cm | 27.29 cm |
| Trial 3 | 27.56 cm | 27.97 cm | 27.53 cm | 27.31 cm |
| Average | 27.54 cm | 27.62 cm | 27.54 cm | 27.30 cm |

The accepted length of the object is 27.55 cm. Which of these students deserves the higher lab grade? Both students 1 and 3 have values close to the accepted value, if you just consider their average values. (The average, found by summing the individual measurements and dividing by the number of measurements, is normally considered to be more useful than any individual value.) Both students 1 and 3 have made *accurate* determinations of the length of the object. The average values determined by students 2 and 4 are not very close to the accepted value, so their values are not considered to be accurate.

However, if you examine the individual determinations for students 1 and 3, you notice a great deal of variation in the measurements of student 1. The measurements don't agree with each other very well; their precision is low even though the accuracy is good. The measurements by student 3 agree well with each other; both precision and accuracy are good. Student 3 deserves a higher grade than student 1.

Neither student 2 nor student 4 has average values close to the accepted value; neither determination is very accurate. However, student 4 has values that agree closely with each other; the precision is good. This student probably had a consistent error in his or her measuring technique. Student 2 had neither good accuracy nor precision. The accuracy and precision of the four students is summarized below.

| | Accuracy | Precision |
|---|---|---|
| Student 1 | High | Low |
| Student 2 | Low | Low |
| Student 3 | High | High |
| Student 4 | Low | High |

Usually, measurements with a high degree of precision are also somewhat accurate. Because the scientists or students don't know the accepted value beforehand, they strive for high precision and hope that the accuracy will also be high. This was not the case for student 4.

So remember, accuracy and precision are not the same thing:

- **Accuracy:** Accuracy describes how closely a measurement approaches an actual, true value.
- **Precision:** Precision, which we discuss more in the next section, describes how close repeated measurements are to one another, regardless of how close those measurements are to the actual value. The bigger the difference between the largest and smallest values of a repeated measurement, the less precision you have.

The two most common measurements related to accuracy are *error* and *percent error:*

- **Error:** Error measures accuracy, the difference between a measured value and the actual value:

  $$\text{Actual value} - \text{Measured value} = \text{Error}$$

- **Percent error:** Percent error compares error to the size of the thing being measured:

  $$\frac{|\text{Error}|}{\text{Actual value}} = \text{Fraction error}$$

  $$\text{Fraction error} \times 100 = \text{Percent error}$$

  Being off by 1 meter isn't such a big deal when measuring the altitude of a mountain, but it's a shameful amount of error when measuring the height of an individual mountain climber.

If you want a simpler all-in-one formula to help you remember percent error, here is all of the above put into one simple-to-use formula:

$$\frac{|\text{Actual value} - \text{Measured value}|}{\text{Actual value}} \times 100 = \text{Percent error}$$

EXAMPLE

**Q.** A police officer uses a radar gun to clock a passing Ferrari at 131 miles per hour (mph). The Ferrari was really speeding at 127 mph. Calculate the error in the officer's measurement.

**A.** –4 mph. First, determine which value is the actual value and which is the measured value:

- Actual value = 127 mph
- Measured value = 131 mph

Then calculate the error by subtracting the measured value from the actual value:

$$\text{Error} = 127 \text{ mph} - 131 \text{ mph} = -4 \text{ mph}$$

**Q.** Calculate the percent error in the officer's measurement of the Ferrari's speed.

**A.** 3.15%. First, divide the error's absolute value (the size, as a positive number) by the actual value:

$$\frac{|-4 \text{ mph}|}{127 \text{ mph}} = \frac{4 \text{ mph}}{127 \text{ mph}} = 0.0315$$

Next, multiply the result by 100 to obtain the percent error:

$$\text{Percent error} = 0.0315 \times 100 = 3.15\%$$

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YOUR TURN

**13** Two people, Reginald and Dagmar, measure their weight in the morning by using typical bathroom scales, instruments that are famously unreliable. The scale reports that Reginald weighs 237 pounds, though he actually weighs 256 pounds. Dagmar's scale reports her weight as 117 pounds, though she really weighs 129 pounds. Whose measurement incurred the greater error? Who incurred a greater percent error?

