CHAPTER 1

The Scope of Portfolio Performance

INTRODUCTION

The notion of "portfolio performance," which is at the core of the present book, is related to the quality of the management process of a financial investment. Even though it is part of the control function, we view its importance as essential for the success of the whole venture. We claim in this chapter that the feedback loop generated by the performance evaluation process has implications for the preparation and execution steps in the process. This motivates the subtitle of our book: "Appraise–Analyze–Act."

In the first section, we decorticate portfolio management by analogy with a production process. The important takeaway of this matching exercise is the identification of performance as a measure of how efficiently the inputs are transformed into an output being the terminal portfolio value. Accordingly, its inputs are *cash outlay, costs, time*, and, most critically, *risk*. According to this point of view, a risk-adjusted performance measure can be defined as an *efficiency indicator* or an *output gap*. These two structures are shared by many performance measures developed throughout the consecutive chapters.

Pursuing on the same logic, we are in a position to explain the structure of the rest of the book. It is split into four parts. The first two are dedicated to the identification and interpretation of the many portfolio performance measures that exist nowadays. This is the "appraisal" dimension. The second one, namely "analysis," mostly tries to capture and understand the drivers of performance. This is the subject matter of the third part of the book. Finally, we dare a forward-looking dimension through the "act" motto. In the fourth part of the book, we use performance measures as a real adjuvant to the investor's or the manager's decision-making process. In doing so, we close the loop of the portfolio management cycle.

For the purpose of the book's technical developments, we need to clarify a certain number of notions. The third section is therefore dedicated to laying out some basic definitions regarding returns, risk, and benchmarks. Specifically, we briefly describe the different rules for compounding and averaging returns and for incorporating cash flows into the calculation of returns. We also review the economic and probabilistic foundations of risk, elaborate on some specific risks associated with combining assets into a portfolio, and present some axioms for relevant risk measures. Finally, we stress the relevance of portfolio benchmarking and discuss some practical issues related to a comparative assessment of portfolio performance.

1.1 FROM PORTFOLIO MANAGEMENT TO PORTFOLIO PERFORMANCE

Different reference books or summary articles devoted to portfolio performance address the scope of their subject matter in various ways. Since our book will be released perhaps decades later than some of them, we do not have the ambition to reinvent the wheel. Nevertheless, our critical reading of our peers' introductory chapters reveals very diverse standpoints and structuring of thoughts. There are some disagreements on the terms, the objectives, or even the definitions surrounding the notion of portfolio performance. Since our main goal is to propose a guided tour of the matter, it is necessary that we restart from a blank page and, notwithstanding the existence of many insightful contributions in the same field, come up with our own vocabulary and structure.

We first attempt to understand where the question of portfolio performance evaluation arises in the investment decision-making process. Then, we focus on what the notion of performance itself covers in that particular context. This allows us, in a third stage, to provide more explanations about the subtitle that we have chosen for this book: "Appraise–Analyze–Act."

1.1.1 The role of portfolio performance in the investment management process

Like any well-constructed management process, the investment in a financial portfolio generally obeys a three-stage procedure, *prepare, execute, and control*, that repeats itself in a feedback loop. This process leads, in particular, to what Bailey, Richards, and Tierney (2007) call "performance evaluation." From the most general perspective, we can represent it with the chart shown in Figure 1-1, inspired from the process depicted by Maginn, Tuttle, McLeavey, and Pinto (2007).

The sequence presented in Figure 1-1 clearly shows that the preparation phase (left part of the chart) features two complementary exercises. The first one, on the upper side, consists of understanding the investor and translating this knowledge into usable parameters. The outcome of the process is to determine a portfolio policy (very long term) and associated

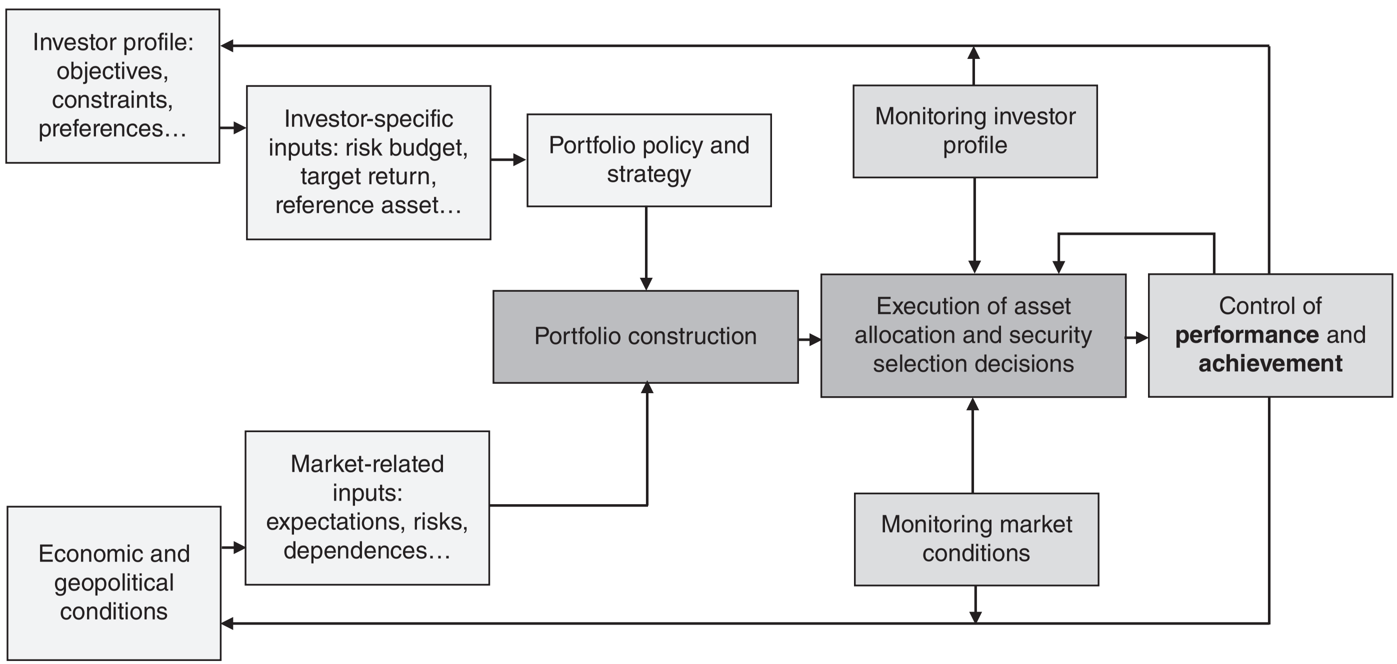

FIGURE 1-1 Steps in the portfolio management process. *Source*: Adapted from Maginn et al. (2007), Exhibit 1-1.

strategy (medium long term).[1] This is also where the benchmark portfolio, if applicable, will be determined.[2] The second module relates to the financial market. It involves the economic intelligence that leads to the calibration of the market parameters that are necessary to parametrize the risk and return properties of financial assets.

The meeting point of the market and investor dimensions is the inception of the actual execution process. It first involves creating a "real" portfolio, made up of stocks, bonds, and other instruments, with the aim of being the best possible allocation corresponding to the inputs. This is the construction phase. This is also where the optimization process is implemented. Then, the portfolio manager starts showing their skills through their decisions. The portfolio composition is regularly updated through the buy and sell trades. This can be performed from the upper level of global asset classes down to the most granular securities selection decisions, with a continuum of decision levels in between. If the portfolio is managed passively, the rebalancing decisions are driven by a mechanical rule. If, however, the manager has an active strategy, then this is the stage where they can show the quality of their decisions.

