1 Introduction

At the time of the writing of this book, and since about the turn of the century, commodities have experienced a continued surge in both price (as shown in Figure 1.1) and interest in a world of increasingly scarce resources and rapid population growth together with demand growth in the rapidly industrialising emerging markets. This book is a practical quantitative analyst's guide to how to get professionally involved in the world of commodities.

While the book is focused primarily on the market in commodity derivatives and the quantitative analysts (quants) who work in these markets, much commodity trading occurs through the vehicle of futures trading on organised exchanges, so I envisage this work will be of use to the traders, quant developers, structurers and finance professionals who work alongside the quants. Though this is first and foremost a practitioners' book, I have attempted to put the material into context with regard to the literature in the area, which I believe will be of tangible benefit to academics and students of financial mathematics from all areas, who are interested in learning more about the fascinating world of commodities.

Although this is a technical book, I have attempted to make it as accessible as possible on several levels. One barrier to making the transition into commodities is the necessary, and unavoidable, jargon, which I have tried to cut through for the benefit of the reader (see the Glossary at the end of the book, for example). This I hope should set the avid reader on course to apply the theory from his or her studies to build the models and systems required to add real value to a commodities desk. Content has been developed using real-world data throughout and has been written in conjunction with both industry professionals and university lecturers in commodities.

Once again, the preferred mathematical background for the derivatives elements of this book is a familiarity with option pricing at the level of Hull (2011) and, ideally, Baxter and Rennie (1996). There is necessarily some overlap with my previous FX book (Clark, 2011) but this book is a standalone work and introduces the various commodity markets

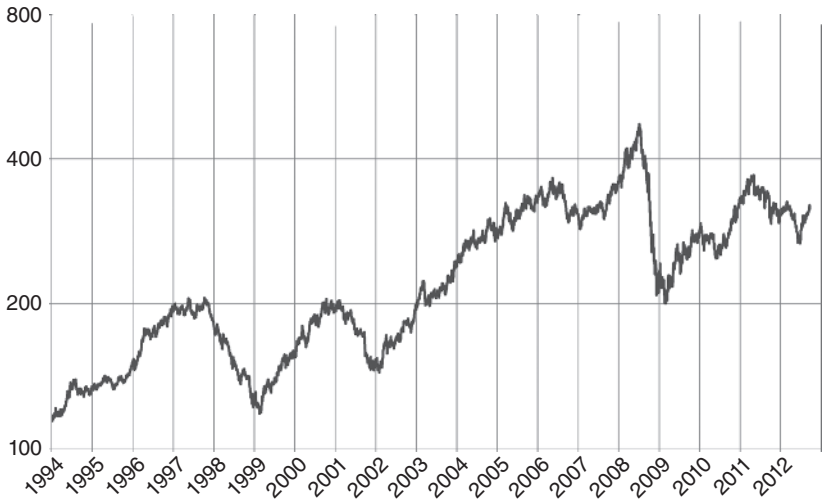


Figure 1.1 Price history of TR/J CRB Commodity Index (1994–2012, log-scale).

and develops the option pricing toolkit for commodity derivatives in a self-contained manner. This book is not purely mathematical, however; it is important to have some familiarity with the physical aspect of the various commodities also, in order to develop intuition and to have credibility when talking with traders (or in interviews). I have therefore attempted to relate the technical machinery back to the practical aspects throughout.

Producers, intermediaries and consumers are all exposed to risk. Additionally, some investors have been increasingly attracted towards speculative investment in some commodities due to the benefits of diversification and perceived underperformance in other asset classes. Finally, the durable physical aspect of some commodities (especially the metals and energy) may appeal to those who have lost money on purely financial investments during the collapse of the dot com equity bubble and the subprime financial crisis thereafter.

As well as trading the physical commodity itself, it is commonplace to trade financial contracts linked to commodities. Simple examples are futures contracts on oil and the base metals; more complicated are option contracts which provide the owner with the right but not the obligation to take profits on price moves in a particular direction. Commodity derivatives also allow participants in the financial markets to spread their risk exposure over the course of a calendar period, and to

"lock-in" profits on a spread between the price of a refined product and an unrefined product.