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**14** Two jewelers were asked to measure the mass of a gold nugget. The true mass of the nugget is 0.856 grams (g). Each jeweler took three measurements. The average of the three measurements was reported as the "official" measurement with the following results:

- **Jeweler A:** 0.863 g, 0.869 g, 0.859 g
- **Jeweler B:** 0.875 g, 0.834 g, 0.858 g

Which jeweler's official measurement was more accurate? Which jeweler's measurements were more precise? In each case, what was the error and percent error in the official measurement?

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# Identifying Significant Figures

*Significant figures* are the number of digits that you report in the final answer of the mathematical problem you're calculating. If we told you that one student determined the density of an object to be 2.3 g/mL and another student figured the density of the same object to be 2.272589 g/mL, we bet that you'd believe that the second figure was the result of a more accurate experiment. You may be right, but then again, you may be wrong. You have no way of knowing whether the second student's experiment was more accurate unless both students obeyed the significant figure convention.

If we ask you to count the number of automobiles that you and your family own, you can do it without any guesswork involved. Your answer may be 0, 1, 2, or 10, but you know exactly how many autos you have. Those numbers are what are called *counted numbers.* If we ask you how many inches are in a foot, your answer will be 12. That number is an *exact number* — it's exact by definition. Another exact number is the number of centimeters per inch, 2.54. In both exact and counted numbers, you have no doubt what the answer is. When you work with these types of numbers, you don't have to worry about significant figures.

Now suppose that we ask you and four of your friends to individually measure the length of an object as accurately as you possibly can with a meter stick. You then report the results of your measurements: 2.67 meters, 2.65 meters, 2.68 meters, 2.61 meters, and 2.63 meters. Which of you is right? You are all within experimental error. These measurements are *measured numbers,* and measured values always have some error associated with them. You determine the number of significant figures in your answer by your least reliable measured number.

REMEMBER

When you report a measurement, you should include digits only if you're really confident about their values. Including a lot of digits in a measurement means something — it means that you really know what you're talking about — so we call the included digits *significant figures.* The more significant figures (sig figs) in a measurement, the more accurate that measurement must be. The last significant figure in a measurement is the only figure that includes any uncertainty, because it's an estimate. Here are the rules for deciding what is and what isn't a significant figure:

- **Any nonzero digit is significant.** So 6.42 contains three significant figures.
- **Zeros sandwiched between nonzero digits are significant.** So 3.07 contains three significant figures.
- **Zeros on the left side of the first nonzero digit are *not* significant.** So 0.0642 and 0.00307 each contain three significant figures.
- **One or more *final zeros* (zeros that end the measurement) used after the decimal point are significant.** So 1.760 has four significant figures, and 1.7600 has five significant figures. The number 0.0001200 has only four significant figures because the first zeros are not final.
- **When a number has no decimal point, any zeros after the last nonzero digit *may* or *may not* be significant.** So in a measurement reported as 1,370, you can't be certain whether the 0 is a certain value or is merely a placeholder.

Be a good chemist. Report your measurements in scientific notation to avoid such annoying ambiguities. (See the earlier section "Using Exponential and Scientific Notation to Report Measurements" for details on scientific notation.)

- **If a number is already written in scientific notation, then all the digits in the coefficient are significant.** So the number $3.5200 \times 10^{-6}$ has five significant figures due to the five digits in the coefficient.

Numbers from counting (for example, 1 kangaroo, 2 kangaroos, 3 kangaroos) or from defined quantities (say, 60 seconds per 1 minute) are understood to have an unlimited number of significant figures. In other words, these values are completely certain.

