1

Overhead Transmission Lines and Their Circuit Constants

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In order to understand fully the nature of power systems, we need to study the nature of transmission lines as the first step. In this chapter we examine the characteristics and basic equations of three-phase overhead transmission lines. However, the actual quantities of the constants are described in Chapter 2.

1.1 Overhead Transmission Lines with *LR* Constants

1.1.1 Three-phase single circuit line without overhead grounding wire

1.1.1.1 Voltage and current equations, and equivalent circuits

A three-phase single circuit line between a point m and a point n with only L and R and without an overhead grounding wire (OGW) can be written as shown in Figure 1.1(a). In the figure, r_g and L_g are the equivalent resistance and inductance of the earth, respectively. The outer circuits I and II connected at points m and n can theoretically be three-phase circuits of any kind.

All the voltages V_a, V_b, V_c and currents I_a, I_b, I_c are vector quantities and the symbolic arrows show the measuring directions of the three-phase voltages and currents which have to be written in the same direction for the three-phases as a basic rule to describe the electrical quantities of three-phase circuits.

In Figure 1.1, the currents I_a, I_b, I_c in each phase conductor flow from left to right (from point m to point n). Accordingly, the composite current $I_a + I_b + I_c$ has to return from right to left (from point n to m) through the earth–ground pass. In other words, the three-phase circuit has to be treated as the set of 'three-phase conductors + one earth circuit' pass.

In Figure 1.1(a), the equations of the transmission line between m and n can be easily described as follows. Here, voltages V and currents I are complex-number vector values:

$$\left.\begin{aligned}
{}_mV_a - {}_nV_a &= (r_a + j\omega L_{aag})I_a + j\omega L_{abg}I_b + j\omega L_{acg}I_c - {}_{mn}V_g \quad ① \\
{}_mV_b - {}_nV_b &= j\omega L_{bag}I_a + (r_b + j\omega L_{bbg})I_b + j\omega L_{bcg}I_c - {}_{mn}V_g \quad ② \\
{}_mV_c - {}_nV_c &= j\omega L_{cag}I_a + j\omega L_{cbg}I_b + (r_c + j\omega L_{ccg})I_c - {}_{mn}V_g \quad ③ \\
\text{where } {}_{mn}V_g &= (r_g + j\omega L_g)I_g = -(r_g + j\omega L_g)(I_a + I_b + I_c) \quad ④
\end{aligned}\right\} \tag{1.1}$$

Handbook of Power Systems Engineering with Power Electronics Applications, Second Edition. Yoshihide Hase.
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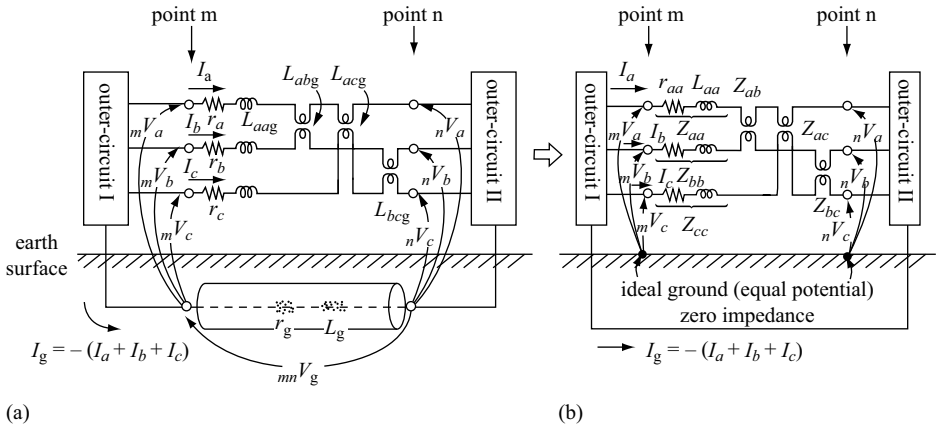


Figure 1.1 Single circuit line with *LR* constants

Substituting ④ into ①, and then eliminating ${}_{mn}V_g$, I_g,

$$\left.\begin{aligned} {}_mV_a - {}_nV_a &= (r_a + r_g + j\omega\overline{L_{aag} + L_g})I_a + (r_g + j\omega\overline{L_{abg} + L_g})I_b + (r_g + jw\overline{L_{acg} + L_g})I_c \quad ⑤ \\ \text{Substituting ④ into ② and ③ in the same way,} & \\ {}_mV_b - {}_nV_b &= (r_g + j\omega\overline{L_{bag} + L_g})I_a + (r_b + r_g + j\omega\overline{L_{bbg} + L_g})I_b + (r_g + j\omega\overline{L_{bcg} + L_g})I_c \quad ⑥ \\ {}_mV_c - {}_nV_c &= (r_g + j\omega\overline{L_{cag} + L_g})I_a + (r_g + j\omega\overline{L_{cbg} + L_g})I_b + (r_c + r_g + j\omega\overline{L_{ccg} + L_g})I_c \quad ⑦ \end{aligned}\right\} \tag{1.2}$$

Now, the original Equation 1.1 and the derived Equation 1.2 are the equivalent of each other, so Figure 1.1(b), showing Equation 1.2, is also the equivalent of Figure 1.1(a).

Equation 1.2 can be expressed in the form of a matrix equation and the following equations are derived accordingly (refer to Appendix B for the matrix equation notation):

$$\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \end{bmatrix}$$

$$= \begin{bmatrix} r_a + r_g + j\omega\overline{L_{aag} + L_g} & r_g + j\omega\overline{L_{abg} + L_g} & r_g + j\omega\overline{L_{acg} + L_g} \\ r_g + j\omega\overline{L_{bag} + L_g} & r_b + r_g + j\omega\overline{L_{bbg} + L_g} & r_g + j\omega\overline{L_{bcg} + L_g} \\ r_g + j\omega\overline{L_{cag} + L_g} & r_g + j\omega\overline{L_{cbg} + L_g} & r_c + r_g + j\omega\overline{L_{ccg} + L_g} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{1.3}$$

$$\equiv \begin{bmatrix} r_{aa} + j\omega L_{aa} & r_{ab} + j\omega L_{ab} & r_{ac} + j\omega L_{ac} \\ r_{ba} + j\omega L_{ba} & r_{bb} + j\omega L_{bb} & r_{bc} + j\omega L_{bc} \\ r_{ca} + j\omega L_{ca} & r_{cb} + j\omega L_{cb} & r_{cc} + j\omega L_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$\equiv \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

where $Z_{aa} = r_{aa} + j\omega L_{aa} = (r_a + r_g) + j\omega(L_{aag} + L_g)$

Z_{bb}, Z_{cc} are written in similar equation forms

and Z_{ac}, Z_{bc} are also written in similar forms (1.4)

Now, we can apply symbolic expressions for the above matrix equation as follows:

$$ {}_m\boldsymbol{V}_{abc} - {}_n\boldsymbol{V}_{abc} = \boldsymbol{Z}_{abc} \cdot \boldsymbol{I}_{abc} \tag{1.5} $$

where

$$ {}_m\boldsymbol{V}_{abc} = \begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix},\ {}_n\boldsymbol{V}_{abc} = \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \end{bmatrix},\ \boldsymbol{Z}_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix},\ \boldsymbol{I}_{abc} = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{1.6} $$

Summarizing the above equations, Figure 1.1(a) can be described as Equations 1.3 and 1.6 or Equations 1.5 and 1.6, in which the resistance r_g and inductance L_g of the earth return pass are already reflected in all these four equations, although I_g and ${}_{mn}V_g$ are eliminated in Equations 1.5 and 1.6. We can consider Figure 1.1(b) as the equivalent circuit of Equations 1.3 and 1.4 or Equations 1.5 and 1.6. In Figure 1.1(b), earth resistance r_g and earth inductance L_g are already included in the line constants Z_{aa}, Z_{ab}, etc., so the earth in the equivalent circuit of Figure 1.1(b) is 'the ideal earth' with zero impedance. Therefore the earth can be expressed in the figure as the equal-potential (zero-potential) earth plane at any point. It is clear that the mutual relation between the constants of Figure 1.1(a) and Figure 1.1(b) is defined by Equation 1.4. It should be noted that the self-impedance Z_{aa} and mutual impedance Z_{ab} of phase a, for example, involve the earth resistance r_g and earth inductance L_g.

Generally, in actual engineering tasks, Figure 1.1(b) and Equations 1.3 and 1.4 or Equations 1.5 and 1.6 are applied instead of Figure 1.1(a) and Equations 1.1 and 1.2; in other words, the line impedances are given as Z_{aa}, Z_{ab}, etc., instead of Z_{aag}, Z_{abg}. The line impedances Z_{aa}, Z_{bb}, Z_{cc} are named '**the self-impedances of the line including the earth–ground effect'**, and Z_{ab}, Z_{ac}, Z_{bc}, etc., are named '**the mutual impedances of the line including the earth–ground effect'**.

1.1.1.2 Measurement of line impedances Z_{aa}, Z_{ab}, Z_{ac}

Let us consider how to measure the line impedances taking the earth effect into account.

As we know from Figure 1.1(b) and Equations 1.3 and 1.4, the impedances Z_{aa}, Z_{ab}, Z_{ac}, etc., can be measured by the circuit connection shown in Figure 1.2(a).

The conductors of the three-phases are grounded to earth at point n, and the phase b and c conductors are opened at point m. Accordingly, the boundary conditions ${}_nV_a = {}_nV_b = {}_nV_c = 0, I_b = I_c = 0$ can be adopted for Equation 1.3:

$$ \begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix} \quad ① \tag{1.7} $$

$$ \therefore \quad {}_mV_a/I_a = Z_{aa}, \qquad {}_mV_b/I_a = Z_{ba}, \qquad {}_mV_c/I_a = Z_{ca} \quad ② $$

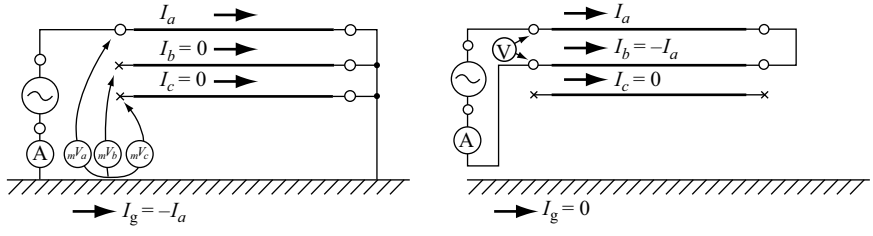


Figure 1.2 Measuring circuit of line impedance

Therefore the impedances Z_{aa}, Z_{ab}, Z_{ac} can be calculated from the measurement results of ${}_mV_a$, ${}_mV_b$, ${}_mV_c$ and I_a.

All the impedance elements in the impedance matrix $\boldsymbol{Z_{abc}}$ of Equation 1.7 can be measured in the same way.

1.1.1.3 Working inductance ($L_{aa} - L_{ab}$)

Figure 1.2(b) shows the case where the current I flows along the phase a conductor from point m to n and comes back from n to m only through the phase b conductor as the return pass. The equation is with boundary conditions $I_a = -I_b = I, \quad I_c = 0, \quad {}_nV_a = {}_nV_b$:

$$\left.\begin{array}{|c|}\hline {}_mV_a \\ \hline {}_mV_b \\ \hline {}_mV_c \\ \hline \end{array} - \begin{array}{|c|}\hline {}_nV_a \\ \hline {}_nV_a \\ \hline {}_nV_c \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline Z_{aa} & Z_{ab} & Z_{ac} \\ \hline Z_{ba} & Z_{bb} & Z_{bc} \\ \hline Z_{ca} & Z_{cb} & Z_{cc} \\ \hline \end{array} \cdot \begin{array}{|c|}\hline I \\ \hline -I \\ \hline 0 \\ \hline \end{array} \right\} \tag{1.8a}$$

Therefore

$$\left.\begin{array}{l} {}_mV_a - {}_nV_a = (Z_{aa} - Z_{ab})I: \text{ voltage drop of the phase a conductor between points m and n} \\ {}_mV_b - {}_nV_b = -(Z_{bb} - Z_{ba})I: \text{ voltage drop of the phase b conductor between points m and n} \end{array}\right\} \text{①}$$
$$\left.\begin{array}{l} V = {}_mV_a - {}_mV_b = \{(Z_{aa} - Z_{ab}) + (Z_{bb} - Z_{ba})\}I \\ V/I = ({}_mV_a - {}_mV_b)/I = (Z_{aa} - Z_{ab}) + (Z_{bb} - Z_{ba}) = \{\text{twice values of working impedance}\} \end{array}\right\} \text{②} \tag{1.8b}$$

Equation 1.8b① indicates the voltage drop of the parallel circuit wires a, b under the condition of the 'go-and-return-current' connection. The current I flows out at point m on the phase a conductor and returns to m only through the phase b conductor, so any other current flowing does not exist on the phase c conductor or earth–ground pass. In other words, Equation 1.8b① is satisfied regardless of the existence of the third wire or earth–ground pass. Therefore the impedance $(Z_{aa} - Z_{ab})$ as well as $(Z_{bb} - Z_{ba})$ should be specific values which are determined only by the relative condition of the phase a and b conductors, and they are not affected by the existence or absence of the third wire or earth–ground pass. $(Z_{aa} - Z_{ab})$ is called the **working impedance** and the corresponding $(L_{aa} - L_{ab})$ is called the **working inductance** of the phase a conductor with the phase b conductor.