This book shows how these derivatives are priced in the industry context. It is a practitioner's guide which introduces commodity options, describing the features of the various commodity markets and what industry professionals need to know when developing option pricing analytics for trading desks and risk departments. What sort of products might one encounter? What typical price quotes might one obtain and have to calibrate a model to? What makes oil options different from precious metal or base metal options, for example.

These questions, and more, are the concern of this book.

1.1 TRADE, COMMERCE AND COMMODITIES

The oldest financial markets are the commodity markets. By a commodity, we mean an undifferentiated physical item that satisfies an economic want or need. The oldest commodity markets are therefore the agricultural and metal markets – wheat, gold, etc. Energy (fossil fuels) is a comparatively recent commodity. Differentiation is what makes gold as a commodity different from gold as jewelry or coinage, for example, where there are particular features that may have particular appeal to the purchaser. In contrast, wheat of a particular grade is wheat, and so long as it is of the promised grade and quality, no one can be expected to care too much about anything apart from the actual physical amount available. That is what makes it a commodity.

As described in Section 7.2 of Clark, Lesourd and Thiéblemont (2001), one can categorise production systems into four various modes of production using Woodward's classification, as described in Table 1.1.

From this we see that commodities are specifically outputs of process production, in that they are standardised (within various grades, typically) and divisible. Note that this has not always been the case, agriculture historically used to be more of a craft production enterprise. Commodities are generally tradeable goods, though there are some instruments that sometimes fall under the heading of commodity derivatives which are not, e.g. freight derivatives, weather derivatives, and other instruments that more closely resemble insurance products.

Within tradeable goods, there are some specific features of commodity markets within the sector of process production. Firstly, these goods generally have a long lead time for production (electricity generation being the notable exception) and generally have a liquid supply and

Table 1.1 Woodward's categorisation of production systems.

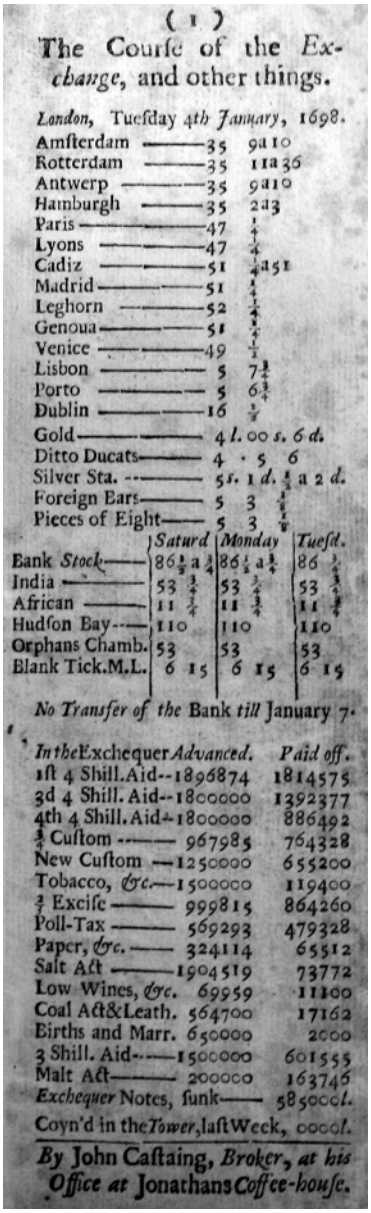
System	Features	Examples
Mass production	Large scale production of an indivisible standardised product	Automobile manufacturing
Process production	Large scale production of a divisible standardised product	Food, chemical manufacturing
Craft production	Small scale production of a nonstandardised product	Fashion
Project production	Production of a unique product to bespoke client specifications	Infrastructure construction (e.g. airports, bridges), building construction

demand market with reference pricing – this means that an equilibrium between supply and demand can in principle be found. In times when supply and demand are greatly out of line, generally inventories and storage come into play, as physical commodities can usually be warehoused subject to storage costs (electricity once again being an exception). This does, however, have the effect of increasing volatility both in historical terms and implied volatility (see Chapter 2) for commodities.