The last stage occurs *a posteriori* from the execution decisions. It is about monitoring and control. The monitoring process can be viewed as a continuous attempt to challenge and improve the way the portfolio is managed. To do so, the manager will seek information from the market and the investor's sides, which generates a feedback loop both at the execution level (by generating trades) and even going back to the preparation level by possibly changing the assumptions underlying the whole portfolio management cycle. The control level occurs at the end of the process. It consists of providing an answer to two types of questions: "*How has the portfolio management process performed?*" and "*To what extent has the outcome achieved its stated objectives?*". If the answers are satisfactory, then the feedback is positive. If not, it may certainly impact the way the mandate is executed but probably some of the assumptions used in the preparatory phase as well. Our scope is related to the first of these two questions.[3]

1.1.2 Performance as an efficiency indicator

1.1.2.1 Efficiency versus effectiveness

The two dimensions of control mentioned in Figure 1-1 are termed "performance" and "achievement." The former concept aims at evaluating to what extent the whole process has delivered an output that makes the best possible use of all the inputs that were put at the disposal of the management unit. It is a measure of the *efficiency* of the portfolio management process and indicates how productive it has been. By contrast, the concept of

[1]The articulation between these two terms for actual portfolio construction is examined in Chapter 13.
[2]See Section 1.3.3 for a detailed discussion on this notion.
[3]We come back to the difference between the meaning of these two questions, which reflect the "performance" and "achievement" components of the control function, in the next subsection.

achievement reflects how successful the portfolio has been: It is a measure of *effectiveness*. There is necessarily an objective that is associated with it.

Consider that we are interested in a specific portfolio P that has been managed according to the principles of Figure 1-1. Adapting the concepts of efficiency (performance) and effectiveness (achievement) to the particular case of the management of a financial portfolio, we can simultaneously translate them under the form of a summary equation:

$$\left\{X_{P,1}, \ldots, X_{P,K}\right\} \underset{\text{efficiency}}{\Rightarrow} Y_P \underset{\text{effectiveness}}{\Longleftrightarrow} Y_P^* \tag{1-1}$$

where $\{X_{P,1}, \ldots, X_{P,K}\}$ is the set of K inputs in the portfolio management process, Y_P is the observed output (that, for simplicity, we consider to be unique at a single point in time), and Y_P^* is the objective set by the target user, who in our case is the investor. The right arrow "⇒" symbolizes the whole transformation process that uses the inputs in order to produce the output, whose quality reflects its efficiency level. The double arrow "⟺" symbolizes the gap between the observed and the desired output. The magnitude of this gap reflects the effectiveness of the process.

It is not the purpose of this section to discuss at length the notion of an efficient production frontier, but we can simply posit that the process aims at consuming the least possible inputs. Thus, the smaller the quantity of each input $X_{P,i}$ producing the same level of output Y_P, the higher efficiency is. A well-performing portfolio management process will thus seek to be as parsimonious as possible for the same result or, equivalently, to produce the largest possible result with a given set of available inputs.

The issue of effectiveness involves setting and disclosing a measurable output objective. The portfolio management process will have achieved its goal if the observed output exceeds this target. It does not matter here whether the inputs have been wisely used or simply wasted as long as the objective is attained. Thus, a process can be effective without being efficient and *vice versa*. Consider a simple example of someone who absolutely needs to obtain $120 at the end of their investment horizon. They have the choice between two mutually exclusive accounts. The first one requires an initial outlay of $100 and delivers a final amount of $115. The second one requires an initial outlay of $115 and delivers a final amount of $120. Because it makes better use of the available money, the first process is more efficient than the second one. However, since the second process allows the investor to attain their objective and not the first one, it is effective and the other one is not.

1.1.2.2 Performance evaluation as a function of inputs and output

Since we are primarily interested in the notion of "portfolio performance," our focus is on the left part of Equation (1-1). Identifying the inputs of the process involves answering a simple question: *What do we want to minimize or even avoid in the hope of obtaining the output Y_P?* The qualitative answer leads to the identification of four types of inputs: cash outlay, costs, time, and risk.

1. *Cash outlay*: This is the initial amount that the investor has committed in order to start the process. Sometimes, this amount is released gradually over the portfolio lifetime, as in the case of private equity investments (see Chapter 12). This outlay can be viewed, as for any investment decision, as a temporary sacrifice of immediate consumption in order to obtain the possibility to consume more in the future. Everything else being equal, any rational investor would like to minimize this outlay to generate a given output.
2. *Costs*: This is the natural extension of the previous item. Fees, charges, taxes, and any other outflow represent an erosion of the final output. Even though, for many managed portfolios, these various costs are deducted from the investor's account, in reality this should be viewed as an add-on to the initial cash outlay.
3. *Time*: The portfolio management process is not instantaneous; it requires a certain amount of time in order to produce its output. In the meantime, the investor's money is frozen and cannot be used for any other purpose, which corresponds to a missed opportunity. The longer they have to wait to obtain their output, the lower their satisfaction.[4]
4. *Risk*: This is the most "controversial" input. We assume that the standard investor who cares about their net worth and commits a substantial amount of their own wealth in a portfolio exhibits risk aversion. This means that, *ceteris paribus*, they prefer to take less risk for the same expected outcome. There are numerous real-life examples where this assumption is violated, but these happen under specific circumstances and can be considered exceptions in the field of portfolio management.[5]

Consequently, we can generically write the set of inputs in the following way:

$$\{X_{P,1}, \ldots, X_{P,K}\} \equiv \{V_{P,0}, C_{P,H}, H, \mathcal{R}_{P,H}\} \tag{1-2}$$

[4]This statement assumes that the opportunity cost of having money committed to an investment is positive. This translates concretely into a positive interest rate. Even though there have been numerous recent examples of periods of negative nominal interest rates for many currencies, we assume that these situations are artificial and do not reflect the representative investor's time-related preferences.
[5]We discuss some of these exceptions in the context of behavioral finance, which is the subject matter of Chapter 10.

where $V_{P,0}$ stands for the initial value of portfolio P at time 0 expressed in currency units (e.g., US Dollar), $C_{P,H}$ is the total cost incurred by the investor during the whole horizon H of the portfolio management process (expressed at time H for simplicity), and $\mathcal{R}_{P,H}$ represents the total risk borne by the investor during this time frame. The symbol "≡" means "identity," i.e., the terms on both sides are identical. The arguments of the right-hand side of Equation (1-2) represent the four inputs, in their order of presentation.

The unequivocal quantitative outcome of the portfolio management process is an amount of money that crystallizes the portfolio value at the end of the period under consideration, including all reinvested interim cash flows received by the investor. This final amount can be distributed to the investor or simply available for liquidation. Thus, if the portfolio has been managed over a given investment horizon H, we can simply identify the output as

$$Y_P \equiv V_{P,H} \tag{1-3}$$

where $V_{P,H}$ stands for the value of the portfolio P at time H.

To address the link between the inputs (Equation (1-2)) and the output (Equation (1-3)), we proceed in two steps. The first one consists of eliminating the scale-related factor in the portfolio management process because most performance measures do not consider this issue explicitly.[6]

$$R_{P,H} \equiv f_{\text{scale}}(V_{P,H}; V_{P,0}, C_{P,H}) = \frac{V_{P,H} - C_{P,H}}{V_{P,0}} - 1 \tag{1-4}$$

where $f_{\text{scale}}(V_{P,H}; V_{P,0}, C_{P,H})$ is the transformation function that relates the output with two of its inputs, and $R_{P,H}$ is the cumulative net (from costs and fees) rate of return of the portfolio.

Similarly, if the time dimension is simultaneously taken into account, another simple and well-known function can be used to assess the efficiency of the portfolio management process:

$$\overline{R}_P \equiv f_{\text{scale+time}}(V_{P,H}; V_{P,0}, C_{P,H}, H) = \sqrt[H]{\frac{V_{P,H} - C_{P,H}}{V_{P,0}}} - 1 \tag{1-5}$$

where $\overline{R}_P$ is the (geometric) average net rate of return of the portfolio. Note that we can simplify this function by simply stating that the return itself reflects the impact of scale on the process. Equation (1-5) can be simply rewritten as $\overline{R}_P \equiv f_{\text{scale+time}}(R_{P,H}; H) = \sqrt[H]{1 + R_{P,H}} - 1$.

Since functions $f_{\text{scale}}(\cdot)$ and $f_{\text{scale+time}}(\cdot)$ explicitly relate the output with (some of) its inputs, the cumulative and average net portfolio returns are undoubtedly performance

[6] The main exception to the principle of scale-free performance measurement is the "first dollar alpha," that explicitly foresees decreasing returns to scale. This issue is discussed in Chapter 18.

measures.[7] This is why, in practice, there are plenty of documents that use the words "returns" and "performance" interchangeably. Nevertheless, this way of making the concepts equivalent does not meet our preference. Our reasoning is that these versions of portfolio returns do not *fully* reflect the efficiency level of the portfolio management process. A very important input is missing: risk.