Furthermore, as the conductors a and b are generally of the same specification (the same dimension, same resistivity, etc.), the impedance drop between m and n of the phase a and b conductors should be the same. Accordingly, the working inductances of both conductors are clearly the same, namely $(L_{aa} - L_{ab}) = (L_{bb} - L_{ba})$.

The value of the working inductance can be calculated from the well-known equation below, which is derived by an electromagnetic analytical approach as a function only of the conductor radius r and the parallel distance s_{ab} between the two conductors:

$$L_{aa} - L_{ab} = L_{bb} - L_{ba} = 0.4605 \log_{10} \frac{s_{ab}}{r} + 0.05 \qquad [\text{mH/km}] \tag{1.9}$$

This is the equation for the working inductance of the parallel conductors a and b, whose deriving process is shown in the section 1.3.1 as of theory of electromagnetism. The equation shows that the working inductance $L_{aa} - L_{ab}$ for the two parallel conductors is determined only by the relative distance between the two conductors s_{ab} and the radius r, so it is not affected by any other conditions such as other conductors or the distance from the earth surface.

The working inductance can also be measured as the value $(1/2)V/I$ by using Equation 1.8b②.

1.1.1.4 Self- and mutual impedances including the earth–ground effect L_{aa}, L_{ab}

Now we evaluate the actual numerical values for the line inductances contained in the impedance matrix of Equation 1.3.

The currents I_a, I_b, I_c flow through each conductor from point m to n and $I_a + I_b + I_c$ returns from n to m through the ideal earth return pass. All the impedances of this circuit can be measured by the method of Figure 1.2(a). However, these measured impedances are experimentally a little larger than those obtained by pure analytical calculation based on the electromagnetic equations with the assumption of an ideal, conductive, earth plane surface.

In order to compensate for these differences between the analytical result and the measured values, we can use an imaginary ideal conductive earth plane at some deep level from the ground surface as shown in Figure 1.3.

In this figure, the imaginary perfect conductive earth plane is shown at the depth H_g, and the three imaginary conductors α, β, γ are located at symmetrical positions to conductors a, b, c, respectively, based on this datum plane.

The inductances can be calculated by adopting the equations of the electromagnetic analytical approach to Figure 1.3.

1.1.1.4.1 Self-inductances L_{aa}, L_{bb}, L_{cc}

In Figure 1.3, the conductor a (radius r) and the imaginary returning conductor α are symmetrically located on the datum plane, and the distance between a and α is $h_a + H_a$. Thus the inductance of conductor a can be calculated by the following equation which is a special case of Equation 1.9 under the condition $s_{ab} \to h_a + H_a$:

$$L_{aag} = 0.4605 \log_{10} \frac{h_a + H_a}{r} + 0.05 \qquad [\text{mH/km}] \tag{1.10a}$$

Conversely, the inductance of the imaginary conductor α (the radius is H_a, because the actual grounding current reaches up to the ground surface), namely the inductance of earth, is

$$L_g = 0.4605 \log_{10} \frac{h_a + H_a}{H_a} + 0.05\ [\text{mH/km}] \fallingdotseq 0.05 \qquad [\text{mH/km}] \tag{1.10b}$$

Therefore,

$$L_{aa} = L_{aag} + L_g = 0.4605 \log_{10} \frac{h_a + H_a}{r} + 0.1 \qquad [\text{mH/km}] \tag{1.11}$$

L_{bb}, L_{cc} can be derived in the same way.

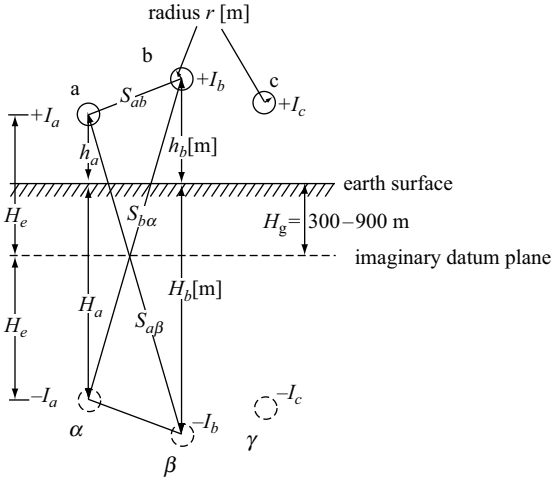


Figure 1.3 Earth–ground as conductor pass

Incidentally, the depth of the imaginary datum plane can be checked experimentally and is mostly within the range of $H_g = 300 - 1000$ m. On the whole H_g is rather shallow, say $300 - 600$ m in the geological younger strata after the Quaternary period, but is generally deep, say $800 - 1000$ m, in the older strata of the Tertiary period or earlier.

1.1.1.4.2 Mutual inductances L_{ab}, L_{bc}, L_{ca}

The mutual inductance L_{ab} can be derived by subtracting L_{aa} from Equation 1.11 and the working inductance $(L_{aa} - L_{ab})$ from Equation 1.9:

$$\begin{aligned} L_{ab} &= L_{aa} - (L_{aa} - L_{ab}) = 0.4605 \log_{10} \frac{h_a + H_a}{s_{ab}} + 0.05 \quad [\text{mH/km}] \\ &\fallingdotseq 0.4605 \log_{10} \frac{s_{a\beta}}{s_{ab}} + 0.05 \quad [\text{mH/km}] \end{aligned} \tag{1.12a}$$

Similarly

$$\begin{aligned} L_{ba} &= 0.4605 \log_{10} \frac{h_b + H_b}{s_{ab}} + 0.05 \quad [\text{mH/km}] \\ &\fallingdotseq 0.4605 \log_{10} \frac{s_{b\alpha}}{s_{ab}} + 0.05 \quad [\text{mH/km}] \end{aligned} \tag{1.12b}$$

where $h_a + H_a = 2H_e \fallingdotseq 2H_g$, and so on.

Incidentally, the depth of the imaginary datum plane $H_g \fallingdotseq H_e = (h_a + H_a)/2$ would be between 300 and 1000 m, while the height of the transmission tower h_a is within the range of 10–100 m (UHV towers of 800–1000 kV would be approximately 100 m or less). Furthermore, the phase-to-phase distance S_{ab} is of order 10 m, while the radius of conductor r is a few centimetres (the equivalent radius r_{eff} of EHV/UHV multi-bundled conductor lines may be of the order of 10–50 cm).

Accordingly,

$$\left.\begin{aligned} &H_a \fallingdotseq H_b \fallingdotseq H_c \fallingdotseq 2H_e \gg h_a \fallingdotseq h_b \fallingdotseq h_c \gg s_{ab} \fallingdotseq s_{bc} \fallingdotseq s_{ca} \gg r, r_{\text{eff}} \\ &s_{a\beta} \fallingdotseq s_{b\alpha} \fallingdotseq h_a + H_a = 2H_e \fallingdotseq h_b + H_b \end{aligned}\right\} \tag{1.13}$$

Then, from Equations 1.9, 1.11 and 1.12,

$$L_{aa} \fallingdotseq L_{bb} \fallingdotseq L_{cc}, \quad L_{ab} \fallingdotseq L_{bc} \fallingdotseq L_{ca} \tag{1.14}$$

1.1.1.4.3 Numerical check

Let us assume conditions $s_{ab} = 10$ m, $r = 0.05$ m, $H_e = (h_a + H_a)/2 \fallingdotseq H_g = 900$ m.

Then calculating the result by Equation 1.11 and 1.12,

$$L_{aa} = 2.20 \,\text{mH/km}, \quad L_{ab} = 1.09 \,\text{mH/km}$$

If $H_e = (h_a + H_a)/2 = 300$ m, then $L_{aa} = 1.98$ mH/km, $L_{ab} = 0.87$ mH/km. As $h_a + H_a$ is contained in the logarithmic term of the equations, constant values L_{aa}, L_{ab} and so on are not largely affected by $h_a + H_a$, neither is radius r nor r_{eff} as well as the phase-to-phase distance s_{ab}. Besides, 0.1 and 0.05 in the second term on the right of Equations 1.9–1.12 do not make a lot of sense.

Further, if transmission lines are reasonably transpositioned, $Z_{aa} \fallingdotseq Z_{bb} \fallingdotseq Z_{cc}$, $Z_{ab} \fallingdotseq Z_{bc} \fallingdotseq Z_{ca}$ can be justified so that Equation 1.3 is simplified into Equation 2.13 of Chapter 2.

1.1.1.5 Reactance of multi-bundled conductors

For most of the recent large-capacity transmission lines, multi-bundled conductor lines ($n = 2 - 8$ per phase) are utilized as shown in Figure 1.4. In the case of n conductors (the radius of

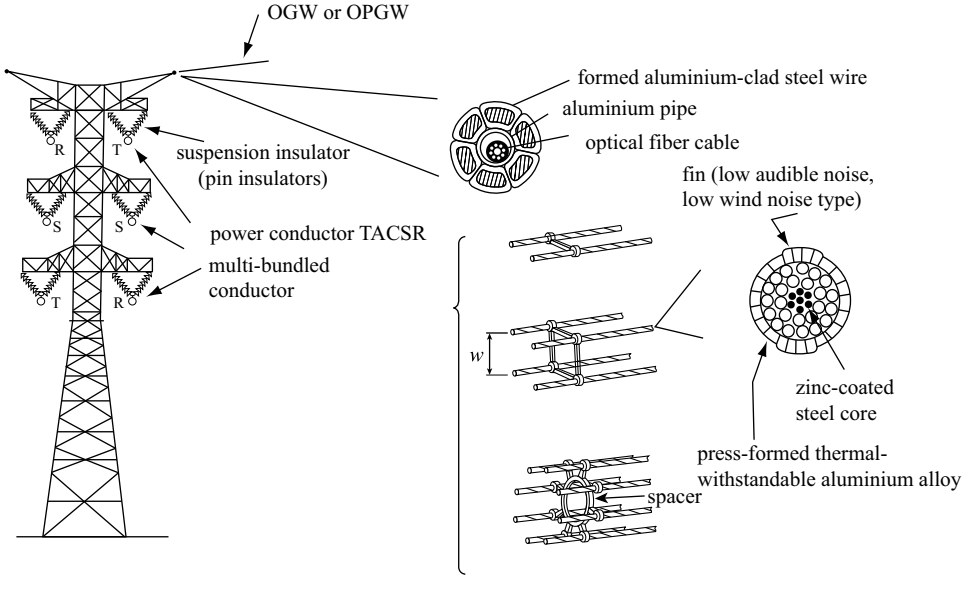


Figure 1.4 Overhead double circuit transmission line

each conductor is r), L_{aag} of Equation 1.10a can be calculated from the following modified equation:

$$\left.\begin{aligned} L_{aag} &= 0.4605\log_{10}\frac{h_a+H_a}{r^{1/n}\times w^{(n-1)/n}}+\frac{0.05}{n} \quad [\text{mH/km}] \\ &\equiv 0.4605\log_{10}\frac{h_a+H_a}{r_{\text{eq}}}+\frac{0.05}{n} \quad [\text{mH/km}] \end{aligned}\right\} \tag{1.15a}$$

where $r_{\text{eq}} = r^{1/n}\times w^{(n-1)/n}$ is the equivalent radius and

w [m] is the geometrical averaged distance of bundled conductors

Refer the Supplement 1 for the introduction of equivalent radius of a multi-bundled conductors. Since the self-inductance L_g of the virtual conductor α given by Equation 1.10b is not affected by the adoption of multi-bundled phase a conductors, accordingly

$$L_{aa} = L_{aag} + L_g = 0.4605\log_{10}\frac{h_a+H_a}{r_{\text{eq}}} + 0.05\left(1+\frac{1}{n}\right) \quad [\text{mH/km}] \tag{1.15b}$$

1.1.1.5.1 Numerical check Using TACSR = 810 mm^2 (see Chapter 2), $2r = 40$ mm and four bundled conductors ($n = 4$), with the square allocation $w = 50$ cm averaged distance

$$\left.\begin{aligned} w &= (w_{12}\cdot w_{13}\cdot w_{14}\cdot w_{23}\cdot w_{24}\cdot w_{34})^{1/6} \\ &= (50\cdot 50\sqrt{2}\cdot 50\cdot 50\cdot 50\sqrt{2}\cdot 50)^{1/6} = 57.24\,\text{cm} \\ r_{\text{eq}} &= r^{1/n}\cdot w^{(n-1)/n} = 2.0^{1/4}\cdot 57.25^{3/4} = 24.7\,\text{cm} \end{aligned}\right\} \tag{1.16}$$

The equivalent radius $r_{\text{eq}} = 24.7$ cm is 12.4 times $r = 2.0$ cm, so that the line self-inductance L_{aa} can also be reduced by the application of bundled conductors. The mutual inductance L_{ab} of Equation 1.12a is not affected by the adoption of multi-bundled conductor lines.