REMEMBER

The number of significant figures you use in a reported measurement should be consistent with your certainty about that measurement. If you know your speedometer is routinely off by 5 miles per hour, then you have no business protesting to a policeman that you were going only 63.2 mph in a 60 mph zone.

EXAMPLE

**Q.** How many significant figures are in the following three measurements?

**(a)** 20,175 yards

**(b)** 1.75 yards

**(c)** 1.750 yards

**A.** a) Five, b) three, and c) four significant figures. In the first measurement, all digits are nonzero, except for a 0 that's sandwiched between nonzero digits, which counts as significant. The coefficient in the second measurement contains only nonzero digits, so all three digits are significant. The coefficient in the third measurement contains a 0, but that 0 is the final digit and to the right of the decimal point, so it's significant.

YOUR TURN

**15** Identify the number of significant figures in each measurement:

**(a)** $76.093 \times 10^{-2}$ meters

**(b)** 0.000769 meters

**(c)** 769.3 meters

**16** In chemistry, the potential error associated with a measurement is often reported alongside the measurement, as in $793.4 \pm 0.2$ grams. This report indicates that all digits are certain except the last, which may be off by as much as 0.2 grams in either direction. What, then, is wrong with the following reported measurements?

**(a)** $893.7 \pm 1$ grams

**(b)** $342 \pm 0.01$ grams

# Doing Arithmetic with Significant Figures

Doing chemistry means making a lot of measurements. The point of spending a pile of money on cutting-edge instruments is to make really good, really precise measurements. After you've got yourself some measurements, you roll up your sleeves, hike up your pants, and do some math.

REMEMBER

When doing math in chemistry, you need to follow some rules to make sure that your sums, differences, products, and quotients honestly reflect the amount of precision present in the original measurements. You can be honest (and avoid the skeptical jeers of surly chemists) by taking things one calculation at a time, following a few simple rules. One rule applies to addition and subtraction, and another rule applies to multiplication and division.

## Addition and subtraction

In addition and subtraction, round the sum or difference to the same number of decimal places as the measurement with the fewest decimal places. For example, suppose you're adding the following amounts:

$$2.675\text{ g} + 3.25\text{ g} + 8.872\text{ g} + 4.5675\text{ g}$$

Your calculator will show 19.3645, but you round off to the hundredths place based on the 3.25, which has the fewest number of decimal places. You round the figure off to 19.36. (See the later section "Rounding off numbers" for the rounding rules.)

## Multiplication and division

In multiplication and division, you report the answer to the same number of significant figures as the number that has the *fewest* significant figures. Remember that counted and exact numbers don't count in the consideration of significant numbers. For example, suppose that you are calculating the density in grams per liter of an object that weighs 25.3573 (six sig figs) grams and has a volume of 10.50 milliliters (four sig figs). The setup looks like this:

$$\frac{25.3573\text{ g}}{10.5\ \cancel{\text{mL}}} \times \frac{1{,}000\ \cancel{\text{mL}}}{1\text{ L}}$$

Your calculator will read 2,414.981000. You have six significant figures in the first number and four in the second number (the 1,000 mL/L doesn't count because it's an exact conversion). You should have four significant figures in your final answer, so round the answer off to 2,415 g/L.

Notice the difference between the two rules. When you add or subtract, you assign significant figures in the answer based on the number of decimal places in each original measurement. When you multiply or divide, you assign significant figures in the answer based on the smallest number of significant figures from your original set of measurements.

TIP

Caught up in the breathless drama of arithmetic, you may sometimes perform multistep calculations that include addition, subtraction, multiplication, and division, all in one go. No problem. Follow the normal order of operations, doing multiplication and division first, followed by addition and subtraction. At each step, follow the simple significant-figure rules, and then move on to the next step.

## Rounding off numbers

Sometimes you have to round numbers at the end of a measurement to account for significant figures. Here are a couple of very simple rules to follow and remember:

- **Rule 1:** If the first number to be dropped is 5 or greater, drop it and all the numbers that follow it, and increase the last retained number by 1.