We therefore generically define a *risk-adjusted performance measure* as a function that connects, explicitly or implicitly, the portfolio return not only with time but also with risk. It is interesting to distinguish two main cases, depending on whether the efficiency level of the portfolio is assessed in absolute terms or relative to an external, but comparable portfolio management process. Accordingly, our definitions are generically written as follows:

$$\mathrm{PM}_P^{\mathrm{abs}} \equiv f_{\mathrm{all}}(R_{P,H}; H, \mathcal{R}_{P,H}) \tag{1-6}$$

$$\mathrm{PM}_P^{\mathrm{rel}} \equiv f_{\mathrm{all}}(R_{P,H}; H, \mathcal{R}_{P,H}, R_{B,H}, \mathcal{R}_{B,H}) \tag{1-7}$$

where $R_{B,H}$ and $\mathcal{R}_{B,H}$ are the return and risk of a "benchmark" portfolio B, respectively, that is supposed to be comparable to portfolio P regarding the portfolio management process.

In general, the most popular types of measures encountered in practice are of two kinds. For the absolute ones (Equation (1-6)), the majority of them involve a ratio: $\mathrm{PM}_P^{\mathrm{abs}} = \frac{\mathcal{P}(R_{P,H})}{\mathcal{R}_{P,H}}$, where $\mathcal{P}(R_{P,H})$ stands for an increasing function of portfolio return. The interpretation as an *efficiency ratio* is quite obvious: The ratio increases with the numerator (output) and decreases with the denominator (input). This is the case for the Sharpe ratio, as discussed in Chapter 3, for instance. For the relative measures (Equation (1-7)), most measures use a differential operator: $\mathrm{PM}_P^{\mathrm{rel}} = \mathcal{P}(R_{P,H}) - \mathcal{P}(R_{B,H})$, where the benchmark risk is implicitly subsumed under its rate of return. This simple expression, equivalent to an *output gap*, covers Jensen's alpha, which is also discussed in Chapter 3.

However, the ways to measure performance are not limited to these simple examples. In total, we list 116 performance measures, summarized in Chapter 11, that correspond to the definition of Equation (1-6) or (1-7). In addition, for some specific cases, some adaptations are also examined in Chapters 12 and 16. However, they all share a common characteristic: The transformation function $f_{\mathrm{all}}(\cdot)$ can be of many different types, but it must account for portfolio risk in one way or another.

A final note is worth mentioning here. Some performance measures mix some elements of efficiency and effectiveness. This is the case of measures that use a notion of reservation rate

[7]Some authors, like Bailey et al. (2007), indeed associate the name "performance measure" with only the rate of return defined in Equations (1-4) or (1-5) and contrast them with the notion of "performance appraisal," which takes into account all inputs in order to assess the skills of the portfolio manager. While we acknowledge the correctness of their point of view, we diverge from their terminology by associating the term (risk-adjusted) "performance measure" to a function that takes all inputs (including risk) into account and refer to "performance appraisal" as the use and interpretation of the measures.

(i.e., minimum acceptable rate of return) or target return (for specific investment goals). They are mostly studied in Chapter 10, which is devoted to the study of performance measures that are adapted to a behavioral finance context.

1.1.3 Why appraise, analyze, and use performance in decision-making

From the above discussion, it appears that our focus is on the last phase of the process. Even though it occurs in the end, after the output of the portfolio management decisions has been observed, Figure 1-1 clearly shows that the control stage is part of a circuit. It can thus be viewed as the beginning of a new cycle in the decision-making process. This statement motivates the whole philosophy that we pursue in this book.

Zooming in on the last part of Figure 1-1 provides a first visual insight into our red wire. Leaving aside the "achievement" function, the cyclical character of the investment management process can be very clearly connected to the three dimensions that are covered throughout the book, as shown in Figure 1-2.

Figure 1-2 clearly shows the interaction between different dimensions involving the use of performance-related tools. It starts with an *ex post* appraisal method, out of which an analysis is carried out in order to gain insights and prepare for potential action for the future.

The retrospective application of performance evaluation consists of appraising how efficiently the portfolio management process has been carried out. This is where we need to put in place a measurement approach that detects how the manager has performed their job with the inputs that they have consumed. Since there are a variety of ways to approach the "risk" input, and also to treat the characteristics of a properly designed benchmark, it is not surprising to realize that this dimension is particularly bushy and requires a lot of attention.

The next step is to properly analyze how the performance has been generated. From a very wide perspective, it can be viewed to encompass the performance measurement stage. Nevertheless, Amenc and Le Sourd (2003) point out that "*Performance analysis is the final stage in*

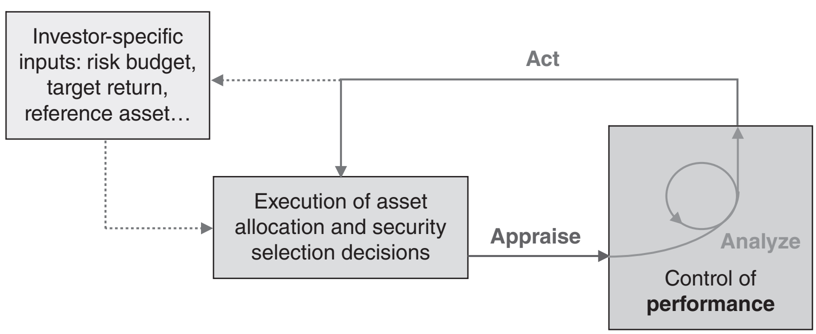

FIGURE 1-2 The feedback loop of performance evaluation.

the portfolio management process. It provides an overall evaluation of the success of the investment management process in reaching its objective and also identifies the individual contribution of each phase to the overall result." We stand at the heart of the control function. Leaving aside the effectiveness check (see above), the key objective is to understand and explain. This means that it might be necessary to (i) account for the specificities of the portfolio that does not lend itself easily to a plain appraisal approach and (ii) use some specific tools, such as attribution models (Chapters 13 and 14), in order to gain insight into the sources of overperformance or underperformance.

Lastly, performance appraisal and analysis can serve as a basis for decision-making. This "act" dimension starts with good communication and understanding from the receiver. Feedback on performance can then be used to revise the investment management process, at least, not only operationally (this is the solid arrow, going back directly to the execution engine) but also potentially strategically. It serves not only as an internal guidance for the manager but also as a selection criterion for the investor. Incidentally, performance can be used to reward, penalize, hire or fire managers. One must ensure that the signal sent by the retrospective evaluation is properly processed because in the end, it is all about people.

1.2 ROLLING OUT THE THREE LAYERS OF PERFORMANCE EVALUATION

The three layers of performance evaluation presented in Figure 1-2 involve dealing with the past (appraise), the present (analyze), and the future (act). Because of the cyclical character of the process, these dimensions are heavily intricated: The future of the current time is also the past of the next control period. Nevertheless, we can identify several themes that more closely correspond to each of these dimensions. The whole book's structure, including all 17 chapters following this introductory one, is designed in accordance with the cyclical view. Each of them is discussed below.

1.2.1 Understanding the past: Performance appraisal

Designing and interpreting performance measures represents the bulk of creative contributions in the field. This is the core ingredient of the controlling function of the portfolio management process. With a specific time window and a full set of information that is considered relevant for the exercise, the purpose of performance appraisal is to detect whether the portfolio manager has displayed investment skill through their past decisions.[8]

The history of research in portfolio management reveals that this issue of appraisal is the first dimension of performance that has retained some attention. The question of whether

[8]Bailey et al. (2007) also associate performance appraisal with the assessment of whether the manager can preserve their skill. We adopt a more granular perspective and associate this predictive assessment with the "act" dimension.

active portfolio managers are able to "beat the market" through a representative stock index is central to the article of Jensen (1968). Since the 1960s, hundreds of researchers and practitioners have designed measures aimed at diagnosing as precisely as possible if, and how, managers have actually been able to manage their portfolio in the most efficient way. This plethora of measurement approaches, many of which are very meaningful, explains why this topic represents a substantial fraction of our material. Figure 1-3 represents the logic underlying our structure.