1.1.1.6 Line resistance

Earth resistance r_g in Figure 1.1(a) and Equation 1.2 can be regarded as negligibly small. Accordingly, the so-called mutual resistances r_{ab}, r_{bc}, r_{ca} in Equation 1.4 become zero. Therefore, the specific resistances of the conductors r_a, r_b, r_c are actually equal to the resistances r_{aa}, r_{bb}, r_{cc} in the impedance matrix of Equation 1.3.

In addition to the power loss caused by the linear resistance of conductors, non-linear losses called the **skin-effect loss** and **corona loss** occur on the conductors. These losses would become progressionally larger in higher frequency zones, so they must be major influential factors for the attenuation of travelling waves in surge phenomena. However, they can usually be neglected for power frequency phenomena because they are smaller than the linear resistive loss and, further, very much smaller than the reactance value of the line, at least for power frequency.

In regard to the bundled conductors, due to the result of the enlarged equivalent radius r_{eq}, the **dielectric strength around the bundled conductors** is somewhat relaxed, so that corona losses can also be relatively reduced. Skin-effect losses of bundled conductors are obviously far smaller than that of a single conductor whose aluminium cross-section is the same as the total sections of the bundled conductors.

1.1.2 Three-phase single circuit line with OGW, OPGW

Most high-voltage transmission lines are equipped with **OGW** (**overhead grounding wires**) and/or **OPGW** (**OGW with optical fibres** for communication use).

In the case of a single circuit line with single OGW, the circuit includes four conductors and the fourth conductor (x in Figure 1.5) is earth grounded at all the transmission towers. Therefore, using the figure for the circuit, Equation 1.3 has to be replaced by the following equation:

$$\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \\ {}_mV_x = 0 \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \\ {}_nV_x = 0 \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{ax} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bx} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cx} \\ Z_{xa} & Z_{xb} & Z_{xc} & Z_{xx} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_x \end{bmatrix} \tag{1.17a}$$

Extracting the fourth row,

$$I_x = -\frac{1}{Z_{xx}}(Z_{xa}I_a + Z_{xb}I_b + Z_{xc}I_c) \tag{1.17b}$$

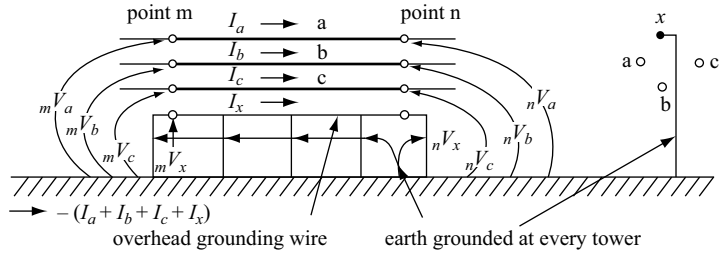


Figure 1.5 Single circuit line with OGW

Substituting I_x into the first, second and third rows of Equation 1.17a,

$$\left.\begin{aligned}
&\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} + \begin{bmatrix} Z_{ax}I_x \\ Z_{bx}I_x \\ Z_{cx}I_x \end{bmatrix} \\
&= \begin{bmatrix} Z_{aa} - \dfrac{Z_{ax}Z_{xa}}{Z_{xx}} & Z_{ab} - \dfrac{Z_{ax}Z_{xb}}{Z_{xx}} & Z_{ac} - \dfrac{Z_{ax}Z_{xc}}{Z_{xx}} \\ Z_{ba} - \dfrac{Z_{bx}Z_{xa}}{Z_{xx}} & Z_{bb} - \dfrac{Z_{bx}Z_{xb}}{Z_{xx}} & Z_{bc} - \dfrac{Z_{bx}Z_{xc}}{Z_{xx}} \\ Z_{ca} - \dfrac{Z_{cx}Z_{xa}}{Z_{xx}} & Z_{cb} - \dfrac{Z_{cx}Z_{xb}}{Z_{xx}} & Z_{cc} - \dfrac{Z_{cx}Z_{xc}}{Z_{xx}} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\
&\equiv \begin{bmatrix} Z'_{aa} & Z'_{ab} & Z'_{ac} \\ Z'_{ba} & Z'_{bb} & Z'_{bc} \\ Z'_{ca} & Z'_{cb} & Z'_{cc} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \\
&\text{where } Z_{ax} = Z_{xa}, \qquad Z_{bx} = Z_{xb}, \qquad Z_{cx} = Z_{xc} \\
&\quad Z'_{aa} = Z_{aa} - \delta_{aa}, \quad Z'_{ab} = Z_{ab} - \delta_{ab} \\
&\quad \delta_{aa} = \frac{Z_{ax}Z_{xa}}{Z_{xx}}, \qquad \delta_{ab} = \frac{Z_{ax}Z_{xb}}{Z_{xx}}
\end{aligned}\right\} \tag{1.18}$$

This is the fundamental equation of the three-phase single circuit line with OGW in which I_x has already been eliminated and the impedance elements of the grounding wire are slotted into the three-phase impedance matrix. Equation 1.18 is obviously of the same form as Equation 1.3, while all the elements of the rows and columns in the impedance matrix have been revised to smaller values with corrective terms $\delta_{ax} = Z_{ax}Z_{xa}/Z_{xx}$ etc.

The above equations indicate that the three-phase single circuit line with OGW can be expressed as a 3×3 impedance matrix equation in the form of Equation 1.18 regardless of the existence of OGW, as was the case with Equation 1.3. Also, we can comprehend that OGW has roles not only to shield lines against lightning but also to reduce the self- and mutual reactances of transmission lines.

1.1.3 Three-phase double circuit line with *LR* constants

The three-phase double circuit line can be written as in Figure 1.6 and Equation 1.19 regardless of the existence or absence of OGW:

$$\begin{bmatrix} {}_mV_a \\ {}_mV_b \\ {}_mV_c \\ {}_mV_A \\ {}_mV_B \\ {}_mV_C \end{bmatrix} - \begin{bmatrix} {}_nV_a \\ {}_nV_b \\ {}_nV_c \\ {}_nV_A \\ {}_nV_B \\ {}_nV_C \end{bmatrix} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{aA} & Z_{aB} & Z_{aC} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bA} & Z_{bB} & Z_{bC} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cA} & Z_{cB} & Z_{cC} \\ Z_{Aa} & Z_{Ab} & Z_{Ac} & Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{Ba} & Z_{Bb} & Z_{Bc} & Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{Ca} & Z_{Cb} & Z_{Cc} & Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \\ I_A \\ I_B \\ I_C \end{bmatrix} \tag{1.19}$$

In addition, if the line is appropriately phase balanced, the equation can be expressed by Equation 2.17 of Chapter 2.

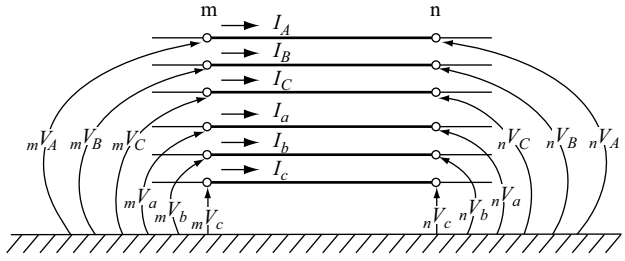


Figure 1.6 Three-phase double circuit line with *LR* constants

1.2 Stray Capacitance of Overhead Transmission Lines

1.2.1 Stray capacitance of three-phase single circuit line

1.2.1.1 Equation for electric charges and voltages on conductors

Figure 1.7(a) shows a single circuit line, where electric charges q_a, q_b, q_c [C/m] are applied to phase a, b, c conductors and cause voltages v_a, v_b, v_c [V], respectively. The equation of this circuit is given by

$$\underbrace{\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}}_{\boldsymbol{v}_{abc}} = \underbrace{\begin{bmatrix} p_{aa} & p_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix}}_{\boldsymbol{p}_{abc}} \cdot \underbrace{\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}}_{\boldsymbol{q}_{abc}} \quad \therefore \boldsymbol{v}_{abc} = \boldsymbol{p}_{abc} \cdot \boldsymbol{q}_{abc} \qquad \text{(1.20a)}$$

where q [C/m], v [V] are instantaneous real numbers

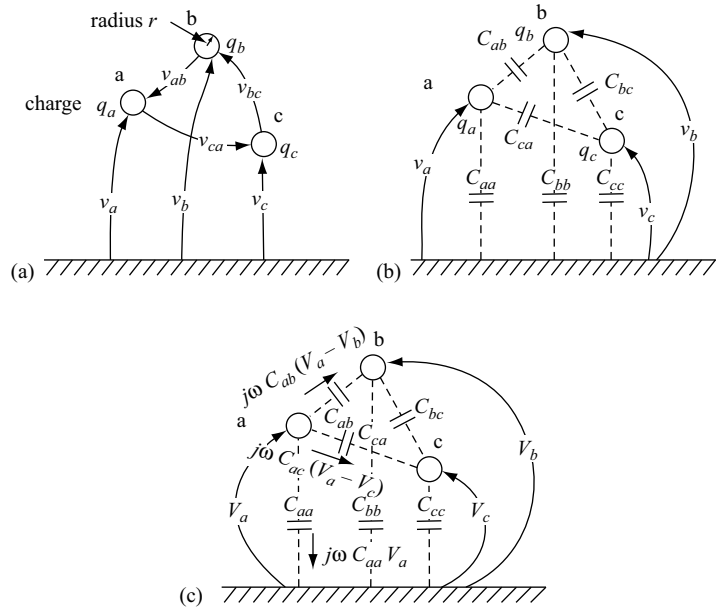


Figure 1.7 Stray capacitance of single circuit line

The inverse matrix equation can be derived from the above equation as

$$\underbrace{\begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}}_{\boldsymbol{q}_{abc}} = \underbrace{\begin{bmatrix} k_{aa} & k_{ab} & k_{ac} \\ k_{ba} & k_{bb} & k_{bc} \\ k_{ca} & k_{cb} & k_{cc} \end{bmatrix}}_{\boldsymbol{k}_{abc}} \cdot \underbrace{\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}}_{\boldsymbol{v}_{abc}} \quad \therefore \boldsymbol{q}_{abc} = \boldsymbol{k}_{abc} \cdot \boldsymbol{v}_{abc} \tag{1.20b}$$

Here, $\boldsymbol{p}_{abc}$ and $\boldsymbol{k}_{abc}$ are inverse 3×3 matrices of each other, so that $\boldsymbol{p}_{abc} \cdot \boldsymbol{k}_{abc} = \mathbf{1}$ ($\mathbf{1}$ is the 3×3 unit matrix; refer to Appendix B).