Basically, commodities are comparatively simple tangible goods created by means of process production, differentiated purely by price and prespecified grades of quality rather than by any specific branding or differentiating factors.

Note the difference between upstream and downstream commodities – upstream means that the commodity in question is used primarily to satisfy industrial demand (e.g. crude oil) and may well be reprocessed, whereas downstream means that the commodity is marketed to end-users, such as gasoline and gold. This is based largely upon the role of the consumer, not the producer.

We can see evidence of early commodity markets. From 1698, a merchant called John Castaing published twice a week a series of pocketbooks called "The Course of the Exchange" (see Figure 1.2) tabulating the price of various stocks and commodities as gold, silver, pieces of eight and so forth, as traded at Jonathan's Coffee House in the City of London. It is worth noticing that commodity trading is linked to trade, in that most commodities are produced or extracted at a distance from where they are finally used. Markets such as Smithfield cattle market in London are an example, with cattle being driven to market from the countryside for consumption in the city. Other historical markets are



(1)

The Courſe of the *Exchange*, and other things.

London, Tueſday 4th *January*, 1698.

Amſterdam	35	9a10
Rotterdam	35	11a36
Antwerp	35	9a10
Hamburgh	35	2a3
Paris	47	
Lyons	47	
Cadiz	51	a51
Madrid	51	
Leghorn	52	
Genoua	51	
Venice	49	
Lisbon	5	7
Porto	5	6
Dublin	16	
Gold	4 *l.* 00 *s.* 6 *d.*	
Ditto Ducats	4 · 5	6
Silver Sta.	5*s.* 1 *d.*	a 2 *d.*
Foreign Ears	5	3
Pieces of Eight	5	3

	Saturd	*Monday*	*Tueſd.*
Bank *Stock*	86 a	86 a	86
India	53	53	53
African	11	11	11
Hudſon Bay	110	110	110
Orphans Chamb.	53	53	53
Blank Tick.M.L.	6 15	6 15	6 15

No Transfer of the Bank *till* January 7.

In the Exchequer *Advanced.*	*Paid off.*
1ſt 4 Shill. Aid -- 1896874	1814575
3d 4 Shill. Aid -- 1800000	1392377
4th 4 Shill. Aid - 1800000	886492
Cuſtom ---- 967985	764328
New Cuſtom -- 1250000	655200
Tobacco, *&c.* -- 1500000	119400
Exciſe ---- 999815	864260
Poll-Tax ---- 569293	479328
Paper, *&c.* ---- 324114	65512
Salt Act ---- 1904519	73772
Low Wines, *&c.* 69959	11100
Coal Act & Leath. 564700	17162
Births and Marr. 650000	2000
3 Shill. Aid ---- 1500000	601555
Malt Act ---- 200000	163745
Exchequer Notes, ſunk ----	585000*l.*
Coyn'd in the *Tower*, laſt Week,	0000*l.*

By John Caſtaing, *Broker, at his Office at* Jonathans *Coffee-houſe.*

Figure 1.2 Castaing's "The Course of the Exchange" (1698).

well known, for example the Forum in Rome, the Agora in Athens and the Roman Forum in London (under the present day site of Leadenhall Market), all of which were markets for cash transactions for immediate delivery.

Obviously these ancient empires did not last, and for a while thereafter markets became more fragmented and localised. Not indefinitely, though. By the 12th century, however, medieval fairs had become increasingly popular in England and France (among other locations), where the markets were preannounced and the so-called *Pieds Poudres*[1] travelled from town to town organising the fairs. What often happened at these travelling fairs is that samples were sold for a cash price, together with a contract for later delivery of merchandise in accordance with the quality of the established sample. In such a manner, the forward market began to take hold.