Figure 1-3 shows how first two parts of the book, which are devoted to the "appraisal" dimension, organize the corresponding nine chapters. The figures featuring in each box correspond to the numbers of remarkable performance measures that we list and analyze further in Chapter 11 (whose scope is explained in the next subsection).

Part I, entitled "Classical Performance Measurement," is entirely dedicated to the in-depth study of the performance measures that were developed in the early days of the Standard Portfolio Theory (SPT) – generally known in the literature as the Modern Portfolio Theory but that we prefer to rename according to a more contemporaneous perspective – introduced by Markowitz (1952), and the consecutive Capital Asset Pricing Model (CAPM) developed by Treynor (1962), Sharpe (1964), Lintner (1965), and Mossin (1966). Whereas Part I comprises four chapters, Figure 1-3 only displays one of them. Due to the historical importance and the popularity of the classical measures, the other chapters discuss their theoretical underpinnings (Chapter 2), the rationale for their choice (Chapter 4), and the difficulties associated with their application (Chapter 5).

Part II investigates the other ways to measure performance – including extensions of the original measures – that coexist in the literature or in market practice, or that we propose ourselves within this book. They can be split into three categories. The first one covers the extensive set of measures that can be viewed as variations from the classical ones (Chapter 6). The second one examines measures that have been developed with a view to reveal how the

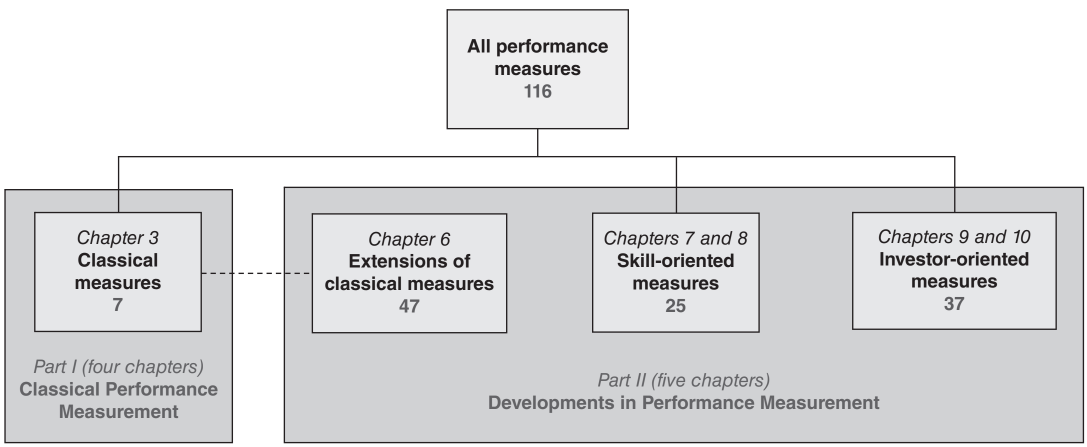

FIGURE 1-3 Structure of the "appraisal" dimension in the book.

portfolio manager has implemented their skills through (i) the identification of their risk factor sensitivities (Chapter 7) or (ii) the uncovering of their market timing skills (Chapter 8). The third one focuses on the investor's preferences and makes a distinction between the assumption of a standard (i.e., fully rational) or a behavioral investor (Chapters 9 and 10, respectively).

The summaries in Tables 1-1 and 1-2 provide a high-level description of each chapter for Parts I and II.

TABLE 1-1 Overviews of the chapters contained in Part I.

Chapter	Title	Objective	Key features
2	**Standard Portfolio Theory and the CAPM**	To develop and explain the theoretical underpinnings of the classical performance measures	• Presentation of the SPT and the CAPM • Practical insights on how to implement the CAPM • Presentation of multifactor extensions
3	**Classical Portfolio Performance Measures**	To present the main classical performance measures based on total risk, systematic risk, and specific risk	• Definition of each measure • Practical and statistical significance criteria • Discussion of strengths and weaknesses
4	**Selecting a Classical Performance Measure**	To discuss the criteria to be considered for the choice of the most suitable classical performance measure	• Categorization of measures on the risk and measurement dimensions • Choice criteria from the normative and the positive standpoints • Identification of how to discriminate managers based on their performance
5	**Pitfalls and Dangers with the Classical Performance Measures**	To identify all the potential issues associated with the practical implementation of classical measures	• Identification of issues related to the SPT and CAPM frameworks • Analysis of the statistical issues related to the sample and the regressions • Discussion of the interpretation issues

TABLE 1-2 Overviews of the chapters contained in Part II.

Chapter	Title	Objective	Key features
6	**The Classical Performance Measures Revisited**	To review relevant alternative performance measures that build on the classical ones	• Presentation of refined or altered versions of the Sharpe ratio • Alternative versions of the other performance measures • Design of risk-adjusted forms of classical measures
7	**Performance Measurement in Multifactor Models**	To examine how performance can be measured when the manager is exposed to several risk factors	• Identification of the different families of multifactor models • Presentation of classical performance measures adapted to a multifactor context • Special cases of multifactor models
8	**Performance Measurement with Market Timing**	To examine how performance can be measured when the manager attempts to time the market	• Measurement of performance with non-linear regressions • Capture of market timing with return-based and holding-based exposures • Roadmap for the choice of market timing appraisal method
9	**Preference-based Performance for the Standard Investor**	To determine how performance must be appraised when the investor's risk aversion is taken into account	• Definition of the structure of the standard investor's preferences • Identification of preference-based measures in the mean–variance framework • Use of standard utility functions for performance measurement
10	**Preference-based Performance for the Behavioral Investor**	To determine how performance must be appraised when the investor's behavioral traits are taken into account	• Definition of the structure of the behavioral investor's preferences • Use of behavioral utility functions for performance measurement • Extensions to ratios of gains over losses • Assessing performance with mental accounts

1.2.2 Explaining the present: Performance analysis

With (at least) 116 ways to assess the past performance of a portfolio manager, anyone would presumably be lost without further guidance. Fortunately, there exist tools in order to make sense of all this information. In Part III, devoted to the analysis of performance-related information and made up of four chapters, we propose some guidance on three dimensions.

First and foremost, we find it necessary to provide the reader with some keys in order to find a way out of the maze of performance measures. Chapter 11, which attempts to perform this task, should be viewed as the bridge between the appraisal and the analysis of performance. This chapter applies different methods to sort the measures in a logical way and concludes by providing a dashboard for the selection of one or several approaches that are suited to the analysis.

The second dimension, addressed in Chapter 12, focuses on the necessary adaptations to faithfully reflect how performance should be analyzed in specific portfolio management contexts. The purpose is not to reinvent performance measures but to adapt the framework to the specificities of particular asset classes that go beyond the traditional equity or global allocation portfolios for which many measures have been designed.

The last two chapters (Chapters 13 and 14) of Part III are dedicated to a set of processes that are generally considered at the core of performance analysis, namely contribution and attribution. Consistent with our philosophy, the purpose of these two chapters is neither to replace the very detailed treatment of some qualitative and highly specialized textbooks on the matter nor to present the sometimes very sophisticated practical systems that are made available to the analyst by premium vendors. Rather, they aim at providing some insight regarding the approach and some useful applications in order to get started with a thorough analysis of the performance drivers in many circumstances.

The key insights of the chapters covered in Part III are reported in Table 1-3.

1.2.3 Preparing the future: Using performance for action

Because of its richness, performance analysis can serve as basis in the decision-making process. There are various ways to consider this matter, but the first element to consider is a proper transmission mechanism from the issuer to the consumer of performance information. This is the reason why Part IV, which is dedicated to the use of performance for action, starts with the issue of the communication of performance-related information in Chapter 15. This involves not only a faithful disclosure but also the possibility for the receiver to verify the correctness and exhaustiveness of information that serves as the basis for their own decision-making process.