Accordingly,

$$\left.\begin{aligned} k_{aa} &= (p_{bb}\,p_{cc} - p_{bc}^2)/\Delta && [\mathrm{F/m}] \\ k_{bb} &= (p_{cc}\,p_{aa} - p_{ca}^2)/\Delta && [\mathrm{F/m}] \\ k_{cc} &= (p_{aa}\,p_{bb} - p_{ab}^2)/\Delta && [\mathrm{F/m}] \\ k_{ab} &= k_{ba} = -(p_{ab}\,p_{cc} - p_{ac}\,p_{bc})/\Delta && [\mathrm{F/m}] \\ k_{bc} &= k_{cb} = -(p_{bc}\,p_{aa} - p_{ba}\,p_{ca})/\Delta && [\mathrm{F/m}] \\ k_{ca} &= k_{ac} = -(p_{ca}\,p_{bb} - p_{cb}\,p_{ab})/\Delta && [\mathrm{F/m}] \\ \Delta &= p_{aa}\,p_{bb}\,p_{cc} + 2p_{ab}\,p_{bc}\,p_{ac} - (p_{aa}\,p_{bc}^2 + p_{bb}\,p_{ca}^2 + p_{cc}\,p_{ab}^2) && [\mathrm{m^3/F^3}] \end{aligned}\right\} \tag{1.20c}$$

where p [m/F] are the **coefficients of the potential** and k [F/m] are the electrostatic **coefficients of static capacity**.

Modifying Equation 1.20b a little,

$$\left.\begin{aligned} q_a &= k_{aa}v_a + k_{ab}v_b + k_{ac}v_c \\ &= (k_{aa} + k_{ab} + k_{ac})v_a + (-k_{ab})(v_a - v_b) + (-k_{ac})(v_a - v_c) && [\mathrm{C/m}] \\ q_b &= (k_{ba} + k_{bb} + k_{bc})v_b + (-k_{bc})(v_b - v_c) + (-k_{ba})(v_b - v_a) && [\mathrm{C/m}] \\ q_c &= (k_{ca} + k_{cb} + k_{cc})v_c + (-k_{ca})(v_c - v_a) + (-k_{ca}) + (v_c - v_b) && [\mathrm{C/m}] \end{aligned}\right\} \tag{1.21}$$

then

$$\left.\begin{aligned} q_a &= C_{aa}v_a + C_{ab}(v_a - v_b) + C_{ac}(v_a - v_c) && [\mathrm{C/m}] \\ q_b &= C_{bb}v_b + C_{bc}(v_b - v_c) + C_{ba}(v_b - v_a) && [\mathrm{C/m}] \\ q_c &= C_{cc}v_c + C_{ca}(v_c - v_a) + C_{cb}(v_c - v_b) && [\mathrm{C/m}] \end{aligned}\right\} \tag{1.22}$$

with q_a, q_b, q_c [C/m], v_b, v_b, v_c [V] and

$$\left.\begin{aligned} C_{aa} &= k_{aa} + k_{ab} + k_{ac} && [\mathrm{F/m}] & \quad C_{ab} &= -k_{ab} && [\mathrm{F/m}] \\ C_{bb} &= k_{ba} + k_{bb} + k_{bc} && [\mathrm{F/m}] & \quad C_{bc} &= -k_{bc} && [\mathrm{F/m}] \\ C_{cc} &= k_{ca} + k_{cb} + k_{cc} && [\mathrm{F/m}] & \quad C_{ca} &= -k_{ca} && [\mathrm{F/m}] \\ C_{ac} &= -k_{ac} && [\mathrm{F/m}] \\ C_{ba} &= -k_{ba} && [\mathrm{F/m}] \\ C_{cb} &= -k_{cb} && [\mathrm{F/m}], \end{aligned}\right\} \tag{1.23}$$

Equations 1.22 and 1.23 are the fundamental equations of stray capacitances of a three-phase single circuit overhead line. Noting the form of Equation 1.22, Figure 1.7(b) can be used for another expression of Figure 1.7(a): C_{aa}, C_{bb}, C_{cc} are the **phase-to-ground capacitances** and $C_{ab} = C_{ba}$, $C_{bc} = C_{cb}$, $C_{ca} = C_{ac}$ are the **phase-to-phase capacitances** between two conductors.

1.2.1.2 Fundamental voltage and current equations

It is usually convenient in actual engineering to adopt current $i(= \mathrm{d}q/\mathrm{d}t)$ [A] instead of charging value q [C], and furthermore to adopt effective (rms: root mean square) voltage and current of complex-number V, I instead of instantaneous value $v(t)$, $i(t)$.

As electric charge $q(t)$ is the integration over time of current i, the following relations can be derived:

$$\left.\begin{aligned}
&q(t) = \int i(t)dt, \quad i(t) = \frac{dq(t)}{dt} \quad ①\\
&i(t) = \mathrm{Re}(\sqrt{2}\cdot I(t)) = \mathrm{Re}(\sqrt{2}\,|I|\cdot \mathrm{e}^{j(\omega t+\theta_1)}) = \sqrt{2}\,|I|\cos(\omega t+\theta_1)\ ②\\
&\mathrm{Re}() \text{ shows the real part of the complex number}(\mathrm{Re}(a+jb)=a).\\
&v(t) = \mathrm{Re}(\sqrt{2}\cdot V(t)) = \mathrm{Re}(\sqrt{2}\,|V|\cdot \mathrm{e}^{j(\omega t+\theta_2)})\\
&\quad = \sqrt{2}\,|V|\cos(\omega t+\theta_2) \qquad ③\\
&\therefore\quad q(t) = \int i(t)dt = \int \mathrm{Re}(\sqrt{2}\,|I|\cdot \mathrm{e}^{j(\omega t+\theta_1)})dt\\
&\quad = \mathrm{Re}(\sqrt{2}\,|I|\cdot \int \mathrm{e}^{j(\omega t+\theta_1)}dt) \qquad \text{(note that, in this book,}\\
&\quad = \mathrm{Re}\left(\sqrt{2}\,|I|\cdot\frac{\mathrm{e}^{j(\omega t+\theta_1)}}{j\omega}\right) = \mathrm{Re}\left(\frac{\sqrt{2}I(t)}{j\omega}\right) \quad ④ \qquad \text{the exponential function}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{will be denoted by 'e').}
\end{aligned}\right\} \tag{1.24}$$

Equation 1.22 can be modified to the following form by adopting Equation 1.24④ and by replacement of $v_a(t) \to \sqrt{2}V_a(t)$ etc.:

$$\left.\begin{aligned}
\mathrm{Re}\left(\frac{\sqrt{2}I_a}{j\omega}\right) &= \mathrm{Re}\{C_{aa}\cdot\sqrt{2}V_a + C_{ab}\cdot\sqrt{2}(V_a-V_b) + C_{ac}\cdot\sqrt{2}(V_a-V_c)\}\\
\mathrm{Re}\left(\frac{\sqrt{2}I_b}{j\omega}\right) &= \mathrm{Re}\{C_{bb}\cdot\sqrt{2}V_b + C_{bc}\cdot\sqrt{2}(V_b-V_c) + C_{ba}\cdot\sqrt{2}(V_b-V_a)\}\\
\mathrm{Re}\left(\frac{\sqrt{2}I_c}{j\omega}\right) &= \mathrm{Re}\{C_{cc}\cdot\sqrt{2}V_c + C_{ca}\cdot\sqrt{2}(V_c-V_a) + C_{cb}\cdot\sqrt{2}(V_c-V_b)\}
\end{aligned}\right\} \tag{1.25}$$

Therefore

$$\left.\begin{aligned}
I_a &= j\omega C_{aa}V_a + j\omega C_{ab}(V_a-V_b) + j\omega C_{ac}(V_a-V_c)\\
I_b &= j\omega C_{bb}V_b + j\omega C_{bc}(V_b-V_c) + j\omega C_{ba}(V_b-V_a)\\
I_c &= j\omega C_{cc}V_c + j\omega C_{ca}(V_c-V_a) + j\omega C_{cb}(V_c-V_b)
\end{aligned}\right\} \tag{1.26a}$$

or, with a small modification,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = j\omega \begin{bmatrix} C_{aa}+C_{ab}+C_{ac} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{ba}+C_{bb}+C_{bc} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{ca}+C_{cb}+C_{cc} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{1.26b}$$

This is the fundamental equation for stray capacitances of a three-phase single circuit transmission line. Also Figure 1.7(c) is derived from one-to-one correspondence with Equation 1.26.

1.2.1.3 Coefficients of potential (p_{aa}, p_{ab}), coefficients of static capacity (k_{aa}, k_{ab}) and capacitances (C_{aa}, C_{ab})

The earth surface can be taken as a perfect equal-potential plane, so that we can use Figure 1.8, in which the three imaginary conductors α, β, γ are located at symmetrical positions of conductors a, b, c, respectively, based on the earth surface plane. By assuming electric charges $+q_a$, $+q_b$, $+q_c$ and $-q_a$, $-q_b$, $-q_c$ per unit length on conductors a, b, c, and α, β, γ respectively, the following voltage equation can be derived:

$$\begin{aligned}
v_a = &\left(\text{voltage of conductor a due to } \pm q_a \text{ of conductor a, } \alpha : 2q_a\log_{\mathrm{e}}\frac{2h_a}{r}\times 9\times 10^9\ [\mathrm{V}]\right)\\
&+\left(\text{voltage of conductor a due to } \pm q_b \text{ of conductor b, } \beta : 2q_b\log_{\mathrm{e}}\frac{s_{a\beta}}{s_{ab}}\times 9\times 10^9\ [\mathrm{V}]\right)\\
&+\left(\text{voltage of conductor a due to } \pm q_c \text{ of conductor c, } \gamma : 2q_c\log_{\mathrm{e}}\frac{s_{a\gamma}}{s_{ac}}\times 9\times 10^9\ [\mathrm{V}]\right) \quad ①
\end{aligned}$$

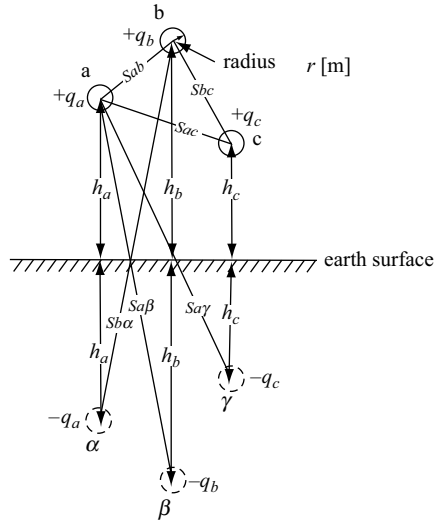


Figure 1.8 Three parallel overhead conductors

Equations for v_b, v_c can be derived in the same way. Then

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} p_{aa} & p_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix} \cdot \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix}$$

$$= 2 \times 9 \times 10^9 \times \begin{bmatrix} \log_e \dfrac{2h_a}{r} & \log_e \dfrac{s_{a\beta}}{s_{ab}} & \log_e \dfrac{s_{a\gamma}}{s_{ac}} \\ \log_e \dfrac{s_{b\alpha}}{s_{ba}} & \log_e \dfrac{2h_b}{r} & \log_e \dfrac{s_{b\gamma}}{s_{bc}} \\ \log_e \dfrac{s_{c\alpha}}{s_{ca}} & \log_e \dfrac{s_{c\beta}}{s_{cb}} & \log_e \dfrac{2h_c}{r} \end{bmatrix} \cdot \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} \text{②} \qquad (1.27)$$

where $s_{a\beta} = s_{b\alpha} = \sqrt{\{s_{ab}^2 - (h_a - h_b)^2\} + (h_a + h_b)^2} = \sqrt{s_{ab}^2 + 4h_a h_b}$.

Refer the section 1.3.2 for the deriving process as of the Equation 1.27 as of theory of electromagnetism.

The equation indicates that the **coefficients of potential** (p_{aa}, p_{ab}, etc.) are calculated as a function of the conductor's radius r, height (h_a, h_b, h_c) from the earth surface, and phase-to-phase distances (s_{ab}, s_{ac}, etc.) of the conductors. p_{aa}, p_{ab}, etc., are determined only by physical allocations of each phase conductor (in other words, by the structure of towers), and relations like $p_{ab} = p_{ba}$ are obvious.