  For example, suppose that you want to round off 237.768 to four significant figures. You drop the 6 and the 8. The 6, the first dropped number, is greater than 5, so you increase the retained 7 to 8. Your final answer is 237.8.

- **Rule 2:** If the first number to be dropped is less than 5, drop it and all the numbers that follow it, and leave the last retained number unchanged.

  If you're rounding 2.35427 to three significant figures, you drop the 4, the 2, and the 7. The first number to be dropped is 4, which is less than 5. The 5, the last retained number, stays the same. So you report your answer as 2.35.

EXAMPLE

**Q.** Express the following sum with the proper number of significant figures:

$$35.7 \text{ miles} + 634.38 \text{ miles} + 0.97 \text{ miles} = ?$$

**A.** 671.1 miles. Adding the three values yields a raw sum of 671.05 miles. However, the 35.7 miles measurement extends only to the tenths place. Therefore, you round the answer to the tenths place, from 671.05 to 671.1 miles.

**Q.** Express the following product with the proper number of significant figures:

$$27 \text{ feet} \times 13.45 \text{ feet} = ?$$

**A.** $3.6 \times 10^2 \text{ feet}^2$. Of the two measurements, one has two significant figures (27 feet) and the other has four significant figures (13.45 feet). The answer is therefore limited to two significant figures. You need to round the raw product, 363.15 feet$^2$. You could write 360 feet$^2$, but doing so may imply that the final 0 is significant and not just a placeholder. For clarity, express the product in scientific notation, as $3.6 \times 10^2$ feet$^2$.

YOUR TURN

**17** Express the answer to this calculation using the appropriate number of significant figures:

$$127.379 \text{ seconds} - 13.14 \text{ seconds} + \left(1.2 \times 10^{-1} \text{ seconds}\right) = ?$$

**18** Express the answer to this calculation using the appropriate number of significant figures:

$$345.6 \text{ feet} \times \left(\frac{12 \text{ inches}}{1 \text{ foot}}\right) = ?$$

**19** Report the difference using the appropriate number of significant figures:

$$\left(3.7 \times 10^{-4} \text{ minutes}\right) - 0.009 \text{ minutes} = ?$$

**20** Express the answer to this multistep calculation using the appropriate number of significant figures:

$$\frac{87.95 \text{ feet} \times 0.277 \text{ feet} + 5.02 \text{ feet} - 1.348 \text{ feet}}{10.0 \text{ feet}} = ?$$

# Qualitative and Quantitative Observations

Observations are an essential part of any scientific discipline. In chemistry you are regularly required to make observations about experiments that you do in the lab. In most cases these observations are going to be made in the form of gathering data or taking measurements based on your experiment. Usually when you see the term *data* you likely assume a number is going to be the result of that data, but data in a chemistry lab can take the form of a numerical measurement or a descriptive observation. Both are completely valid and simply depend on what you are asked to do. It is important that you understand the distinction between these two types of observations.

- **Qualitative Data**: If there is data gathered during an experiment that is based on your observations this data is called qualitative data. This goes for anything that you might be observing during an experiment or anything else you might record based on what you see, feel, or hear. In short, qualitative data does not involve numbers in any way. To help you remember this, simply think of qualitative data as measuring the "quality" of something. It is a subjective observation that you make based on your observations.
- **Quantitative Data**: Any type of data that you gather through numerical measurement is considered quantitative. These measurements are exact and are not subjective in any way; they are absolute and based on a measuring device. In short, anything that is numerically based would be considered quantitative data. A helpful way to remember this is to think of the idea of "quantity" in quantitative. If it has a number in it, it is quantitative data.

EXAMPLE

**Q.** Identify the qualitative pieces of data described in the following statement:

You mix 50 ml of two solutions together in a test tube. Upon mixing the solution changes color and you feel the test tube getting warmer.

**A.** Your observation of a color change and heat being given off by the reaction are examples of qualitative data. The 50 ml is not qualitative in nature, it is quantitative data.