The next two chapters propose a thorough investigation of two important themes belonging to this dimension. The first one, which is the focus of Chapter 16, examines how performance measures themselves can be used as a decision-making tool for several strategies or investor types. This is where we analyze more closely the link between financial and non-financial (i.e., sustainability-driven) performance. The second chapter (Chapter 17) studies the link between past and future performance, generally associated with the term

TABLE 1-3 Overviews of the chapters contained in Part III.

Chapter	Title	Objective	Key features
11	**Navigating the Maze of Portfolio Performance**	To sort and structure the main 116 measures presented in the first two parts	• Structured list and glossary of the 116 measures • Analysis of analytical and statistical sorting procedures • Dashboard for the selection of measures
12	**Performance Design for Specific Asset Classes**	To determine how performance analysis must be adapted to reflect specificities of some portfolio management strategies	• Adaptation of the performance framework for fixed-income portfolios • Specificities of illiquid investments • Design of performance analysis for hedge funds and private equity
13	**The Granular Analysis of Performance**	To introduce the processes of performance contribution and attribution of performance measures	• Dissection of the fundamentals of performance decomposition • Defining performance attribution • Applying attribution to performance ratios
14	**Performance Attribution Methods**	To develop the main practical methods to attribute portfolio performance	• Introduction to the BHB attribution analysis framework for single and multiple periods • Extension of the framework to different portfolio types • Adaptation to statistical attribution analysis

"persistence," extending the scope to other relevant indicators for the investment decision-maker.

The final chapter explores the delicate topics of agency conflicts and illusions regarding performance. It aims at identifying many situations in which performance must be considered with great caution. These situations concern remuneration issues, temptation to artificially embellish the outcome of the portfolio management process, or simply disentangling skill from luck. Chapter 18 can be viewed as a synthetic *vade mecum* for the decision-maker who wishes to avoid being mistaken throughout the process.

The contents of Part IV are summarized in Table 1-4.

TABLE 1-4 Overviews of the chapters contained in Part IV.

Chapter	Title	Objective	Key features
15	**Disclosing and Verifying Portfolio Performance**	To identify the challenges of properly communicating and understanding performance-related information	• Main features of the Global Investment Performance Standards (GIPS) • Discussion of the challenges of effective performance communication • Introduction to fund rating and portfolio analysis systems
16	**Applications of Performance in Investment Decisions**	To use performance-related approaches as an aid in the investment decision-making process	• Determination of investment universe and strategy design using performance analysis • Use of performance criteria for specific typologies of investors • Reconciliation of ESG criteria and financial performance
17	**Performance and Predictability**	To analyze the link between past and future performance-related characteristics	• Discussion of the drivers of predictability in portfolio management • Identification of absolute persistence in performance • Methods for studying relative persistence
18	**Agency Issues and Illusion of Performance**	To distinguish the cases in which performance should not be considered at face value for decision-making	• Examination of the implications of performance measure in agency conflicts • Design principles for adequate performance measurement • Distinction between skill and luck in performance

1.3 RETURNS, RISK, AND BENCHMARKS

This section is dedicated to the definition and some technical clarifications regarding the three major ingredients of portfolio performance appraisal, namely the return, its associated risk, and the benchmark.

1.3.1 Returns

A portfolio return seems to be the most intuitive economic notion related to performance. However, the precise notion of a return requires a few specifications that we examine below.

1.3.1.1 Compounding

Using the notations introduced in Section 1.1, the gross and net returns on portfolio P between dates t_1 and t_2 are given by

$$R^{\text{gross}}_{P,t_2} = \frac{V_{P,t_2}}{V_{P,t_1}} \tag{1-8}$$

$$R^{\text{net}}_{P,t_2} = \frac{V_{P,t_2} - V_{P,t_1}}{V_{P,t_1}} = R^{\text{gross}}_{P,t_2} - 1 \tag{1-9}$$

For clarity of exposition, we will work with net returns whenever possible, and the notation $R_{P,t}$ will by default refer to a net return at the period ending at time t. For instance, the presentation of the STP in Chapter 2 relies on net returns.

Returns can be compounded discretely or continuously, depending on how the passing of time is envisioned. This distinction yields two different methods for calculating cumulative returns over several periods. The discrete compounding of \$1 over N (equally spaced) periods yields

$$(1 + R_{P,t_1})(1 + R_{P,t_2}) \ldots (1 + R_{P,t_N}) = \prod_{i=1}^{N} (1 + R_{P,t_i}) \tag{1-10}$$

where all the R_{P,t_i} are periodic returns (i.e., expressed in reference to the duration of the period). The continuous compounding of \$1 over N (equally spaced) periods yields

$$e^{R^c_{P,t_1}} e^{R^c_{P,t_2}} \ldots e^{R^c_{P,t_N}} = \prod_{i=1}^{N} e^{R^c_{P,t_i}} = \exp\left(\sum_{i=1}^{N} R^c_{P,t_i} \right) \tag{1-11}$$

where superscript "c" indicates that the R^c_{P,t_i} are continuously compounded.

Equations (1-10) and (1-11) do not just reflect two compounding conventions. They also involve two different types of returns: Discretely compounded (or, simply, “discrete”) returns take on values above −1, whereas continuous returns are real values.

1.3.1.2 Average returns

Another issue that arises when performance is evaluated over several periods is how to compute average returns. There are two methods for averaging returns whose use depends on the context of application.

When an investor aims at assessing the average financial profitability of their investment, they should use the *geometric average* return given by

$$\overline{R}^{\text{geo}}_{P,t_N} = \sqrt[N]{\prod_{i=1}^{N}(1 + R_{P,t_i})} - 1 \tag{1-12}$$

The geometric average return reflects the average economic conditions under which the portfolio owner or manager actually invests their money. Suppose the end-of-month values of portfolio P (with initial cash outlay 100) turn out to be 102 in January, 99 in February, and 103 in March. The periodic net returns are $\left(\frac{102}{100} - 1\right) = 2\%$, $\left(\frac{99}{102} - 1\right) = -2.94\%$, and $\left(\frac{103}{99} - 1\right) = 4.04\%$ for the first, second, and third months, respectively. Over the three-month period, the portfolio has experienced an average monthly return of $\overline{R}^{\text{geo}}_{P,t_N} = \sqrt[3]{(1 + 2\%)(1 - 2.94\%)(1 + 4.04\%)} - 1 \approx 0.99\%$. Indeed, compounding the initial investment at this rate over three months yields the same terminal value: $100(1 + 0.99\%)^3 \approx 103$.

For statistical purposes, it might be relevant to use the *arithmetic average* return defined as

$$\overline{R}^{\text{ari}}_{P,t_N} = \frac{1}{N}\sum_{i=1}^{N}(1 + R_{P,t_i}) - 1 \tag{1-13}$$

To extrapolate the portfolio profitability, one may wish to generate scenarios for future periodic returns. If one assumes that the observed past returns represent independent draws from an identical (not necessarily known) distribution, then one can consider that, on average, the investor should expect their portfolio to yield $\overline{R}^{\text{ari}}_{P,t_N}$ for the upcoming periods. Using the same numerical example, we obtain $\overline{R}^{\text{ari}}_{P,t_N} = \frac{1}{3}(1 + 2\% + 1 - 2.94\% + 1 + 4.04\%) - 1 \approx 1.03\%$.

Note that, by construction, $\overline{R}_{P,t_N}^{\text{geo}} < \overline{R}_{P,t_N}^{\text{ari}}$, and the financial explanation for this difference is the compounding of interests over several periods that the geometric average captures while the arithmetic average does not.

1.3.1.3 Multiperiod returns with interim cash flows

During their management, portfolios often receive additional deposits or experience partial withdrawals. These interim cash inflows and outflows clearly affect the value of the portfolio. However, they should not interfere with the calculation of the value created by the portfolio manager. To this end, one should use *time-value weighted returns* defined as geometric average returns with periodic returns given by

$$R_{P,t_i} = \frac{V_{P,t_i} - (V_{P,t_{i-1}} + \text{CF}_{t_i})}{V_{P,t_{i-1}} + \text{CF}_{t_i}} \tag{1-14}$$

where CF_{t_i} denotes the net cash flow affecting portfolio P between dates t_{i-1} and t_i. Other similar methods are available (such as the modified Dietz method). A more detailed discussion on this issue can be found in Chapter 15.