As trade routes opened up, transport of commodities over greater distances became viable and quite profitable – the silk route across Central Asia, the spice trade with East India, transport of silver from the mines of South America to Spain and Portugal, etc. Other more localised examples certainly existed: the speculation in tulips in the Netherlands from 1634–37, for example. In parallel, specialised markets developed in the major cities and replaced the trade fairs, these centers being called exchanges or bourses.[2]

Relative scarcity in one region and abundance in another drives the markets to develop trade routes – a modern example being the continuing shipment of crude oil from the Middle East to Asia, Europe and the Americas. A trade route has to have a start and a finish and, at the destination end, markets developed around the construction of large city docks, warehouses and markets such as the docks in the City of London (both around London Bridge and the geographically named docks around Canary Wharf) and similarly in other cities – Paris, New York, Chicago and so on. Relatively sophisticated systems for landing bonded cargos and making payments of the various customs duties were part of this grand system of commerce, all being part of the trade in the physical commodity.

Financial markets, however, take a different view and allow participants to trade with different time horizons. Consider a bond – one can deposit a certain amount of cash and obtain the principal at some fixed

[1] Men of dusty feet.

[2] We can thank an 18th-century Bruge innkeeper named Van der Beurs for this name.

time in the future, together with a series of coupon payments. Separating cashflows over different time horizons is one of the features of the financial markets, and one that certainly applies to commodities.

The simplest and earliest financial instruments are the forward and futures contracts (see Chapter 2), which allow buyers and sellers to fix a price today for purchase of a commodity at some prespecified time in the future. These have been around a surprisingly long time. As well as the OTC forward transactions at medieval trade fairs, we can go back to around 1730 to see forward contracts on rice trading at the Dojima Rice Market in Osaka (cash trading in rice on that market dates back only some 30 years earlier). Interestingly, there was also trading on edible oils, cotton and precious metals in Dojima, but the volumes were completely dwarfed by those for rice.

Commodity markets in the USA date back to 1750 or so, but it was not until the early 1800s that futures trading in the USA really took off. Much of this growth can be linked to the urban growth of Chicago itself, a rapidly growing city which served as a grain terminal for the fertile lands of the Midwest. As anyone who has visited knows, transport in the region can be easily disrupted by inclement weather, and this was undoubtedly much more the case in the early 1800s. Snow and rain made transport exceedingly difficult and wagonloads of grain were often brought in on plank roads made of wooded boards. Once in Chicago, there were still problems with inadequate warehouse storage, which immediately led to the unfortunate state of affairs where gluts sometimes had to be disposed of in the street if a buyer could not be found. One problem is that harvests are generally gathered in late autumn/early winter, when the problems with transportation are getting worse – just when you need to get the goods to market (basically around Thanksgiving holiday in North America, which lines up with harvest season).

Forward contracts were first used by river merchants who received corn from farmers but had to store it themselves until spring, when the corn had dried sufficiently and the rivers and canals were free of ice to make transportation possible. The first recorded forward contract on corn was made on 13 March 1851, for 3,000 bushels of corn to be delivered to Chicago in June at a one cent discount to the March price. Wheat contracts developed subsequently, and tended to be sold for delivery forward to millers and exporters east of Chicago.

In 1848, 82 merchants founded the Chicago Board of Trade (CBOT), a centralised location for trading forward contracts on agricultural commodities. In early days, there was no standardisation with respect

to quality or delivery time, and it was not infrequent that merchants and traders reneged on their commitments. In 1865, therefore, the CBOT developed futures contracts which were standardised with regard to quality, quantity and time and place of delivery for the underlying commodity. Later that year, a margining system was set up also to mitigate against the underlying counterparty risk.

The choice of months may seem arbitrary, but in fact was very much by design. March was chosen to coincide with the end of winter, when transportation became feasible again. May was a preferred delivery month due to old-crop oats and wheat, harvested in the previous summer. December was chosen for new-crop corn (harvested in the autumn) and also because it was the final month before inclement weather would make attempts at winter transportation foolhardy.

From futures it was only a small step to introducing options on futures – these were introduced in 1984 as options on soybean futures and 1985 saw the introduction of options on corn futures contracts.

The CBOT has since merged with other exchanges including the Chicago Mercantile Exchange (CME), and now encompasses the New York Mercantile Exchange (NYMEX) as part of the CME group, showing that the history of trading in financial products in commodity markets is a long and continuing one.

For those who are interested, a great deal of further historical information about the development of the commodities and futures markets can be found in CBOT (1989).

It is financial contracts such as these, based on the various commodities, together with more modern contracts, such as options, that will be the focus of this book.