**Q.** Identify the quantitative observations that can be made from the following statement:

You take the mass of a sample of iron and find that it is 56 grams. The iron is shinny and feels smooth to the touch.

**A.** 56 grams of iron is quantitative data. The shiny appearance of iron and the way it feels are qualitative.

YOUR TURN

21 Identify the quantitative and qualitative data described in the passage below:

You are doing an experiment where you react lead nitrate with potassium iodide. You carefully measure out 1.5 grams of lead nitrate and 3.5 grams of potassium iodide and record the mass of each. You add 100 ml of water to a beaker and then add both substances into the beaker and mix them together. Upon mixing them the solution turns a bright yellow color.

# Practice Questions Answers and Explanations

1 **$2\times10^5$.** Move the decimal point immediately after the 2 to create a coefficient between 1 and 10. Because you're moving the decimal point five places to the left, multiply the coefficient, 2, by the power $10^5$.

2 **$8.0736\times10^4$.** Move the decimal point immediately after the 8 to create a coefficient between 1 and 10. You're moving the decimal point four places to the left, so multiply the coefficient, 8.0736, by the power $10^4$.

3 **$2\times10^{-5}$.** Move the decimal point immediately after the 2 to create a coefficient between 1 and 10. You're moving the decimal point five places to the right, so multiply the coefficient, 2, by the power $10^{-5}$.

4 **690.3.** You need to understand scientific notation to change the number back to regular decimal form. Because $10^2$ equals 100, multiply the coefficient, 6.903, by 100. This moves the decimal point two places to the right.

5 **$1.1\times10^6$.** First, multiply the coefficients: $2.2\times5.0=11$. Then multiply the powers of 10 by adding the exponents: $10^9\times10^{-4}=10^{9+(-4)}=10^5$. The raw calculation yields $11\times10^5$, which converts to the given answer when you express it in scientific notation.

6 **$3.0\times10^{-7}$.** The ease of math with scientific notation shines through in this problem. Dividing the coefficients yields a coefficient quotient of $9.3/3.1=3.0$, and dividing the powers of 10 (by subtracting their exponents) yields a quotient of $10^{-5}/10^2=10^{-5-2}=10^{-7}$. Marrying the two quotients produces the given answer, already in scientific notation.

7 **1.82.** First, convert each number to scientific notation: $5.2\times10^1$ and $3.5\times10^{-2}$. Next, multiply the coefficients: $5.2\times3.5=18.2$. Then add the exponents on the powers of 10: $10^{1+(-2)}=10^{-1}$. Finally, join the new coefficient with the new power: $18.2\times10^{-1}$. Expressed in scientific notation, this answer is $1.82\times10^0=1.82$. (***Note:*** Looking back at the original numbers, you see that both factors have only two significant figures; therefore, you should round your answer to match that number of sig figs, making it 1.8. See the previous sections "Identifying Significant Figures" and "Doing Arithmetic with Significant Figures" for details.)

8 **$3.99\times10^{-4}$.** First, convert each number to scientific notation: $8.09\times10^{-3}$ and $2.03\times10^1$. Then divide the coefficients: $8.09/2.03=3.99$. Next, subtract the exponent in the denominator from the exponent in the numerator to get the new power of 10: $10^{-3-1}=10^{-4}$. Join the new coefficient with the new power: $3.99\times10^{-4}$. Finally, express gratitude that the answer is already conveniently expressed in scientific notation.

9 **$545\times10^{-6}$.** Because the numbers are each already expressed with identical powers of 10, you can simply add the coefficients: $398+147=545$. Then join the new coefficient with the original power of 10.

10 **$6.402\times10^5$.** Because the numbers are each expressed with the same power of 10, you can simply subtract the coefficients: $7.685-1.283=6.402$. Then join the new coefficient with the original power of 10.