In those cases where the value of the portfolio cannot be marked to market on a frequent basis such as, for instance, investments in illiquid segments (e.g., private equity or real estate), one must resort to *money-value weighted returns*. These can be calculated as an internal rate of return (IRR) over the whole period of evaluation. That is, the IRR is the solution to

$$V_{P,t_0} = \sum_{i=1}^{N} \frac{\text{CF}_{t_i}}{(1+\text{IRR})^i} + \frac{V_{P,t_N}}{(1+\text{IRR})^N} \tag{1-15}$$

The IRR reflects the average periodic growth rate of the investment, but its determination is biased by the timing of cash flows.

1.3.2 Risk and risk measure in portfolio performance

As discussed in Section 1.1.2, risk is one of the most important inputs in the assessment of portfolio performance, and it turns out to be one of the most difficult to define and measure. The very notion of risk is related to danger,[9] and one fundamental assumption in economics and finance is that investors are non-satiable (they always prefer more to less) and risk averse.

[9]The word "risk" allegedly stems from "*resecum*" (literally "something that cuts" in Latin), which designated a reef and, by extension, the hazard threatening supplies shipped by boat.

Since the semantic distinction made by Frank Knight in 1921, it is commonly admitted that risk, in contrast to uncertainty, is measurable. Thus, the characterization of risk in performance appraisal is subject to a twin problem: One needs to rely on a definition of risk and a corresponding *risk measure*. We briefly review the major approaches followed in the finance literature to provide a representation of risk. Then, we touch upon the specific additional issues while dealing with *portfolio risk*. Finally, we present some axioms related to measuring risk and provide examples of some commonly used risk metrics.

In what follows, the representations and measures of risk refer, without any loss in generality, to *total risk*. Depending on the investment context, they can also be applied to systematic risk or idiosyncratic risk. These alternative specifications will yield different dimensions for portfolio performance appraisal and, accordingly, different performance metrics (see Chapters 3 and 6).

1.3.2.1 Representations of risk

1. The utility-based representation

In neo-classical economics, the formal treatment of decision under uncertainty relies on the expected utility theory. Consider an investment universe comprised of N risky assets with (stochastic) gross returns $R_{i,H}$ over horizon ($i = 1, \ldots, N$). Endowed with a utility function $U(\cdot)$ assumed to be monotonically increasing (for non-satiability) and concave (for risk aversion), the rational investor allocates the amount x_i in asset i, and solves the following optimization problem:

$$\max_{x_i} \mathrm{E}[U(V_{P,H})] \qquad \text{(1-16)}$$
$$\text{s.t.} \sum_{i=1}^{N} x_i = V_{P,0}$$

where $V_{P,H} = V_{P,0} + \sum_{i=1}^{N} x_i R_{i,H}$ stands for the value of the portfolio P at time H.

To avoid resorting to utility functions, the utility maximization problem above can be reformulated as risk minimization. More specifically, the measure of risk $\mathcal{R}_{P,H}$ is said to be *consistent* with the order relation induced by the utility function $U(\cdot)$ if Equation (1-16) is equivalent to (see Ortobelli, Svetlozar, Rachev, Stoyanov, Fabozzi, and Biglova, 2005, for more details):

$$\min_{x_i} \mathcal{R}_{P,H}(V_{P,H}) \qquad \text{(1-17)}$$
$$\text{s.t.} \sum_{i=1}^{N} x_i = V_{P,0} \text{ and } \mathrm{E}(V_{P,H}) \geq m$$

where m denotes a minimal requirement on expected terminal wealth.

The formulation (1-17) of portfolio investment puts the risk measure at the center of the problem. Alternatively, the utility maximization program of Equation (1-16) offers another representation of risk through the notion of *stochastic dominance*.[10] Indeed, it has been shown (see, e.g., Levy, 1992) that having $\mathrm{E}[U(V_{P_1,H})] \geq \mathrm{E}[U(V_{P_2,H})]$ for two portfolios P_1 and P_2 and for any increasing, concave utility function (where the inequality is strict for at least one particular $U(\cdot)$) is equivalent to portfolio P_1 being second-order stochastically dominating portfolio P_2.

2. Probabilistic representation

The representation of risk through stochastic dominance is particularly insightful because it reflects the idea that the risk associated with the investment problem (1-16) can be directly measured from the distribution of terminal wealth $V_{P,H}$.

The probabilistic approach also represents a shift in the focus on risk: While the utility function is inherently related to the investor's attitude toward risk (a subjective measure), the distribution of terminal wealth (or, equivalently, return) can be objectively constructed from the data using, for instance, statistical inference or parametric modeling. Nevertheless, the subjective assessment of risk is not entirely ruled out in the probabilistic approach. Once the distribution of returns is determined, the investor must decide which aspects of that distribution they like or dislike.

In this regard, the investor's attitude toward risk can be conveniently subsumed within their tastes for the different moments of the return distribution. It is usually acknowledged that rational investors enjoy higher odd moments and lower even moments. As a matter of fact, a non-satiable investor likes a higher expected terminal wealth, all else being equal. A risk-averse investor dislikes the variance surrounding that terminal wealth. However, even if the attitude toward the third moment is usually a preference for positive skewness, such behavior may not hold for all agents in all markets. The fourth moment (also known as kurtosis) represents the variance of the variance and indicates the likelihood of extreme events, which investors typically dislike. It is conceptually difficult to associate investors' behavioral traits with moments of order higher than four.[11]

It therefore appears that, for a given return distribution, a measure of risk can be designed by penalizing to various extents the exposure to variance, negative skewness, and kurtosis. A technical limitation associated with this approach is the aggregation over time. When portfolio performance must be evaluated over several periods, the return expectation and variance grow proportionally with time under mild assumptions, but no similar rule exists regarding skewness and kurtosis.

[10] Stochastic dominance is a preference rule between two outcomes that involves the shapes of their cumulative distribution functions (CDFs). See Chapter 16 for a formal analysis.

[11] A more detailed analysis of the structure of investors' preferences is carried out in Chapter 9.

3. Downside and upside risks

An important question affecting the representation of risk is whether the investor should only worry about the potential losses or if they should find a way to trade off the exposures to gains and losses. There are, in fact, three ways to address this dilemma. The first one (and oldest one in the development of portfolio theory) is to penalize upside and downside wealth variations equally. This only makes sense if the return distribution is symmetric (like the normal distribution, for example). Such behavior leads to adopting a risk measure such as the standard deviation.

A second attitude is to focus exclusively on downside risk. Assuming investment portfolios are made of long positions in assets, risk exposure is then assessed from the shape of the left tail of the return distribution. Note that this behavior can be motivated by subjective reasons (such as aversion to regret) or by objective reasons (avoidance of costly frictions such as illiquidity or bankruptcy). Examples of corresponding risk measures[12] include the negative semi-variance, the Value-at-Risk (VaR), the Conditional Value-at-Risk (CVaR), and Young's (1998) minimax criterion.

Finally, an investor with a more comprehensive view might want to adjust their appreciation of downside risk with the potential for gains. This leads to performance metrics combining gains and losses (typically in the form of a ratio), that are examined in a greater detail in Chapter 10.

1.3.2.2 Risks of portfolios

The discussion up to this point has considered the risk affecting the terminal wealth $V_{P,H}$ as a whole (i.e., as if it were composed of a single asset). However, since P is a portfolio made of many different assets, its overall risk is the sum of all individual risks of its components. As such, the portfolio risk is impacted by the *diversification* effect and by *correlation risk*.

Diversification refers to the gradual elimination of idiosyncratic risk as the number of assets in the portfolio increases. Indeed, correlation among many assets tends to cancel out shocks in returns that go in opposite directions. When this mechanism is applied on a large scale, mostly co-movements in asset returns affect the portfolio value, and such exposure to covariance risk can be interpreted as systematic risk. However, diversification not only reduces total risk but also alters the moments of the portfolio return distribution and thereby the nature of risk that the investor is exposed to.

To illustrate this, Figure 1-4 shows the evolution of the portfolio return standard deviation (σ), skewness (s), and kurtosis (k) as a function of the number of assets (N). The individual

[12] Ortobelli et al. (2005) refer to them as "safety-first" risk measures.