1.2 ADAPTING TO COMMODITIES AS AN ASSET CLASS

Many of the readers of this book may well not have commodities as their only area of interest, it is quite possible that they will have some exposure to other asset classes such as equities, fixed income and foreign exchange. This book attempts to demonstrate how some of the techniques which are commonly seen in option pricing in those asset classes can be used for commodity option pricing.

The first thing to note is the immense variety of different types of commodities that can be encountered. We can do worse than survey the various commodity indices to get an idea of what generally falls within the realm of commodities.

Thomson Reuters/Jefferies CRB Index

The TR/J CRB index dates back to 1957, when the prices of 28 commodities were tabulated in the 1958 CRB Commodity Year Book. It is the oldest of the commodity indices.

The index currently is comprised of the prices of 19 commodities. We have already seen a historical chart of the TR/J CRB index from 1994 to mid-2012 in Figure 1.1 at the start of the book.

S&P GSCI Index

The GSCI index comprises 24 commodities from various commodity sectors – energy, industrial metals, agricultural products, livestock products and precious metals. It is the second oldest of the commodity indices, and dates back to 1992.

The GSCI is calculated on a production-weighted basis and is comprised of physical commodities that are the subject of active, liquid futures markets. The weightings are far from equal, energy makes up over 78% of the basket and precious metals slightly under 2%. In fact the top six components by weighting are easily the six commodities within the energy category, with corn, wheat, copper and aluminium making up the remaining top ten.

DJ UBS Index

The Dow Jones-UBS commodities index is composed of commodities traded on the exchanges, with the exception of aluminium, nickel and zinc, which are traded on the London Metal Exchange (copper is traded on COMEX).

RICI Index

The Rogers International Commodity Index (RICI) is a measure of the price action of commodities on a worldwide basis. It currently includes 37 commodities weighted according to their importance in international trade, from WTI crude oil (21%), Brent crude oil (14%), Corn and Wheat (4.75% each) down to Milk (0.15%).

1.2.1 Classification of Commodities into Sub-categories

One can see from Tables 1.2 to 1.5 that commodities can be broadly categorised into three major types – energy, metals and agricultural commodities – being further divisible into subtypes. We also include power (electricity) and a fifth "other" category for commodities which do not neatly fall into the groupings above, such as lumber and all the contracts to be discussed in Chapter 9. Figure 1.3 shows a useful categorisation we can use to place the various commodities we shall encounter into various subtypes.

Table 1.2 Composition of Thomson Reuters/Jefferies CRB Index.

Energy	Industrial metals	Precious metals	Softs	Grains & seeds	Livestock
WTI crude oil	Al	Au	Cocoa	Corn	Live cattle
Heating oil	Cu	Ag	Coffee	Wheat	Lean hogs
Unleaded gas	Ni		Cotton	Soybeans	
Natural gas			Orange juice		
			Sugar		

Table 1.3 Composition of S&P GSCI Index.

Energy	Industrial metals	Precious metals	Softs	Grains & seeds	Livestock
WTI crude oil	Al	Au	Cotton	Wheat	Live cattle
Brent crude oil	Cu	Ag	Sugar	Red wheat	Feeder cattle
Unleaded gas	Pb		Coffee	Corn	Lean hogs
Heating oil	Ni		Cocoa	Soybeans	
Gas oil	Zn				
Natural gas					

Table 1.4 Composition of Dow Jones–UBS Commodities Index.

Energy	Industrial metals	Precious metals	Softs	Grains & seeds	Livestock
Natural gas	Cu	Au	Coffee	Corn	Live cattle
WTI crude oil	Al	Ag	Cotton	Soybeans	Lean hogs
Brent crude oil	Zn		Sugar	Wheat	
Unleaded gasoline	Ni			Kansas wheat	
Heating oil				Soybean meal	
				Soybean oil	

Table 1.5 Composition of Rogers International Commodity Index.