11 **$40.16\times10^{-3}$ (or an equivalent expression).** First, convert the numbers so they each use the same power of 10: $2.06\times10^{-3}$ and $38.1\times10^{-3}$. Here, we use $10^{-3}$, but you can use a different power as long as the power is the same for each number. Next, add the coefficients: $2.06+38.1=40.16$. Finally, join the new coefficient with the shared power of 10.

12 **$89.21 \times 10^2$ (or an equivalent expression).** First, convert the numbers so each uses the same power of 10: $93.52 \times 10^2$ and $4.31 \times 10^2$. Here, we've picked $10^2$, but any power is fine as long as the two numbers have the same power. Then subtract the coefficients: $93.52 - 4.31 = 89.21$. Finally, join the new coefficient with the shared power of 10.

13 **Reginald's measurement incurred the greater magnitude of error, and Dagmar's measurement incurred the greater percent error.** Reginald's scale reported with an error of 256 pounds − 237 pounds = 19 pounds, and Dagmar's scale reported with an error of 129 pounds − 117 pounds = 12 pounds. Comparing the *magnitudes* of error, you see that 19 pounds is greater than 12 pounds. However, Reginald's measurement had a percent error of $(19 \text{ pounds} / 256 \text{ pounds}) \times 100 = 7.42\%$, while Dagmar's measurement had a percent error of $(12 \text{ pounds} / 129 \text{ pounds}) \times 100 = 9.30\%$.

14 Jeweler A's official average measurement was 0.864 grams, and Jeweler B's official measurement was 0.856 grams. You determine these averages by adding up each jeweler's measurements and then dividing by the total number of measurements, in this case three. Based on these averages, Jeweler B's official measurement is more accurate because it's closer to the actual value of 0.856 grams.

However, Jeweler A's measurements were more precise because the differences between A's measurements were much smaller than the differences between B's measurements. Despite the fact that Jeweler B's average measurement was closer to the actual value, the *range* of his measurements (that is, the difference between the largest and the smallest measurements) was 0.041 grams ($0.875 \text{ g} - 0.834 \text{ g} = 0.041 \text{ g}$). The range of Jeweler A's measurements was 0.010 grams ($0.869 \text{ g} - 0.859 \text{ g} = 0.010 \text{ g}$).

This example shows how low-precision measurements can yield highly accurate results through averaging of repeated measurements. In the case of Jeweler A, the error in the official measurement was $0.864 \text{ g} - 0.856 \text{ g} = 0.008 \text{ g}$. The corresponding percent error was $(0.008 \text{ g} / 0.856 \text{ g}) \times 100 = 0.9\%$. In the case of Jeweler B, the error in the official measurement was $0.856 \text{ g} - 0.856 \text{ g} = 0.000 \text{ g}$. Accordingly, the percent error was 0%.

15 The correct number of significant figures is as follows for each measurement: **a) 5, b) 3,** and **c) 4.**

16 The number of significant figures in a reported measurement should be consistent with your certainty about that measurement.

a. "$893.7 \pm 1$ grams" is an improperly reported measurement because the reported value, 893.7, suggests that the measurement is certain to within a few tenths of a gram. The reported error is known to be greater, at ±1 gram. The measurement should be reported as "$894 \pm 1$ grams."

b. "$342 \pm 0.01$ grams" is improperly reported because the reported value, 342, gives the impression that the measurement becomes uncertain at the level of grams. The reported error makes clear that uncertainty creeps into the measurement only at the level of hundredths of a gram. The measurement should be reported as "$342.00 \pm 0.01$ grams."

17 **114.36 seconds.** The trick here is remembering to convert all measurements to the same power of 10 before comparing decimal places for significant figures. Doing so reveals that $1.2 \times 10^{-1}$ seconds goes to the hundredths of a second, despite the fact that the measurement contains only two significant figures. The raw calculation yields 114.359 seconds, which rounds properly to the hundredths place (taking significant figures into account) as 114.36 seconds, or $1.1436 \times 10^2$ seconds in scientific notation.