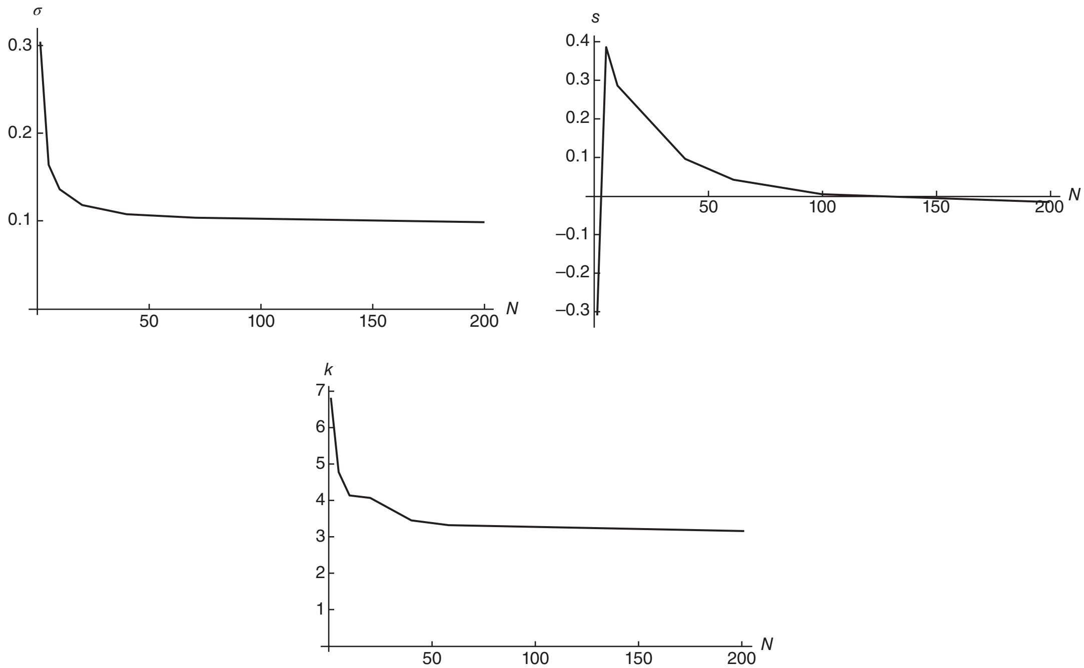

FIGURE 1-4 Example of diversification on portfolio return standard deviation σ, skewness s, and kurtosis k.

asset returns are generated by a Cornish–Fisher expansion that accommodates slight deviations from normality.[13] To simplify the exposition, all individual asset returns display similar characteristics (same σ, s, and k) and are linked by the same pairwise correlation coefficient (equi-correlation). More specifically, these returns are slightly negatively skewed, leptokurtic (i.e., the tails of their distribution are fatter than the normal tails), and they are positively correlated with one another.

The example displayed in Figure 1-4 is only one among many different situations depending on the characteristics of the individual assets. It, however, highlights some common effects of diversification. First, the standard deviation rapidly decreases with N and stabilizes at some incompressible level, its amount of systematic risk. Second, a similar phenomenon applies to the kurtosis: The exposure to some extreme outcomes cannot be fully diversified away. These two diversification effects should be welcomed by the risk-averse investor. However, the behavior of skewness is more complex. The individual negative skewness is quickly flipped into a positive value for a small number of assets. Then, the portfolio skewness erodes with portfolio size and can ultimately revert to negative values. Depending on the investor's preferences, the combined impact of diversification on portfolio skewness and kurtosis may or may not be favorably perceived.

[13]See Chapter 6 for a detailed presentation of the Cornish–Fisher expansion.

Another source of risk engendered by the constitution of portfolios is correlation risk. Early asset pricing models have assessed correlation risk with return covariance and defined systematic risk accordingly. However, standard statistical tools such as the covariance only measure a linear form of correlation. In reality, the way asset returns move together can be more difficult to fathom, and specific techniques are required to gauge the dependence between extreme returns. This is of practical importance because the joint realization of extreme events (such as defaults) can create a high level of stress on the portfolio – sometimes referred to as *tail risk*.

Some methods of portfolio performance appraisal have incorporated measures of dependence. One basic approach is to look at pairwise rank correlations between asset returns.[14] A more advanced approach is the study of the multivariate joint distribution of asset returns through their marginal behavior and dependence structure. That approach uses *copulas* to measure highly non-linear forms of dependence that manifest in tail risk.

1.3.2.3 Axioms for a risk measure

Given the discussion above, several scholars have tried to develop a list of desirable properties that a risk measure should display. To date, one of the most widely accepted axiomatic approaches for the definition of a risk measure is that of Artzner, Delbaen, Eber, and Heath (1999). A risk measure can be characterized as a function $\mathcal{R}_{P,H}$ that assigns a non-negative real number (a score) to the random variable "the value of portfolio P at horizon H." Artzner et al. (1999) posit that $\mathcal{R}_{P,H}$ is a *coherent* risk measure if it satisfies the following properties:

- *Subadditivity*: For two portfolios P_1 and P_2, $\mathcal{R}_{P,H}(P_1 + P_2) \leq \mathcal{R}_{P,H}(P_1) + \mathcal{R}_{P,H}(P_2)$. This property implies that the risk score should reflect the benefits of diversification.
- *Positive homogeneity*: For any positive scalar c, $\mathcal{R}_{P,H}(cP) = c\mathcal{R}_{P,H}(P)$. This property implies that any leveraged portfolio should increase the risk score in proportion to the leverage factor.
- *Translation invariance*: For any scalar c and denoting by R_f the periodic risk-free rate, $\mathcal{R}_{P,H}(P + c(1 + R_f)^H) = \mathcal{R}_{P,H}(P) - c$. This property implies that any investment in the risk-free asset should diminish the risk score by the invested amount.
- *Monotonicity*: If in all states of the world, $P_1 \leq P_2$, then $\mathcal{R}_{P,H}(P_2) \leq \mathcal{R}_{P,H}(P_1)$. This property implies that the risk measure should focus on the downside risk.

Alternatively, the first two properties can be replaced with a convexity condition, i.e., for two portfolios P_1 and P_2 and any scalar c between 0 and 1, $\mathcal{R}_{P,H}(cP_1 + (1 - c)P_2) \leq c\mathcal{R}_{P,H}(P_1) + (1 - c)\mathcal{R}_{P,H}(P_2)$. The convexity property being less stringent than combined subadditivity and

[14] The main corresponding statistical tool is Kendall's tau discussed in Chapter 17.

positive homogeneity, a risk measure that complies with convexity, translation invariance and monotonicity is said to be weakly coherent (Carr, Geman, and Madan, 2001).

This list of axioms serves as a guideline for a proper assessment of risk. As we will see, several (but not all) portfolio performance metrics rely on risk measures that meet some (but not all) of these criteria. Table 1-5 (inspired by Ortobelli et al., 2005) lists some of the classic risk measures used in portfolio performance appraisal and checks their conformity with coherence.

Note also that all the risk measures listed in this table are consistent with second-order stochastic dominance (Section 1.3.2.1), except for the Value-at-Risk.

In a recent simulation study on nearly 200 000 portfolios of US stocks, Righi and Borenstein (2018) examine the rankings of portfolio strategies that solve either a risk minimization problem such as Equation (1-17) or a risk-adjusted value maximization problem such as Equation (1-16). The optimization problems are solved using 11 popular risk measures. The authors' major finding is that, despite some variations in the portfolio rankings, there is no risk measure that is clearly preferable to the others, in the sense that it allows us to achieve a significantly better optimum. This result does not mean that risk measures are pointless. Rather, it calls for a robust analysis of risk in the appraisal of portfolio performance.

1.3.3 Benchmarks

The concept of a benchmark is pervasive in the asset management literature. The reason is that for most investors, performance must be evaluated in relative terms.[15] Indeed, the comparative approach in appraising an (active) investment strategy reflects the scarcity of investable

TABLE 1-5 Properties of some common risk measures (adapted from Ortobelli et al., 2005).