Energy	Industrial metals	Precious metals	Softs	Grains & seeds	Livestock & others
WTI crude oil	Al	Au	Cotton	Corn	Live cattle
Brent crude oil	Cu	Ag	Coffee	Wheat	Lean hogs
Natural gas	Pb	Pt	Cocoa	Soybeans	Milk class III
RBOB gasoline	Zn	Pd	Sugar	Soybean oil	Lumber
Heating oil	Ni		White sugar	Kansas wheat	
Gas oil	Sn		Orange juice	Milling wheat	
			Rubber	Rapeseed	
				Rice	
				Soybean meal	
				Oats	

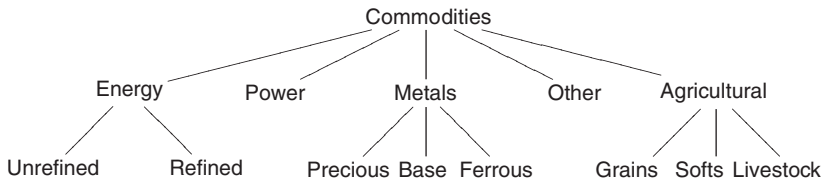


Figure 1.3 Taxonomic classification of commodities.

Energy commodities broadly can be separated into *primary* or unrefined energy products such as crude oil, coal, natural gas and nuclear fuels, which are generally obtained and traded in a standardised but unrefined form for further processing, and *secondary* or refined energy products such as residual oil, fuel oils, heating oil, diesel fuel, RBOB gasoline (US) or petrol (UK), jet kerosene and liquefied petroleum gas (LPG) which are sold to end-users. There are intermediate energy products which do not fit so neatly into this categorisation, such as ethylene and naphtha, but these are of minor importance compared to the refined and unrefined energies.

Not usually included with energy but clearly related to it is **power**, by which we mean electricity. This differs markedly from the other energy commodities in that it cannot easily be stored on an industrial scale – there is such a thing as a "barrel" of electricity, but we probably know it as a battery. This greatly affects the volatility and the pricing of electricity derivatives, as we shall discuss in Chapter 7.

Metals broadly can be separated into three major categories: (i) *precious metals* such as gold, silver and various platinum group metals; (ii) *base metals*, which are industrial non-ferrous metals such as copper, aluminium, lead, nickel, tin and zinc; and finally (iii) *ferrous*, by which we mean iron ore, iron, rolled iron, or steel or refined steel products such as rolled products, flat products, wires, bars and beams.

Additionally to these three categories, but of comparatively minor importance, we can include specialist non-ferrous metals such as chromium, molybdenum, titanium, tantalum, vanadium and the rare earth metals such as scandium, yttrium and the lanthanides. Many of these have niche applications in technology, e.g. tantalum is used in capacitors for mobile phones, and the rare earth metal neodymium is used in the rare-earth magnets found in precision guided munitions. While interesting, there are no derivatives markets on these, so we shall not need to discuss them any further in this book.

Agricultural derivatives can be similarly categorised into three major categories: (i) *softs* such as the main four: cocoa, coffee, sugar and rubber (Savaiko, 1985) but also including orange juice, tea, cotton, jute, wool, jute and hides; (ii) *grains and seeds* such as wheat, corn, soybeans and soybean products, rice and oats; and (iii) *livestock* such as live cattle, feeder cattle, lean hogs and pork bellies. It is interesting that the soft commodities are mostly of tropical or subtropical origin.

Finally, we reserve the **other** derivatives designation for those which are hard to categorise into the preceding four classes, such as lumber, milk, freight and weather derivatives, carbon emissions, bandwidth derivatives and water (potentially).

1.3 CHALLENGES IN COMMODITY MODELS

1.3.1 Futures

Readers who come to commodities from other asset classes, like equity or FX, are typically used to the concept of a spot price process, the price one pays today to take either immediate delivery or delivery at the spot date (usually two good business days later) of the investment asset. While this is true for precious metals, it is not true for most of the other commodities discussed in the book, which trade for forward delivery at agreed times in the future, in the futures markets. The reason for this is clearly evident from our discussion in Section 1.1, it relates to the uncertainty in matching supply and demand for goods that have a long lead time to produce and deliver.

What this means from a modelling perspective is that we need to regard the futures contracts as the underlying stochastic variables which need to be modelled – either individually, or as a coupled system. This of course bring us to the next complication.