18 **$4.147 \times 10^3$ inches.** Here, you have to recall that defined quantities (1 foot is defined as 12 inches) have unlimited significant figures. So your calculation is limited only by the number of significant figures in the measurement 345.6 feet. When you multiply 345.6 feet by 12 inches per foot, the feet cancel, leaving units of inches:

$$(345.6\ \cancel{ft}) \times \left(\frac{12\text{ in.}}{1\ \cancel{ft}}\right) = 4{,}147.2\text{ in.}$$

The raw calculation yields 4,147.2 inches, which rounds properly to four significant figures as 4,147 inches, or $4.147 \times 10^3$ inches in scientific notation.

19 **–0.009 minutes.** Here, it helps to convert all measurements to the same power of 10 so you can more easily compare decimal places in order to assign the proper number of significant figures. Doing so reveals that $3.7 \times 10^{-4}$ minutes goes to the hundred-thousandths of a minute, and 0.009 minutes goes to the thousandths of a minute. The raw calculation yields –0.00863 minutes, which rounds properly to the thousandths place (taking significant figures into account) as –0.009 minutes, or $-9 \times 10^{-3}$ minutes in scientific notation.

20 **2.80 feet.** Following standard order of operations, you can do this problem in two main steps.

Following the rules of significant-figure math, the first step yields $24.4\text{ feet} + 5.02\text{ feet} - 1.348\text{ feet}$. Each product or quotient contains the same number of significant figures as the number in the calculation with the fewest number of significant figures.

After completing the first step, divide by 10.0 feet to finish the problem:

$$\frac{28.03\text{ ft}}{10.0\text{ ft}} = 2.803\text{ ft} = 2.80\text{ ft}$$

You write the answer with three sig figs because the measurement 10.0 feet contains three sig figs, which is the smallest available between the two numbers.

21 The masses of each compound, 1.5 grams of lead nitrate and 3.5 grams of potassium iodide, are both quantitative data. The volume of water added, 100 ml, is quantitative data. The fact that these are measured quantities involving numbers make them quantitative pieces of data. The observation that the solution turned a bright yellow color upon adding the substances to the beaker and mixing is qualitative data. This is due to it being an observation you are making rather than a measured quantity you are determining.

If you're ready to test your skills a bit more, take the following chapter quiz that incorporates all the chapter topics.

# Whaddya Know? Chapter 1 Quiz

Quiz time! Complete each problem to test your knowledge on the various topics covered in this chapter. You can then find the solutions and explanations in the next section.

1. Convert 56000 to scientific notation.

2. Convert 780 to scientific notation.

3. Convert 0.0032 to scientific notation

4. Convert 0.000000098 to scientific notation.

5. Solve the following: $(9.2 \times 10^{6})(7.5 \times 10^{-2})$

6. Solve the following: $\frac{(3.10 \times 10^{-4})}{(1.33 \times 10^{7})}$

7. Solve the following: $(2.5 \times 10^{3}) + (8.1 \times 10^{2})$

8. Solve the following: $(8.55 \times 10^{-2}) - (4.2 \times 10^{-3})$

9. The actual mass of a rock is 5.6 g. A student decides to find the mass of this rock 5 times.

   Upon completing this task the masses they recorded are: 5.6 g, 5.7 g, 5.3 g, 5.4 g, 5.8 g, 5.4 g, 5.6 g.

   Classify their measurement results as accurate, precise, both, or neither.

10. A student does an experiment and determines the boiling point of water to is 104°C. The actual/known value of water's boiling point is 100°C. What is the student's percent error?

11. Calculate the correct number of significant figures in the following numbers:

    **a.** 0.0005

    **b.** 1000.44

    **c.** 100

    **d.** 100.0

12. Solve the following problem and write the answer with the correct number of significant figures:

    $$456 \text{ cm} \times 2.0 \text{ cm}$$

13. Solve the following problem and write the answer with the correct number of significant figures:

    $$83000 \text{ m} \div 4.550 \text{ m}$$

# Answers to Chapter 1 Quiz

1 $5.6 \times 10^4$. You need to move the decimal point 4 places to the left to create a coefficient between 1 and 10. This means the exponent used will be positive.