In conclusion, the coefficients of potential (p_{aa}, p_{ab}, etc.), the **coefficients of static capacity** (k_{aa}, k_{ab}, etc.) and the capacitance (C_{aa}, C_{ab}, etc.) are calculated from Equations 1.27, 1.20 and 1.23, respectively. Again, all these values are determined only by the physical allocation of conductors and are not affected by the applied voltage.

1.2.1.4 Stray capacitances of phase-balanced transmission lines

Referring to Figure 1.8, a well-phase-balanced transmission line, probably by transposition, can be assumed. Then

$$\left.\begin{aligned} &h \equiv h_a \fallingdotseq h_b \fallingdotseq h_c, \qquad s_{ll} \equiv s_{ab} = s_{ba} \fallingdotseq s_{bc} = s_{cb} \fallingdotseq s_{ca} = s_{ac} \\ &s_{a\beta} \fallingdotseq s_{b\alpha} \fallingdotseq s_{a\gamma} \fallingdotseq s_{c\alpha} \fallingdotseq s_{b\gamma} \fallingdotseq s_{c\beta} \end{aligned}\right\} \tag{1.28}$$

$$\left.\begin{aligned} &p_s \equiv p_{aa} \fallingdotseq p_{bb} \fallingdotseq p_{cc} \\ &p_m \equiv p_{ab} = p_{ba} \fallingdotseq p_{ac} = p_{ca} \fallingdotseq p_{bc} = p_{cb} \end{aligned}\right\} \tag{1.29}$$

Accordingly, Equation 1.20 can be simplified as follows:

$$\left.\begin{aligned} \Delta &= p_s^3 + 2p_m^3 - 3p_s p_m^2 \\ &= (p_s - p_m)^2 (p_s + 2p_m) \\ k_s &\equiv k_{aa} \fallingdotseq k_{bb} \fallingdotseq k_{cc} \fallingdotseq (p_s^2 - p_m^2)/\Delta = \frac{p_s + p_m}{(p_s - p_m)(p_s + 2p_m)} \\ k_m &\equiv k_{ab} = k_{ba} \fallingdotseq k_{ac} = k_{ca} \fallingdotseq k_{bc} = k_{cb} = -(p_m p_s - p_m^2)/\Delta \\ &= \frac{-p_m}{(p_s - p_m)(p_s + 2p_m)} \\ k_s + 2k_m &= \frac{1}{p_s + 2p_m} \end{aligned}\right\} \tag{1.30}$$

and from Equation 1.23

$$\left.\begin{aligned} C_s &\equiv C_{aa} \fallingdotseq C_{bb} \fallingdotseq C_{cc} = k_s + 2k_m = \frac{1}{p_s + 2p_m} \\ C_m &\equiv C_{ab} = C_{ba} \fallingdotseq C_{ac} = C_{ca} \fallingdotseq C_{bc} = C_{cb} = -k_m \\ &= \frac{p_m}{(p_s - p_m)(p_s + 2p_m)} = \frac{p_m}{p_s - p_m} \cdot C_s \end{aligned}\right\} \tag{1.31}$$

and from Equation 1.27

$$\left.\begin{aligned} p_s &\equiv p_{aa} \fallingdotseq p_{bb} \fallingdotseq p_{cc} = 2 \times 9 \times 10^9 \log_e \frac{2h}{r} \quad [\text{m/F}] \quad ① \\ p_m &\equiv p_{ab} \fallingdotseq p_{bc} \fallingdotseq p_{ca} = 2 \times 9 \times 10^9 \log_e \frac{s_{b\alpha}}{s_{ll}} \quad [\text{m/F}] \\ &\fallingdotseq 2 \times 9 \times 10^9 \log_e \frac{\sqrt{s_{ll}^2 + (2h)^2}}{s_{ll}} \\ &= 2 \times 9 \times 10^9 \log_e \left\{ 1 + \left(\frac{2h}{s_{ll}} \right)^2 \right\}^{1/2} \quad [\text{m/F}] \quad ② \end{aligned}\right\} \tag{1.32}$$

where generally

$$h > s_{ll}, \left(\frac{2h}{s_{ll}} \right)^2 \gg 1$$

and

$$\therefore\ p_m \fallingdotseq 2 \times 9 \times 10^9 \log_e \frac{2h}{s_{ll}} \quad [\text{m/F}] \quad ②'$$

Substituting p_s, p_m from Equation 1.32 into Equation 1.31,

$$\left.\begin{aligned} C_s &= \frac{1}{p_s + 2p_m} \fallingdotseq \frac{1}{2\times 9\times 10^9\left(\log_e\frac{2h}{r}+2\log_e\frac{2h}{s_{ll}}\right)} = \frac{1}{2\times 9\times 10^9\log_e\frac{8h^3}{rs_{ll}^2}} \\ &= \frac{0.02413}{\log_{10}\frac{8h^3}{rs_{ll}^2}}\times 10^{-9}[\mathrm{F/m}] = \frac{0.02413}{\log_{10}\frac{8h^3}{rs_{ll}^2}}[\mu\mathrm{F/km}] \qquad ① \\ &\qquad (\text{zero-sequence capacitance}) \end{aligned}\right\}$$

while

$$\left.\begin{aligned} \frac{p_m}{p_s - p_m} &\fallingdotseq \frac{\log_e\frac{2h}{s_{ll}}}{\log_e\frac{2h}{r}-\log_e\frac{2h}{s_{ll}}} = \frac{\log_{10}\frac{2h}{s_{ll}}}{\log_{10}\frac{s_{ll}}{r}} \\ \therefore C_m = C_s\cdot\frac{p_m}{p_s - p_m} &\fallingdotseq C_s\cdot\frac{\log_{10}\frac{2h}{s_{ll}}}{\log_{10}\frac{s_{ll}}{r}} = \frac{0.02413}{\log_{10}\frac{8h^3}{rs_{ll}^2}}\cdot\frac{\log_{10}\frac{2h}{s_{ll}}}{\log_{10}\frac{s_{ll}}{r}}[\mu\mathrm{F/km}] \qquad ② \end{aligned}\right\} \tag{1.33}$$

In conclusion, a well-phase-balanced transmission line can be expressed by Figure 1.9(a) and Equation 1.26b is simplified into Equation 1.34, where the stray capacitances C_s, C_m can be calculated from Equation 1.33:

$$\underbrace{\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}}_{\boldsymbol{I}_{abc}} = j\omega \underbrace{\begin{bmatrix} C_s+2C_m & -C_m & -C_m \\ -C_m & C_s+2C_m & -C_m \\ -C_m & -C_m & C_s+2C_m \end{bmatrix}}_{\boldsymbol{C}_{abc}} \cdot \underbrace{\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}}_{\boldsymbol{V}_{abc}} \tag{1.34}$$

$$\therefore \boldsymbol{I}_{abc} = j\omega\boldsymbol{C}_{abc}\cdot\boldsymbol{V}_{abc}$$

Incidentally, Figure 1.9(a) can be modified to Figure 1.9(b), where the total capacitance of one phase $C \equiv C_s + 3C_m$ is called the **working capacitance** of single circuit transmission lines, and can be calculated by the following equation:

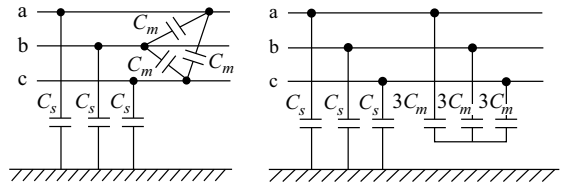
$$\left.\begin{aligned} C &\equiv C_s + 3C_m = (k_s + 2k_m) + 3(-k_m) = k_s - k_m = \frac{1}{p_s - p_m} \\ &= \frac{1}{2\times 9\times 10^9\left(\log_e\frac{2h}{r}-\log_e\frac{2h}{s_{ll}}\right)} = \frac{1}{2\times 9\times 10^9\log_e\frac{s_{ll}}{r}} \quad [\mathrm{F/m}] \\ &= \frac{0.02413}{\log_{10}\frac{s_{ll}}{r}} \quad [\mu\mathrm{F/km}] \quad (\text{positive sequence capacitance}) \qquad ① \end{aligned}\right\} \tag{1.35}$$

In case of multi-bundled (n) conductor lines, the radius r is replaced by the equivalent radius r_{eff},

$$r_{\mathrm{eq}} = r^{1/n}\times w^{(n-1)/n} \quad [\mathrm{m}] \qquad ②$$

where w is the geometrical averaged distance between bundled conductors.

Refer the Supplement 1 for the introduction of equivalent radius of a multi-bundled conductors.



(a) single circuit line (b) single circuit line

Figure 1.9 Stray capacitances of single circuit overhead line (well balanced)

1.2.1.4.1 Numerical check Taking the conditions conductor radius $r = 0.05$ m, averaged phase-to-phase distance $s_{ll} = 10$ m and average height $h = 60$ m, then by Equations 1.33 and 1.35, we have

$$C_s = 0.00436\,\mu\text{F/km},\ C_m = 0.00204\,\mu\text{F/km}\ \text{and}\ C = C_s + 3C_m = 0.01048\,\mu\text{F/km}$$

1.2.2 Three-phase single circuit line with OGW

Four conductors of phase names a, b, c, x exist in this case, so the following equation can be derived as an extended form of Equation 1.26a:

$$I_a = j\omega C_{aa}V_a + j\omega C_{ab}(V_a - V_b) + j\omega C_{ac}(V_a - V_c) + j\omega C_{ax}(V_a - V_x) \tag{1.36a}$$

where $V_x = 0$, because OGW is earth grounded at every tower. Accordingly,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = j\omega \begin{bmatrix} C_{aa} + C_{ab} + C_{ac} + C_{ax} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{ba} + C_{bb} + C_{bc} + C_{bx} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{ca} + C_{cb} + C_{cc} + C_{cx} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \tag{1.36b}$$

This matrix equation is again in the same form as Equation 1.26b. However, the phase-to-ground capacitance values (diagonal elements of the matrix $\boldsymbol{C}$) are increased (the value of C_{ax} is increased for the phase a conductor, from $C_{aa} + C_{ab} + C_{ac}$ *to* $C_{aa} + C_{ab} + C_{ac} + C_{ax}$).

1.2.3 Three-phase double circuit line

Six conductors of phase names a, b, c, A, B, C exist in this case as is shown in Figure 1.10, so the following equation can be derived as an extended form of Equation 1.26a:

$$\begin{aligned} I_a = j\omega[&C_{aa}V_a + C_{ab}(V_a - V_b) + C_{ac}(V_a - V_c) + C_{aA}(V_a - V_A) + C_{aB}(V_a - V_B) \\ &+ C_{aC}(V_a - V_C)] \end{aligned} \tag{1.37a}$$

Then

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_A \\ I_B \\ I_C \end{bmatrix} = j\omega \begin{bmatrix} C_{aa}+C_{ab}+C_{ac}+C_{aA}+C_{aB}+C_{aC} & -C_{ab} & -C_{ac} & -C_{aA} & -C_{aB} & -C_{aC} \\ -C_{ba} & C_{ba}+C_{bb}+C_{bc}+C_{bA}+C_{bB}+C_{bC} & -C_{bc} & -C_{bA} & -C_{bB} & -C_{bC} \\ -C_{ca} & -C_{cb} & C_{ca}+C_{cb}+C_{cc}+C_{cA}+C_{cB}+C_{cC} & -C_{cA} & -C_{cB} & -C_{cC} \\ -C_{Aa} & -C_{Ab} & -C_{Ac} & C_{AA}+C_{AB}+C_{AC}+C_{Aa}+C_{Ab}+C_{Ac} & -C_{AB} & -C_{AC} \\ -C_{Ba} & -C_{Bb} & -C_{Bc} & -C_{BA} & C_{BA}+C_{BB}+C_{BC}+C_{Ba}+C_{Bb}+C_{Bc} & -C_{BC} \\ -C_{Ca} & -C_{Cb} & -C_{Cc} & -C_{CA} & -C_{CB} & C_{CA}+C_{CB}+C_{CC}+C_{Ca}+C_{Cb}+C_{Cc} \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_A \\ V_B \\ V_C \end{bmatrix} \tag{1.37b}$$

It is obvious that the double circuit line with OGW can be expressed in the same form.