1.3.2 Correlation

Looking ahead to Chapter 5, let us take WTI crude oil as an example. At any point in time, we have a sequence of futures contracts with maturity dates spaced out monthly, the one with the shortest time to maturity being known as the *prompt future*. Generally, the entire forward curve for a commodity moves in a coupled manner, so we expect a high degree of correlation between the various components, e.g. between the prompt future and the next futures contract one month further out. This is quite clearly visible both in Figure 1.4(a), where we show the historical price

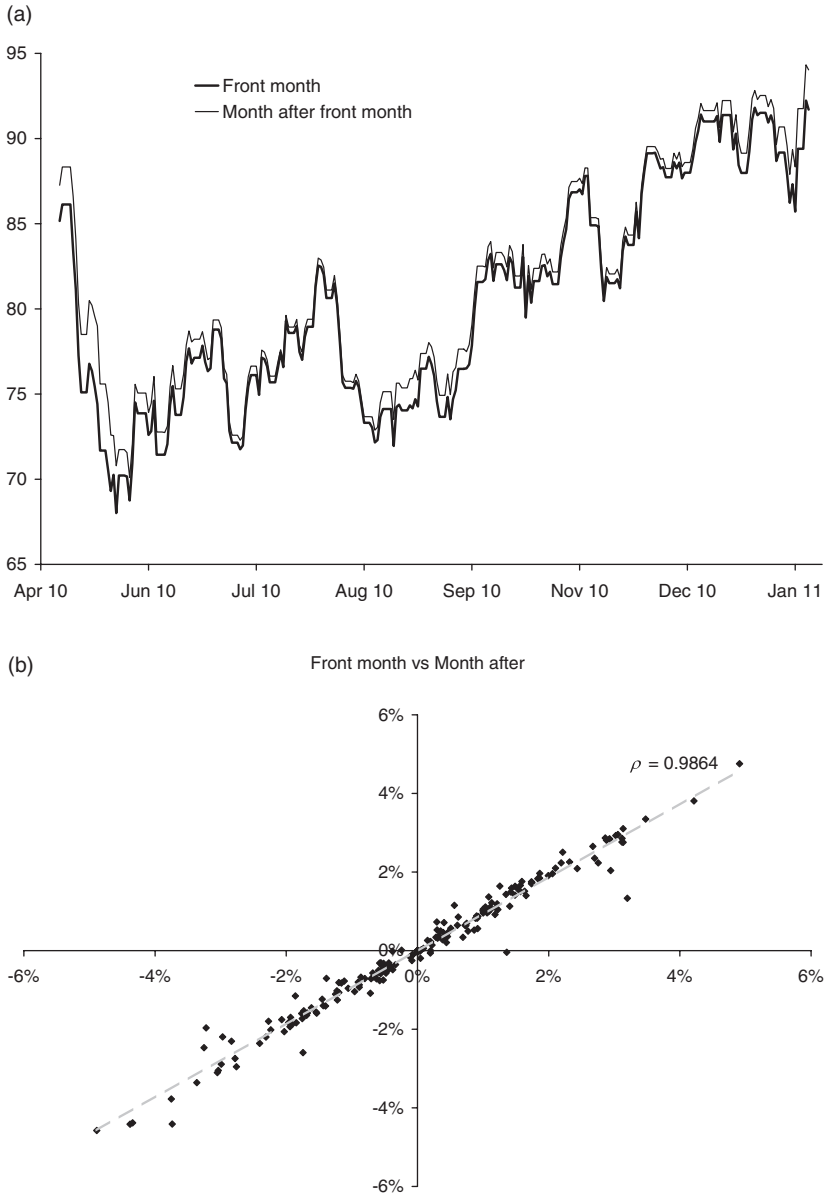
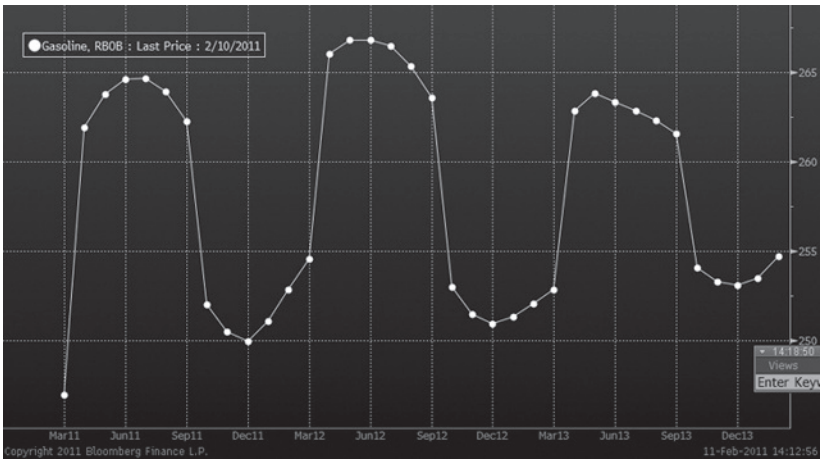
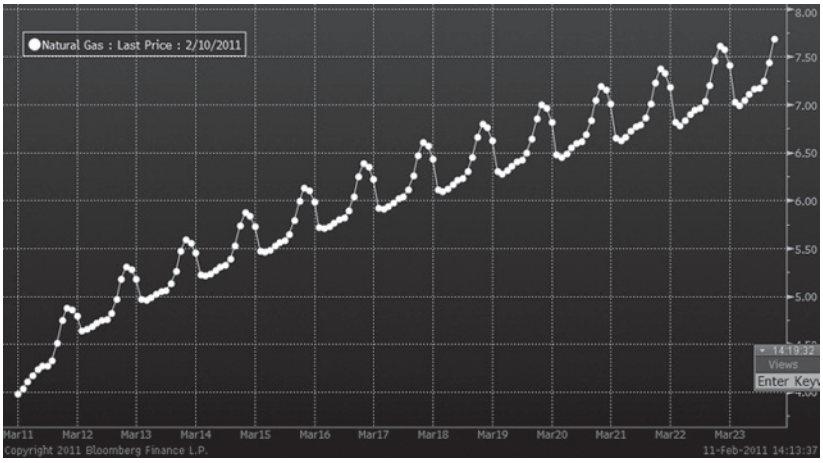