Risk measure	Subadditivity	Positive homogeneity	Translation invariance	Monotonicity	Convexity
Standard deviation	✓	✓	X	X	✓
Mean absolute difference	✓	✓	X	X	✓
Value-at-Risk (VaR)	X	✓	✓	✓	X
Conditional VaR	✓	✓	✓	✓	✓
Minimax criterion	✓	✓	✓	✓	✓

[15]Notable exceptions are hedge funds that design and market investment strategies with the goal of generating performance in absolute terms.

resources and the associated opportunity cost. Simply put, the investor wants to know not only "*How much money have I made?*" but also "*How much money could I have made considering the (less expensive, less time-consuming, passive) alternatives?*"

The assessment of performance therefore requires the proper identification of a benchmark. We first address the gap between the ideal market reference in theory and the available benchmarks in practice. Then, we briefly elaborate on the qualities that a relevant benchmark should have.

1.3.3.1 The market portfolio and the benchmark

From the onset of its development, the SPT has established that efficient financial markets would reward only those active asset managers that can do better than what everybody else obtains with buy and hold strategies. In equilibrium, the positions of all these passive investors result in the so-called *market portfolio*, i.e., the sum of all investable assets weighted by their market capitalization. Therefore, in theory at least, the market portfolio turns out to be the proper benchmark for active investors.

Unfortunately, the market portfolio is hardly observable in practice. Given the complex investment universe, it is virtually impossible (or excessively too costly) to replicate all positions in all investable assets.[16] Thus, we are left to resort to more or less imperfect proxies for benchmarking (such as market indices or ETFs).

These approximations of the market portfolio lead to several practical challenges for portfolio performance evaluation. Chapter 5 provides a thorough discussion of these issues in the context of a one factor asset pricing model. In a multifactor framework, the discrepancy between the analytical benchmark underlying the multifactor model and the self-reported benchmark adds another layer of difficulty in finding the correct proxy for performance assessment (see Chapter 7).

The imperfect identification of a benchmark also has implications for performance attribution, whose goal is to adequately recognize the talents of portfolio managers (ability to select securities and/or market timing skills). This is analyzed in Chapter 13.

1.3.3.2 Some desirable properties for a benchmark

Even though the choice of a relevant benchmark is a case-by-case decision impacted, among other things, by the type of strategy, investment style, and asset classes, there are some

[16] It might be more feasible if the universe is restricted to an asset class. However, then the question is whether the strategy should be benchmarked with such a restriction in the first place.

fundamental properties that a benchmark should display. We shall emphasize four of them and claim that a benchmark should be:

- *Transparent*: The construction method for the benchmark should be accessible and clear to everyone (the investor, the portfolio manager, the portfolio manager's supervisor, any external auditor, ...).
- *Investable*: The benchmark should represent a feasible alternative for passive investment. This condition also applies to the position sizes that the manager can achieve.
- *Representative*: A significant proportion of assets managed in the portfolio should be present in the benchmark.
- *Manipulation proof*: The portfolio manager should behave as a price taker for all the constituents of the benchmark.

Related to transparency, Hamilos and Ribando (2016) advocate that investors should understand the implications of the benchmark construction method. They argue that benchmarks never truly reflect a passive investment, but rather implicit allocation decisions made by the benchmark provider. They cite the example that market-cap-weighted indices overweight high momentum securities (see also, Treynor, 2005, for a related point).

In terms of representativeness, Bailey (1992) further points out that the benchmark turnover should be low (for a sound comparison over the long run), and that the correlation between the manager's performance against the market and the benchmark's performance against the market should be high.

With numerical experiments to support their claim, Amenc, Goltz, and Lodh (2012) note that beyond the selection of constituents, the weighting scheme of the benchmark also has an impact on the assessment of strategies.

Key Takeaways and Equations

- The management of a financial portfolio, analogously with any transformation process, follows a three-stage procedure: *prepare, execute, and control*. A feedback loop is triggered by the controlling step, which combines the verification of the performance and the achievement of the process.
- Portfolio performance corresponds to the concept of efficiency of the process, whereas the achievement of initial objectives indicates the process's effectiveness. These concepts are summarized by the following equation:

$$\left\{X_{P,1}, \ldots, X_{P,K}\right\} \underset{\text{efficiency}}{\Rightarrow} Y_P \underset{\text{effectiveness}}{\Longleftrightarrow} Y_P^* \qquad (1\text{-}1)$$

(*continued*)

(*continued*)

- The inputs of the process are mainly fourfold: cash outlay, costs, time, and risk:

$$\{X_{P,1}, \ldots, X_{P,K}\} \equiv \{V_{P,0}, C_{P,H}, H, \mathcal{R}_{P,H}\} \tag{1-2}$$

  whereas the output is generally considered to be the final portfolio value:

$$Y_P \equiv V_{P,H} \tag{1-3}$$

- The rate of return is a measure of performance, but it does not account for risk. The two main versions of a *risk-adjusted performance measure* are generically written as

$$\mathrm{PM}_P^{\mathrm{abs}} \equiv f_{\mathrm{all}}(R_{P,H}; H, \mathcal{R}_{P,H}) \tag{1-6}$$

$$\mathrm{PM}_P^{\mathrm{rel}} \equiv f_{\mathrm{all}}(R_{P,H}; H, \mathcal{R}_{P,H}, R_{B,H}, \mathcal{R}_{B,H}) \tag{1-7}$$

  - The first version is usually written in the form $\mathrm{PM}_P^{\mathrm{abs}} = \frac{\mathcal{P}(R_{P,H})}{\mathcal{R}_{P,H}}$ and is interpreted as an *efficiency ratio*.
  - The second version is usually written as $\mathrm{PM}_P^{\mathrm{rel}} = \mathcal{P}(R_{P,H}) - \mathcal{P}(R_{B,H})$ and is interpreted as an *output gap*.
- We consider the feedback loop surrounding the performance evaluation process as a circuit that connects the preparation, execution, and control stages. Consequently, it constitutes the red wire of the book:
  - *Performance appraisal* refers to understanding the past. Parts I and II of the book are dedicated to this dimension. It results in the identification of 116 measures analyzed through nine chapters.
  - *Performance analysis* corresponds to explaining the present. It features in particular the very important performance attribution analysis, which can take many forms, studied in Part III.
  - *Acting upon performance* involves preparing the future. Part IV proposes different applications of this point of view, including the use performance in predictive analyses.
- For the purpose of the book's technical developments, we need to define some key concepts. These involve the *returns*, *risk* and its measurement, as well as the identification of *benchmarks*.

- Returns can be expressed in discrete or continuous time, leading to slightly different compounding rules. Their mean can be computed with a *geometric average* (that reflects the actual compounding over several periods)

$$\overline{R}^{\text{geo}}_{P,t_N} = \sqrt[N]{\prod_{i=1}^{N}(1 + R_{P,t_i})} - 1 \tag{1-8}$$

  or with an *arithmetic average* (that reflects, from a statistical standpoint, the compounding conditions to be expected):

$$\overline{R}^{\text{ari}}_{P,t_N} = \frac{1}{N}\sum_{i=1}^{N}(1 + R_{P,t_i}) - 1 \tag{1-9}$$

- In the presence of significant interim cash flows, the actual portfolio growth rate should be measured with *time-weighted returns*. If the value of the portfolio cannot be updated regularly, an internal rate of return can be calculated.
- Risk can be apprehended from the expected utility theory or from a probabilistic approach. In the latter case, the attitude toward risk can be fully captured from the asset return distribution. A simplified approach consists of gauging the preference toward the *moments of the distribution*.
- Aggregating assets into portfolios exposes the investor to the effects of *diversification* (affecting the moments of the return distribution) and to *correlation risk*.
- Many risk measures have been used in the field of portfolio performance appraisal. *Consistency* (with respect to second-order stochastic dominance) and *coherence* (with respect to the axioms of Artzner et al., 1999) help appreciate the quality of a risk measure.
- The wedge between the unobservable market portfolio and the benchmark used in practice is a source of errors in performance appraisal. Several chapters elaborate on how to mitigate the impact of these errors.
- Educated investors should understand the limitations in any benchmark, not only in relation to the strategy under assessment but also inherently associated with its construction method.

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