The case of a well-transposed double circuit line is as shown in Figure 1.9(b):

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_A \\ I_B \\ I_C \end{bmatrix} = j\omega \begin{bmatrix} C_s+2C_m+3C'_m & -C_m & -C_m & -C'_m & -C'_m & -C'_m \\ -C_m & C_s+2C_m+3C'_m & -C_m & -C'_m & -C'_m & -C'_m \\ -C_m & -C_m & C_s+2C_m+3C'_m & -C'_m & -C'_m & -C'_m \\ -C'_m & -C'_m & -C'_m & C_s+2C_m+3C'_m & -C_m & -C_m \\ -C'_m & -C'_m & -C'_m & -C_m & C_s+2C_m+3C'_m & -C_m \\ -C'_m & -C'_m & -C'_m & -C_m & -C_m & C_s+2C_m+3C'_m \end{bmatrix} \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_A \\ V_B \\ V_C \end{bmatrix}$$

$$\begin{aligned} &C_s \equiv C_{aa} \fallingdotseq C_{bb} \fallingdotseq C_{cc} \fallingdotseq C_{AA} \fallingdotseq C_{BB} \fallingdotseq C_{CC} && \text{: one phase-to-ground capacitance} \\ &C_m \equiv C_{ab} \fallingdotseq C_{bc} \fallingdotseq \cdots \fallingdotseq C_{AB} \fallingdotseq C_{BC} \fallingdotseq \cdots && \text{: capacitance between two conductors of the same circuit} \\ &C'_m \equiv C_{aA} \fallingdotseq C_{bC} \fallingdotseq \cdots \fallingdotseq C_{Aa} \fallingdotseq C_{Bb} \fallingdotseq \cdots && \text{: capacitance between two conductors of a different circuit} \end{aligned} \tag{1.38}$$

Above, we have studied the fundamental equations and circuit models of transmission lines and the actual calculation method for the L, C, R constants. Concrete values of the constants are investigated in Chapter 2.

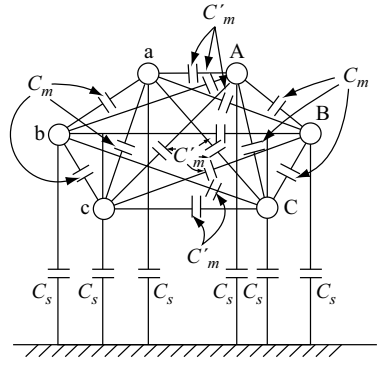


Figure 1.10 Stray capacitance of double circuit line (well balanced)

1.3 Working Inductance and Working Capacitance

The Equation 1.9 for working inductance and Equation 1.35 for working capacitance as well as Equation 1.27 for capacitive induced voltage were briefly shown in the previous sections. Now we introduce these equations and examine what these equations mean from the physical viewpoint of electromagnetism.

1.3.1 Introduction of working inductance

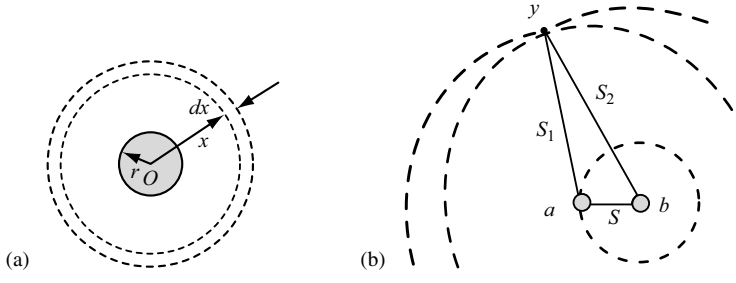


Figure 1.11

1.3.1.1 Introduction of self-inductance L_{aa} of a straight conductor

As is shown in Figure 1.11, one conductor a (radius r) is laid out straight in an area of **permeability** $\mu = \mu_s \cdot \mu_0$ (μ_0 is **permeability in vacuum** space and μ_s is **relative permeability** and $\mu_s = 1.0$ in vacuum space). If current i flows through conductor a, concentric circular magnetic paths are composed in a conductor section as well as in outer space, and the central point O of the conductor a is also the central point of induced concentric magnetic paths. The concentric magnetic paths in the outer space of the conductor a is examined first. A thin concentric magnetic ring path at point x from O with length $2\pi x$ and width dx can be imaged. The magnetic resistance R of the ring path is proportional in the length of the ring path $2\pi x[m]$, and is inversely proportional in the sectional area$1 \times dx[m^2]$. Namely,

$$R = \frac{2\pi x}{\mu dx}\ [\mathrm{A \cdot turn/Wb}] \quad \text{where } x \geq r \tag{1.39a}$$

where $\mu = \mu_s \cdot \mu_0$:the permeability of the ring path

μ_0: permeability in vacuum space ($\mu_0 = 4\pi \times 10^{-7}$ by MKS rational unit system)

μ_s: relative permeability ($\mu_s = 1.0$ in vacuum space)

The reason that μ_0 is $4\pi \times 10^{-7}$ in MKS rational unit system is discussed later in section 1.3.4. If current i[A] is flowed through the conductor (or if electromotive force i[A · turn] is charged in the conductor), flux $d\varphi$ is produced through the ring path with sectional depth dx and

$$d\varphi = \frac{i}{R} = \frac{\mu \cdot i}{2\pi x} \cdot dx \quad [\mathrm{Wb}] \tag{1.39b}$$

The linking flux number $d\psi$ is

$$d\psi = 1 \cdot d\varphi = \frac{\mu \cdot i}{2\pi x} \cdot dx \tag{1.39c}$$

Therefore the total linking flux ψ_{out} of the space from the conductor surface (radius $r[m]$) to point S is

$$\begin{aligned} \psi_{out} &= \int_r^S d\psi_{out} = \int_r^S 1 \cdot d\varphi = \int_r^S \frac{\mu \cdot i}{2\pi x} \cdot dx = \left[\frac{\mu \cdot i}{2\pi} \cdot \log_e x\right]_r^s \\ &= \frac{\mu \cdot i}{2\pi} \log_e \frac{\mathrm{S}}{\mathrm{r}} = \left(\frac{\mu_s \cdot \mu_0}{2\pi} \log_e \frac{\mathrm{S}}{\mathrm{r}}\right) \cdot i \end{aligned} \tag{1.39d}$$

Next, linking flux number ψ_{in} in the conductor section is examined. If current $i[A]$ is flowed through the conductor, the current within space of diameter $x[m]$ is

$$i_x = i \cdot \frac{x^2}{r^2} \quad [\mathrm{A}] \quad \text{where } r \geq x \geq 0 \tag{1.40a}$$

The intensity of magnetic field at the ring path with length $2\pi x$ and width dx which is x distant from point O in the radial direction is:

$$H = \frac{i_x}{2\pi x} \ [\mathrm{A \cdot turn/m}] \tag{1.40b}$$

The flux density is:

$$B = \mu_{cond} \cdot \mu_0 \cdot H = \frac{\mu_{cond} \cdot \mu_0 \cdot i_x}{2\pi x} = \frac{\mu_{cond} \cdot \mu_0 \cdot i \cdot x}{2\pi r^2} \ \left[\mathrm{Wb/m^2}\right] \tag{1.40c}$$

where μ_{cond} is the relative permeability of the conductor
The flux at the x distant ring path with dx width is:

$$d\varphi = B \cdot (1 \times dx) = Bdx = \frac{\mu_{cond} \cdot \mu_0 \cdot i \cdot x}{2\pi r^2} \cdot dx \ [\mathrm{Wb}] \tag{1.40d}$$

The turn number of the conductor within circle of radius x can be considered x^2/r^2, then the linking flux number is

$$\begin{aligned} \psi_{in} &= \int_0^r d\psi_{in} = \int_0^r i_x \cdot d\varphi = \int_0^r \frac{\mu_{cond} \cdot \mu_0 \cdot i \cdot x}{2\pi r^2} \cdot \frac{x^2}{r^2} \cdot i dx = \int_0^r \frac{\mu_{cond} \cdot \mu_0}{2\pi r^4} \cdot i \cdot x^3 dx \\ &= \frac{\mu_{cond} \cdot \mu_0}{2\pi r^4} \cdot \left[\frac{1}{4}x^4\right]_0^r \cdot i = \frac{\mu_{cond} \cdot \mu_0}{2\pi r^4} \cdot \left(\frac{1}{4}r^4\right) \cdot i = \frac{\mu_{cond} \cdot \mu_0}{8\pi} \cdot i \end{aligned} \tag{1.40e}$$

As the result of all the above Equations (1.39d)(1.40e), total linking flux numbers which is produced by current i of the conductor a and interlink with the current i itself between the area of conductor a to outer space point S is:

$$\psi_{total} = \psi_{out} + \psi_{in} = \left(\frac{\mu_s \cdot \mu_0}{2\pi} \log_e \frac{S}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi}\right) \cdot i \tag{1.41a}$$

As the definition of inductance is the linking flux number per 1A, or $L = \psi/i$, then

$$L_{aa} = \frac{\psi_{total}}{i} = \frac{\mu_s \cdot \mu_0}{2\pi} \log_e \frac{S}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi} \tag{1.41b}$$

or $L_{aa} = \left(0.4605\mu_s \log_{10} \frac{S}{r} + 0.05\mu_{cond}\right) \times 10^{-6} \ [\mathrm{H/m}] = 0.4605\mu_s \log_{10} \frac{S}{r} + 0.05\mu_{cond} \ [\mathrm{mH/km}]$

where μ_0: permeability of vacuum space, and $\mu_0 = 4\pi \times 10^{-7}$ by MKS rational unit system

(1.41c)

This is the self inductance of the conductor a, and the equation correspond with Equation (1.10a).

1.3.1.2 Introduction of working-inductance $L_{aa} - L_{ab}$ of two conductors

In next, working inductance $L_{aa} - L_{ab}$ of two conductors a and b is examined. (refer Figure 1.11(b)).

Two conductors a and b (radius r) are lay out in parallel with distance S and the current i [A] go out on the conductor a and come back from b, or current i [A] flows in a and current $-i$ [A] flows in b. Now, we image an arbitrary point $y(S_1, S_2)$, which is S_1 distant from a and S_2 distant from b, and the point y is far distant from both conductors a and b, namely, $S_1 \cong S_2 \gg S$.

Current $i\,[A]$ of conductor a produces concentric flux of conductor a and all these flux interlink with the current i, so that linking flux number is given by Equation (1.41a). That is again,

$$\psi_{aa} = \left(\frac{\mu_s \cdot \mu_0}{2\pi}\log_e\frac{S_1}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi}\right) \cdot i \quad \text{where } \mu = \mu_{cond} \cdot \mu_0 \tag{1.42a}$$

Next, current $-i\,[A]$ of conductor b produces concentric flux of conductor b. Among these flux, linking flux to which current i of conductor a links with can be calculated by accumulating $d\psi_{ab}$ from S to S_2. That is,

$$\begin{aligned} -\psi_{ab} &= \int_S^{S_2} d\psi_{ab} = \int_S^{S_2} (-i)d\varphi = \int_S^{S_2} \frac{\mu(-i)}{2\pi x} \cdot dx = \left[\frac{\mu(-i)}{2\pi}\log_e x\right]_S^{S_2} \\ &= \left(\frac{\mu}{2\pi}\log_e\frac{S_2}{S}\right) \cdot (-i) \end{aligned} \tag{1.42b}$$

The total linking flux number of current i of conductor a is the sum of $\psi_{aa}$and $-\psi_{ab}$, and reminding $S_1 \cong S_2 >> S$

$$\begin{aligned} \psi_{aa} - \psi_{ab} &= \left(\frac{\mu}{2\pi}\log_e\frac{S_1}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi}\right) \cdot i + \left(\frac{\mu}{2\pi}\log_e\frac{S_2}{S}\right) \cdot (-i) \\ &= \left(\frac{\mu}{2\pi}\log_e\frac{S}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi}\right) \cdot i \end{aligned} \tag{1.42c}$$

The definition of inductance is linking flux numbers per 1Ampere, that is $L = \psi/i$, then

$$L_{aa} - L_{ab} = \frac{\psi_{aa} - \psi_{ab}}{i} = \frac{\mu}{2\pi}\log_e\frac{S_1}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi} = \frac{\mu_s \cdot \mu_0}{2\pi}\log_e\frac{S}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi} \tag{1.42d}$$

Now, we have introduced general equation of working inductance $L_{aa} - L_{ab}$.
The Equation (1.42d) is modified a little by putting $\mu_0 = 4\pi \times 10^{-7}$ as of MKS rational unit system.

$$\begin{aligned} L_{aa} - L_{ab} &= \left(2\mu_s \log_e\frac{S}{r} + \frac{\mu_{cond}}{2}\right) \cdot 10^{-7}\ [\mathrm{H/m}] \\ &= \left(0.4605\log_{10}\frac{S}{r} + 0.05\mu_{cond}\right) \times 10^{-6}\ [\mathrm{H/m}] = 0.4605\log_{10}\frac{S}{r} + 0.05\mu_{cond}\ [\mathrm{mH/km}] \end{aligned} \tag{1.42e}$$

This is the working inductance of two conductors lay out through three dimensional vacuum space, and is of course the same with Equation (1.9). In case of vacuum space or air space $\mu_s = 1.0$ and μ_{cond} is the permeability of aluminum or copper and is $\mu_{cond} = 1$.