Figure 1.4 (a) Time series of WTI crude oil (front month and month after front month). (b) Correlation of logreturns on WTI crude oil (front month vs month after front month).



(a)



(b)

Figure 1.5 (a) Futures curve for RBOB gasoline – © 2013 Bloomberg Finance L.P. All rights reserved. Used with permission.
(b) Futures curve for natural gas – © 2013 Bloomberg Finance L.P. All rights reserved. Used with permission.

chart for the two adjacent WTI futures contracts, and in Figure 1.4(b) where the correlation between daily logreturns is graphed, revealing a historical correlation of 0.9864.

Correlations such as these are very high and certainly need to be handled accurately, as for many commodity options (e.g. spread options) the value is very sensitive to the correlation.

1.3.3 Seasonality

We can see little evidence of seasonality in the price of WTI crude oil in Figure 1.4(a) above. This is to be expected, as the overall supply and demand for crude oil is relatively constant throughout the year. The same is not true of various refined products, however. Demand for gasoline in the US peaks markedly in the so-called "driving season" from late May to early September, and demand for heating oil peaks in the winter, both of which are reflected in peaks and troughs in the futures curves for forward delivery. We can see clear visual evidence of this in Figure 1.5(a) and Figure 1.5(b).

We need to use the price information embedded in the entire futures curve in order to price options on such commodities correctly, which will depend on the nature of the model used.

1.3.4 American and Asian Features

A final complicating factor with commodity options is that many of the quoted options are either American straddles or Asian (average rate) options, which are more complicated to deal with than straightforward Europeans. Not only that, but the average rate options are generally exposed to an average of the prompt future over the course of a period, during which the prompt future rolls from one contract to the next. This will all be discussed in Chapter 2, where we attempt to build the technical foundations for commodity option pricing. In subsequent chapters, we shall relate this technical toolkit to each of the various sub-categories described in Section 1.2.1.