2 $7.8 \times 10^2$. You need to move the decimal point 2 places to the left to create a coefficient between 1 and 10. This means the exponent used will be positive.

3 $3.2 \times 10^{-3}$. You need to move the decimal point 3 places to the right to create a coefficient between 1 and 10. This means the exponent used will be negative.

4 $9.8 \times 10^{-8}$. You need to move the decimal point 8 places to the right to create a coefficient between 1 and 10. This means the exponent used will be negative.

5 $6.9 \times 10^5$. To solve this problem you will do two steps. First multiply the coefficients: $9.2 \times 7.5$. This gives you 69. Next, add the exponents $6 + -2.0$. This gives you 4. Put it back together and you get $69 \times 10^4$. You then need to simplify that by moving the decimal one place over to the left, increasing the exponent by one, giving you $6.9 \times 10^5$ as the final answer.

6 $2.33 \times 10^{-11}$ To solve this problem you will do two steps. First divide the coefficients: $3.10 \div 1.33$. This gives you 2.33. Next, subtract the exponents $-4 - 7$. This gives you $-11$. Put it back together and you get $2.33 \times 10^{-11}$.

7 $3.31 \times 10^3$. To solve this problem you need to convert one of the numbers so both have the same exponent. Convert $8.1 \times 10^2$ to $.81 \times 10^3$ and then add $2.5 + .81$. This gives you 3.31. Once you have done this, add the $\times 10^3$ back in and you have your correct answer.

8 $8.13 \times 10^{-2}$. To solve this problem you need to convert one of the numbers so both have the same exponent. Convert $4.2 \times 10^{-3}$ to $0.42 \times 10^{-2}$ and then subtract $8.55 - 0.42$. This gives you 8.13. Once you have done this add the $\times 10^{-2}$ back in and you have your correct answer.

9 Accurate and precise. These results can be considered both accurate and precise. The measurements from each trial are closely grouped together. No single result is an obvious outlier from the data set. This makes it precise. Since the results are also close to actual value of the measurement, the data is considered to be accurate.

10 4% error. The actual value of the boiling point of water is given as 100°C and the measured value by the student is 104°C. Plug those numbers into the percent error formula as shown below to determine your answer.

$$\frac{|100°\text{C} - 104°\text{C}|}{100°\text{C}} \times 100 = 4\% \text{ error}$$

11
a. 1 significant figure. None of the zeros in the number are final zeroes so they are not significant. The 3 is the only significant figure in this number.

b. 6 significant figures. All of the numbers are significant in this measurement. The nonzero numbers are significant. The zeros are also significant as they are between other significant figures.

c. 1 significant figure. The 1 is the only significant figure. The 2 trailing/final zeros are not significant because even though they are final zeros they are not after a decimal point as well.

d. 4 significant figures. All figures are significant. The one is a number so it is significant. The zero after the decimal point is significant because it is a final zero after the decimal point. The 2 middle zeroes are significant because they are between 2 other significant figures.

12 910 cm. If you solve this problem you will find the exact answer when calculated is 912 cm. Since you are multiplying you must round this answer to have the same number of significant figures as the lowest number of significant figures from the measurements you used when doing this calculation. 456 has 3 significant figures. 2.0 has 2 significant figures. This means your answer must have 2 significant figures.

13 18000 m. If you solve this problem you will find the exact answer when calculated is 18241.75824 m. That is quite a number. Since you are dividing you must round this answer to have the same number of significant figures as the lowest number of significant figures from the measurements you used when doing this calculation. 83000 has 2 significant figures. 4.550 has 4 significant figures. This means your answer must have 2 significant figures. This is a good problem to remember that you have to pay careful attention to sig figs. Even though 83000 has more digits, it has fewer significant figures than 4.550, so make sure when you are rounding problems to keep good track of significant figures.