1.3.2 Introduction of working capacitance

Now referring to Figure 1.12(a), we introduce working capacitance of two parallel conductors a and b (radius r) with the same lay out of that in the previous section. Supposing the case in that the conductor a is charged by $+q\,[\mathrm{C/m}]$ and b is charged by $-q\,[\mathrm{C/m}]$, and the condition of point y is examined which is S_1, S_2 distant from the conductors a and b. Because the conductor radius r is quite small (S_1, S_2 $>> r$), it can be presumed that the charges $+q$ and $-q$ are allocated at the center pin points of the conductors a and b. The intensity of electric field $U_{ya}[\mathrm{V/m}]$ at point y caused by $+q\,[\mathrm{C/m}]$ of conductor a and $U_{yb}[\mathrm{V/m}]$ caused by $-q[\mathrm{C/m}]$ of conductor b are:

$$U_{ya} = \frac{q}{2\pi\varepsilon \cdot S_1}\ [\mathrm{V/m}],\quad U_{yb} = \frac{-q}{2\pi\varepsilon \cdot S_2}\ [\mathrm{V/m}] \tag{1.43}$$

where $\varepsilon = \varepsilon_s \cdot \varepsilon_0$: permittivity of the circuit field

ε_0: permittivity of vacuum space and $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ by MKS rational unit system
ε_s: relative permittivity (1.44)

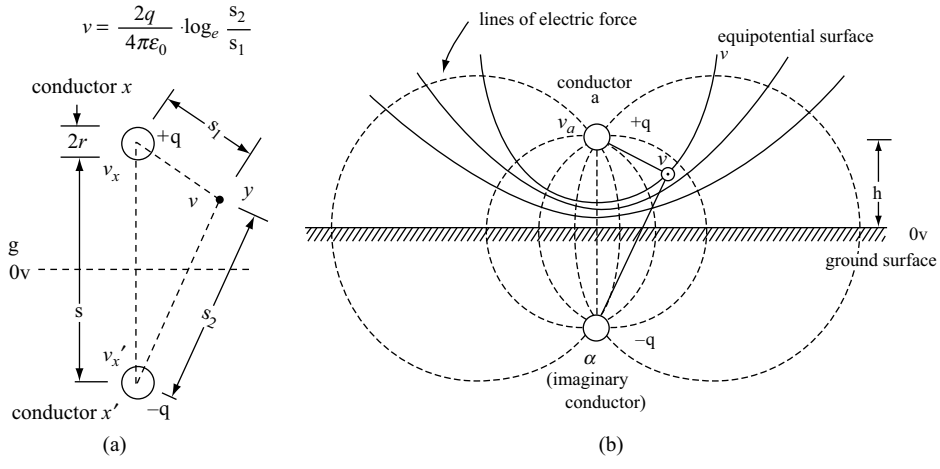


Figure 1.12

The electric potential at the mid-point which is the same distance from the two conductors (the point of $S_1 = S_2 = S/2$) should be obviously zero, then,

$$v_y = \int_{S_1}^{S/2} U_a dS_1 + \int_{S_2}^{S/2} U_b dS_2 = \int_{S_1}^{S/2} \frac{1}{2\pi\varepsilon S_1} \cdot q dS_1 + \int_{S_2}^{S/2} \frac{1}{2\pi\varepsilon S_2} \cdot (-q) dS_2$$

$$= \left(\left[\frac{1}{2\pi\varepsilon} \cdot \log_e S_1 \right]_{S_1}^{S/2} - \left[\frac{1}{2\pi\varepsilon} \cdot \log_e S_2 \right]_{S_2}^{S/2} \right) \cdot q = \left(\frac{1}{2\pi\varepsilon} \cdot \log_e \frac{S_2}{S_1} \right) \cdot q$$

$$\therefore\ v_y = \left(\frac{1}{2\pi\varepsilon_s \cdot \varepsilon_0} \cdot \log_e \frac{S_2}{S_1} \right) \cdot q \quad \text{where} \quad \varepsilon = \varepsilon_s \cdot \varepsilon_0 \tag{1.45a}$$

then

$$v_y = \frac{2q}{\varepsilon_s} \log_e \frac{S_2}{S_1} \times 9 \times 10^9 \ [\text{V}] \tag{1.45b}$$

where

$$\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \quad \text{by MKS rational unit system} \tag{1.46}$$

The Equation of surface potential v of the conductor a is given by $S_1 \rightarrow r$ $S_2 \rightarrow S$ as a special case of (1.45a).

$$v_a = \left(\frac{1}{2\pi\varepsilon_s \cdot \varepsilon_0} \cdot \log_e \frac{S}{r} \right) \cdot q \tag{1.47a}$$

$$v_a = \frac{2q}{\varepsilon_s} \log_e \frac{S}{r} \times 9 \times 10^9 \ [\text{V}] \quad \text{by MKS rational unit system} \tag{1.47b}$$

The capacitance C_a from conductor a (or b) to the zero potential plane (neutral plane) at the mid-point of conductors a and b is given by:

$$C_a = \frac{q}{v_a} = \frac{1}{\frac{1}{2\pi\varepsilon_s \cdot \varepsilon_0} \cdot \log_e \frac{S}{r}} \tag{1.48a}$$

Applying MKS rational unit system by Equation (1.46) and decimal logarithm,

$$C_a = \frac{\varepsilon_s}{2 \times 9 \times 10^9 \log_e \frac{S}{r}} = \frac{0.02413\varepsilon_s}{\log_{10} \frac{S}{r}} \times 10^{-9} \ [\mathrm{F/m}] = \frac{0.02413\varepsilon_s}{\log_{10} \frac{S}{r}} \ [\mu\mathrm{F/km}] \tag{1.48b}$$

The Equations (1.45a)(1.47a)(1.48a) explain natural physics whose forms are not affected by selection of any measuring unit system, and Equations (1.45b)(1.47b)(1.48b) are the expression by MKS rational unit system based on Equation (1.46).

Now , let us compare the Figure 1.12(a) and (b). The potential of neutral plane g is zero, so that the plane can be equated with earth ground, and therefore Figure 1.12(a) and (b) are equivalent of each other. In other words, theory of transmission line can be treated by a set of real conductor a with charge $+q$ and imaginary conductor α with charge $-q$. Needless to say Equation (1.48b) corresponds to Equation (1.35).

Furthermore, if we change space distance S_1, S_2 from the conductors a and α but by keeping S_2/S_1 as of constant value, v_y of Equation (1.45a,b) should be kept unchanged. So Equation (1.45) gives **equipotential lines** as is shown in Figure 1.12(b).

1.3.3 Special properties of working inductance and working capacitance

The equation of working inductance and working capacitance were introduced in the previous section. These are again:

$$L_{aa} - L_{ab} = \frac{\mu_s \cdot \mu_0}{2\pi} \log_e \frac{S}{r} + \frac{\mu_{cond} \cdot \mu_0}{8\pi} \tag{1.42d}$$

$$C_a = \frac{q}{v_a} = \frac{1}{\frac{1}{2\pi\varepsilon_s \cdot \varepsilon_0} \cdot \log_e \frac{S}{r}} \tag{1.48a}$$

Also **permeability** and **permittivity** were explained through the deriving process, and these are again, by our MKS rational unit system:

μ_0: permeability of vacuum space, and $\mu_0 = 4\pi \times 10^{-7}$

ε_0: permittivity of vacuum space, and $\dfrac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ (1.49)

Now, let us examine furthermore about the above equations. The right side second term $\mu_{cond} \cdot \mu_0/8\pi$ of Equation (1.42d) is of linking flux number in narrow conductor section, so that it can be ignored when phenomena of wide space is examined. Then, working inductance $L_{aa} - L_{ab}$ and working capacitance C_a relate of each other as follows:

$$\frac{1}{\sqrt{(L_{aa} - L_{ab}) \cdot (C_a)}} = 1 \Bigg/ \sqrt{\left(\frac{\mu_s \cdot \mu_0}{2\pi} \log_e \frac{S}{r}\right)\left(\frac{1}{\frac{1}{2\pi\varepsilon_s \cdot \varepsilon_0} \log_e \frac{S}{r}}\right)} = \frac{1}{\sqrt{\mu_s \cdot \mu_0 \times \varepsilon_s \cdot \varepsilon_0}}$$

In case of vacuum space $\mu_s = 1.0$ and $\varepsilon_s = 1.0$, then

$$\therefore \quad \frac{1}{\sqrt{(L_{aa} - L_{ab}) \cdot (C_a)}} = \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}} \equiv c_0 \quad \text{(the constant value)} \tag{1.50}$$

By MKS unit system

$$\begin{aligned} c_0 &\equiv \frac{1}{\sqrt{\mu_0 \cdot \varepsilon_0}} = \frac{1}{\sqrt{(4\pi \times 10^{-7}) \cdot (1/(4\pi \times 9 \times 10^9))}} = 3 \times 10^8 \ [\mathrm{m/sec}] \\ &= 300{,}000 \ \mathrm{km/sec} \end{aligned} \tag{1.51}$$

Now it was found that $1/\sqrt{(L_{aa}-L_{ab})\cdot(C_a)}$ always comes to $1/\sqrt{\mu_0\cdot\varepsilon_0}$ which takes constant value c_0 unconditionally. From the physical viewpoint, if current flow through a straight conductor lay out in three dimensional vacuum space, it would be accompanied by magnetic line with permeability μ_0 and electric line of force with permittivity ε_0. Furthermore, $1/\sqrt{\mu_0\cdot\varepsilon_0}$ takes constant value c_0 unconditionally.

In fact, Equation (1.50)(1.51) are the climax of the conclusion which was presented by James C Maxwell in 1873 in his famous paper (refer Coffee break 5). The constant c_0 is of a value with dimension of 'distance/time' or 'velocity'. With these conclusive equations, Maxwell presumed as follows

i) electromagnetic wave would exist and it can propagate through 'vacuum space without 'ether',

ii) The propagating velocity of the wave is always constant value $1/\sqrt{\mu_0\cdot\varepsilon_0}=c_0$, and it would be 300,000km/sec if it is measured by MKS rational unit system. This was the time that electromagnetic wave was discovered theoretically by Maxwell. He also presumed by analogy that light from the sun must be also a kind of wave having the same velocity 300,000km/sec.

1.3.4 MKS rational unit system and the various MKS practical units in electrical engineering field

1.3.4.1 MKS rational unit system

We discuss about fundamentals of **MKS rational unit system** as the last subject of this chapter.

The velocity of electromagnetic wave c_0 is an universal unchanged constant, and the value is $c_0\equiv 1/\sqrt{\mu_0\cdot\varepsilon_0}=3\times10^8\,[\mathrm{m/sec}]$ if measured by MKS unit system. In next, c_0 is the unchanged value 300,000km/sec, so that $\mu_0\cdot\varepsilon_0$ is also unchanged value. In other words, we can freely determine either one of μ_0 or ε_0 as methods of unit system selection although $\mu_0\cdot\varepsilon_0=1/c_0^2$ is unchanged value. Namely, if one value is given to one of μ_0 and ε_0 as its definition, the another should be defined dependently to satisfy the above equation.

Therefore, μ_0 and ε_0 are defined as follows by MKS rational unit system.

$$\mu_0=4\pi\times10^{-7}\ [\mathrm{H/m}] \tag{1.52a}$$

$$\varepsilon_0=\frac{1}{(4\pi\times10^{-7})\cdot c_0^2}=\frac{1}{4\pi\times9\times10^9} \tag{1.52b}$$

$$c_0\equiv\frac{1}{\sqrt{\mu_0\cdot\varepsilon_0}}=3\times10^8\,[\mathrm{m/sec}] \tag{1.52c}$$

Now, we go back to the historical story of MKS rational unit system.

Famous **Coulomb's laws** for force by electric charge and for force by magnetic pole can be described by **Gaussian unit system** and by MKS rational unit system as follows:

	Coulomb's law by electric charge q_1, q_2	Coulomb's law by magnetic pole $m_1,\ m_2$
Gaussian unit system	$F=\dfrac{q_1\cdot q_2}{r^2}$	$F=\dfrac{m_1\cdot m_2}{r^2}$
MKS rational unit system	$F=\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{q_1\cdot q_2}{r^2}$	$F=\dfrac{1}{4\pi\mu_0}\cdot\dfrac{m_1\cdot m_2}{r^2}\ \left[\mathrm{V/m^2}\right]$

(1.53)

where ε_0 and μ_0 are defined by Equation (1.52a,b) by MKS rational unit system.

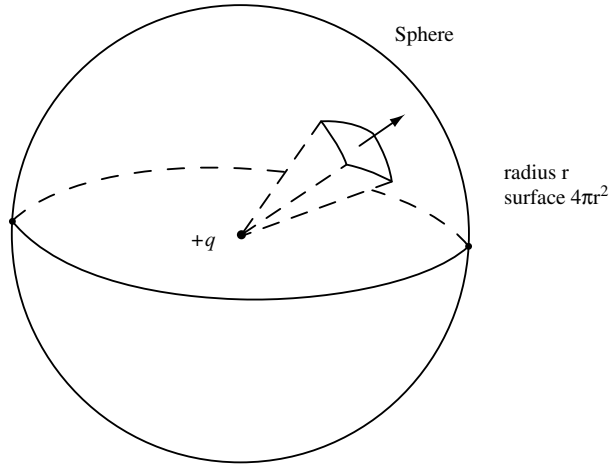


Figure 1.13

In order to compare both unit systems, we imagine a hollow sphere as shown in Figure 1.13. If the radius is r, the surface area is $4\pi r^2$ regardless of unit system. If electric charge $+q = 1.0$ is placed at the center point of the sphere, the electric line of force would be radiated uniformly towards the sphere surface. Now, we are free to count the total numbers of the radiated **line of force**. And then, the number is counted as 1(one) by Gaussian unit system, 4π by **CGS(cm, gr, sec) rational unit system**, $4\pi \times 10^{-7}$ by MKS rational unit system.

By Gaussian unit system, the expression of Coulomb's law is simple; however the number of line of force per unit area would become $1/4\pi r^2$. By CGS unit system, total number is 4π, and then the number per unit area at the surface is $1/r^2$ which means we can count the line of force per unit area by the equation without 4π. Generally by CGS rational unit system, we can escape from the inconvenience of 4π or $2\sqrt{\pi}$ by removing them in the related equations in counting various physical quantities, while on the other hand 4π or $2\sqrt{\pi}$ are always included in equations based on Gaussian unit system.

MKS rational unit system has the same concept with the CGS rational unit system except that m instead of cm and kg instead of g are adopted. In conclusion, μ_0 and ε_0 are defined by Equation (1.52b)(1.52c) by MKS rational unit system because of the above reason.

Hereunder is an comparison of MKS rational unit system and CGS rational unit system in regard with force F and energy,

$$1\,\text{Neuton} = 10^5\,\text{dyne}$$

$$energy = (force) \cdot (distance) = \underbrace{(\text{kg} \cdot \text{m/sec}^2)}_{Neuton} \cdot (\text{m}) = \underbrace{(\text{g} \cdot \text{cm/sec}^2)}_{dyne} \cdot (\text{cm}) \cdot 10^7 \qquad (1.54)$$

The digits number are different by10^5 times for force and by 10^7 times for energy.

1.3.4.2 Practical MKS units for electrical engineering physics

A conspectus of various electrical practical units is explained in brief as the last part of this chapter.

The meter unit system was established in 1875 and then unit system based on three fundamental units $m, kg,$ sec were popularized all over the world. In 1951, *Ampere* was added as the forth fundamental unit, and the **expanded MKSA unit system** was authorized, which means various units for electrical physics were officially combined with various units for Newton physics. After this year, $Kelvin(K)$ for temperature and $Candela(cd)$ for light intensity were added, and then in 1960, the **International unit system** (SI: International System of Units) was established which includes **seven fundamental units** as shown in Table 1.1. This is today's **Expanded MKS unit system**. All other units

Table 1.1 Fundamental units by International unit system (SI)

Quantity	Name	Symbol
Distance	meter	m
Weight	kilogram	kg
Time	second	sec
Current	ampere	A
Thermodynamic temperature	Kelvin	K
Molecule volume	mol	mol
Light intensity	candela	cd

Table 1.2 Definition of various derived units in electrical physics

$$
\begin{aligned}
\text{Neuton} &= \text{m} \cdot \text{kg}/\text{sec}^2 \\
\text{Pascal} &= \text{Newton}/\text{m}^2 \\
\text{Joule} &= \text{Newton} \cdot \text{m} \\
\text{Watt} &= \text{Joule}/\text{sec} = \text{Neuton} \cdot \text{m}/\text{sec} \\
\text{Volt} &= \text{Watt}/\text{Ampere} \\
\text{Ohm} &= \text{Volt}/\text{Ampere} \\
\text{Weber} &= \text{Volt} \cdot \text{sec} \\
\text{Tesla} &= \text{Weber}/\text{m}^2 \\
\text{Henry} &= \text{Weber}/\text{Ampere} \\
\text{Coulomb} &= \text{Ampere} \cdot \text{sec} \\
\text{Farad} &= \text{Coulomb}/\text{Volt}
\end{aligned}
$$

except these seven units are defined dependently as the **derived units** from seven fundamental units. Further, useful derived units are defined with proper unit names. As an example, the unit for electric charge $\pm q$ is counted as time-integration of *Ampere* then having unit value of *Ampere* · sec. Therefore new unit name *Coulomb* is defined for the derived unit *Ampere* · sec. In other words $C = A \cdot \text{sec}$ is a derived unit defined with proper unit name. Table1.2 shows various derived units having proper defined unit names in electrical physics.

1.4 Supplement: Proof of Equivalent Radius $r_{eq} = r^{1/n} \cdot w^{n-1/n}$ for a Multi-bundled Conductor

The equivalent radius $r_{eq} = r^{1/n} \cdot w^{n-1/n}$ of a multi-bundled conductor in Equations (1.15a) and (1.35②) can be proved as follows.

1.4.1 Equivalent radius for inductance calculation

One phase n-bundled conductor is examined where (n: number of conductors, r: radius of each conductor, w: averaged distance between two conductors, h: height above ground level. As all the elemental conductors are well balanced, the equation bellow is derived as of analogy to Equation (1.3).

$$
\begin{bmatrix} {}_r v_1 \\ {}_r v_2 \\ \cdot \\ \cdot \\ {}_r v_n \end{bmatrix} - \begin{bmatrix} {}_s v_1 \\ {}_s v_2 \\ \cdot \\ \cdot \\ {}_s v_n \end{bmatrix} = j\omega \begin{bmatrix} L_s & L_m & & & L_m \\ L_m & L_s & & & L_m \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ L_m & L_m & & & L_s \end{bmatrix} \cdot \begin{bmatrix} i_1 \\ i_2 \\ \cdot \\ \cdot \\ i_n \end{bmatrix} \tag{1}
$$

If the voltage and current of the bundled-conductor are v and i, the voltage and current of each elemental conductor is v and i/n, then.

$$\begin{bmatrix} {}_rv \\ {}_rv \\ \\ {}_rv \end{bmatrix} - \begin{bmatrix} {}_sv \\ {}_sv \\ \\ {}_sv \end{bmatrix} = j\omega \begin{bmatrix} L_s & L_m & L_m \\ L_m & L_s & L_m \\ \\ L_m & L_m & L_s \end{bmatrix} \cdot \begin{bmatrix} i/n \\ i/n \\ \\ i/n \end{bmatrix} \tag{2}$$

Then we have

$${}_rv - {}_sv = j\omega\{L_s + (n-1)L_m\} \cdot \left(\frac{1}{n}\right) \cdot i \tag{3}$$

where $$L_s = 0.4605 \log_{10} \frac{h+H}{r} + 0.05 \tag{4a}$$

$$L_m = 0.4605 \log_{10} \frac{h+H}{w} + 0.05 \tag{4b}$$

If the above bundled-conductor is equivalent with a single conductor with radius r_{eq} and arranged at the same height h, and is charged with the same v and i,

$${}_rv - {}_sv = j\omega L_{eq} \cdot i \tag{5}$$

where $$L_{eq} = 0.4605 \log_{10} \frac{h+H}{r_{eq}} + 0.05 \tag{6}$$

As the Equation (3) and (5) should be equal, then

$$L_{eq} = \{L_s + (n-1)L_m\}\left(\frac{1}{n}\right) \tag{7}$$

therefore

$$0.4605 \log_{10} \frac{h+H}{r_{eq}} + 0.05 = \left\{ \left(0.4605 \log_{10} \frac{h+H}{r} + 0.05\right) + (n-1)(0.4605 \log_{10} \frac{h+H}{w} + 0.05) \right\} \left(\frac{1}{n}\right)$$

then,

$$0.4605 \log_{10} \frac{h+H}{r_{eq}} + 0.05 = 0.4605 \log_{10} \frac{h+H}{r^{1/n} \cdot w^{(n-1)/n}} + 0.05 \tag{8}$$

therefore

$$r_{eq} = r^{1/n} \cdot w^{(n-1)/n}. \tag{9}$$

This is the same with Equation (1.15a).

1.4.2 Equivalent radius of capacitance calculation

If the voltage and charge of a n-bundled conductor is v and $+q$, the charge of each elemental conductor is $+q/n$. Then the following equation is derived in analogy with Equation (1.27).

$$\begin{aligned} v &= 2(q/n) \log_e \frac{2h}{r} \times 9 \times 10^9 + \sum_1^{n-1} 2(q/n) \log_e \frac{2h}{w} \times 9 \times 10^9 \\ &= 2q \left\{ \log_e \left(\frac{2h}{r}\right)^{1/n} + \log_e \left(\frac{2h}{r}\right)^{(n-1)/n} \right\} \times 9 \times 10^9 \\ &= 2q \cdot \log_e \frac{2h}{r^{1/n} \cdot w^{(n-1)/n}} \times 9 \times 10^9 \end{aligned} \tag{10}$$

If the above n-bundled conductor is equivalent to a single conductor with radius r_{eq} and the same height h, and is charged with the same v and $+q$,

$$v = 2q \log_e \frac{2h}{r_{eq}} \times 9 \times 10^9 \tag{11}$$

Comparing the both equations, the equation below is derived.

$$r_{eq} = r^{1/n} \cdot w^{(n-1)/n}. \tag{12}$$

This is the same with Equation (9), and of course with Equation (1.35②).

Now above all, inductance as well as capacitance of multi-bundled conductors can be calculated by applying equivalent radius given by Equation (9) or (12). This is the proof of Equation (1.15a) and (1.35②).

Coffee break 1: Electricity, its substance and methodology

The new steam engine of **James Watt** (1736–1819) ushered in the great dawn of the Industrial Revolution in the 1770s. Applications of the steam engine began to appear quickly in factories, mines, railways, and so on, and the curtain of modern mechanical engineering was raised. The first steam locomotive, designed by **George Stephenson** (1781–1848), appeared in 1830.

Conversely, electrical engineering had to wait until Volta began to provide 'stable electricity' from his voltaic pile to other electrical scientists in the 1800s. Since then, scientific investigations of the unseen electricity on one hand and practical applications for **telegraphic communication** on the other hand have been conducted by scientists or electricians simultaneously, often the same people. In the first half of the nineteenth century, the worth of electricity was recognized for telegraphic applications, but its commercial application was actually realized in the 1840s. Commercial telegraphic communication through wires between New York and Boston took place in 1846, followed at Dover through a submarine cable in 1851. However, it took another 40 years for the realization of commercial applications of electricity as the replacement energy for steam power or in lighting.